Advancing the Theory of NuclearReactions with Rare Isotopes: From the Laboratory to the Cosmos
Final report from Michigan State University[1]
MSU PI: Filomena Nunes
June 2010 – June 2015

Abstract

The mission of the Topical Collaboration on the Theory of Reactions for Unstable iSotopes (TORUS) was to develop new methods to advance nuclear reaction theory for unstable isotopes—particularly the (d,p) reaction in which a deuteron, composed of a proton and a neutron, transfers its neutron to an unstable nucleus. After benchmarking the state-of-the-art theories, the TORUS collaboration found that there were no exact methods to study (d,p) reactions involving heavy targets; the difculty arising from the long-range nature of the well known, yet subtle, Coulomb force. To overcome this challenge, the TORUS collaboration developed a new theory where the complexity of treating the long-range Coulomb interaction is shifted to the calculation of so-called form-factors. An efficient implementation for the computation of these form factors was a major achievement of the TORUS collaboration. All the new machinery developed are essential ingredients to analyse (d,p) reactions involving heavy nuclei relevant for astrophysics, energy production, and stockpile stewardship.

1 Report on Research

Mission

The TORUS topical collaboration's goal was to develop methods to improve the accuracy and reliability of calculations of neutron and proton capture rates, make it possible to extract relevant structure information from one-nucleon transfer reactions, and advance methods for determining important reaction cross sections from indirect measurements.

The first component of TORUS focuses on improving the descriptions of direct reactions. This part of the project is performed by the MSU and Ohio groups, in close collaboration. In particular, we employ few-body techniques to develop an advanced treatment of breakup channels during transfer reactions, especially for transfers of nucleons to weakly-bound or unbound (continuum) states.

Science Highlights

1. Benchmark the Continuum Discretized Coupled Channel (CDCC) method again the current implementation of the Faddeev AGS method (FAGS): Uphadhyay, Nunes and Deltuva, Phys.

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Rev. C 85, 054621 (2012).

2. Implementation of the Coulomb distorted nuclear form factors: Upadhyay et al., Phys. Rev. C 90, 014615 (2014)
3. With the Ohio group on the work, develop a stand-alone code for partial wave Coulomb wave functions in momentum space: Eremenko, Upadhyay, et al., Computer Physics Communications, 187, 195 (2015)
4. Collaboration with the Ohio group on the development of separable optical potentials: Lhophe et al., Phys. Rev C. 90, 061602 (2014); Phys. Rev C 88, 064608 (2013).

1.1 Breakup and transfer within CDCC

Upadhyay and Nunes, in collaboration with Deltuva

One of the most well established theories for direct nuclear reactions is the Continuum Discretized Coupled Channels (CDCC) method. It includes breakup to all orders by discretizing the projectile continuum into bins. Alternatively, the 3-body problem can be solved exactly within the Faddeev momentum space integral formalism (here denoted FAGS) which explicitly includes breakup and transfer channels to all orders.

With the aim of quantifying the accuracy of CDCC in computing elastic, breakup and transfer cross sections and establish a range of validity, we completed a systematic comparison of CDCC and Faddeev. Our test cases consist of: i) d+^{10}Be at $E_{\rm d} = 21.4, 40.9$ and 71 MeV; ii) d+^{12}C at $E_{\rm d} = 12$ and 56 MeV; and iii) d+^{48}Ca at $E_{\rm d} = 56$ MeV. These reactions were chosen to match available experimental data however the goal of the project was to understand the limitations of CDCC and therefore no fine tuning of interactions was performed. This work has been published [21]. As we summarize below, our results pose important constraints on the validity of CDCC when applied to deuteron induced reactions, as well as the current implementation of FAGS, and call for a better description of the reaction dynamics.

We compute elastic scattering, transfer cross sections to the ground state of the final system, as well as breakup observables. In CDCC, elastic scattering and breakup cross sections are obtained directly from the S-matrix, while transfer is calculated replacing the exact three-body wavefunction by the CDCC wavefunction in the exact post-form T-matrix.

Our CDCC/FAGS comparisons show no immediate correlation between elastic, transfer or breakup. In other words, finding agreement for the elastic for a given target and beam energy does not imply agreement in breakup or transfer. Indeed, these processes are sensitive to different parts of configuration space and therefore, only by looking at elastic, transfer and breakup simultaneously, can the CDCC method be thoroughly tested.

Overall, and regardless of the beam energy, CDCC is able to provide a good approximation to FAGS for *elastic* scattering. The inclusion of a neutron-nucleus bound state in the FAGS1 calculations [21] introduces small modifications mostly are backward angles. Only for d+^{12}C at 12 MeV we found stronger discrepancies in the elastic angular distribution between CDCC and Faddeev-AGS.

