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MODEL BASED APPROACH TO UXO IMAGING USING THE TIME DOMAIN ELECTROMAGNETIC METHOD

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ABSTRACT

Time domain electromagnetic (TDEM) sensors have emerged as a field-worthy technology for UXO detection in a variety of geological and environmental settings. This success has been achieved with commercial equipment that was not optimized for UXO detection and discrimination. The TDEM response displays a rich spatial and temporal behavior which is not currently utilized. Therefore, in this paper we describe a research program for enhancing the effectiveness of the TDEM method for UXO detection and imaging. Fundamental research is required in at least three major areas: (a) model based imaging capability *i.e.* the forward and inverse problem, (b) detector modeling and instrument design, and (c) target recognition and discrimination algorithms. These research problems are coupled and demand a unified treatment. For example: (i) the inverse solution depends on solution of the forward problem and knowledge of the instrument response; (ii) instrument design with improved diagnostic power requires forward and inverse modeling capability; and (iii) improved target recognition algorithms (such as neural nets) must be trained with data collected from the new instrument and with synthetic data computed using the forward model. Further, the design of the appropriate input and output layers of the net will be informed by the results of the forward and inverse modeling. A more fully developed model of the TDEM response would enable the joint inversion of data collected from multiple sensors (e.g. TDEM sensors and magnetometers). Finally, we suggest that a complementary approach to joint inversions is the statistical recombination of data using principal component analysis. The decomposition into principal components is useful since the first principal component contains those features that are most strongly correlated from image to image.

1.0 INTRODUCTION

The utility of the TDEM method for UXO detection can be improved by recording and interpreting more attributes of the scattered wavefield. The scattered wavefield is due to currents induced in the target conductor by a primary time varying magnetic field. The wavefield is not measured directly, instead, voltages or emf's are measured in receiver coils. The voltages are induced according to Faraday's law by the time varying scattered fields. Measurements are performed in the time domain after the termination of the primary current waveform in the transmitter loop. The target conductor and ground response signals can be separated since currents from the latter decay much more rapidly than currents in conductors with the large and positive conductivity contrasts that are typical of UXO. The residual effect of the primary field can be separated since it is present only for a very short time after the termination of the current waveform. TDEM sensors (e.g. the Geonics Ltd. EM61) that have been deployed in the field over the last several years for UXO detection measure the induced voltages in only one time gate. In addition, the transmitter and receiver coils of this sensor are all in the horizontal plane. The imaging and discrimination capability of TDEM sensors could be significantly enhanced by recording measurements in receiver coils oriented in three orthogonal directions, and in many different time gates. However, effective use of these measurements would depend on the availability of an appropriate forward and inverse theory with which to interpret them. It would also be desirable to develop a formal and rational basis for designing an instrument that would provide the maximum diagnostic power for broad classes of targets. Finally, the formal inversion approach is adequate for post-processing of data, but complementary to this, it would be desirable to develop and implement an inference technique that is field robust and can operate in real-time or near real-time.

Thus, there are three major problem areas that we consider in this paper: (a) model based imaging capability *i.e.* the forward and inverse problem, (b) detector modeling (including instrument response) and instrument design, and (c) target recognition and discrimination algorithms. We describe a theoretical approximation developed by Habashy *et al.* (1993) that can be used to compute rapidly and accurately secondary fields for targets with large conductivity contrasts. This work can provide a basis for the forward and inverse theory required to address problem area (a). For problem area (b) we describe a formalism that can be used to improve the design of

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geophysical surveys and instruments. The classical geophysical approaches to inverse theory for complicated problems often requires hand-on interaction (*i.e.* selection of model-norm constraints, optimal point on the trade-off curve between variance reduction and model norm, starting guess, etc.). The classical approaches provide essential insight and quantitative criteria for instrument design and target classification, but are not ideally suited for the real-time results required in the field due to lack of robustness. Therefore, for problem area (c) we discuss the use of neural nets for pattern recognition and anomaly classification. Also, we consider a statistical technique known as principal component analysis that can be used to produce compressed input training data for the net, and which may prove to be of utility for joint interpretation of data sets from differing sensor types. In section 2 we discuss the forward and inverse problem; instrument design is considered in section 3, pattern recognition approaches are examined in section 4, and issues in data processing and data representation are considered in section 5.

