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Relations Between Structural and Dynamic Thermal Characteristics of Building Walls

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RELATIONS BETWEEN STRUCTURAL AND DYNAMIC THERMAL CHARACTERISTICS OF BUILDING WALLS

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Abstract: The effect of internal thermal structure on dynamic characteristics of walls is analyzed. The concept of structure factors is introduced and the conditions they impose on response factors are given. Simple examples of multilayer walls, representing different types of thermal resistance and capacity distribution, are analyzed to illustrate general relations between structure factors and response factors. The idea of the „thermally equivalent wall", a plane multilayer structure, with dynamic characteristics similar to those of a complex structure, in which three-dimensional heat flow occurs, is presented.

1. RELATIONSHIPS BETWEEN RESPONSE FACTORS AND THERMAL STRUCTURE FACTORS FOR WALLS

The reason to study relationships between structural and dynamic thermal characteristics of building walls was for us the following problem: How to modify response factors (or transfer function coefficients) for plane walls, which are used in computerized energy calculations, to take into account effects of thermal bridges?

The simplest method to be suggested here is of course just to solve the steady state heat transfer problem for the overall resistance of a wall with imperfections and then generate effective response factors multiplying response factors of the perfect, primary wall, by the resulting correction factor. This would be accurate for light walls, for which storage effects are insignificant. Calculating separately response factors for a variety of wall elements with thermal bridges and including them into existing programs would be rather troublesome. We propose a better method, which is simple, but at the same time accurate.

Imperfections in plane walls not only change their steady state thermal resistance, but also modify the dynamic properties - which in simulations may be represented by response factors. To take into account this effect it is necessary to have the appropriate mathematical tool - the formal relationships between structural and dynamic characteristics for walls. Such relationships follow from the asymptotic formulae for the heat flow across the surfaces of the separated wall element, due to temperature difference on its both sides [2].

Consider the heat flow through an element of a building envelope, of complex material and geometrical structure, that is embedded in a plane wall, homogeneous in every cross section, parallel to its surfaces. It is assumed that all material properties: thermal conductivity k, density ρ and specific heat c are constant in time. The element, together with its immediate neighborhood, is represented by the region D, bounded by an inner surface, facing room temperature T_i, an outer surface, facing environmental temperature T_e and adiabatic surfaces at the cuts which separate it from those parts of the wall, where the heat flow can be considered as one-dimensional.

Let $\theta(\mathbf{r})$ be a dimensionless temperature for the steady-state heat conduction problem in D, with boundary conditions $T_i = 0$ and $T_e = 1$. For a plane wall θ is given as $R_{i\text{-}x}/R_T$ /see [1, 4, 5]/, where R_T is the total resistance for heat transmission through a wall and $R_{i\text{-}x}$ is the resistance from the point x in a wall to the internal environment.

For T_i and T_e constant for time $t > 0$, and zero initial conditions, the asymptotic expressions for the total heat flow across the inner and outer surface in the direction of the outside normal, Q_{ni} and Q_{ne}, are as follows [1, 2, 3, 4]:

$$Q_{ni}(t) \Rightarrow \frac{t}{R_T}\left[T_e - T_i\right] - T_i\, C\varphi_{ii} - T_e\, C\varphi_{ie} \tag{1}$$

$$Q_{ne}(t) \Rightarrow \frac{t}{R_T}\left[T_i - T_e\right] - T_i\, C\varphi_{ie} - T_e\, C\varphi_{ee} \tag{2}$$

where R_T is the total thermal resistance of the element, calculated for the steady state heat flow, C is the total thermal capacity, whereas the quantities φ_{ii}, φ_{ee}, φ_{ie} are given by:

$$\varphi_{ii} = \frac{1}{C}\int_D dv\rho c\,(1-\theta)^2 \quad , \qquad \varphi_{ee} = \frac{1}{C}\int_D dv\rho c\,\theta^2 \quad , \tag{3}$$

$$\varphi_{ie} = \frac{1}{C}\int_D dv\rho c\,\theta(1-\theta) \quad , \tag{4}$$

Dimensionless quantities φ_{ii}, φ_{ie}, φ_{ee} constitute, together with the total thermal resistance R_T and capacity C, the basic thermal characteristics of the separated wall element, which can be determined experimentally in the heat transfer processes with steady initial and final states of heat flow [1, 2]. They are defined as the thermal structure factors. For a plane wall they depend directly on the thermal structure, determined by the capacity and resistance profiles along its thickness. For an element in which three-dimensional heat flow occurs, this dependence is indirect through the reduced temperature distribution. To calculate effectively thermal structure factors, as well as total resistance, one has to solve the steady state heat transfer problem.