The comparison of CDCC and Faddeev-AGS for *transfer* cross sections is consistent with the

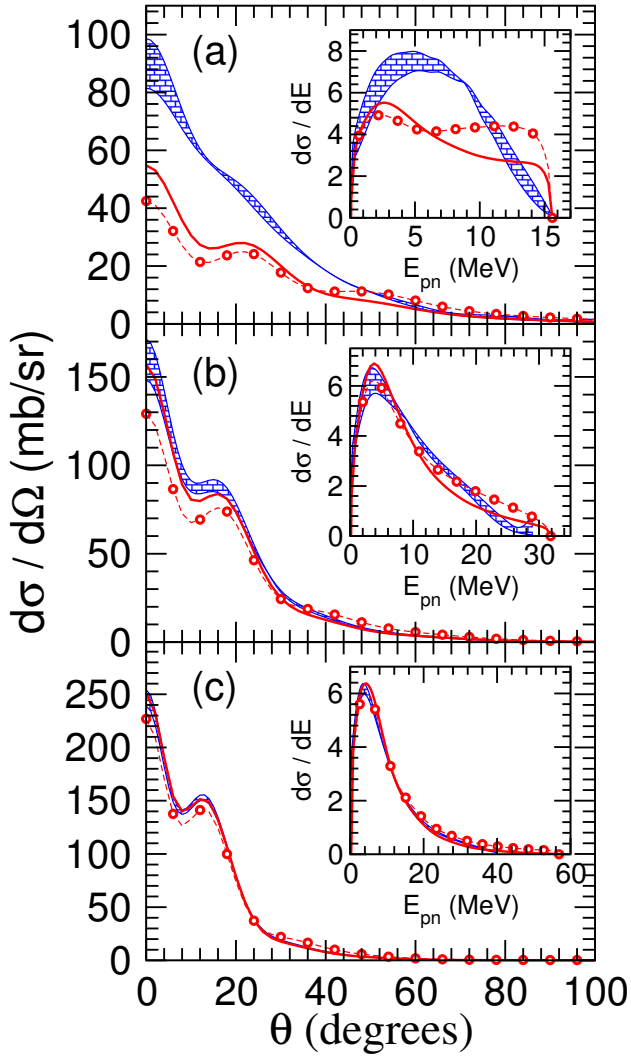

Figure 1: Breakup distributions for ^{10}Be (d, pn) ^{10}Be reaction at: (a) $E_{\rm d}$ = 21 MeV, (b) $E_{\rm d}$ = 40.9 MeV and (c) $E_{\rm d}$ = 71 MeV. Results for CDCC (hashed bar), FAGS(solid) and FAGS1 (circles).

results presented in [16]. We found CDCC to be a very good approximation of FAGS1 at reactions around 10 MeV/u, but not so good for larger beam energies. What became clear from our study is that, for loosely bound s-wave dominated projectiles, CDCC does not improve the description of transfer when compared to the adiabatic model (ADWA). Because CDCC is computationally expensive, ADWA should be the preferred tool. Why, at larger beam energies, CDCC is not able to improve the description of breakup when compared to ADWA has to do with the fact that at higher energies the adiabatic approximation works very well.

Breakup observables predicted by CDCC are at its best for the higher beam energies explored in this work. To reduce the technical challenges of the problem, we ignore the Coulomb interaction in the breakup comparison. Also, we use exactly the same Hamiltonian (CDCC and FAGS) to remove any ambiguity.

In Fig.1 we present the results for the angular distribution as a function of the c.m. angle of the pn system following the breakup on ^{10}Be at the three energies of choice. At the lowest energy, we find that CDCC does not reproduce FAGS, even taking into account the error estimated by

model space truncation. At the higher energies, this discrepancy is removed. The insets of Fig.1 contain the corresponding energy distributions as functions of the proton-neutron relative energy E_{pn}. Again, a very large discrepancy is found at 21.4 MeV while fair agreement between CDCC and FAGS is obtained at the higher energies.

Similar conclusions can be drawn from the comparison of breakup angular and energy distributions for reactions on ^{12}C. Despite the large error bar in the CDCC predictions, there is a striking mismatch between CDCC and FAGS in both magnitude and shape of the breakup cross sections at 12 MeV. These discrepancies disappear at the higher energy. Agreement is obtained between CDCC and FAGS for the breakup of deuterons on ^{48}Ca at 56 MeV.

Although the calculations themselves represent a challenge, understanding the reason for the differences turns out to be even more challenging. We explored the effects of including the nA bound state in the transfer channel with FAGS1 (dotted circles). By comparing FAGS and FAGS1 we conclude that the effects of transfer are not negligible on breakup, particularly at low energies.

In addition we looked at the various components in the Faddeev approach. Strong contributions from the proton and neutron Faddeev components, not explicitly included in the CDCC expansion, are present when the proton-neutron relative energies are large. At low energy, the energy distribution is broad, the breakup to scattering states with large proton-neutron relative energy is important and therefore CDCC does not perform well. One possible solution to this shortcoming may be to use the CDCC wavefunction in a T-matrix that probes only short distances between the proton and neutron, instead of its asymptotic form.

1.2 Coulomb in momentum space without screening

Upadhyay and Nunes, in collaboration with OU

One of the most challenging aspects of solving the three-body problem for nuclear reactions is the repulsive Coulomb interaction. While for light nuclei, often the Coulomb interaction is a small correction to the problem, this is certainly not the case for intermediate mass and heavy systems [15]. Over the last decade many theoretical efforts have focused on advancing the theory for (d,p) (e.g. [14, 1]) and testing existing methods (e.g. [5, 22]). Currently, the most complete implementation of the theory is provided by the Lisbon group [2], which solves the Faddeev equations in momentum space written in the plane wave basis (the so-called AGS equations for Alt, Grassberger and Sandhas). The method introduced in [2] treats the Coulomb interaction with a screening and renormalization procedure as detailed in [4, 3]. While the current implementation of the AGS with screening is computationally effective for light systems, as the charge of the nucleus increases, technical difficulties arise in the screening procedure [15]. Indeed, for most of the new exciting nuclei to be produced at the Facility of Rare Isotope Beams, the current method is not adequate. One then has to explore solutions to the nuclear reaction three-body problem where Coulomb is treated without screening.