2.0 THE FORWARD AND INVERSE PROBLEM OF ELECTROMAGNETIC SCATTERING

Background

The most difficult challenge in modeling TDEM sensor responses is the computation of the secondary EM fields scattered by the unexploded ordnance (UXO) target. Many of the techniques developed for forward and inverse scattering computations are notoriously CPU intensive. Techniques based on linearization of the field variables are simpler and faster but limited in the range of problems they can solve. For example, these techniques can only be applied to problems that display small conductivity contrasts. Iterative techniques have been devised to overcome these problems, but there is no guarantee of convergence, and the approach requires repeated solutions of the forward problem which can be very expensive. Many of the computational techniques are based on the finite element method, finite differences, or integral equation approaches. These techniques can accurately simulate EM field behavior for wide frequency ranges and for a diverse variety of material properties so long as the discretization is sufficiently fine and the computational requirements do not exceeded the available resources. The principal disadvantage of these techniques is that they require the repeated inversion of a large stiffness matrix. This is a particularly worrisome problem for inverse scattering since in minimizing a given cost function or objective criteria, the same operation must be performed again and again.

A review of many of these computational techniques can be found in Volakis and Kempel (1995). A summary and comparison of computer codes that have been developed for geophysical problems can be found in Smith and Paine (1995). These codes suffer from the computational requirements described above. Some break down for the conductivity contrasts that are characteristic of the UXO problem, others are limited by the source-receiver geometries they can model, and others can model only a single conductor. Furthermore, all of these codes are proprietary, and would require many man-years to develop from scratch.

The Born Approximation and the Extended Born Approximation

The Born approximation is a scattering method that is widely used in acoustics, seismology, quantum mechanics, and electromagnetics (e.g. Wu and Toksoz, 1987; Zhou, 1989; Zhou *et al.*, 1993). The key to this approximation is that the electric field that exists within the scatter is modeled by the electric field that would exist in the homogeneous background medium. This formulation is advantageous since the forward and inverse problem can be posed simply and computed rapidly. The disadvantage is that the conductivity contrast of the scatterer with the background medium is required to be small, whereas the conductivity contrast due to metallic components of UXO can be quite large. For example, the conductivity of steel is $\sim 10^7$ Siemens/m whereas the ground conductivity of soils range from $\sim 10^{-3}$ to 10^0. This contrast causes both the amplitude and phase of the electric field induced in the metallic components of UXO to differ greatly from the background field thus vitiating the use of the approximation. The Born approximation has found useful application in environmental EM problems where the conductivity contrasts are not nearly so large. Recently, Habashy *et al.* (1993) and Torres-Verdin and Habashy (1994) have developed an approximation called the *Extended Born Approximation*. Their formulation extends the range of validity of the Born approximation to very large conductivity contrasts, yet retains many of the numerical and analytical advantages of the Born approximation. Their work provides the means to simulate accurately the electric field internal to the conductivity distribution without having to invert the large, often full, stiffness matrix that results from solving integral-equation or finite-difference schemes.

The Inverse Problem

The Extended Born Approximation renders practical the nonlinear EM inversion problem, and has been of demonstrated utility for geophysical inverse problems (Habashy *et al.* 1995). Although the estimators to compute the internal electric field are weak nonlinear functions in conductivity, they are generally much faster to compute than the full forward problem, and are almost as efficient as the Born and Rytov approximations. The enhanced accuracy of these new estimators and the advantages described above make their application to low frequency three-dimensional inverse problems in ordnance detection and imaging ideal. Forward and inverse problems based on these estimators can be applied to scatterers with many geometries including spherical objects, rectangular parallelepipeds, rectangular cylinders, thin plates, etc. Complex models can be constructed using these simple building blocks.

In the inversion process we attempt to infer information about the properties and location of an object from measurements of a scattered field. Here, the scattered field is the secondary magnetic field induced in the scatter by the primary magnetic field. Instead of measuring the field directly, we measure the time varying flux of the field which is recorded as a time dependent electromotive force (emf) in the receiver coil. The emf is a functional of the secondary fields and the recording geometry; the exact dependence can be obtained from Faraday's law which states that

$$\nabla x \boldsymbol{E} = -(1/c)\, \partial_t \boldsymbol{B}. \qquad (2.1)$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic field, and c is the speed of light. The emf is given by the integral of the above over the area of the receiver coil. By applying Stokes law we obtain

$$emf = \int \boldsymbol{E} \bullet \boldsymbol{dl} = -(1/c)\, \partial_t \int \boldsymbol{B} \bullet \boldsymbol{dA} \qquad (2.2)$$

so that the emf can be expressed either as the line integral of the electric field along the loop of the receiver coil, or as the time derivative of the magnetic flux across the aperture of the receiver coil. The observed emf values constitute the data used in the inversion. According to the Extended Born Approximation, the integral equation that governs the total electric field $\boldsymbol{E}$ in the frequency domain is written