General rules, concerning magnitudes of the structure factors, may be deduced immediately from the form of the integral expressions (3), (4). Keep in mind that, in general, in steady heat flow through an element composed of different materials, most severe temperature gradient occurs in regions of small conductivity, whereas in those of large conductivity, temperature it is almost constant.

The quantity φ_{ii} is comparatively large if most of the thermal mass is located near the interior surface of an element, whereas most of the resistance resides in its outer part, located near the exterior surface; the opposite holds for φ_{ee}. The upper limits on φ_{ii} and φ_{ee} are 1, the lower is 0. For elements that are internally symmetric $\varphi_{ii} = \varphi_{ee}$. Since φ_{ie} is *the integral over the volume of the element of the expression $\theta(1-\theta)$ multiplied by $\rho c/C$* and $\theta(1-\theta)$ attains its maximum of 1/4 at $\theta = 1/2$, φ_{ie} attains its maximum, equal to 1/4, when the whole thermal mass of negligible resistance is in the center and when the whole resistance is distributed symmetrically on both sides of the center. For a homogeneous wall $\varphi_{ii}=\varphi_{ee}=1/3$, $\varphi_{ie}=1/6$.

Structure factors of multilayer walls are affected by differentiation of thermal parameters of individual layers and their arrangement. This is illustrated by a simple example of a wall composed in six different ways of two layers of heavyweight concrete ($k = 1.73$ W/mK, $\rho = 2240$ kg/m^3, $c= 0.838$ kJ/kgK) and two layers of insulation ($k = 0.043$ W/mK, $\rho = 91$ kg/m^3, $c = 0.838$ kJ/kgK) of the same thickness /Fig. 1/.The structure factors are given in Table 1.

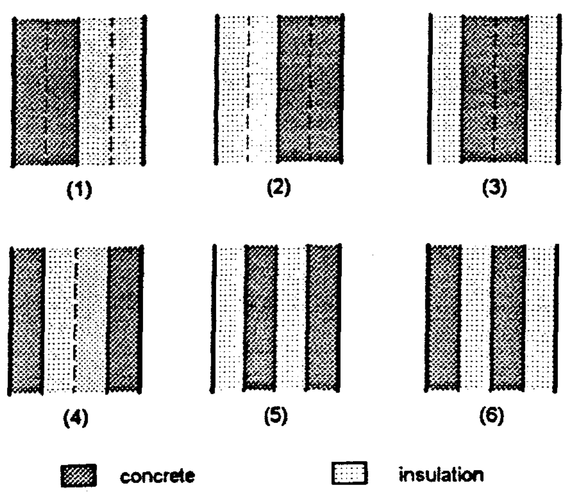

Figure 1 Different types of four-layer walls composed of concrete and insulation

Table 1 Thermal structure factors for different types of four-layer walls represented in Fig.1

Wall	φ_{ie}	φ_{ii}	φ_{ee}
1	0,018	0,950	0,014
2	0,018	0.014	0,950
3	0,247	0,253	0,253
4	0,012	0,488	0,488
5	0,130	0,605	0,136
6	0,130	0,136	0,605

In programs for energy simulation in building design, heat flow rates across the wall's surfaces are modelled with the use of ~~the~~ so called response factors. A response factor with the number m represents the response of a linear system to the unit triangular temperature pulse with the base width 2Δ, at the discrete time moment $m\Delta$.

Let $H_{ii}(m\Delta)$, $H_{ee}(m\Delta)$ and $H_{ie}(m\Delta)$ denote the normalized response factors corresponding to the three different heat transfer modes. The heat flow rates $\dot{Q}_{ni}(n\Delta)$ and $\dot{Q}_{ne}(n\Delta)$, across the internal and external surface of the element, in the directions of the outside normals, as functions of the room and environment temperature history, are represented in terms of the response factors in the following way:

$$\dot{Q}_{ni}(n\Delta)=\frac{1}{R_T}\left\{\sum_{m=0}^{n-1}T_e[(n-m)\Delta]H_{ie}(m\Delta)-\sum_{m=0}^{n-1}T_i[(n-m)\Delta]H_{ii}(m\Delta)\right\} \tag{5}$$

$$\dot{Q}_{ne}(n\Delta)=\frac{1}{R_T}\left\{\sum_{m=0}^{n-1}T_i[(n-m)\Delta]H_{ie}(m\Delta)-\sum_{m=0}^{n-1}T_e[(n-m)\Delta]H_{ee}(m\Delta)\right\} \tag{6}$$

For the asymptotic compatibility of (5) and (6) with the steady state heat flow solution it is necessary that response factors satisfy the condition:

$$\sum_{m=0}^{\infty} H_{ii}(m\Delta) = \sum_{m=0}^{\infty} H_{ie}(m\Delta) = \sum_{m=0}^{\infty} H_{ee}(m\Delta) = 1 \tag{7}$$