This is precisely what is done in [14]. Mukhamedzhanov *et al.* derived a theory for (d,p) whereby the AGS equations are casted in the Coulomb distorted wave representation, instead of the plane wave basis. For a practical implementation of the theory of [14], one needs to be able to accurately compute the Coulomb distorted form factors used as a basis for the theory. This was the focus of the MSU activities during 2013.

First we addressed the form of the Coulomb wave function in momentum space. A closed analytic form for the Coulomb function in momentum space was first derived in [6]. The procedure starts with the Fourier transform of the Coulomb wave function in coordinate space, which results in:

$$\Psi^C_{\vec{p}}(\vec{q}) = -4\pi \exp^{-\eta\pi/2} \Gamma(1+i\eta) \lim_{\gamma\to+0} \frac{d}{d\gamma} \left\{ \frac{[q^2-(p+i\gamma)^2]^{i\eta}}{[\gamma^2+(\vec{p}-\vec{q})^2]^{1+i\eta}} \right\}. \tag{1}$$

Next, one performs the partial wave decomposition, and after some non-trivial mathematical manipulations, obtains the expression:

$$\psi^C_{l,p}(q) \;=\; -\frac{2\pi\, e^{\eta\pi/2}}{pq} \lim_{\gamma\to+0} \frac{d}{d\gamma} \left\{ \left[\frac{q^2-(p+i\gamma)^2}{2pq}\right]^{i\eta} (\zeta^2-1)^{-i\frac{\eta}{2}}\, Q^{i\eta}_l(\zeta) \right\}. \tag{2}$$

Here p is the magnitude of a fixed asymptotic momentum and $\zeta = (p^2+q^2)/2pq$. The Sommerfeld parameter is given as $\eta = Z_1 Z_2 e^2 \mu/p$ with $Z_1 Z_2 e^2$ being the total charge and μ the reduced mass of the two-body system under consideration. The spherical function $Q^{i\eta}_l(\zeta)$ is expressed in terms of Hypergeometic functions ${}_2F_1$, depending on angular momentum l, the strength of the Coulomb potential η and the dynamic variable related to the momenta $\zeta = (p^2+q^2)/2pq$:

$$\begin{aligned} Q^{i\eta}_l(\zeta) \;=\; & \frac{e^{-\pi\eta}}{2} \Bigg\{ \Gamma(i\eta) \left(\frac{\zeta+1}{\zeta-1}\right)^{\frac{i\eta}{2}} {}_2F_1\left(-l, l+1;\, 1-i\eta;\, \frac{1-\zeta}{2}\right) \\ & + \Gamma(-i\eta) \frac{\Gamma(l+1+i\eta)}{\Gamma(l+1-i\eta)} \left(\frac{\zeta-1}{\zeta+1}\right)^{\frac{i\eta}{2}} {}_2F_1\left(-l, l+1;\, 1+i\eta;\, \frac{1-\zeta}{2}\right) \Bigg\}. \end{aligned} \tag{3}$$

These ${}_2F_1$ have tricky conditions, hidden in the spherical function $Q^{i\eta}_l$. The spherical functions in Eq.(3) are valid under two conditions: (a) $|\arg(\zeta \pm 1)| < \pi$ and (b) $|1-\zeta| < 2$, *i.e.*, $-1 < \zeta < 3$. Since ζ is a positive quantity, it always satisfies condition-(a). However, condition-(b) is in some physical situations not satisfied, giving rise to spurious behavior of the function in the region outside the valid range, namely when p is very small or very large. In those situations we have to consider alternate expansions.

The $Q^{i\eta}_l(\zeta)$ can also be expanded for $|\zeta| > 1$ as:

$$Q^{i\eta}_l(\zeta) \;=\; \frac{e^{-\pi\eta}\Gamma(l+i\eta+1)\Gamma(\frac{1}{2})}{2^{l+1}\,\Gamma(l+\frac{3}{2})} \frac{(\zeta^2-1)^{\frac{i\eta}{2}}}{\zeta^{l+i\eta+1}}\, {}_2F_1\left(\frac{l+i\eta+2}{2}, \frac{l+i\eta+1}{2};\, l+\frac{3}{2};\, \frac{1}{\zeta^2}\right). \tag{4}$$

Eq. (4) is well-behaved at low and high momenta, where the original expression Eq. (3) is ill defined. Eq. (3) is valid around the singularity. Thus, it is important to switch to the appropriate expansion depending on the value of ζ. Since no code was available for computing momentum-space Coulomb wavefunctions, we have prepared a stand-alone code that computes the partial wave Coulomb wave function in momentum space and published it in computational physics communication [7].

Next we turn our attention to the form factors. The Coulomb distorted nuclear form factors, required for the Faddeev AGS equations for a three-body system consisting of a deuteron and a

nucleus [14], for which the interactions are given as separable forces of arbitrary rank, are integrals over a nuclear formfactor $u_l(q)$ and the Coulomb wave function $\psi^C_{l,p}(q)$

$$u^C_l(p) = \int_0^\infty \frac{dq\ q^2}{2\pi^2}\ u_l(q)\ (\psi^C_{l,p})^\star(q). \tag{5}$$

The nuclear formfactors should be chosen according to the physical properties of the two-body system under consideration. While for the neutron-proton interaction traditionally a superposition of Yamaguchi formfactors is used [9], for the interaction between neutrons or protons and a nucleus, separable forms of phenomenological optical potentials [11] should be employed. Our formulation for calculating the integrals of Eq. (5) is general.