$$\boldsymbol{E}(\boldsymbol{r}) \cong \boldsymbol{E}_b(\boldsymbol{r}) + i\, \omega\, \mu_o \int_{Vs} \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}') \bullet \boldsymbol{\Gamma}(\boldsymbol{r}') \bullet \boldsymbol{E}_b(\boldsymbol{r}')\, \Delta\sigma(\boldsymbol{r}')d\boldsymbol{r}' \qquad (2.3)$$

where $\boldsymbol{G}$ is the Green's function tensor, $\boldsymbol{E}_b(\boldsymbol{r})$ is the background electric field, $\Gamma\ (\boldsymbol{r}')$ is the so-called depolarization tensor and contains weak nonlinear dependence on the scalar Green's function and the anomalous conductivity $\Delta\sigma$, and ω is the frequency. In the integral equation the total electric field $\boldsymbol{E}$ is represented as the sum of the background field $\boldsymbol{E}_b(\boldsymbol{r})$ and the scattered field (the integral terms). The scattered field is generated by the scattering currents (and charges) induced inside the scatterer by the interaction of the total electric field $\boldsymbol{E}(\boldsymbol{r})$ with variation of the conductivity within the scatterer and host medium. The objective of the inverse problem is to recover $\Delta\sigma$ from the observed emf's.

In general, the inverse scattering problem is non-unique and the scattered field is nonlinearly related to the scattering object. The nonlinearity complicates the solution of both the forward and inverse problems. In essence, the nonlinear dependence of the scattered field on the properties of the scattering object is due to the mutual interaction between the induced currents. The non-uniqueness arises due to incomplete data coverage and because in order to stabilize the inversion matrix it is often necessary to incorporate damping or smoothing constraints which lead to low pass images of the actual object.

In addition to imaging, a benefit of constructing the inverse solution is the ability to analyze the efficiency of the *antenna array i.e.*, the space-time distribution of TDEM observations of the scattered field. The antenna array of a TDEM detector is defined by the orientation, location, and size of the transmitter and receiver coils, the distribution and width of time windows of observation, and system parameters such as base frequencies, transmitter waveforms, etc. This leads to the design considerations in the presented in the next section.

3.0 OVERVIEW OF OPTIMIZATION PRINCIPLES FOR SURVEY AND INSTRUMENT DESIGN

General considerations

The design of any geophysical survey or instrument e.g. seismic, electromagnetic, gravity, etc., is fundamental to a successful outcome. The typical goal of maximum target resolution is often in conflict with practical requirements. Thus, survey/instrument design is a classic problem in optimization theory (*i.e.* given quantifiable objective criteria, what is the optimal choice of instrument or survey design parameters?). There are a number of questions that should be addressed in the design process: (1) What is the optimal trade-off between redundancy of field measurements (which we seek to minimize) versus accurate characterization of the target? (2) How should *a – priori* information e.g. statistically characterized noise environments, geophysical measurements from other sensors, known geology, target class, etc., be incorporated into the design process? (3) What objective criteria in the inverse problem are most appropriate to consider e.g. resolution of sub-surface voxels, delineation of boundaries, covariance between estimated model parameters, effect of design parameters on noise amplification, stability of inversion matrices, etc.? In this section we discuss the general problem of geophysical survey and instrument design, and present criteria that can be used to evaluate the efficiency of such designs. The survey/instrument design can be adjusted until an objective function defined by the criteria is maximized. This process depends on the ability to compute the forward and inverse solutions. In addition, since the objective function may be topologically complex, it is necessary to use an optimization technique capable of locating the globally optimum solution. We show that the smallest singular value of the design matrix constitutes a useful objective function, and we suggest that genetic algorithms are well-suited for discovering the optimal solution.

Barth and Wunsch (1990) have shown that inverse theory can be usefully applied to the experiment design problem. They considered the optimal deployment of sources and hydrophones for an ocean acoustic tomography experiment. Their design approach applies equally well to the design of geophysical sensors. The inverse solution determines which data contribute most to the resolution of the model parameters that characterize the problem. For example, in the resistivity inverse problem, the inverse solution determines (1) which of the measurable field quantities, and which set of transmitter-receiver separations and transmitter frequencies best resolve the conductivity and thickness values of an earth model, and (2) which combinations of data optimally mitigate noise effects. The singular value decomposition (SVD) of the design matrix provides valuable diagnostics of the experimental design. It is generally the case that the utility of the design is improved by adjusting the survey parameters to increase the condition number or the size of the smallest singular value of the design matrix. Thus, to optimize a survey design, one attempts to maximize an objective function given, for example, by the smallest singular value. Clearly, this is a strongly nonlinear problem, and cannot be solved with classical techniques. However, as mentioned above, genetic algorithms can be usefully applied to this problem

