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**INPUT RELEGATION CONTROL FOR
GROSS MOTION OF A KINEMATICALLY
REDUNDANT MANIPULATOR**

M. A. Unseren

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ABSTRACT

This report proposes a method for resolving the kinematic redundancy of a serial link manipulator moving in a three-dimensional workspace. The underspecified problem of solving for the joint velocities based on the classical kinematic velocity model is transformed into a well-specified problem. This is accomplished by augmenting the original model with additional equations which relate a new vector variable quantifying the redundant degrees of freedom (DOF) to the joint velocities. The resulting augmented system yields a well specified solution for the joint velocities. Methods for selecting the redundant DOF quantifying variable and the transformation matrix relating it to the joint velocities are presented so as to obtain a minimum Euclidean norm solution for the joint velocities. The approach is also applied to the problem of resolving the kinematic redundancy at the acceleration level. Upon resolving the kinematic redundancy, a rigid body dynamical model governing the gross motion of the manipulator is derived. A control architecture is suggested, which according to the model, decouples the Cartesian space DOF and the redundant DOF.

1. INTRODUCTION

The historical trend of minimality in design is manifested in the majority of present day robotic manipulators including their control systems [1]. It is argued in [1] that this historical trend has impeded, to some extent, the development of new theory and new computational and analytical tools needed for the design of future robotic systems. Two of the attributes proposed in [1] for the design and control of future robots are the enlargement of the input and state spaces of the manipulator such that it has a larger number of degrees of freedom (DOF) than the minimum required. The additional DOF enable several tasks to be accomplished simultaneously during motion of the manipulator. The concept of relegation of control which involves assigning specific tasks to subsets of the control inputs to a manipulator system with enlarged state and input spaces is proposed as a particular type of generalized, nonlinear decoupling control in [1]. The control philosophy framework [1] is presented in a conceptual, qualitative manner.

A kinematically redundant manipulator [2,3,4,5,6] is a classical example of an actuated mechanical system possessing enlarged input and state spaces. This report formalizes analytically and mathematically the input relegation control philosophy [1] to the decoupling of the nonredundant and redundant DOF during gross motion of a kinematically redundant manipulator. The approach to resolving the kinematic redundancy as well as deriving a rigid body dynamical model and a control architecture for the manipulator are based on a general method for modeling and controlling constrained mechanical systems [7,8,9].

In the previous work [7,8,9], the motion of the dynamical system studied was restricted by a set of k bilateral constraint equations imposed on n generalized velocities which describe the configuration of the system, where $k < n$. Thus the problem of solving the constraint equations to obtain generalized velocities which satisfy the constraints is underspecified and there are infinitely many solutions. $(n - k)$ new scalar variables (termed pseudovelocities in [7,9]) consisting of linear combinations of the generalized velocities were introduced, which, together with the original constraints, yielded a well specified solution for the generalized velocities.

The Jacobian matrix which transforms the joint velocities to obtain the Cartesian velocities of a point, link, or end effector located on a kinematically redundant manipulator is rectangular with fewer rows than columns. Thus the problem of solving the kinematic velocity transformation equations for the joint velocities is underspecified and there are infinitely many solutions. Interestingly, the problems of resolving the redundancy of a manipulator and of determining generalized velocities which satisfy the constraint equations in a constrained mechanical system appear to be intimately related. It will be investigated here if the approach in [7,8,9] can be extended to the problem of modeling a kinematically redundant manipulator.

The report is organized as follows. A general procedure for resolving the kinematic redundancy is first presented. Several techniques for applying the procedure are discussed. A rigid body dynamical model is derived which consists of separate equations of motion for the nonredundant and redundant DOF. Finally, a decoupled control architecture is developed based on the separated form of the model.

2. PROBLEM STATEMENT AND SYSTEM DESCRIPTION

The problem is to resolve the kinematic redundancy of a serial-link manipulator moving in a three-dimensional workspace. The manipulator has a stationary base and contains N single DOF joints. Upon resolving the kinematic redundancy, a rigid body dynamical model and a control architecture are developed. The kinematical and dynamical models of the redundant manipulator are assumed to be known. The configuration of the system is shown in Fig. 1.

2.1 SYSTEM VARIABLES AND COORDINATE FRAMES

The joint positions of the manipulator are the generalized coordinates describing the configuration of the system. The system variables include the generalized coordinates, velocities, accelerations, and the generalized input forces (i.e., the joint torques) applied to the joint actuators.

As shown in Fig. 1, the coordinate frame (X_k, Y_k, Z_k) is assigned to the k^{th} link of the manipulator, where $k = 0, 1, \dots, N$. The origin of the moving (X_N, Y_N, Z_N) coordinate system is located at the centerpoint of the end effector. The (X_0, Y_0, Z_0) coordinate frame is the stationary base reference system.

2.2 KINEMATIC MANIPULATOR MODEL

The kinematic model for a serial-link manipulator which relates the Cartesian velocities of a point, link, or end effector to the joint velocities is given by:

$$\dot{x} = J(q) \dot{q}. \quad (1)$$

The joint positions of the manipulator defined by the vector $q = [q_1, q_2, \dots, q_N]^T$ are the generalized coordinates, where superscript T denotes a transposition. The Cartesian velocities of a point, link, or end effector on the manipulator are defined by the $(M \times 1)$ vector \dot{x} . It is assumed that $M < N$. For the case where the Cartesian translational and rotational velocities of one of the rigid links (e.g., the $k = N^{th}$ link is the rigid body end effector) are to be controlled, $M = 6$ and $\dot{x} = [(v_0^k)^T, (\omega_0^k)^T]^T$. The (3×1) vector v_0^k is the translational velocity of the k^{th} link at the point coinciding with the origin of the (X_k, Y_k, Z_k) coordinate system. The (3×1) vector ω_0^k is the angular velocity of the k^{th} link. v_0^k and ω_0^k are expressed in the base coordinates. For the case where only the Cartesian translational velocity of a point on the manipulator is to be controlled, then $M = 3$ and $\dot{x} = v_0^k$. In Eq. (1), $J(q)$ is the $(M \times N)$ manipulator Jacobian matrix. $J(q)$ is assumed to possess full rank M .

The corresponding kinematic manipulator model at the acceleration level is obtained by differentiating Eq. (1) in:

$$\ddot{x} = J \ddot{q} + \dot{J} \dot{q}. \quad (2)$$

The $(M \times N)$ matrix $\dot{J} = (\partial J / \partial q) \dot{q}$ in Eq. (2) is a function of (q, \dot{q}) .