Another set of conditions, derived in [1, 2], follows from the compatibility of (5), (6) with the asymptotic formulae (1), (2):

$$-\frac{\Delta}{R_T C}\sum_{m=1}^{\infty} mH_{ii}(m\Delta) = \varphi_{ii} \quad , \qquad -\frac{\Delta}{R_T C}\sum_{m=1}^{\infty} mH_{ee}(m\Delta) = \varphi_{ee} \tag{8}$$

$$\frac{\Delta}{R_T C}\sum_{m=1}^{\infty} mH_{ie}(m\Delta) = \varphi_{ie} \tag{9}$$

Equations (8), (9) represent the relationships between the dimensionless structural and dynamic thermal characteristics of the wall: φ_{ii}, φ_{ie}, φ_{ee} the ratio of the time interval Δ and time constant $R_T C$ and the dimensionless normalized response factors $H_{ii}(m\Delta)$, $H_{ee}(m\Delta)$, $H_{ie}(m\Delta)$.

The conclusions which may be derived immediately from the equations (8), (9), concerning the effect of structure factors on response factors, are the following [1, 2]. Response factors $H(m\Delta)$, with $m \geq 1$, reflect storage effects, giving heat fluxes after the time of duration of the triangular temperature impulse. Large values of the structure factor, corresponding to a given heat flux response mode, indicate that response factors with number $m \geq 1$, are comparatively large; conversely, small values of the structure factor indicate that they are comparatively small. However equations (8) and (9) must be satisfied simultaneously with (7), which states that the sum of all response factors must be equal to 1. Therefore the larger are the values of $H(m\Delta)$ for $m \geq 1$, the smaller is the value $H(0)$ and vice versa. Taking into account that $H(m\Delta)$ should be relatively smooth functions of the number m, one can expect that response factors corresponding to small values of structure factors decay relatively quickly whereas those corresponding to large values of structure factors decay relatively slowly.

For the exterior building walls the values of φ_{ie} are most important - as they have an effect on their thermal stability towards the ambient temperature variations.

To demonstrate the effect of structure on dynamic properties of a plane wall, normalized response factors $H_{ie}(n\Delta)$ were calculated for the set of symmetric three-layer walls, with thickness in proportion 1:2:1, with the same values of resistance R, capacity C, and thermal diffusivity common to all layers, but a different ratio of resistance and capacity for the inner and outer layers /Fig.2/. Time constant RC = 50 h, Δ=1h. The values of structure factor φ_{ie}, decrement factor df and time lag τ of the heat flux for harmonic oscillations of the time period 24 h, are collected in Table 2. df is defined as the ratio of the heat flux amplitude, due to the outdoor temperature harmonic oscillations of unit amplitude, and stationary value $1/R_T$.

The plots of response factors in Figure 2 clearly illustrate the fact that structure factors have an essential influence on dynamic thermal behaviour of a wall. Walls characterized by small values of the structure factor φ_{ie} comparatively quickly transfer thermal responses, whereas those with larger values of φ_{ie} delay thermal responses. The response, in the form of the heat flux at the surface, due to a thermal impulse at the opposite surface, in the case of a wall with φ_{ie} close to zero, is comparatively large, increases and disappears relatively quickly. In the case of a wall with φ_{ie} close to the maximum possible value of 1/4, the response is smaller and slowly decreases - however it increases more quickly than for a homogeneous wall. At the same time Table 2 shows that damping effects of the harmonic heat flux oscillations increase with φ_{ie} /however the time lag τ has its maximum below φ_{ie}=1/4/.

Table 2 Decrement factors and time lags for symmetric three-layer walls of different structure factors φ_{ie}; *RC=50 h*

φ_{ie}	*df*	τ [h]
0.011	0.998	0.530
0.040	0.972	1.989
0.110	0.777	5.129
0.167	0.562	6.751
0.210	0.419	6.922
0.248	0.301	5.143

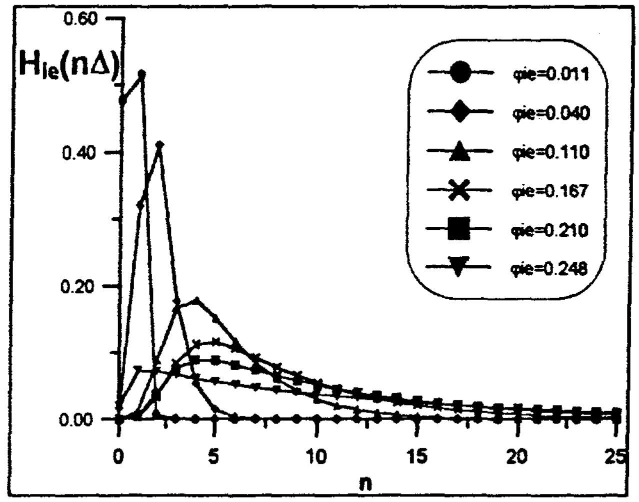

Figure 2. Normalized response factors $H_{ie}(n\Delta)$ *for three-layer walls of RC=50h, the same thermal diffusivity and different structure factors*