Technical Discussion - Objective Criteria

The choice of the objective function is one of the most important elements in survey/instrument design. The design will depend on the target features of interest, and on how the data will be processed and interpreted. A design that yields good performance for one set of criteria may perform less well for another. Regardless of the specific goal, objective criteria can always be framed in terms of very general functionals that are easily computed during the inversion process. Criteria can be defined in terms of subsurface volume element and boundary resolution, model parameter covariance and trade-off, noise sensitivity, matrix stability, and others.

Barth and Wunsch (1990) have shown that singular value decomposition is a useful way of computing many of the objective criteria of interest (see also an early contribution by Glenn and Ward, 1976). To demonstrate this we consider the canonical inverse problem

$$\boldsymbol{G}\Delta\boldsymbol{m} = \boldsymbol{d}, \qquad (3.1)$$

where $\boldsymbol{G}$ is the design matrix, $\Delta\boldsymbol{m}$ is the vector of unknown model parameters, and $\boldsymbol{d}$ is the data vector. $\boldsymbol{G}$ is dependent on the physics of the particular problem and on the detailed properties of the survey/instrument design. It is easy to see how equations (2.2) and (2.3) can be combined to yield this type of inverse problem. If there are no noise or other error terms in the data, a fully determined system would give rise to perfect resolution. In most cases however, the matrix $\boldsymbol{G}$ is singular, and instead of determining the model parameters, one is forced to estimate them. Using the SVD of $\boldsymbol{G}$, one particular estimate for the model parameters is

$$\Delta\boldsymbol{m} = \boldsymbol{V}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}\boldsymbol{d} \qquad (3.2)$$

where in the usual way $\boldsymbol{U}$, $\boldsymbol{V}$, and Λ are such that $\boldsymbol{G} = \boldsymbol{U}\boldsymbol{L}\boldsymbol{V}^T$ (Aki and Richards, 1980). $\boldsymbol{U}$ and $\boldsymbol{V}$ span, respectively, the data and model spaces, and $\boldsymbol{\Lambda}$ is the diagonal matrix of singular values λ_i for $1 \leq i \leq N_m$, where N_m is the number of model parameters. If the rank of $\boldsymbol{G}$ is less than N_m (the rank corresponding to the number of non-zero singular values λ_i for $1 \leq i \leq$

p), then there is a null space and the vector of model parameters breaks up into a piece Δm_p which is determined by G and a piece which lies in the null space Δm_0 about which no information is available. The matrix V also decomposes into V_p made up of the first p columns of V and V_0 which is constructed from the columns $(p+1)$ through of V. Then all solutions for Δm are now of the form $\Delta m = \Delta m_p + V_0 \alpha$ where $\Delta m_p = V_p \Lambda_p^{-1} U_p d$ and α is a matrix of arbitrary coefficients. Different estimators of Δm make different choices for the formally indeterminate values of α, ranging from setting them to zero (for the minimum mean square solution) to employing *a priori* statistical information about the solution. From Aki and Richards (1980), the error covariance matrix is

$$<\Delta m\, \Delta m^T> = \sigma_n^2\, V \Lambda^{-2}\, V^T \qquad (3.3)$$

where we have assumed for simplicity that data errors are uncorrelated with standard deviation σ_n. Obviously, the covariance of the solution becomes large when λ_i is small. Associated with these entries will be certain elements of the estimate Δm which will be poorly determined, possibly unacceptably so. Some workers have eliminated eigenvectors with small eigenvalues to keep the covariance below a certain level. This, however, reduces the number p of non-zero eigenvectors, degrading the resolution in model and data spaces. The resolution matrix of the model parameters is VV^T so that the estimated model parameters Δm_{est} are related to the true model parameters by $\Delta m_{est} = VV^T\, \Delta m$. The trace of the resolution matrix VV^T equals the rank of G, and, therefore, truncation of singular values will lead to deterioration in the resolution matrix.

