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Theoretical and Computational Investigation of High-Brightness Beams

**Final Report**

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**Submitted by**

Chiping Chen
Principal Investigator
Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139

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## Executive Summary

Theoretical and computational investigations of adiabatic thermal beams have been carried out in parameter regimes relevant to the development of advanced high-brightness, high-power accelerators for high-energy physics research and for various applications such as light sources. Most accelerator applications require high-brightness beams. This is true for high-energy accelerators such as linear colliders. It is also true for energy recovery linacs (ERLs) and free electron lasers (FELs) such as x-ray free electron lasers (XFELs). The breakthroughs and highlights in our research in the period from February 1, 2013 to November 30, 2013 were: a) Completion of a preliminary theoretical and computational study of adiabatic thermal Child-Langmuir flow (Mok, 2013); and b) Presentation of an invited paper entitled "Adiabatic Thermal Beams in a Periodic Focusing Field" at Space Charge 2013 Workshop, CERN, April 16-19, 2013 (Chen, 2013). In this report, an introductory background for the research project is provided. Basic theory of adiabatic thermal Child-Langmuir flow is reviewed. Results of simulation studies of adiabatic thermal Child-Langmuir flows are discussed.

## Background

In the past several years, we developed paraxial kinetic and warm-fluid equilibrium theories for a thermal charged-particle beam in a periodic focusing field (Samokhvalova, Zhou and Chen, 2007 and 2009; Samokhvalova, 2008; Zhou, Samokhvalova and Chen, 2008). While the kinetic and warm-fluid beam equilibrium theories are equivalent to the warm-fluid equilibrium theory, the kinetic beam equilibrium theory provides detailed information about the charged-particle distribution in phase space. We found good agreement between the theories and the recent experimental measurements of the electron density profiles at University of Maryland Electron Ring (UMER) (Bernal, Quinn, Reiser and O'Shea, 2002; Zhou, Samokhvalova and Chen, 2008; Samokhvalova, 2008). We found evidence for the control of chaotic particle motion using adiabatic thermal beams (Wei and Chen, 2011). We confirmed the theoretical predictions of the adiabatic thermal beam equilibrium by means of particle-in-cell (PIC) simulations (Barton, Field, Lang and Chen, 2012; Chen, Barton, Field and Lang, 2012; Chen, Akylas, Barton, et al, 2012). Our discovery of the adiabatic thermal beams has filled a major gap in the understanding of periodically focused beams, making an important contribution to beam physics.

Despite these advances, we still need to gain our understanding of charged-particle sources which would enable the particle accelerator community to improve beam brightness. Recently, we have begun to re-examine the Child-Langmuir flow theoretically (Chen, Rizzato and Pakter, 2011) and computationally (Mok, 2013; Chen, 2013), taking into account thermal effects.

## Theory of Adiabatic Thermal Child-Langmuir Flow

Recently, there are renewed interests in the research and development of high-brightness dc thermionic electron or ion guns. The renewed interests arise from a) applications of high-brightness dc electron guns in x-ray free electron laser applications, and b) utilizations of high-brightness dc electron guns in the realization of adiabatic thermal beam equilibria in periodic focusing fields.

The one-dimensional (1D) cold-fluid Child-Langmuir (C-L) flow is an important aspect of dc thermionic gun theory. It corresponds to the cold-fluid equilibrium of a charged-particle flow between two plates with an electrostatic potential bias.

The 1D adiabatic thermal C-L flow we have studied corresponds to a warm-fluid equilibrium of a charged-particle flow between two plates with an electrostatic potential bias under the adiabatic equation of state (Chen, Rizzato and Pakter, 2011). A warm-fluid description of the 1D adiabatic thermal C-L flow has been developed. A self-consistent Poisson equation has been derived. In the limit of zero temperature, the self-consistent Poisson equation recovers the corresponding Poisson equation for the cold-fluid C-L flow. An effort has been initiated to validate the theory via self-consistent simulation. Good agreement has been found between the approximation of the adiabatic equation of state and self-consistent simulation.

