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Comments on an Analytical Thermal Agglomeration for Problems with Surface Growth

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Disclaimer

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Up until Dec 2016, the thermal agglomeration was very "heuristic", and as such, difficult to define. The general scheme was as follows (per the general definition of the problem below):

- $D_a = 20D_r$,
- $\frac{R_a}{D_a} = \frac{R_r}{D_r}$,
- $v_a = v_r$,
- The power p_a is chosen so that volumetric energy density was maintained + whatever "heuristic effects" are necessary to make the results well-behaved.

The lack of predictability became problematic, and the current notes represent the first real attempt to systematize the specification of the agglomerated process parameters.

First, consider a mathematical description of the problem: given a *reference* process, which is taken to be the process at the physical scale at which it actually occurs, defined by the 4-tuple $\{D_r, R_r, v_r, p_r\}$, where D_r is depth of the material layer of interest, R_r is the radius of a (notionally cylindrical) volumetric heat source, v_r is the velocity of the heat source, and p_r is the power of the heat source, the "agglomeration" refers to a different process, which will be represented as $\{D_a, R_a, v_a, p_a\}$. In a very real sense, we now have four unknowns ($\square_a$), for which we need four equations in order to determine. For the sake of the current discussion, the first two of the constraints described above will be maintained more-or-less as described, in order to allow the user control over the degree of geometric agglomeration, in particular,

$$D_a = n_D D_r, \tag{1}$$
$$R_a = n_R R_r; \tag{2}$$

indeed, these wholly specify D_a and R_a, leaving v_a and p_a to be determined.

An initial analysis of the reference problem may prove fruitful . . . Consider the case of a single track of the heat source, with depth D_r, heat source radius R_r, and an arbitrary length L, and a larger array of single tracks, with the end cross section specified via D_a and R_a, such that they form a $n_D \times n_R$ grid of single tracks, which is still just L long; we will refer to this larger volume hereafter as V_a. Introducing the relevant times, it is clear that it takes $t_r = \frac{L}{v_r}$ for the heat source to traverse a single track. If we were to build up V_a *trackwise*, it would take $t_{a,r} = (n_D \times n_R)t_r$ time to do so. Likewise, the heat source imparts $E_r = p_r\, t_r$ energy into each single track, and building up V_a trackwise would result in $E_{a,r} = (n_D \times n_R)\, E_r = (n_D \times n_R)p_r\, t_r$. At this point, we note the following:

- One obvious notion of agglomeration is the one of keeping the time to transform the larger volume V_a equal to the time required to build it up trackwise, *i.e.*, $t_a = t_{a,r}$. In this sense, the part-scale time scale is as realistic as possible. For this case,

$$v_a = \frac{L}{t_{a,r}} = \frac{L}{(n_D \times n_R)t_r} = \frac{1}{n_D \times n_R} v_r, \tag{3}$$

and, for the constraint of constancy of energy over V_a, the agglomeration of the energy results in

$$E_a = E_{a,r}, \tag{4}$$
$$p_a t_a = (n_D \times n_R) p_r \, t_r, \tag{5}$$
$$p_a t_{a,r} = (n_D \times n_R) p_r \, t_r, \tag{6}$$
$$p_a (n_D \times n_R) t_r = (n_D \times n_R) p_r \, t_r, \tag{7}$$
$$p_a = p_r. \tag{8}$$

- Another notion of agglomeration would be to let $t_a = t_r$, which would result in $v_a = v_r$. In this sense, the pointwise time scale is maintained, but the time over which the volume V_a is built up is significantly reduced. In this case, the equivalence of the energy results in

$$E_a = E_{a,r}, \tag{9}$$
$$p_a t_a = (n_D \times n_R) p_r \, t_r, \tag{10}$$
$$\implies$$
$$p_a = (n_D \times n_R) p_r. \tag{11}$$

- It is noted that there are other possible criteria with respect to which to agglomerate the energy; I was originally using constancy of the volumetric energy density, resulting in

$$p_a = V_a \frac{p_r}{V_p}. \tag{12}$$

  As an example, for a typical use case of $n_D = n_R = 20$, for these two different agglomerations, $p_a = 400 \, p_r$ (case 1, derived herein) and $p_a = 8000 \, p_r$ (case 2, with $L = 1 \, mm$, which was the method I previously used, and was "heuristic"). This might have had something to do with the massive overheating . . . :\

- Of course, any reasonable (*i.e.*, positive) value of t_a could be chosen, but the obvious choices are $t_a \in [t_r, t_{a,r}]$; values on the interior of the interval would result in both $v_a \neq v_r$ and $p_a \neq p_r$. If we consider t_a parameterized by the scalar $\alpha \in [0, 1]$, where

$$\log(t_a) = \log(t_r) + \alpha \left(\log(t_{a,r}) - \log(t_r) \right), \tag{13}$$

  so that $\alpha = 0 \implies t_a = t_r$, $\alpha = 1 \implies t_a = t_{a,r}$, and $\alpha = 0.5 \implies t_a = \sqrt{t_r \, t_{a,r}}$ (*i.e.*, the geometric mean), then, simply,

$$v_a = \frac{L}{t_a} = v_r \frac{t_r}{t_a}, \tag{14}$$

  and

$$E_a = E_{a,r}, \tag{15}$$
$$\implies$$
$$p_a = p_r \frac{t_{a,r}}{t_a} \tag{16}$$

It is noted that if this new agglomeration works, it seems sensible to change the controls of the heat sources to only allow sensible behavior. In this case, I would imagine the input parameters being the reference process parameters D_r, R_r, v_r, p_r, the agglomerated geometry D_a, R_a, and maybe the value α, in the case that different values of t_a are reasonable for different analysis cases.

Now, consider an example. Consider the reference process, defined by $D_r = 0.050\,mm$, $R_r = 0.025\,mm$, $v_r = 1600\,mm/s$, and $p_r = 250\,W$. Next, assume that we choose $D_a = 1.0\,mm$ and $R_a = 0.5\,mm$, which results in $n_D = 20$ and $n_R = 20$. It is noted that the two characteristic times are $t_r = 6.25e{-}04\,s$ and $t_{a,r} = (20 \times 20)t_r = 0.25\,s$. If we chose $t_a = t_{a,r}$, then $v_a = 4.0\,mm/s$, and $p_a = 250\,W$.

One final point is worthy of comment . . . It is the case that, to maintain a particular time scale over the complete *layer cycle*, *i.e.*, to maintain the time between the beginning of layers n and $n+1$, not only must the process be agglomerated, but the inter-layer time must be agglomerated as well. For the particular constraint of maintaining the total time between layers precisely, the following relation must be satisfied:

$$n_D \left(t_{layer,r} + t_{interlayer,r}\right) = t_{layer,a} + t_{interlayer,a}. \tag{17}$$

For the case of $\alpha = 1$, the time to consolidate any volume of material is maintained, *i.e.*,

$$n_D t_{layer,r} = t_{layer,a}, \tag{18}$$

which would result in

$$n_D t_{interlayer,r} = t_{interlayer,a}. \tag{19}$$

Obviously, other values of α would require different agglomerated interlayer times, as would different constraints on the total time between layers.