Noise processes represent another important factor. In the presence of noise the minimum variance estimator is a useful solution. The estimator which minimizes the trace of the covariance matrix and the covariance of this solution are given, respectively, by $\Delta m = RG^T(GRG^T + \sigma_n I)^{-1}\, d$ and has covariance $<\Delta m\, \Delta m^T> = V\Lambda(\Lambda^2 + \sigma_n^2/\sigma_m^2\, I)^{-2}\, \Lambda V^T$ where R is the *a priori* covariance matrix associated with the true model parameters, and σ_m is the standard deviation of the model parameters. Although the instability of equation (3.2) is reduced by eliminating small singular values, it is now important to consider the relative magnitudes of the small singular values to that of the noise term in above covariance estimate. Although the design matrix G may be such that the system is fully determined, its smaller singular values may be so small that they become negligible when compared to the noise level in the measurements. In this case, it is usually better to reduce the rank of G. Truncation of small singular values leads to a more stable estimate for the model parameters, but at the price of resolution loss.

In designing an instrument or survey for a given target class, it would seem reasonable then to try to make the rank of G equal to the rank of the system (*i.e.* N_m), and the magnitude of the smallest singular value in the spectrum of G as large as possible. If the smallest singular value is large enough, so that it is not noise dominated, then truncation would not be necessary. The covariance (eq. 3.3) for the estimate in equation (3.2) would be well behaved, and the model resolution would not be compromised. The model estimate and its covariance in the presence of noise would also not be noise dominated. Therefore, if we desire to design a system which is fully determined, then we choose the objective function F to equal the smallest singular value λ_{Nm}. The singular values of G, in turn, are related to how the geophysical fields sample the subsurface, which is determined by the distribution of the sources and receivers and instrumental parameters. If the underdetermined case is considered when perfect resolution of all the model parameters is not required, the objective function is modified to $F = \lambda_p$, where p is less than N_m and equal to the rank of G which is desired. There is yet another approach to this problem suggested by Snieder and Curtis (1995). They point out that although the ill-conditioning of the design matrix is often computed in terms of the condition number (the ratio of the largest to the smallest singular value, and which can therefore be infinite), a more useful measure of conditioning is given by $\Theta = N_m\, \lambda_1 / \Sigma\, \lambda_i$, where λ_1 is the largest singular value, and the sum is taken over all singular values. The sum in the denominator is equal to the trace of G while λ_1 may be estimated using the power method. Hence Θ may be calculated swiftly even for large, non-sparse matrices.

As shown above, the model resolution is sensitive to data error. Thus, estimates of data error can be used to asses which data attribute best resolves a given structure, and the generalized inverse can be used to determine which data contribute the most to model resolution. Study of the eigenvectors that compose V provides insight into model parameter correlations and measurement correlations which can be exploited for improving the design of an experiment or instrument. Although the most intuitive objective function may be the one which yields minimization of the mean square error in synthetic experiments using an *a priori* model, the diagnostic criteria defined above yield other important measures that evaluate the quality of the solution such as model covariance, matrix stability, model resolution, etc.

Numerical Optimization Method

Once the objective criteria are established and can be computed numerically, it is necessary to search efficiently for the survey or instrument design that yields the highest objective function valuation, and which does not exceed the available experimental resources. In general, the survey/instrument designs can be: (1) completely random, (2) structured to the extent that the controlling parameters vary in a well defined and likely discrete manner, or (3) confined to a limited number of fixed designs. The objective function will display highly nonlinear dependence on the parameters that characterize the survey/instrument design and this presents numerical problems. In fact, the problem is so strongly nonlinear that all classical optimization techniques such as the Newton-Raphson or conjugate gradient methods would fail completely. These methods can yield solutions that are local maxima rather than the global maximum. Another problem with the classical approaches is that they required partial derivatives of the objective function with respect to the survey/instrument parameters, and the derivatives can be difficult, if not impossible to evaluate. For example, the singular value of the design matrix is clearly a strongly nonlinear function of the components of the matrix. These problems can be addressed in principle through the use of genetic algorithms, and this was the approach used, for example, by Barth (1992) and Hernandez *et al.* (1995). Although, the geophysical problems are likely to be more computationally intensive than the oceanographic problems described in the above references, the use of genetic algorithms remains appropriate.

Genetic algorithms draw inspiration from the optimization process that forms the basis of biological evolution. Evolution is a process whereby a biological species defined by order 1000-10000 genes is modified to optimally fit the present environment. Geophysical survey/instrument optimization problems can be defined by a similar number of parameters. The most serious challenge is the volume of the phase space in which to search and the complexity of the optimization surface in that space. For example, if the dimensionality of the parameter space is of order 100 with just a few possible states for each parameter, say 10, then the number of points in the phase space approaches 10^{100}! An essential feature of genetic algorithms is their ability to bypass the inefficient component of the phase space.