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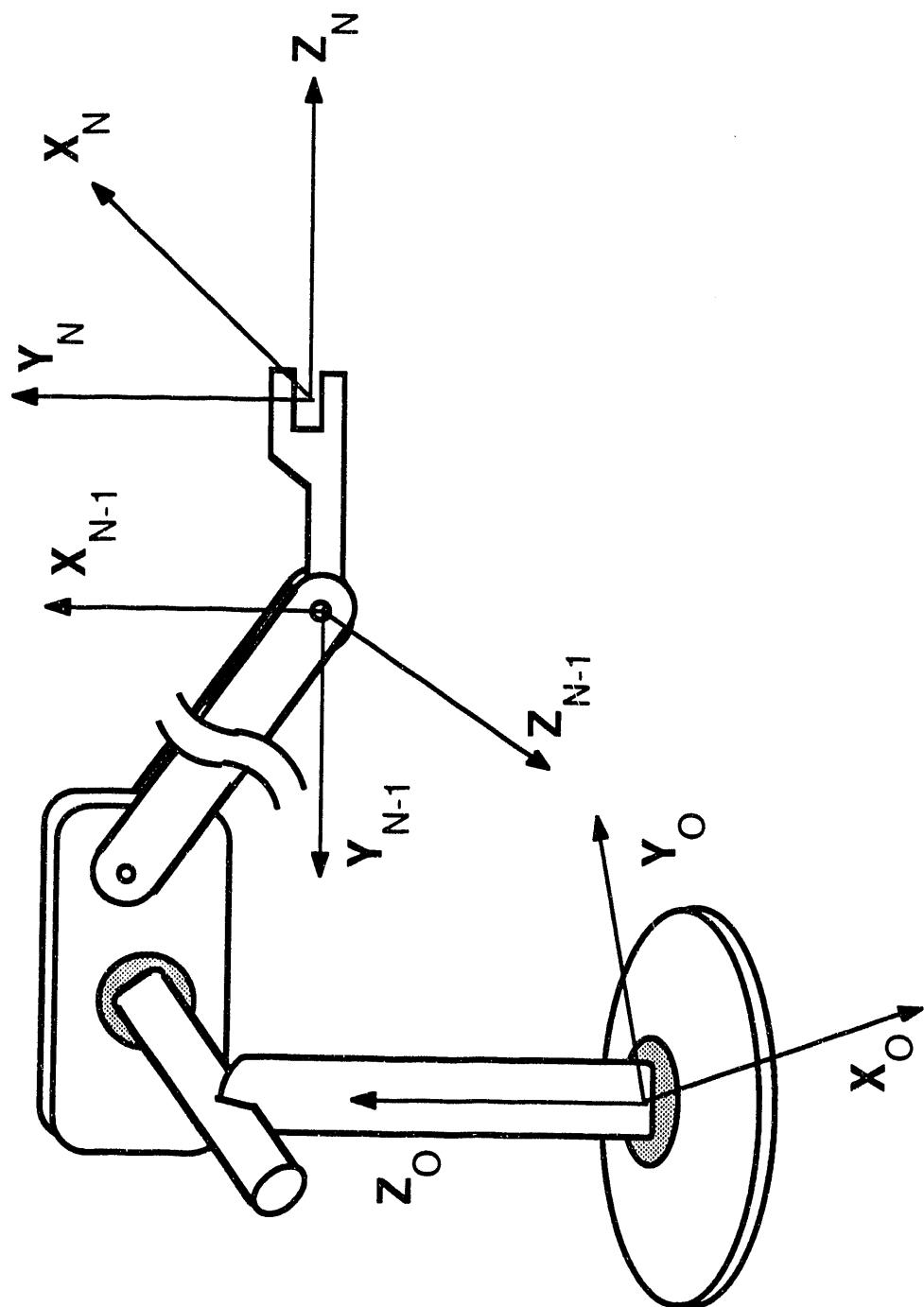


Fig. 1. System configuration and coordinate system assignment.

2.3 DYNAMIC MANIPULATOR MODEL

The dynamical model in the joint space for the manipulator is given in Lagrange's formulation by:

$$\tau = D(q) \ddot{q} + C(q, \dot{q}) + G(q). \quad (3)$$

The generalized input forces (i.e., the joint torques) are denoted by the vector $\tau = [\tau_1, \tau_2, \dots, \tau_N]^T$. The $(N \times N)$ symmetric, positive definite, inertia matrix is $D(q)$, and the Coriolis and centripetal forces are described by the $(N \times 1)$ vector $C(q, \dot{q})$. The gravity forces acting on the manipulator are described by the $(N \times 1)$ vector $G(q)$.

The problem of resolving the kinematic redundancy based on the kinematic model given by Eq. (1) is discussed next.

3. RESOLUTION OF KINEMATIC REDUNDANCY

The problem is to solve Eq. (1) for the generalized velocities \dot{q} given the Cartesian velocities \dot{x} and the generalized coordinates. The solution to the inverse kinematics is underspecified because the number of unknown generalized velocities (N) exceeds the number of equations (M) in the kinematic model. Equation (1) has infinitely many solutions for \dot{q} .

Motivated by the previous work [7,8,9], a new vector variable $\epsilon = [\epsilon_1, \epsilon_2, \dots, \epsilon_{N-M}]^T$ is introduced to resolve the kinematic redundancy. The number of scalar elements contained in ϵ is equal to the number of redundant DOF contained in the system, namely ($N - M$). It is defined by:

$$\epsilon = B(q) \dot{q} . \quad (4)$$

The $((N - M) \times N)$ matrix $B(q)$ in Eq. (4) is selected so that the composite $(N \times N)$ matrix $(J^T(q), B^T(q))^T$ is nonsingular. It is convenient to partition the inverse of $(J^T(q), B^T(q))^T$ into two matrices:

$$\begin{bmatrix} J(q) \\ B(q) \end{bmatrix}^{-1} = [\Pi(q), \Sigma(q)] \quad (5)$$

where $\Pi(q)$ is a $(N \times M)$ matrix and $\Sigma(q)$ a $(N \times (N - M))$ matrix. Equation (5) implies that $J\Pi = I_{M \times M}$, $J\Sigma = 0_{(M \times (N - M))}$, $B\Pi = 0_{((N - M) \times M)}$, $B\Sigma = I_{((N - M) \times (N - M))}$ and $(\Pi J + \Sigma B) = I_{N \times N}$. (Here $I_{r \times r}$ denotes an $(r \times r)$ identity matrix and $0_{r \times s}$ an $(r \times s)$ matrix of zeros). The choice of matrix $B(q)$ and vector ϵ by the designer is somewhat arbitrary. Several examples are provided later in this section.

Differentiating Eq. (4) establishes a relation between $\dot{\epsilon}$ and \ddot{q} :

$$\dot{\epsilon} = B \ddot{q} + \dot{B} \dot{q} . \quad (6)$$

The $((N - M) \times N)$ matrix $\dot{B} = (\partial B / \partial q) \dot{q}$ in Eq. (6) is a function of (q, \dot{q}) .