The main challenge in computing the integral of Eq. (5) is its oscillatory singularity for $p = q$, of the form

$$S(p-q) = \lim_{\gamma\to+0} \frac{1}{(p-q+i\gamma)^{1+i\eta}}. \tag{6}$$

This type of singularity cannot be numerically evaluated by familiar principal value subtractions but rather needs to be treated using the scheme proposed by Gel'fand and Shilov [8]. The essence of this scheme is to subtract as many terms as needed of the Laurent expansion in a small region around the pole so that the oscillations around the pole become small, and the value of the integral around the pole can be estimated analytically.

To our knowledge, this work [23] represents the first attempt to numerically obtain matrix elements with relatively high values of charges involved in the Coulomb distorted basis in momentum space. Given the challenge of accurately calculating the partial wave Coulomb wave functions as well as handling their oscillating singularity, it is critical to demonstrate the numerical accuracy of our computations $u^C_l(p)$ of Eq. (5). For this reason, we performed many tests of the Coulomb distorted nuclear formfactors for the separable Yamaguchi interaction as used in Refs. [14].

1.3 Tests with a Yamaguchi Formfactor

Upadhyay and Nunes, in collaboration with OU

Using a Yamaguchi formfactor as a test case has the advantage that calculations can be performed not only numerically but also semi-analytically, in our case using the *Mathematica* ® [17] software. The Coulomb distorted formfactors, $u^{C,Y}_l(p)$ calculated as integral over the Coulomb wave function given in Eq.(2) and the Yamaguchi formfactor from [14] are depicted in Fig. 2, where our numerical results (labeled FortY) are compared with those from *Mathematica* ® [17] (labeled MathY). The top panels concern protons on ^{12}C and the bottom panels refer to protons on ^{208}Pb. On the right (left) we show the real (imaginary) parts of $u^{C,Y}_l(p)$. Both $l = 0$ and $l = 4$ are shown.

As shown in Fig. 2, the Coulomb distorted Yamaguchi formfactors obtained with our numerical implementation agree perfectly well with the results obtained with *Mathematica*®. To achieve this level of agreement in the form factors, we first compared the accuracy of our numerical implementation of the Coulomb wave functions with the corresponding results provided by *Mathematica*®. The agreement found was in the order of 10 significant figures. Next, we compared the accuracy of the integration given by Eq. (5) and found that our numerical calculation agreed with the

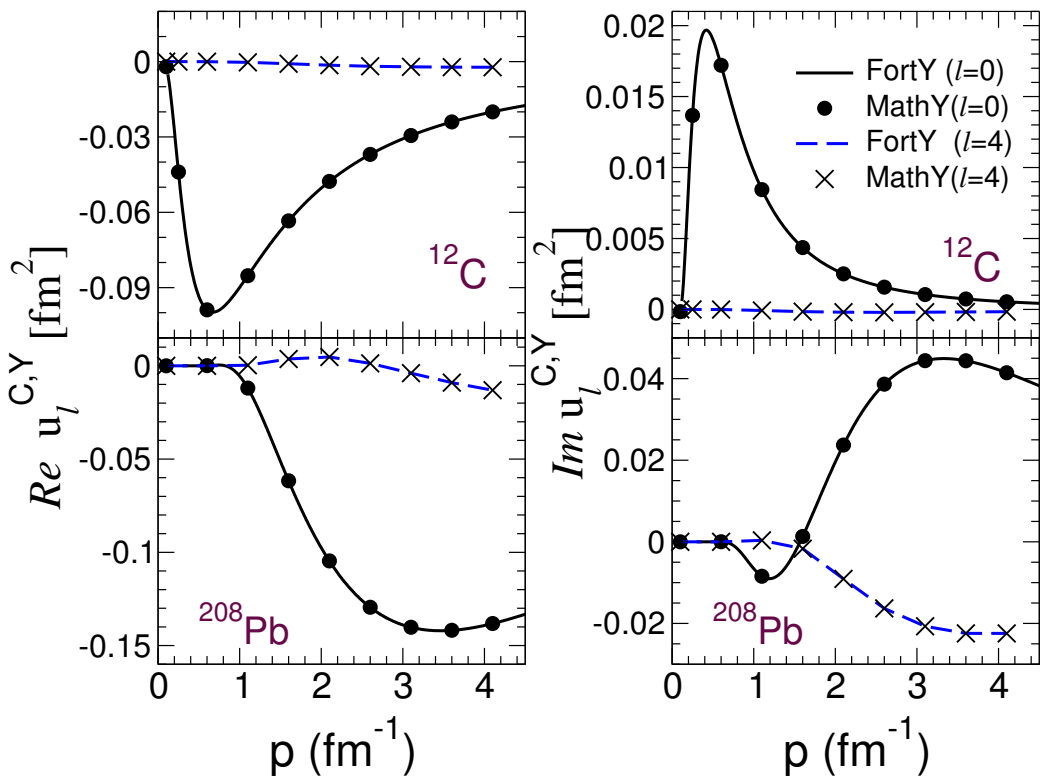

Figure 2: (Color online) The partial wave Coulomb formfactors $u_l^C(p)$ obtained with a Yamaguchi interaction as a function of the external momentum p for selected angular momenta l. Comparison between our numerical evaluation (solid lines) and the *Mathematica*® [17] results (symbols).

corresponding *Mathematica*® calculation for about 6 significant figures. This demonstrates that our numerical implementation of the Coulomb wave functions, integration and regularization techniques, provides a reliable method for calculating form factors involving Coulomb wave functions in momentum space.