In their most basic implementation, genetic algorithms make use of the following simplified version of the biological evolutionary process. First, the model parameters are coded in binary form. The algorithm then starts with a randomly chosen population of models called chromosomes. Second, the fitness values of these models are measured by some fitness criteria such as agreement between data and prediction or simply the minimum or maximum of an objective function. Third, the three genetic processes of *selection*, *crossover*, and *mutation* are performed upon the models in sequence. In *selection*, models are copied in proportion to their fitness values based on a probability define by the value of the objective function divided by the sum of the objective functions over all models. In *crossover*, an operator picks a crossover site between selected pairs of chromosomes and exchanges, based on a crossover probability, the bits between the two models. In *mutation*, a bit is changed at random based on mutation probability, and is applied to the models to maintain diversity. After execution of the above processes, the new models are compared to the previous generation and accepted based on an update probability. The procedure is then repeated until convergence is reached (*i.e.* when the fitness of all the models becomes very close to one another). Useful discussions on the use of genetic algorithms can be found in Charbonneau (1995) and Sen and Stoffa (1995).

Once the TDEM sensor is optimally designed for prescribed target classes, one can expect the performance of target recognition algorithms to improve, which leads us to the next section.

4.0 TARGET RECOGNITION ALGORITHMS

TDEM sensors display a rich spectrum of responses to metallic conductors. The response is dependent on the properties of the scatterer, the nature of the excitation, and on the manner in which the scattered field is measured and processed. As discussed in section 2, the detector response can be characterized mathematically and the inverse problem can be formally posed. This approach is useful since it allows one to discover the types of data and the instrument parameters that usefully constrain the target. In practice, formal inverse problems can be made robust for well-defined and well-rehearsed problems. However, for actual field deployments in which there are highly variable field and noise conditions, and real-time or near real-time imaging requirements, the implementation in the near future of an inversion process that does not require interfacing from the expert user would be rather difficult. (We expect, however, that post-processing with the formal inverse method will be practical). Accurate estimation of model parameters using any kind of sensor (not just TDEM) can be a very difficult problem. Ambiguity arises through insufficient or noisy data. This leads to non-uniqueness that can only be resolved by *a priori* constraints such as positivity, compactness, smoothness, small model norm, etc. Further, any inversion is limited by the accuracy and/or assumptions of the theory used to interpret the data. Lastly, ambiguity arises from the nature of the non-linear inversion itself, *i.e.* the iterative inversion procedure may not converge to the correct solution. These types of problems motivate the use of neural networks for the UXO inverse problem. We suggest in the following that the complete solution of the formal forward and inverse problems informs the development and routine use of the neural network approach.

Raiche (1991) presents an excellent overview of the application of neural nets to problems in geophysical inversion. He describes the shortcomings of the classical techniques including some of the points mentioned above. Raiche describes a paradigm for an automated geophysical system which works like a noise tolerant, interpolating associative memory; *i.e.* given raw geophysical data from one or more types of surveys, it will output a representation of Earth properties which gave rise to the input data structure. He goes on to list the desirable attributes that the system should have. Since these are precisely the attributes that a field robust TDEM detector should have, we quote directly from his paper:

- The system as a whole should be able to extract and classify features of the input data structure, and establish rules associating their interrelationship with observed Earth structure models.
- It should be able to utilize data sets from different geophysical methods by establishing rules governing the relative weighting of various data features.
- It should be able to learn from experience. In this case the experience will comprise all of the numerical and analogue model and field data curves along with whatever interpretations have been input to the system.
- It should be able to infer (develop) a suitable noise model so that the shape of the class boundaries will encompass distorted members of the same class. In other words, it should be capable of mapping input data into a nonlinear space with a metric which minimizes the effect of observed noise.
- It should be stable; i.e., given the same input data sets, it should infer consistent interpretations. It should not 'forget' or distort prototypes.
- It should be capable of inferring model structures not necessarily contained in the training set; i.e., it must be capable of interpolating within the metric established by noisy prototypes.
- It should not be bound by local error minima traps when comparing field data with forward model data from the final model.
- It should be computationally efficient.

The design of an inversion process based on a neural net that displays the above desirable properties is a challenging problem. Broadly speaking, there are three major problem areas that must be addressed, and we refer to these as Task 1, Task 2, and Task 3.

Task 1: The first task is to choose an appropriate interpretation space for the inversion results. By this we mean the characterization of the net's output layer. A very general characterization would be the representation of the subsurface with discrete 3D voxels; each node on the output layer would correspond to a single voxel. The classifier would then assign properties to each voxel. As Raiche (1991) points out, an alternative would be to implement a two-stage output process. The first stage would classify the data as arising from a specific model class; e.g., ant-personnel mine, anti-tank mine, etc. The second stage would consist of estimating the parameters associated with the model class, e.g. depth of burial, location, orientation, etc.