We have considered the non-relativistic 1D C-L flow under the influence of a finite temperature profile between two conducting plates located at $z=0$ and $z=d$. The adiabatic warm-fluid equations in cgs units are:

$$mnV\frac{\partial V}{\partial z}=-qn\frac{\partial \varphi}{\partial z}-\frac{\partial p}{\partial z}, \tag{1}$$

$$\frac{\partial nV}{\partial z}=0, \tag{2}$$

$$\partial^2\varphi/\partial z^2=-4\pi qn, \tag{3}$$

$$\frac{\partial}{\partial z}\left(\frac{p}{n^3}\right)=0, \tag{4}$$

$$p=k_B nT, \tag{5}$$

where $n$, $V$, $\varphi$, $p$ and $T$ are the equilibrium density, flow velocity, electrostatic potential, longitudinal pressure and temperature profiles, respectively, $k_B$ is the Boltzmann constant, and $m$ and $q$ are the rest mass and charge of the charged particle, respectively.

Equation (4) is the one-dimensional adiabatic equation of state. It is a statement of entropy conservation. For any small segment of the flow, the square of the normalized rms thermal emittance, $\varepsilon_n^2=\gamma^2\beta^2\varepsilon^2$, is proportional to $p/n^3$ or $T/n^2$, where $\varepsilon$ is the unnormalized rms thermal emittance, $\gamma=(1-V^2/c^2)^{-1/2}\cong 1$ is the relativistic mass factor, and $\beta=V/c$ with $c$ being the speed of light in vacuum. Therefore, the adiabatic equation of state assures that the normalized rms thermal emittance of any small segment of the flow is conserved.

It is readily shown from Eqs. (1), (4) and (5) that

$$\frac{\partial}{\partial z}\left(\frac{1}{2}mV^2+q\varphi+\frac{3p}{2n}\right)=0, \tag{6}$$

Equation (6) is an important conservation law in addition to charge conservation, i.e., $J=qnV=const.$ and entropy conservation, i.e., $p/n^3=const.$ The solution to Eq. (6) is

$$\frac{1}{2}mV^2+q\varphi+\frac{3k_B}{2q^2}\frac{T}{n^2}\frac{J^2}{V^2}=C, \tag{7}$$

where the constant $C$ is defined by

$$C=\frac{1}{2}mV^2(0)+\frac{3}{2}k_BT_c, \tag{8}$$

$T_c$ is the emitter temperature, and use has been made of the boundary condition $\varphi(0) = 0$.

At $C = C_{crit} = 3k_B T_c$, we have

$$V(0) = \left(\frac{3k_B T_c}{m}\right)^{1/2}, \quad (9)$$

$$n(0) = \frac{J}{q\left(\frac{3k_B T_c}{m}\right)^{1/2}}, \quad (10)$$

$$V = \left[\frac{-q\varphi + 3k_B T_c}{m} + \left[\left(\frac{-q\varphi}{m}\right)\left(\frac{-q\varphi + 6k_B T_c}{m}\right)\right]^{1/2}\right]^{1/2}, \quad (11)$$

$$\frac{\partial^2 \varphi}{\partial z^2} = \frac{-4\pi J}{\left[\frac{-q\varphi + 3k_B T_c}{m} + \left[\left(\frac{q\varphi}{m}\right)\left(\frac{q\varphi - 6k_B T_c}{m}\right)\right]^{1/2}\right]^{1/2}}. \quad (12)$$

The boundary conditions for Poisson's equation (12) are

$$\varphi(0) = 0 = \varphi'(0) \quad (13)$$

and

$$\varphi(d) = \Phi_d, \quad (14)$$

where prime denotes the derivative with respect to $z$. Note that $\varphi'(0) = 0$ is the condition for space-charge-limited emission.

Figure 1 shows $\varphi / \Phi_d$ versus $z/d$ for a) cold-fluid C-L flow and b) adiabatic thermal C-L flow with $\hat{J} \equiv J / J_{CL} = 2.6$ and $\hat{T}_c \equiv k_B T_c / (-q\Phi_d) = 0.1$, where $J_{CL} \equiv \left(\sqrt{2}mc^3 / 9\pi q d^2\right)\left(-q\Phi_d / mc^2\right)^{3/2}$ is the current density of the cold-fluid C-L flow.