Equations (1) and (4) can be solved for \dot{q} , and Eqs. (2) and (6) for \ddot{q} :

$$\dot{q} = \Pi \dot{x} + \Sigma \epsilon , \quad (7)$$

$$\ddot{q} = \Pi \ddot{x} + \Sigma \dot{\epsilon} - (\Pi \dot{J} + \Sigma \dot{B}) (\Pi \dot{x} + \Sigma \epsilon) \quad (8)$$

in which Eq. (5) has been invoked. In Eqs. (7) and (8), \dot{q} and \ddot{q} have been expressed as functions of the variables $\{\dot{x}, \epsilon, q\}$ and $\{\dot{x}, \ddot{x}, \epsilon, \dot{\epsilon}, q\}$, respectively. Substituting the right hand side of Eq. (7) into the kinematic velocity model (Eq. 1) reveals that the expression $(J\Sigma\epsilon)$ identically vanishes regardless of the value of ϵ , since $(J\Sigma) = 0_{(M \times (N - M))}$. Likewise, substituting the right hand side of Eq. (7) into Eq. (4) reveals that the expression $(B\Pi\dot{x})$ identically vanishes regardless of the value of \dot{x} , since $(B\Pi) = 0_{((N - M) \times M)}$. Therefore the physical motion of the point, link, or end effector being controlled in Cartesian space $\{\dot{x}\}$ is independent of vector ϵ .

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The preceding general development demonstrates that it is possible to produce equations of the form of Eqs. (7) and (8) by introducing additional Eqs. (4) and (6) which quantify the behavior of the redundant DOF. However, it does not present a criteria for selecting ϵ . Moreover, the development does not indicate an operational procedure for determining matrix $B(q)$. This matrix is not unique, and several methods for constructing it as well as a method for determining ϵ are discussed next.

3.1 SELECTION OF ϵ AND $\dot{\epsilon}$

The achievement or fulfillment of additional tasks or goals other than controlling the motion of a point or link on the manipulator (i.e., \dot{x}) to track a reference trajectory will be relegated to the $(N - M)$ redundant DOF. The specific additional task or goal is pertinent to the selection of ϵ and $\dot{\epsilon}$. To illustrate the quantification of the redundant DOF, analytical expressions for ϵ and $\dot{\epsilon}$ are developed in this section which lead to minimum Euclidean norm solutions for the generalized velocities and accelerations, respectively. Starting with redundancy resolution at the velocity level, ϵ is selected to minimize $\|\dot{q}\|^2$ using Eq. (7):

$$\|\dot{q}\|^2 = (\Pi \dot{x} + \Sigma \epsilon)^T (\Pi \dot{x} + \Sigma \epsilon) . \quad (9)$$

Taking the partial derivative of Eq. (9) with respect to ϵ and equating the result to zero give:

$$\Sigma^T \Sigma \epsilon + \Sigma^T \Pi \dot{x} = 0_{((N-M) \times 1)} . \quad (10)$$

Since Σ has full rank $(N - M)$, then matrix $(\Sigma^T \Sigma)$ is positive definite, symmetric, and therefore nonsingular. Thus Eq. (10) can be solved for ϵ :

$$\epsilon = -(\Sigma^T \Sigma)^{-1} \Sigma^T \Pi \dot{x} . \quad (11)$$

It is important to note that matrices Π and Σ are unknown quantities whereas \dot{x} is known on the right-hand side of Eq. (11). To determine ϵ using Eq. (11), the designer first chooses a matrix $B(q)$ which immediately leads to the determination of $[\Pi(q), \Sigma(q)]$ by Eq. (5). Sufficient information is now available to calculate ϵ . Substituting the right-hand side of Eq. (11) into Eq. (7) and grouping terms give:

$$\dot{q} = \left(I_{N \times N} - \Sigma (\Sigma^T \Sigma)^{-1} \Sigma^T \right) \Pi \dot{x} . \quad (12)$$

Equation (12) provides a minimum Euclidean norm solution for the generalized velocities as a function of the variables $\{q, \dot{x}\}$. It is straightforward to verify that Eqs. (11) and (12) satisfy Eqs. (1) and (4). The generalized accelerations, if desired, are obtained by first solving for $\dot{\epsilon}$ by differentiating Eq. (11). \ddot{q} may be determined by differentiating Eq. (12) or by applying Eq. (8). The generalized coordinates are obtained by numerical integration.

In summary, given any $B(q)$ such that $(J^T(q), B^T(q))^T$ is nonsingular, a value for ϵ can be determined using Eq. (11) which yields the minimum norm solution for \dot{q} . It should be mentioned that it is invalid to substitute $J(q)\dot{q}$ for \dot{x} on the right-hand side of Eq. (11) in the preceding optimization procedure because \dot{q} is unknown. There would be insufficient information available to solve for ϵ . Moreover, this would

incorrectly imply that $B = -(\Sigma^T \Sigma)^{-1} \Sigma^T \Pi J$ based on the definition of ϵ in Eq. (4). Indeed, this expression for B does not satisfy the matrix relations given in Eq. (5).

For redundancy resolution at the acceleration level, the redundant DOF quantified by ϵ are determined to minimize $\|\ddot{q}\|^2$ using Eq. (8):

$$\begin{aligned} \|\ddot{q}\|^2 = & \left\{ \ddot{x}^T \Pi^T + \dot{\epsilon}^T \Sigma^T - (\Pi \dot{x} + \Sigma \epsilon)^T (\Pi \ddot{J} + \Sigma \dot{B})^T \right\} \left\{ \Pi \ddot{x} + \Sigma \dot{\epsilon} \right. \\ & \left. - (\Pi \ddot{J} + \Sigma \dot{B}) (\Pi \dot{x} + \Sigma \epsilon) \right\} . \end{aligned} \quad (13)$$

Taking the partial derivative of Eq. (13) with respect to $\dot{\epsilon}$, equating the result to zero, and rearranging yield a solution for $\dot{\epsilon}$:

$$\dot{\epsilon} = -(\Sigma^T \Sigma)^{-1} \Sigma^T \left[\Pi \ddot{x} - (\Pi \ddot{J} + \Sigma \dot{B}) (\Pi \dot{x} + \Sigma \epsilon) \right] . \quad (14)$$

In Eq. (14), the quantities $\{\Pi, \Sigma\}$ are unknown. To determine $\dot{\epsilon}$ using Eq. (14), the designer chooses matrix B which immediately yields $[\Pi(q), \Sigma(q)]$ by Eq. (5). The time derivatives of $\{J, B\}$ are then obtained. Given an initial value for ϵ (at the starting time) or using its last known value (e.g., from the previous sampling period), sufficient information is now available to compute $\dot{\epsilon}$.