The appropriate choice of output node classification can benefit from thorough consideration of the forward and inverse problem. This follows from the fact that the solutions to most practical inverse problems are non-unique. There are some properties of the unknown model that simply cannot be retrieved from the data (Aki and Richards, 1980). Furthermore, it is generally not possible to obtain point estimates of the model. Instead, the inverse solution can only retrieve certain averages of the model, and this defines fundamental limits of resolvability. The formal solution of the inverse problem defines what model properties can be retrieved. It is these properties that should be used to characterize the output layer. Further, as discussed in section 3, the inverse solution determines which components of the data contribute most to the resolution of the model. Such knowledge can be used to tailor the input layer. Finally, an important benefit of developing the forward model is that training data can be computed synthetically for arbitrary UXO targets and instrument parameters. This can yield significant cost savings over data training data collected in the field.

Task 2: A crucial task is the development of the appropriate representation for the input geophysical data. The input data should be of minimal dimensionality and represented in such a way as to optimize feature extraction. We suggest in section 5 that the principal component transform can be used to achieve some of these objectives. It can be applied to data sets from multiple sensors (e.g. TDEM sensors and magnetometers), or to a data set in which many realizations of the same signal in a noisy environment are available.

Task 3: An important task is the development of a mapping algorithm capable of transforming a suitable representation of the input data into a description of the UXO target. There are many existing neural network architectures and training algorithms to choose from, and it is likely that the best algorithm can only be discovered by experiment.

5.0 SIGNAL PROCESSING AND DATA REPRESENTATION

In this section we consider the utility of principal component analysis for TDEM processing. The principal component transform is a versatile tool. Although we present here how it may be used to enhance the signal to noise ratios of TDEM responses, we have also used it successfully for spatial processing of geophysical data sets collected with different sensor types. The mathematical treatments for these two different cases are identical. In the following we consider the TDEM response at successive multiple time gates to represent a time series which can be thought of as a vector. Likewise, a two-dimensional spatial image can also be represented as a vector.

Data collected with TDEM sensors can be evaluated statistically since many recordings are obtained for a single location in space. This is due to the base frequency of the waveform which can vary from a few Hz to a 1000 Hz, or more. For a TDEM instrument that performs measurements in more than one time gate, each recording will correspond to a time series. There will generally be correlations among each realization but noise is always present. However, by using the covariance properties, it must be possible to extract those features that are coherent across the data sets, to facilitate noise rejection, and to segregate uncorrelated features. A particular transformation known as the Karhunen-Loeve transformation achieves these objectives. The Karhunen-Loeve transformation is also known as the principal component transformation, the eigenvector transformation, or the Hotelling transformation. Discussions of this class of transform can be found in Fukunaga (1972), Ready and Wintz (1973), and Andrews and Hunt (1977). This method can be applied equally well to a set of spatial images, or to a set of time series, and essentially represents a form of 'smart stacking'. One way to achieve the decomposition into principal components is by performing an eigenvalue-eigenvector decomposition of the covariance matrix, and then rotating the original input data vectors using the eigenvector matrix. We show in the following that the same effect may be achieved by constructing the singular value decomposition of the input data matrix. The advantage of the SVD approach is that it greatly illuminates the meaning of the principal component transformation, and it is computationally efficient and stable.

In essence, the transform decomposes the input data set of time series into an orthogonal set of time series that we call eigenseries. The eigenseries associated with the largest singular value of the data matrix includes those features with the greatest variance and that are most strongly coherent across the original input time series. The eigenseries associated with the intermediate singular values rejects those features in the input data sets that are highly correlated as well as highly uncorrelated. The eigenseries associated with the smallest singular value contains those features that are least correlated across the data sets, and often includes a strong noise component. Thus, the principal time series corresponding to the largest singular value represents the desired result of a smart stack.