In order to explore the importance of the region around the singularity, we have performed additional calculations where we removed a region $p \in [q-\Delta, q+\Delta]$ around the pole $p = q$ from the integral of Eq. (5). In Fig. 3 we show the absolute value of the relative difference between the results $u_l^{C,Y}(p,\Delta)$, obtained removing the pole region, and the full integral $u_l^{C,Y}(p)$, i.e. the quantity

$$\mathbf{D}(\Delta) = \frac{|u_l^{C,Y}(p) - u_l^{C,Y}(p,\Delta)|}{|u_l^{C,Y}(p)|} \tag{7}$$

for fixed values of q. We choose $p = 0.6$ fm^{-1} ($E_{c.m.} = 8.1$ MeV) for ^{12}C and $p = 1.1$ fm^{-1} ($E_{c.m.} = 7.5$ MeV) for ^{208}Pb, as examples. For each of these values of p the nuclear formfactor is far from any node. In Fig. 3 the calculations of the above defined quantity $\mathbf{D}(\Delta)$ are shown as function of Δ for p+^{12}C (top), and for p+^{208}Pb (bottom), for the $l = 0$ (dot-dashed lines) and $l = 4$ (dashed lines). In case of ^{12}C we find that the relative difference is always around 10% or larger, independent of the Δ used and independent of the partial wave. Expectedly, the situation

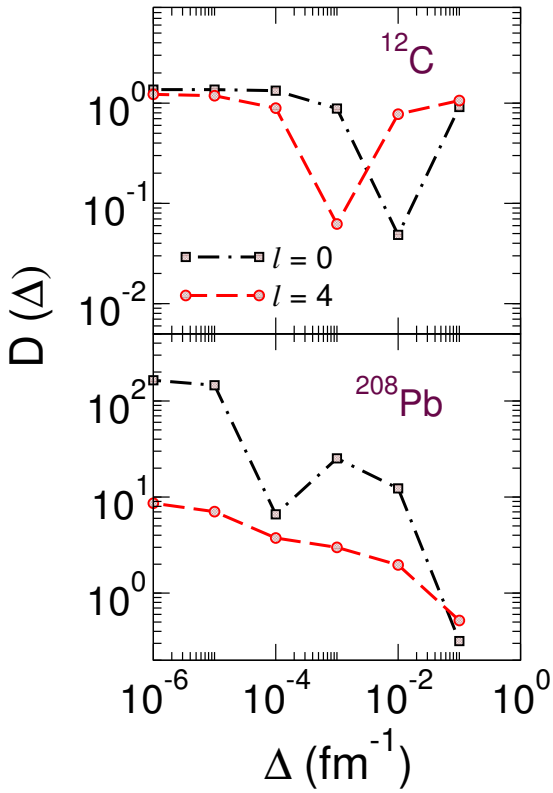

Figure 3: (Color online) The relative difference $\mathbf{D}(\Delta)$ of the exactly calculated integral of Eq. (5) and the integral without including the region $\pm\Delta$ around the pole as a function of Δ for p+^{12}C at $p = 0.6$ fm^{-1} (top) and p+^{208}Pb at $p = 1.1$ fm^{-1} (bottom). Shown are l=0 (dot-dashed) and l=4 (dashed) partial waves.

for ^{208}Pb is worse, discrepancies are about two orders of magnitude for $l = 0$ and one order of magnitude for $l = 4$. The demonstration given in Fig. 3 emphasizes the importance of the pole region.

After finalizing the tests with the Yamaguchi, the codes developed at MSU were ported to OU, for implementation of the realistic interactions using the separable nuclear form factors based on optical potentials [11]. The results with the realistic interactions are shown in the OU report. All details on the implementation of Eq.(5) and the physical cases studied were published in [23]. This work enabled us to understand the general features we obtain for the Coulomb distorted form factors.

1.4 Collaboration with Ohio

After the departure of Dr Upadhyay in the Summer of 2013, funds were directed to Ohio University for hiring the postdoc Dr. Eremenko and a graduate student (Hlophe). MSU continued involved in the projects through a close collaboration with Ohio Univeristy. Nunes participated in weekly skype meeting with the Ohio group, discussing all details of the project, reviewing materi-

als and providing alternate calculations for testing the new developments. While Hlophe focused on the development of optical potentials in separable form, Eremenko finalized the work on the momentum-space Coulomb wavefunction code, and focused on the AGS equations. Nunes was also cowriter of the papers [11, 10]. More details on this part of the project can be found in the report from Ohio University.