Substituting the right-hand side of Eq. (14) into Eq. (8), rearranging terms, and simplifying give:

$$\ddot{q} = \left(I_{N \times N} - \Sigma (\Sigma^T \Sigma)^{-1} \Sigma^T \right) \Pi \left(\ddot{x} - \dot{J} (\Pi \dot{x} + \Sigma \epsilon) \right) . \quad (15)$$

Equation (15) provides a minimum norm solution for the generalized accelerations as a function of the variables $\{q, \epsilon, \dot{x}, \ddot{x}\}$. It is straightforward to verify that Eqs. (14) and (15) satisfy Eqs. (2) and (6). The solution for ϵ is obtained by numerically integrating Eq. (14). The generalized coordinates and velocities are obtained by numerically integrating Eq. (15).

3.2 SELECTION OF B SUCH THAT ϵ IS A SUBSET OF \dot{q}

In our first approach, we choose B to relegate the control of the redundant DOF to $(N - M)$ of the N joints. In this case B is a constant matrix consisting of ones and zeros. The approach is illustrated by examples.

Example 1: The problem is to control the Cartesian translational and rotational motions of the end effector of the CESARm research manipulator [6,10,11,12] operating in a three-dimensional workspace, as shown in Fig. 2. In this configuration CESARm has a single degree of redundancy with $N = 7$, $M = 6$, and $\dot{x} = [(v_0^7)^T, (w_0^7)^T]^T$. Suppose we relegate the control of the redundant DOF to the third (upper arm roll) joint, i.e., $\epsilon = \dot{q}_3$. The choice is reasonable and logical. Indeed, it is the inclusion of that joint which makes CESARm redundant. B is defined as:

$$B = [0, 0, 1, 0, 0, 0, 0] \quad . \quad (16)$$

Given the (6×7) Jacobian matrix $J(q)$ defined in Eq. (1) and the choice of vector B in Eq. (16), the (7×7) composite matrix $(J^T(q), B^T)^T$ can be inverted symbolically using the method of inverse by partitioning [13]:

$$\Pi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \left\{ J \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \right\}^{-1} , \quad (17)$$

$$\Sigma = \left(I_{7 \times 7} - \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \left\{ J \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \right\}^{-1} J \right) B^T$$

$$= (I_{7 \times 7} - \Pi J) B^T \quad (18)$$

where Π is a (7×6) matrix, Σ is a (7×1) vector and, here again, $I_{r \times r}$ denotes an $(r \times r)$ identity matrix. Upon calculating $\Pi(q)$ and $\Sigma(q)$, ϵ and \dot{q} are obtained using Eqs. (11) and (12), respectively, when it is desired to minimize $\|\dot{q}\|^2$.

Example 2: The problem is to control the Cartesian translational motion of the end effector of the CESARm research manipulator at its centerpoint. In this configuration CESARm is a spatial mechanism with four degrees of redundancy ($N = 7$, $M = 3$, $\dot{x} = v_0^7$). A practical application of such a configuration is having the end effector trace a circle where its orientation is of no consequence. We choose $\epsilon = [\dot{q}_3, \dot{q}_5, \dot{q}_6, \dot{q}_7]^T$, thus:

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad . \quad (19)$$

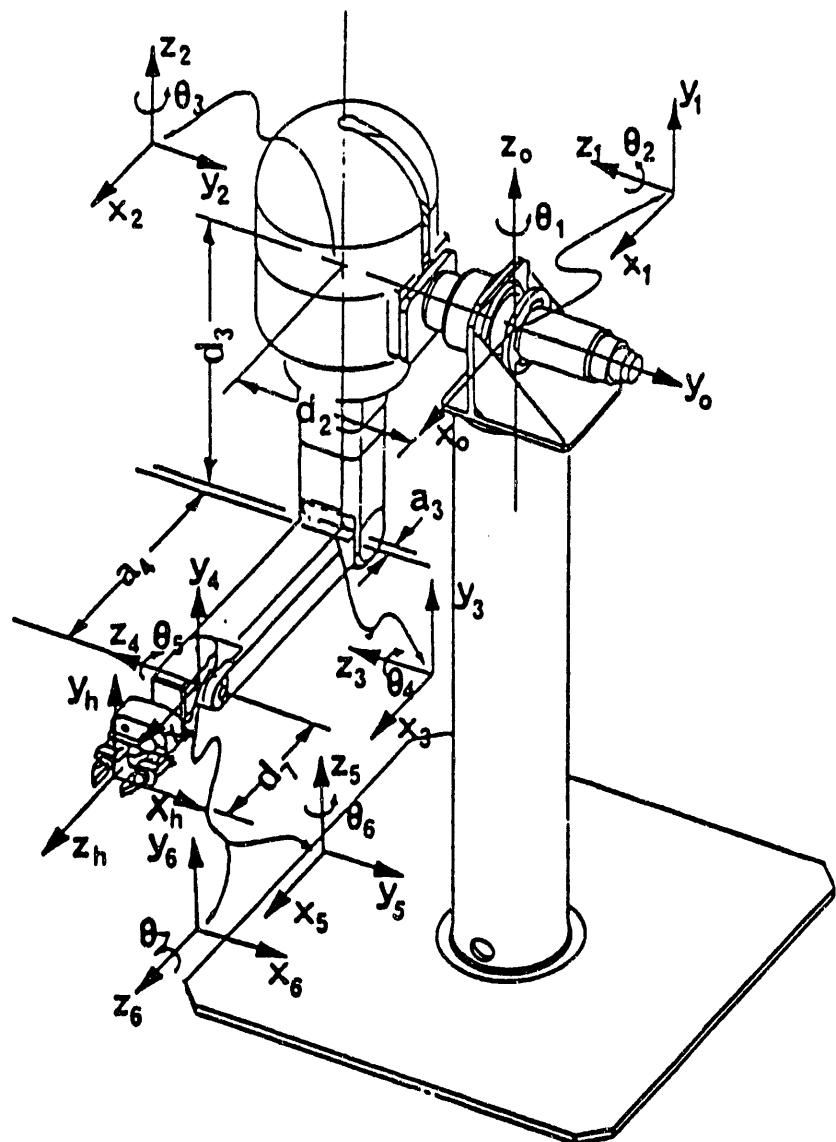


Fig. 2. System configuration and coordinate system assignment for the CESARm research manipulator.

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Given the (3×7) matrix $J(q)$ defined in Eq. (1) and matrix B defined by Eq. (19), the (7×7) matrix $(J^T(q), B^T)^T$ can be inverted symbolically [13]:

$$\Pi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left\{ J \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}^{-1}, \quad (20)$$

$$\Sigma = \left(I_{7 \times 7} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left\{ J \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}^{-1} J \right) B^T$$

$$= (I_{7 \times 7} - \Pi J) B^T \quad (21)$$

where Π and Σ are (7×3) and (7×4) matrices, respectively.