2. THE „THERMALLY EQUIVALENT WALL" CONCEPT

Thermal structure factors φ_{ii}, φ_{ie}, φ_{ee}, defined by the integrals (3), (4), together with total thermal resistance R_T and capacity C, determine, to a great extent, the dynamic thermal properties of a wall element - through the conditions (8), (9) they impose on response factors. Those conditions, however, do not determine the response factors in a unique way, but rather play the role of constraints. Nevertheless one may expect that walls with the same total thermal resistance, capacity and structure factors, have similar dynamic characteristics - response factors - even if they are quite different in details. This leads to the concept of the „thermally equivalent wall" /see [2]/, simple structure which has the same type of dynamic thermal behaviour as a more complex one and may be used as its substitute in one-dimensional energy simulations that are common.

To demonstrate the possibility of replacement of a complex structure by a simple, thermally equivalent plane wall, an example was analyzed of a cuboidal element, composed of heavyweight concrete and EPS foam, presented in Fig.3. The central part of the element constitutes the thermal bridge in the three-layer wall, which is here called the „primary wall". Modifying properly total resistance and capacity, and their partition between layers, gives equivalent wall [2].

Table 3 Static thermal characteristics of the wall element with a thermal bridge, represented in Fig. 3, of the equivalent wall and primary wall

STRUCTURE	RA [m^2K/W]	C/A [kJ/m^2K]	φ_{ie}	R_1/R	C_1/C
Thermal bridge	0.5902	269.909	0.10628	---	---

Equivalent wall	0.5902	269.909	0.10628	0.23676	0.47410
Primary wall	1.6216	307.409	0.02388	0.04776	0.49759

Static thermal characteristics of the element are given in Table 3, normalized response factors $H_{ie}(n\Delta)$, at $\Delta = 1$ h, are presented in Fig.4. R_l, C_l in Table 3 denote the resistance and capacity of the outer parts of the structures, of thickness 4 in, A is the area of the transverse cross section.

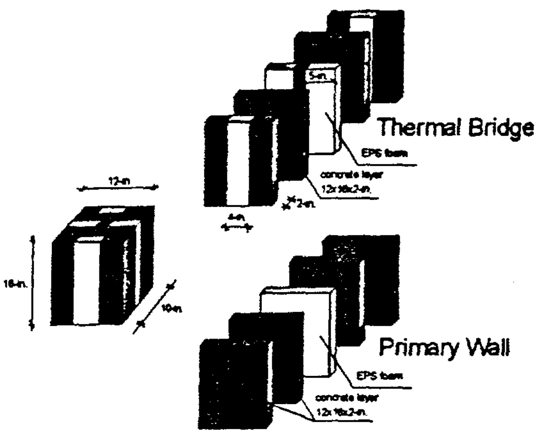

Figure 3 *The wall element representing a composition of two different types of concrete and insulation arrangement*

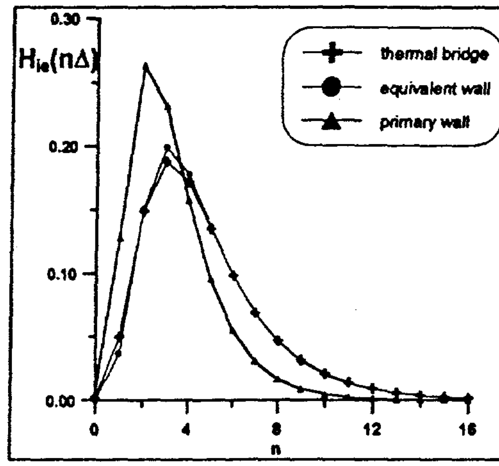

Figure 4 *Normalized response factors $H_{ie}(n\Delta)$ for the wall element with a thermal bridge represented in Fig.3*

The essentially different arrangement of thermal mass and resistance in the thermal bridge region, as compared with the primary wall, causes significant variation of the structure factor φ_{ie}. It is increased, due to the translocation of thermal mass to the center and resistance to the outer parts of the wall. The course of the normalized response factors $H_{ie}(n\Delta)$ is also significantly modified; they decay more slowly with n. Most important: the response factors for the equivalent wall, which is closer to a homogeneous one than the primary wall, are almost identical with those for the wall element with the thermal bridge. Therefore using the response factors of the equivalent wall, as the substitute of response factors for the element with the thermal bridge, gives much better approximation than multiplying the response factors of the primary wall by the correction factor, to take into account just the change of its resistance.

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