2 Other efforts

Nunes and others at MSU

The work of the TORUS postdoc, Neelam Upadhyay, in close collaboration with the OU team, were central contribution of MSU to TORUS during the years (2010-2013). However, there are other projects, that connect to the research developed in TORUS, and that provide levarage to TORUS. Below we provide a short list of those efforts most relevant to TORUS:

- Luke Titus, an MSU PhD student, in collaboration with Nunes, has been investigating nucleon-nucleus non-local potentials [20]. Titus has developed a code to solve the scattering problem with non-local interactions and generalized the adiabatic wave formalism to include non-local interactions in the calculation of transfer reactions. Systematic studies both in DWBA and ADWA demonstrate the importance of including non-locality in the interactions explicitly when computing transfer cross sections. Luke Titus is funded partly by NNSA and partly by NSF.

- Nunes was involved in the interpretation of the GRETINA data taken at NSCL to study ^{56}Ni(d,n)^{57}Cu [12].

- Nunes collaborated with Fred Sarazin from Colorado (and his former student Duane Smalley) on the analysis of TRIUMF data for the reaction ^{12}C(^{6}He,^{4}He)^{14}C, as well as the writing of the paper [19].

- Nunes collaborated with Kate Jones from University of Tennessee (and her former student Kyle Schmitt) on the analysis of the ORNL data on ^{10}Be(d,p)^{11}Be, as well as the writing of the paper [18].

- Amy Lovell, a PhD students working with Nunes, is working on uncertainty quantification in reaction theory [13].

- Jimmy Rotureau, a visiting assistant professor at MSU, is collaborating with Nunes (MSU) and Hagen (ORNL) on constructing a microscopic optical potential from the ab-initio couple cluster Green's functions.

3 Deliverables

Table 1: MSU Publications and Presentations for work completed

	2011	2012	2013	2014	2015	total
Publications	8	7	3	6	3	27
Talks	11	7	7	5	3	33

Publications

1. Improved description of ^{34,36,46}Ar(p,d) transfer reactions
   F. M. Nunes, A. Deltuva and June Hong, Phys. Rev. C 83, 034610 (2011).

2. Asymptotic Normalization Coefficients From the $^{14}\mathrm{C}(\mathbf{d},\mathbf{p})^{15}\mathrm{C}$ Reaction.
   A. M. Mukhamedzhanov, V. Burjan, M. Gulino, Z. Hons, V. Kroha, M. McCleskey, J. Mrzek, N. Nguyen, F. M. Nunes, . Pisko?, S. Romano, M. L. Sergi, C. Spitaleri, and R. E. Tribble; Phys. Rev. C 84, 024616 (2011).

3. Transfer reactions and the dispersive optical model
   N. B. Nguyen, S. J. Waldecker, F. M. Nunes, R. J. Charity, and W. H. Dickhoff; Phys. Rev. C 84, 044611 (2011).

4. Adiabatic approximation versus exact Faddeev method for (d,p) and (p,d) reactions
   F. M. Nunes and A. Deltuva; Phys. Rev. C 84, 034607 (2011).

5. Direct reaction measurements with a ^{132}Sn radioactive ion beam
   K. L. Jones, F. M. Nunes, *et al.*; Phys. Rev. C 84, 034601 (2011).

6. Asymptotic normalization of mirror states and the effect of couplings
   L. J. Titus, P. Capel, and F. M. Nunes, Phys. Rev. C 84, 035805 (2011).

7. Are present reaction theories for studying rare isotopes good enough?
   F. M. Nunes, P. Capel, R.J. Charity, A. Deltuva, W.Dickhoff, H. Esbensen, R.C. Johnson, N.B. Nguyen, N.J. Upadhyay, S.J. Waldecker, Proceedings CGS14, World Scientific.

8. One-neutron halo structure by the ratio method
   P. Capel, R. C. Johnson, F. M. Nunes, Physics Letters B **705**, 112 (2011).

9. Testing the continuum discretized coupled channel method for deuteron induced reactions
   N.J. Upadhyay, A. Deltuva and F.M. Nunes, Phys. Rev. C 85, 054621 (2012).

10. Halo nucleus ^{11}Be : a spectroscopic study via neutron transfer
    K. Schmidt *et al.*; Phys. Rev. Lett. 108, 192701 (2012).

11. Comparing non-perturbative models of the breakup of neutron-halo nuclei
    P. Capel, H. Esbensen and F.M. Nunes; Phys. Rev. C 85, 044604 (2012).

12. Low-temperature triple-alpha rate in a full three-body model
    N.B. Nguyen, F.M. Nunes, I.J. Thompson and E.F. Brown, Phys. Rev. Lett. 109, 141101 (2012).

13. The triple-alpha reaction
    F.M. Nunes, N.B. Nguyen and E. Brown, Proceedings for the International Conference on Nuclei in the Cosmos, Cairns, August 2012, PoS *in press* (2012).

14. Status of reaction theory for studying rare isotopes
    F.M. Nunes and N.J. Upadhyay, Proceedings for the International Conference HITES, New Orleans, June 2012, J. Phys. Conf. Ser. 403, 012029 (2012); arXiv: 1209.2691 [nucl-th].

15. Reaction theory for studying rare isotopes
F.M. Nunes, J. Liu, N.B. Nguyen, L. Titus and N.J. Upadhyay, Proceedings for 13th International Conference on Nuclear Reaction Mechanisms, Varenna, June 2012, CERN Proceeding Series (2012).

16. The ratio method: A new tool to study one-neutron halo nuclei,
Capel, P. and Johnson, R. C. and Nunes, F. M. Phys. Rev. C 88, 044602 (2013).