The principal benefit of choosing B in this way is that the inversion of the $(N \times N)$ matrix $(J^T(q), B^T)^T$ is reduced to inverting the $(M \times M)$ matrix consisting of M columns of the Jacobian J . Moreover, by choosing B as a constant matrix, the problem of deriving an analytical expression for \dot{B} is avoided.

Interestingly, the particular \dot{q}_p and homogeneous \dot{q}_h solutions to Eq. (1) proposed in [6] are just special cases of the components $\{\Pi \dot{x}\}$ and $\{\Sigma \epsilon\}$ of the general solution for \dot{q} proposed in Eq. (7). Indeed, by selecting B as a constant matrix such that ϵ is a subset of \dot{q} , the results of [6] have been obtained here as shown in Example 1 for manipulators with one degree of redundancy. For the case of multiple degrees of redundancy, a procedure is suggested in [6] which is equivalent to transforming the $(N - M)$ linearly independent columns of the orthogonal complement Σ so they become mutually orthogonal using the Gram-Schmidt procedure [14]. Such techniques are numerically intensive and may produce computed vectors that are far from orthogonal [14]. The case of picking B such that ϵ is a subset of the generalized velocities as presented here is motivated in part by its simplicity. The columns of Σ obtained by the method are only linearly independent. The benefits of orthogonalizing the columns of Σ as suggested in [6] are not clearly demonstrated. Besides, this would require recalculating B to ensure that Eq. (5) is satisfied in the method presented here. Thus the minimum Euclidean norm solution for \dot{q} given in Eq. (12) differs from that in [6] when $N - M > 1$. The problem of resolving the kinematic redundancy at the acceleration level is not addressed in [6].

3.3 CHOOSING B ORTHOGONAL TO THE ROWS OF J

This approach to selecting $B(q)$ is inspired in part by the recent work on the modeling and force control of the constrained nonholonomic motion of a platform with multiple, steerable wheels [15]. The partitioned inverse in Eq. (5) only requires that J and B be full rank matrices, and further that the rows of B be linearly independent of the rows of J . Here a method is suggested for choosing $B(q)$ to be orthogonal to the rows of $J(q)$. It is assumed that the serial-link manipulator has a single redundant DOF($N = M + 1$), e.g., the CESARm research manipulator in its most general configuration with $M = 6$ and $N = 7$ [6,10,11,12]. Thus ϵ is a scalar and $B(q)$ is an N -dimensional row vector.

Expanding along the last row of $(J^T(q), B^T(q))^T$ using the Laplace expansion [13], the determinant of $(J^T(q), B^T(q))^T$ is:

$$\det \begin{bmatrix} J(q) \\ B(q) \end{bmatrix} = B(q) \Delta \quad (22)$$

where $\det []$ denotes the determinant of $[]$. In Eq. (22), $\Delta = (\Delta_{N1}, \Delta_{N2}, \dots, \Delta_{NN})^T$ is the vector of cofactors, where Δ_{Ni} is the cofactor of the i th element of the N th row of $(J^T(q), B^T(q))^T$. Vector Δ is a function of the generalized coordinates, but not of the elements of B .

The general class of $(N \times 1)$ orthogonal complements Σ of J is defined as the last column of $\{(J^T(q), B^T(q))^T\}^{-1}$:

$$\Sigma = \frac{1}{\det \begin{bmatrix} J(q) \\ B(q) \end{bmatrix}} \Delta \quad (23)$$

where a specific Σ is obtained by the designer's choice of B .

We select B^T to maximize the determinant of $[J^T(q), B^T(q)]^T$ subject to the constraint of normalizing the Euclidean norm of B^T to a constant value, i.e., $\|B^T\|^2 = \mu$, where the positive scalar μ is a normalization factor selected by the designer. We introduce the Lagrangian \mathcal{L} :

$$\mathcal{L}(B^T, \lambda) = \Delta^T B^T + \lambda (B B^T - \mu) \quad (24)$$

in which Eq. (22) has been invoked. Taking the partial derivative of \mathcal{L} with respect to the vector B^T and scalar λ , respectively, and equating the results to zero provide the necessary optimality conditions:

$$\Delta + 2\lambda B^T = 0_{N \times 1} , \quad (25)$$

$$B B^T - \mu = 0 . \quad (26)$$

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Equation (25) can be solved for B^T :

$$B^T = -\frac{1}{2\lambda} \Delta \quad . \quad (27)$$

Eliminating $\{B, B^T\}$ from Eq. (26) using the right-hand side of Eq. (27) and rearranging terms give:

$$\lambda^2 = \frac{1}{4\mu} \Delta^T \Delta \quad . \quad (28)$$

which yields a solution for λ :

$$\lambda = -\frac{1}{2} \sqrt{\frac{\Delta^T \Delta}{\mu}} \quad . \quad (29)$$

where the negative sign in the general solution $\pm\frac{1}{2}\sqrt{(\Delta^T \Delta)/\mu}$ for λ has been selected to maximize $\det[J^T(q), B^T(q)]^T$. Indeed, it is easy to see that $\partial^2 \mathcal{L}/\partial B^T = 2\lambda I_{N \times N}$ is negative semidefinite when λ is defined by Eq. (29).

Backsubstituting Eq. (29) into Eq. (27) and transposing yield the solution for B :

$$B = \frac{\sqrt{\mu} \Delta^T}{\sqrt{\Delta^T \Delta}} \quad . \quad (30)$$

Substituting the right-hand side of Eq. (30) into Eq. (22) reveals that:

$$\det \begin{bmatrix} J(q) \\ B(q) \end{bmatrix} = \sqrt{\Delta^T \Delta \mu} \quad . \quad (31)$$

A relationship between B and Σ can be obtained from Eqs. (23) and (31):

$$\Sigma = \frac{B^T}{\mu} \quad . \quad (32)$$

Since $J\Sigma = 0_{M \times 1}$, then B is orthogonal to the rows of J . Furthermore, postmultiplying the matrix identity $\Pi J + \Sigma B = I_{N \times N}$ by J^T gives:

$$\Pi J J^T = J^T \quad . \quad (33)$$

Since J has full rank M , then matrix $(J J^T)$ is symmetric, positive definite, and therefore nonsingular. Thus Eq. (33) can be solved for the right inverse Π of J :

$$\Pi = J^T (J J^T)^{-1} \quad . \quad (34)$$

It is immediately evident that the columns of Π are orthogonal to the columns of Σ :

$$\Pi^T \Sigma = 0_{M \times 1} \quad . \quad (35)$$

The general solution for \dot{q} given in Eq. (7) becomes:

$$\dot{q} = J^T (J J^T)^{-1} \dot{x} + \frac{\Delta}{\sqrt{\Delta^T \Delta \mu}} \epsilon \quad . \quad (36)$$