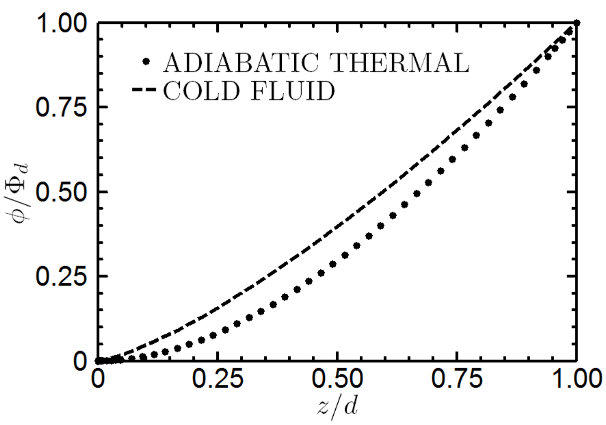


Fig. 1 Plots of the normalized potential $\varphi / \Phi_d$ versus the normalized distance $z/d$ for a) cold-fluid C-L flow with $\hat{J} = 1$ and $\hat{T}_c = 0$ and b) adiabatic thermal C-L flow with $\hat{J} = 2.6$ and $\hat{T}_c = 0.1$ (from Chen, Rizzato and Pakter, 2011).

**Simulation Studies of Adiabatic Thermal Child-Langmuir Flows**

Since the presentation initial results at the 2nd International Particle Accelerator Conference (Chen, Rizzato and Pakter, 2011), we have developed a MatLab code, named One-Dimensional Adiabatic Thermal Child-Langmuir (ATCL1D) for simulation studies of one-dimensional 1D adiabatic thermal Child-Langmuir flows. Several undergraduate researchers (Kevin Burdge, Adrian Jimenez-Galindo, Trung V. Phan, and Tenzin Rabga) initiated the efforts on the development of the Matlab code, and a graduate student (Rachel V. Mok) has completed the code development, carried simulation studies, and written a master thesis summarizing this work (Mok, 2013). Results of the simulation studies have been discussed in an invited paper entitled "Adiabatic Thermal Beams in a Periodic Focusing Field" at Space Charge 2013 Workshop, CERN, April 16-19, 2013 (Chen, 2013).

As an example, Figs. 2-4 show the results of a simulation at the end of one transit time using $2\times 10^6$ charged sheets for the choice of parameters corresponding to: $k_B T_c/(-q\Phi_d)=0.01$ and $J/J_{CL}=1.4886$. Here, $\hat{t}=(-q\Phi_d/2md^2)^{1/2}$ is a dimensionless time, and $\hat{t}_{final}=1$. In the simulation, 2 million charged sheets are used. Considerable care has been taken in the loading of the charged-particle sheets initially as well as in the analysis of the simulation data.

The simulation results shown in Fig. 2-4 validate theoretical predictions for a very low voltage between the emitter and the collector. In obtaining these results, 2 million charged sheets have been used.

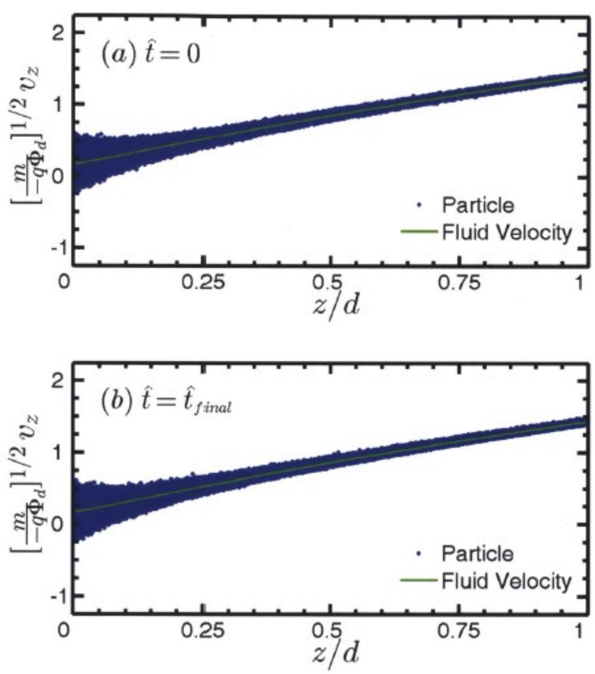


Fig. 2 Initial and final phase space distributions of charged sheets as obtained from a simulation at a) $\hat{t}=0$ and b) $\hat{t}_{final}=1$. The solid curve is the fluid velocity predicted by theory, whereas the dots are the positions and velocities of charged sheets (from Mok, 2013).