The Singular Value Decomposition Approach to the Principal Component Transform

It turns out that the principal components obtained from the Karhunen-Loeve transformation are identical to the vectors of the matrix U computed in a singular value decomposition (SVD) of the data matrix X. Here, we take the matrix X to be of dimension (NxM), where N is the number of rows and corresponds to the total number of time series points, and M is the number of columns, and is simply equal to the number of time series. The singular value decomposition of the matrix X is written as the product of three matrices:

$$X = UWV^T \qquad (5.1)$$

or in index notation

$$X = \sum_{i=1}^{r} w_i\, u_i v_i^T \qquad (5.2)$$

where r is the rank of X, u_i is the ith eigenvector of XX^T, v_i is the ith eigenvector of X^TX, and w_i is the ith singular value of X. A detailed derivation of these eigenvector relationships and the SVD itself can be found in Aki and Richards (1980) (also see Press *et al.*, 1986). The singular values w_i can be shown to be the positive square roots of the eigenvalues of the covariance matrices XX^T or X^TX (see Lanczos, 1961, for a derivation). The eigenvalues are always real and positive due to the positive definite nature of covariance matrices. In equation (5.2), the factor $u_i v_i^T$ is an NxM matrix of unitary rank which we call the ith eigenimage of X (e.g. Andrews and Hunt, 1977). Owing to the orthogonality of the eigenvectors, eigenimages form an orthogonal basis for the representation of X. As can be seen from the form of equation (5.2), the contribution to the construction of X of the eigenimage associated with a given singular value w_i is proportional to that singular value's magnitude. Since the singular values are always ordered in decreasing magnitude, the greatest contributions in the representation of X are contained in the first few eigenimages.

In our application, X represents the recorded time series from many different cycles of the primary waveform at the same location in space. Now suppose that all M time series are linearly independent, *i.e.*, no time series may be represented in terms of a linear combination of the other M-1 time series. In this case, X is of full rank M and all the singular values w_i are different from zero. Hence, the perfect reconstruction of X requires all eigenimages. On the other hand, in the case where all M time series are equal to within a scale factor, all images are linearly dependent; X is of rank one and may be perfectly represented by the first eigenimage $w_1 u_1 v_1^T$. In the general case, depending on the linear dependence which exists among the images, X may be reconstructed from only the first few eigenimages. In this case, the data may be considered to be composed of time series which show a high degree of series-to-series correlation. If only p ($p < r$) eigenimages are used to approximate X, a reconstruction error e is given by

$$\varepsilon = \|X_p - X\|^2$$

$$= (UW_pV^T - UWV^T)\,(UW_pV^T - UWV^T)^T$$

$$= \sum_{k=p+1}^{r} w_k^2 \qquad (5.3)$$

where W_p is a diagonal matrix of singular values in which the singular values w_k in the range $p+1 \le k \le r$ have been set to zero.

We can now define band-pass X_{BP}, low-pass X_{LP}, and high-pass X_{HP} images in terms of the ranges of singular values used in the reconstruction. The band-passed image is reconstructed by rejecting highly correlated as well as highly uncorrelated images and is given by

$$X_{BP} = \sum_{i=p}^{q} w_i\, u_i v_i^T \qquad 1 < p \le q < r. \qquad (5.4)$$

The summation for the low-pass image X_{LP} is from $i=1$ to p-1 and for the high-pass image X_{HP} is from $i=q+1$ to r. It may be simply shown that the fraction of energy which is contained in a reconstructed image X_{BP} is given by E, where

$$E = \sum_{i=p}^{q} (w_i)^2 \;\Big/\; \sum_{i=1}^{r} (w_i)^2 \qquad (5.5)$$

The Karhunen-Loeve matrix of X is the matrix that contains the principal components and is denoted by K. K is an $(N \times M)$ matrix, and is given by

$$K = XV. \qquad (5.6)$$

This matrix multiplication represents a rotation of the input data X into the eigenvector frame ($v_1, v_2, \dots v_M$). We note that the vector components of K (*i.e.*, $k_1, k_2, \dots, k_M$) are mutually orthogonal. Using the singular value decomposition of X, we may write

$$K = UWV^TV \qquad (5.7)$$

which, upon taking advantage of the identity $V^TV = I$ becomes

$$K = UW. \qquad (5.8)$$

Recalling that ($v_1, v_2, \dots v_M$) are the eigenvectors of X^TX, we now see that U is the signal matrix X after both projecting into the frame of the eigenvectors of the covariance matrix and normalization by division with the singular values.

6.0 Conclusions

We have described a research program to improve the utility of the TDEM method for UXO detection and imaging. The key component of our approach is to develop the forward and inverse solutions for the TDEM signals. This is a challenging problem due to the large conductivity contrasts characteristic of UXO.

A quantitative theory would provide the basis for interpreting the spatial-temporal complexity of the TDEM response, and would provide information that could be used to improve instrument design and the performance of target recognition algorithms based on neural nets. Finally, we suggested that the principal component transform provides a formal empirical basis for jointly interpreting responses from multiple sensor types, and for improving the signal to noise ratio of data from single sensor types.

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