When the redundant DOF are relegated to achieve a minimum Euclidean norm solution for \dot{q} , we substitute Eq. (35) into Eq. (11) to yield $\epsilon = 0$. Thus Eq. (36) simplifies to $\dot{q} = J^T (J J^T)^{-1} \dot{x}$ which is the classic Moore Penrose pseudoinverse solution for the generalized velocities [2,3]. Furthermore, the minimum Euclidean norm solution for the generalized accelerations given by Eq. (15) simplifies to:

$$\ddot{q} = J^T (J J^T)^{-1} \left(\ddot{x} - J \left(J^T (J J^T)^{-1} \dot{x} + \frac{\Delta}{\sqrt{\Delta^T \Delta \mu}} \epsilon \right) \right) \quad . \quad (37)$$

Example 3: The problem is to control the Cartesian translational motion of the centerpoint of the wrist of the CESARm research manipulator as shown in Fig. 2. In this configuration CESARm has a single redundant DOF with $M = 3$, $N = 4$, and $\dot{x} = v_0^4$. The matrix $(J^T, B^T)^T$ can be expanded to reveal its component elements:

$$\begin{bmatrix} J \\ B \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} & j_{13} & j_{14} \\ j_{21} & j_{22} & j_{23} & j_{24} \\ j_{31} & j_{32} & j_{33} & j_{34} \\ b_{11} & b_{12} & b_{13} & b_{14} \end{bmatrix} \quad . \quad (38)$$

Expanding along the fourth row of matrix $(J^T, B^T)^T$ defined in Eq. (38), the vector of cofactors $\Delta (= [\Delta_{41}, \Delta_{42}, \Delta_{43}, \Delta_{44}]^T)$ is given by:

$$\Delta = \begin{bmatrix} -j_{12}(j_{23}j_{34} - j_{24}j_{33}) + j_{13}(j_{22}j_{34} - j_{24}j_{32}) - j_{14}(j_{22}j_{33} - j_{23}j_{32}) \\ j_{11}(j_{23}j_{34} - j_{24}j_{33}) + j_{13}(j_{24}j_{31} - j_{21}j_{34}) + j_{14}(j_{21}j_{33} - j_{23}j_{31}) \\ -j_{11}(j_{22}j_{34} - j_{24}j_{32}) + j_{12}(j_{21}j_{34} - j_{24}j_{31}) - j_{14}(j_{21}j_{32} - j_{22}j_{31}) \\ j_{11}(j_{22}j_{33} - j_{23}j_{32}) + j_{12}(j_{23}j_{31} - j_{21}j_{33}) + j_{13}(j_{21}j_{32} - j_{22}j_{31}) \end{bmatrix} \quad . \quad (39)$$

The Jacobian matrix for this particular configuration of CESARm is defined in Appendix A. Substituting for j_{lm} in Eq. (39) using the appropriate equation from Appendix A yields the solution for the cofactors Δ_{4i} ($i = 1, 2, 3, 4$) as a function of the lower four joint angles of CESARm:

$$\begin{aligned} \Delta_{41} = & - \{ a_3 a_4^2 c_3 c_4 s_4 + a_3^2 a_4 c_3 s_4 + a_4^2 d_3 c_3 c_4^2 \\ & + a_3 a_4 d_3 c_3 c_4 \} \quad , \end{aligned} \quad (40)$$

$$\begin{aligned}
\Delta_{42} = & a_3 a_4^2 s_2 s_3 c_4 s_4 - a_4^2 d_2 s_2 c_4 s_4 + a_3^2 a_4 s_2 s_3 s_4 \\
& - a_3 a_4 d_2 s_2 s_4 + a_4^2 d_3 s_2 s_3 c_4^2 + a_4^2 d_2 c_2 c_3 c_4^2 \\
& + a_3 a_4 d_3 s_2 s_3 c_4 + a_3 a_4 d_2 c_2 c_3 c_4 , \tag{41}
\end{aligned}$$

$$\begin{aligned}
\Delta_{43} = & - \{ a_4^2 d_2 c_2 c_3 s_3 c_4 s_4 - a_3 a_4^2 c_2 c_3 c_4 s_4 + a_4^2 d_3 s_2 c_4 s_4 \\
& + a_3 a_4 d_2 c_2 c_3 s_3 s_4 + a_4 d_2 d_3 s_2 s_3 s_4 - a_3^2 a_4 c_2 c_3 s_4 \\
& - a_3 a_4 d_3 s_2 s_4 + a_4^2 d_2 s_2 s_3 c_4^2 - a_4^2 d_3 c_2 c_3 c_4^2 \\
& - a_3 a_4^2 s_2 c_4^2 - a_3 a_4 d_3 c_2 c_3 c_4 - a_4 d_3^2 s_2 c_4 \\
& - a_4^2 d_2 s_2 s_3 + a_3 a_4^2 s_2 \} , \tag{42}
\end{aligned}$$

$$\begin{aligned}
\Delta_{44} = & a_4^2 d_2 s_2 c_3 c_4 s_4 + a_3 a_4 d_2 s_2 c_3 s_4 - a_4^2 d_2 c_2 c_3^2 c_4^2 \\
& - 2 a_3 a_4 d_2 c_2 c_3^2 c_4 - a_4 d_2 d_3 s_2 c_3 c_4 - a_3^2 d_2 c_2 c_3^2 \\
& - a_3 d_2 d_3 s_2 c_3 \tag{43}
\end{aligned}$$

where $c_i = \cos(q_i)$, $s_i = \sin(q_i)$ and where $\{a_3, a_4, d_2, d_3\}$ are constant Denavit-Hartenberg link length parameters defined in Appendix A.

In [4], the orthogonal complement was selected to be $\Sigma = \Delta$ for M even and $\Sigma = -\Delta$ for M odd when $N - M = 1$. Although not discussed, this implies that $\det[J^T, B^T]^T = \pm 1$. It is not clear why the sign of the determinant is dependent on M being even or odd. The approach presented here selects B so as to maximize $\det[J^T, B^T]^T$ subject to the constraint $\|B^T\|^2 = \mu$, in addition to satisfying $BJ^T = 0_{1 \times M}$. In Appendix B symbolic expressions for $\det[J^T, B^T]^T$ and Σ are derived as functions of B for the case $N - M = 2$. They are the two-redundant-DOF counterparts to Eqs. (22) and (23). Additionally, a method for choosing B to set $\det[J^T, B^T]^T$ to a desired reference value is discussed.

3.4 SELECTING B BY APPLICATION OF THE ZERO-EIGENVALUE MATRIX THEOREM

An approach to selecting $B(q)$ is now suggested which is an application of the zero-eigenvalue matrix theorem [14,16,17,18]. Here we only assume that ($N > M$). With this approach, $B(q)$ will again be chosen to be orthogonal to the rows of $J(q)$.