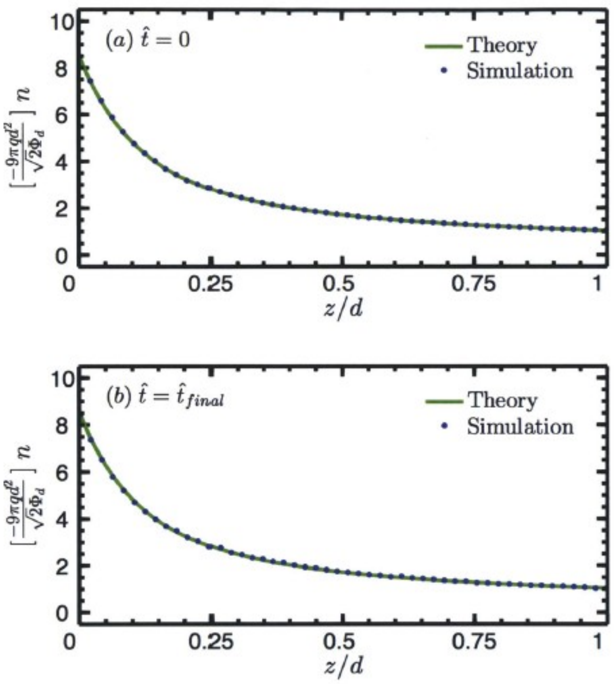


Fig. 3 Initial and final density profiles versus the normalized distance $z/d$ as obtained from a simulation at a) $\hat{t} = 0$ and b) $\hat{t}_{final} = 1$. The solid curve is the density predicted by theory, whereas the dotted curve is from the simulation (from Mok, 2013).

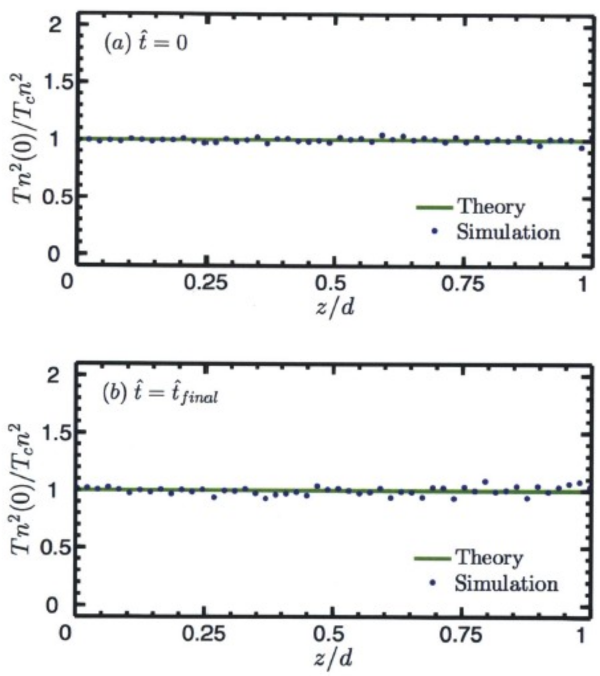

Fig. 4 Initial and final profiles of $Tn^2(0)/T_c n^2$ versus the normalized distance $z/d$ as obtained from a simulation at a) $\hat{t}=0$ and b) $\hat{t}_{final}=1$. The solid line is predicted by theory, whereas the dotted line is from the simulation (from Mok, 2013).

More simulation studies are needed in order to test the theory at moderate and high voltages between the emitter and the collector. The number of charged sheets must be increased by many orders of magnitude in such simulation studies.

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