17. Separable representation of phenomenological optical potentials of Woods-Saxon type,
L. Hlophe, Ch. Elster, R. C. Johnson, N. J. Upadhyay, F. M. Nunes, G. Arbanas, V. Eremenko, J. E. Escher, and I. J. Thompson (TORUS Collaboration), Phys. Rev. C, 88, 064608 (2013).

18. Energy dependence for reactions of a ^{10}Be beam with protons and deuterons,
K.T. Schmitt et al., Phys. Rev. C 88, 064612, (2013).

19. Nuclear Theory and Science of the Facility for Rare Isotope Beams,
A.B Balantekin, J. Carlson, D.J. Dean, G.M. Fuller, R.J. Furnstahl, M. Hjorth-Jensen, R.V.F. Janssens, Bao-An Li, W. Nazarewicz, F.M. Nunes, W.E. Ormand, S. Reddy, B.M. Sherrill, M. Phys. Lett. A 29, 1430010 (2014).

20. Two-neutron transfer reaction mechanisms for 12C(6He,4He)14C using a realistic 3body 6He model
D. Smalley et al, Phys. Rev. C 89, 024602 (2014).

21. Testing the Perey effect
L. Titus and F.M. Nunes, Phys. Rev. C 89, 034609 (2014).

22. Determining the rp-process flow through ^{56}Ni: resonances in ^{57}Cu(p,γ)^{58}Zn identified with GRETINA,
C. Langer *et al.*, Phys. Rev. Lett. 113, 032502 (2014).

23. Coulomb in momentum space without screening,
N.J. Upadhyay, V. Eremenko, L. Hlophe, F.M. Nunes, Ch. Elster, G. Arbanas, J.E. Escher, and I.J. Thompson (TORUS Collaboration), Phys. Rev. C 90, 014615 (2014).

24. Revisiting Surface-Integral Formulations for One-Nucleon Transfers to Bound and Resonances,
J.E. Escher, I.J. Thompson, G. Arbanas, Ch. Elster, V. Eremenko, L.Hlophe, F.M. Nunes; Phys. Rev. C 89, 054605 (2014).

25. Coulomb wavefunctions in momentum space,
V. Eremenko, N.J. Upadhyay, I.J. Thompson, Ch. Elster, F.M. Nunes, G. Arbanas, J.E. Escher, L. Hlophe (TORUS Collaboration), Comp. Phys. Comm. 187 (2015) 195.

26. Systematic uncertainties in direct reaction theories,
A.E. Lovell and F.M. Nunes, J. Phys. G 42, 034014(2015).

27. Separable representation of proton-nucleus optical potentials,
L. Hlophe, V. Eremenko, Ch. Elster, F.M. Nunes, G. Arbanas, J.E. Escher, and I.J. Thompson (TORUS Collaboration). Phys. Rev. C 90, 061602 (R).

Presentations

1. *Improving the theory for transfer reactions*, Invited talk by F.M. Nunes, INPC July 2010, Vancouver, Canada.

2. *Where did matter come from?*, Colloquium by F.M. Nunes, September 2010 University of Michigan Dearborn, Dearborn, U.S.A.

3. *Reaction theory for studying rare isotopes: the missing piece of the puzzle*, Colloquium by F.M. Nunes, 13 October 2010, University of Notre Dame, South Bend, U.S.A.

4. *Some remarks on reaction theory for Rare Isotopes*, Contribution by F.M. Nunes, 19 January at FUSTIPEN inauguration 2011, Caen, France.

5. *Reaction theory for studying rare isotopes: the missing piece of the puzzle*, Colloquium by F.M. Nunes, 24 February 2011, Michigan State University, East Lansing, U.S.A.

6. *Reactions with deuterons within the CDCC formalism*, Contributed Talk by Neelam Upadhyay, APS meeting, April 2011.

7. *Uncertainties from theory on transfer reactions*, Invited Talk by Filomena Nunes, ECT* Workshop on Transfer and Knockout Reactions, Trento, Italy, May 9, 2011

8. *Advancing the theory of transfer reactions*, Invited Talk by Filomena Nunes, UNEDF annual meeting, East Lansing, 21st June 2011

9. The TORUS project, Invited Talk by Filomena Nunes, FRIB theory workshop, INT, Seattle, 8th August 2011

10. *Are present reaction theories for studying rare isotopes good enough?*, Invited Talk by Filomena Nunes, CGS14, Guelph, 2nd September 2011

11. *Testing formalisms for deuteron breakup and transfer reactions*, Contributed Talk by Neelam Upadhyay, DNP meeting, East Lansing, 28th October 2011.

12. *Lectures on FRIB physics*, Invited Lectures by Filomena Nunes, Lattice QCD summer school, INT, August 2012

13. *Status of reaction theory for studying rare isotopes*, Invited Talk by Filomena Nunes, 13th International conference on Nuclear Reactions Mechanisms, Varenna, 11-15 June 2012

14. *Status of reaction theory for studying rare isotopes*, Invited Talk by Filomena Nunes, HITES, New Orleans, 4-7 June 2012

15. *Status of reaction theory for (d,p) reactions*, Invited Talk by Filomena Nunes, Colloquium at University of Connecticut, 6th April 2012

16. *Reaction theory for exotic nuclei*, Invited Seminar by Neelam Upadhyay, Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai, India May 2012.

17. *Reaction theory for exotic nuclei*, Invited Seminar by Neelam Upadhyay, Centre for Excellence in Basic Sciences, University of Mumbai, Mumbai, India, May 2012.