Let us first consider the properties of the $(N \times N)$ matrix $(J^T J)$. It is proven in Appendix C by analytical techniques that $(J^T J)$ is a singular matrix and that its rank is equal to M . Furthermore, it is shown that $(N - M)$ of the eigenvalues of $(J^T J)$ are identically zero. Let the $(N \times 1)$ vector Y_i ($i = 1, 2, \dots, N - M$) denote the i th eigenvector of $(J^T J)$ corresponding to the i th zero eigenvalue. Then αY_i , where α is any nonzero scalar, is also an eigenvector of $(J^T J)$ associated with the i th zero eigenvalue. This indeterminacy can be removed by further requiring that $\|Y_i\|^2 = 1$. It is straightforward to verify that:

$$J^T J [Y_1, Y_2, \dots, Y_{N-M}] = 0_{(N \times (N-M))} \quad . \quad (44)$$

Premultiplying Eq. (44) by Π^T yields:

$$J [Y_1, Y_2, \dots, Y_{N-M}] = 0_{(M \times (N-M))} \quad (45)$$

in which Eq. (5) has been invoked. Equation (45) reveals an obvious choice for matrix B whose rows are orthogonal to those of J :

$$B = \begin{bmatrix} Y_1^T \\ Y_2^T \\ \vdots \\ Y_{N-M}^T \end{bmatrix} \quad . \quad (46)$$

Given matrix B as defined in Eq. (46), suppose matrix Σ is chosen as:

$$\Sigma = B^T (B B^T)^{-1} \quad (47)$$

where for computational reasons it should be mentioned that the main diagonal elements of $(B B^T)$ are all ones. It is easy to see that $J \Sigma = 0_{(M \times (N-M))}$ and $\Pi^T \Sigma = 0_{(M \times (N-M))}$, where matrix Π is defined by Eq. (34).

When picking vector ϵ to minimize $\|\dot{q}\|^2$ as shown in Eqs. (9) and (11), it is easy to see that $\epsilon = 0_{(N-M) \times 1}$ and $\dot{q} = J^T (J J^T)^{-1} \dot{x}$ when B is defined by Eq. (46). The minimum Euclidean norm solution for the generalized accelerations can be obtained by substituting the above results into Eq. (15). Since analytical methods for obtaining B such that $J B^T = 0_{(M \times (N-M))}$ have been developed in the preceding section for $N - M = 1$, the zero-eigenvalue matrix method is particularly useful for the case of $N - M \geq 2$. It is a numerical approach that requires determining the eigenvectors Y_i using various computational iterative algorithms [18,19].

3.5 THE VECTOR CROSS PRODUCT METHOD FOR PLANAR MANIPULATORS

In this section a method is proposed for determining a full rank orthogonal complement of the Jacobian matrix for the inverse kinematics of a serial-link planar manipulator with revolute joints. The planar manipulator shown in Fig. 3 contains N revolute joints, where $N > 2$. Only the translational motion of the end effector is to be specified and controlled. Therefore, the planar system contains $(N - 2)$ redundant DOF. Let the (2×1) vectors p_0^N ($= [p_{0x}^N, p_{0y}^N]^T$) and v_0^N ($= [v_{0x}^N, v_{0y}^N]^T$) denote the Cartesian translational position and velocity of the end effector in the base coordinate system. p_0^N is related to the joint positions as follows:

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$$p_0^N = \begin{bmatrix} l_1 c(q_1) + l_2 c(q_1 + q_2) + \dots + l_N c(q_1 + q_2 + \dots + q_N) \\ l_1 s(q_1) + l_2 s(q_1 + q_2) + \dots + l_N s(q_1 + q_2 + \dots + q_N) \end{bmatrix} \quad (48)$$

where l_i is the constant length of link i and $c(\cdot) = \cos(\cdot)$, $s(\cdot) = \sin(\cdot)$.

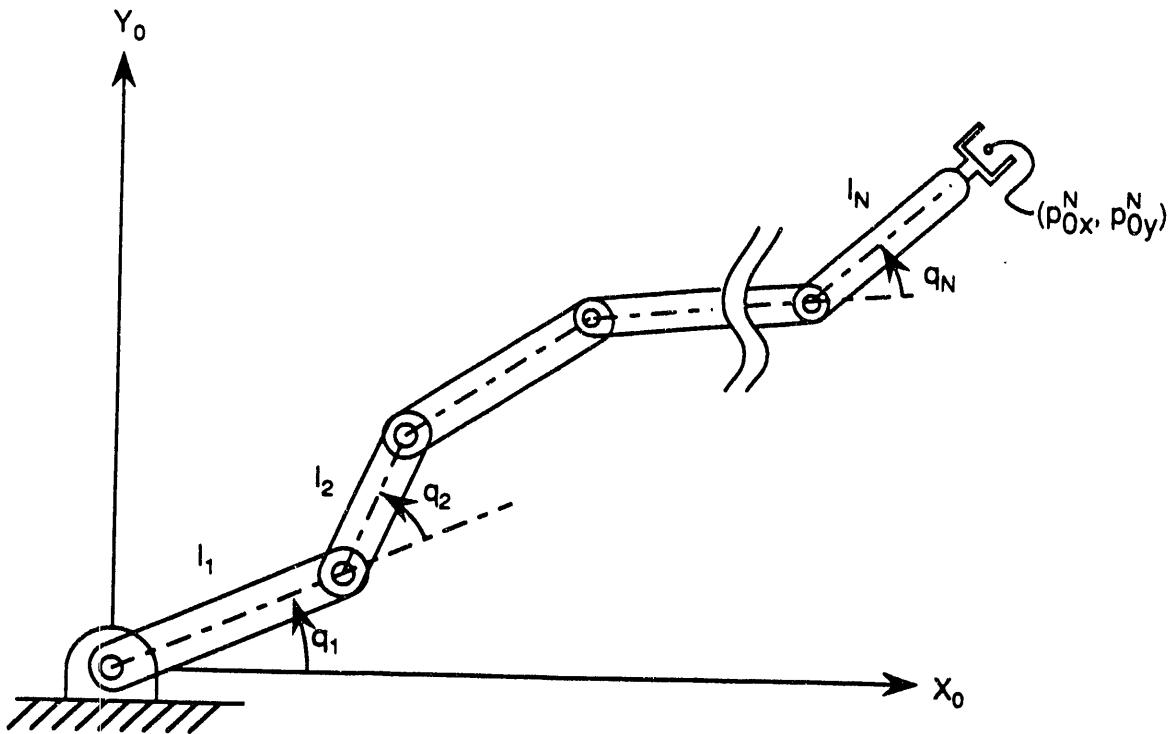


Fig. 3. Planar revolute redundant manipulator.