18. *Comparing CDCC, Faddeev and Adiabatic models*; Talk by Neelam Upadhyay, TORUS Second Year Review, NSCL, Michigan State University, East Lansing, USA, June 2012.

19. *Updates on FRIB and FRIB theory*, Invited Talk by Filomena Nunes, NUCLEI collaboration meeting, Bloomington, June 2013

20. *Lectures on Reaction Theory*, Invited Lectures by Filomena Nunes, TALENT course 6, Caen, 1-20 July 2013

21. *Overview of Nuclear Theory*, Invited Talk by Filomena Nunes, Physics of Atomic Nuclei Program, East Lansing, August 2013

22. *Quantifying the limits of the (d,p) reaction theories* Interview Talk by Neelam Upadhyay, National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, 28 February 2013

23. *The (d,p) reaction theories* & *their limitations* Interview Talk by Neelam Upadhyay, Department of Physics Astronomy, Louisiana State University, Baton Rouge, 8 March 2013

24. *Limitations of (d,p) reaction theory* Talk by Neelam Upadhyay, Stewardship Science Academic Alliance (SSAA) Meeting, Lawrence Livermore National Laboratory, Livermore, 18-19 March 2013

25. *Methods for Vertex Integrals of Coulomb Potentials* Talk by Neelam Upadhyay, TORUS Third Year Review, Lawrence Livermore National Laboratory, Livermore, 11-12 June 2013

26. *Theoretical developments in the study of deuteron induced reactions*, Invited Talk by Filomena Nunes, Nuclear structure and reactions: EXperimental and Ab-initio theoretical perspectives, TRIUMF, 18-21 Feb 2014

27. *FRIB theory: a broad perspective*, Invited Talk by Filomena Nunes, Colloquium, University of Washington St. Louis, October 2014

28. *Theory of nuclear reactions*, Invited Talk by Filomena Nunes, Exotic Beam Summer School, Oak Ridge, July 2014

29. *Recent reaction theory results and plans*, Invited Talk by Filomena Nunes, Center of Excellence RIBSS retreat, East Lansing, June 2014

30. *Theory opportunities with Facility for Rare Isotope Beams*, Invited Talk by Filomena Nunes, Colloquium at Colorado School of Mines, March 2014

31. *Theory opportunities with the Facility for Rare Isotope Beams*, Invited Talk by Filomena Nunes, Western Michigan University, February 2015

32. *Six challenges: reaction theory for heavy unstable nuclei*, Invited Talk by Filomena Nunes, Institute for Nuclear Theory, March 2015

33. *One nucleon transfer reactions and the optical potential*, Invited Talk by Filomena Nunes, 14th International Conference on Nuclear Reaction Mechanisms , Varenna, Italy, June 2015

4 Staff, visitors and travel

MSU Post Doc:
The TORUS project has funding for one postdoc for the duration of 4 years. For the initial 3 years of the collaboration, the postdoc was located at MSU. Neelam Upadhyay developed an initial project, comparing CDCC and Faddeev [21]. Following that detailed study, Neelam's works focused on the numerical implementation of the momentum-space Coulomb distorted wave representation. As a first task, the implementation of the partial wave Coulomb wave function in momentum space was performed. Next, the Coulomb distorted form factors discussed in Section 1.2 were implemented both in fortran90 and mathematica, for testing the codes.

Dr. Upadhyay moved onto another postdoc position at Louisiana State University in August 2013, and has since returned back to India.

Mini-workshop at MSU organized by Nunes and Elster
On November 10 and 11, 2011 a Mini-workshop on the Separabilization of two-body interactions was held at MSU with Profs. Ron Johnson (Surrey), George Rawitscher (U. Connecticut), and Scott Bogner (MSU) as invited guests. In addition postdoctoral researcher N. Upadhyay and graduate students N.B. Nguyen and L. Titus participated in the workshop. During this short workshop the pros and cons of various approaches to representing nucleon-nucleon and nucleon-nucleus interactions by functional approximations, either Sturmian or separable, was discussed.

Mini-workshop at MSU organized by Nunes
On October 18th 2012, a Mini-workshop on AGS equations and implementation, was held at MSU with Deltuva (Lisbon), Elster (Ohio), Hlope (Ohio), Nunes (MSU), Titus (MSU), and Upadhyay (MSU). Discussions included non-local interactions, separabalizing optical potentials, the momentum-space Coulomb distorted representation and details on the AGS equations as implemented by the Lisbon group.

Other Visitors:
Antonio Fonseca (University of Lisbon) visited MSU in November 2010. He is an expert in few-body methods and in particular the AGS method.
Seth Waldecker (from Washington University in St Louis) visited MSU in January 2011. He was a student of Wim Dickhoff and working on the dispersion optical model.
Ron Johnson (Surrey) visited Nunes at MSU during May 2013. During that period the group from OU came to MSU for a two day TORUS meeting, for discussions on various topics including the work on the EST separable potentials [11].

Travel:
On a yearly basis, both the MSU postdoc associated with TORUS and the MSU PI presented the work at conferences, including the Division of Nuclear Physics Fall meeting. In addition, all members of collaboration got together for a yearly retreat in June. All but one year, this retreat was hosted by MSU. In June 2013, the retreat was held at LNLL. Both Nunes and Upadhyay travelled to LLNL (June 2013) for that TORUS retreat. More details on these visits can be found in the full report.

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