Differentiating Eq. (48) provides a relation between the joint velocities and the Cartesian translational velocities of the end effector:

$$v_0^N = J(q) \dot{q} \quad (49)$$

where $q = [q_1, q_2, \dots, q_N]^T$. In Eq. (49), the $(2 \times N)$ Jacobian matrix $J(q)$ is defined by:

$$J = \begin{bmatrix} j_{11}, & j_{12}, & j_{13}, & \dots, & j_{1N} \\ j_{21}, & j_{22}, & j_{23}, & \dots, & j_{2N} \end{bmatrix} \quad (50)$$

where its elements are defined by:

$$j_{11} = -l_1 s(q_1) - l_2 s(q_1 + q_2) - \dots - l_N s(q_1 + q_2 + \dots + q_N) , \quad (51)$$

$$j_{12} = -l_2 s(q_1 + q_2) - \dots - l_N s(q_1 + q_2 + \dots + q_N) , \quad (52)$$

⋮

$$j_{1N} = -l_N s(q_1 + q_2 + \dots + q_N) , \quad (53)$$

$$j_{21} = l_1 c(q_1) + l_2 c(q_1 + q_2) + \dots + l_N c(q_1 + q_2 + \dots + q_N) , \quad (54)$$

$$j_{22} = l_2 c(q_1 + q_2) + \dots + l_N c(q_1 + q_2 + \dots + q_N) , \quad (55)$$

⋮

$$j_{2N} = l_N c(q_1 + q_2 + \dots + q_N) . \quad (56)$$

The problem is to determine the $(N \times (N - 2))$ orthogonal complement Σ of the matrix J such that $J\Sigma = 0_{(2 \times (N-2))}$. We propose the “vector cross product method” to obtain Σ . The $(k - 2)$ th column of Σ is determined by taking the vector cross product of two (3×1) vectors $\{v_{1k}, v_{2k}\}$ whose components consist of subsets of the rows of the Jacobian. v_{1k} and v_{2k} are defined by:

$$v_{1k} = [j_{11}, j_{12}, j_{1k}]^T , \quad (57)$$

$$v_{2k} = [j_{21}, j_{22}, j_{2k}]^T \quad (58)$$

where $k = 3, 4, \dots, N$.

The cross product between v_{1k} and v_{2k} may be expressed in a matrix-column vector notation [13]:

$$v_{rsltk} = \begin{bmatrix} 0, & -j_{1k}, & j_{12} \\ j_{1k}, & 0, & -j_{11} \\ -j_{12}, & j_{11}, & 0 \end{bmatrix} \begin{bmatrix} j_{21} \\ j_{22} \\ j_{2k} \end{bmatrix} = \begin{bmatrix} j_{12} j_{2k} - j_{1k} j_{22} \\ j_{1k} j_{21} - j_{11} j_{2k} \\ j_{11} j_{22} - j_{12} j_{21} \end{bmatrix} \quad (59)$$

where (3×1) vector v_{rsltk} is the resultant of the cross product.

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Let $(N \times 1)$ vector $\Sigma_{(k-2)}$ denote the $(k-2)$ th column of Σ ($k = 3, 4, \dots, N$). It is defined by:

$$\Sigma_{(k-2)} = \begin{bmatrix} [1, 0, 0] v_{rsltk} \\ [0, 1, 0] v_{rsltk} \\ 0_{((k-3) \times 1)} \\ [0, 0, 1] v_{rsltk} \\ 0_{((N-k) \times 1)} \end{bmatrix} . \quad (60)$$

By invoking the definition of v_{rsltk} in Eq. (59), it is seen that $J \Sigma_{(k-2)} = 0_{2 \times 1}$.

Example 4: Suppose $N = 5$. The orthogonal complement of the (2×5) Jacobian J is given by:

$$\Sigma = \begin{bmatrix} j_{12}j_{23} - j_{13}j_{22} & j_{12}j_{24} - j_{14}j_{22} & j_{12}j_{25} - j_{15}j_{22} \\ j_{13}j_{21} - j_{11}j_{23} & j_{14}j_{21} - j_{11}j_{24} & j_{15}j_{21} - j_{11}j_{25} \\ j_{11}j_{22} - j_{12}j_{21} & 0 & 0 \\ 0 & j_{11}j_{22} - j_{12}j_{21} & 0 \\ 0 & 0 & j_{11}j_{22} - j_{12}j_{21} \end{bmatrix} \quad (61)$$

in which Eqs. (59) and (60) have been applied. Matrix Σ can be expressed as a function of the five joint angles $\{q_1, q_2, \dots, q_5\}$ by applying Eqs. (51)–(56) with $N = 5$.

Given $\Sigma = [\Sigma_1, \Sigma_2, \dots, \Sigma_{(N-2)}]$, suppose we choose the $((N-2) \times N)$ matrix $B(q)$ such that:

$$B = (\Sigma^T \Sigma)^{-1} \Sigma^T . \quad (62)$$

Then it is easy to verify that $J B^T = 0_{(2 \times (N-2))}$ and that Π is defined by Eq. (34).

In [4], a set of equations of the form of Eq. (4) was introduced with the restriction that $\epsilon = 0_{((N-M) \times 1)}$. Equations (1) and (4) were combined and solved for \dot{q} . The square matrix $(J^T, B^T)^T$ was referred to as the extended Jacobian. Matrix B was obtained by taking the Lie derivative of a scalar objective function to be optimized in the independent directions of the null space of J to yield $J B^T = 0_{(M \times (N-M))}$. The problem of determining the null space of the Jacobian, that is, the orthogonal complement Σ , is not addressed for the case of $N - M > 1$ in [4]. In the approach used here, the components of ϵ can be nonzero. The general development leading to Eqs. (7) and (8) only requires that the rows of B be linearly independent of the rows of J . As a special case it is shown that B can be selected to be orthogonal to the rows of J , which results in $\epsilon = 0_{((N-M) \times 1)}$ when minimizing the Euclidean norm $\|\dot{q}\|^2$. In Section 3.2 it has been shown that matrices Π and Σ can be determined analytically thus avoiding the numerical inversion of $(J^T, B^T)^T$ for calculating their values, which was not addressed in [4].

The general solution for \ddot{q} given by Eq. (8) obtained when resolving the kinematic redundancy at the acceleration level is useful in the development of

a dynamical model and control architecture for the gross motion of a redundant manipulator. This is discussed next.

4. RIGID BODY DYNAMICAL MODEL AND CONTROL ARCHITECTURE

A dynamical model for the entire system is formed by first combining the joint space dynamical model of the manipulator with the inverse kinematic solutions derived in the previous section. The model is then transformed to separate it into two distinct sets of equations of motion. Additionally, a control architecture of the input relegation decoupling type [1] is suggested based on the model.

To determine a rigid body model, the generalized velocities and accelerations are eliminated from the joint space dynamical model in Eq. (3) using Eqs. (7) and (8):

$$D \{ \Pi \ddot{x} + \Sigma \dot{\epsilon} \} = \tau + D \{ \Pi \dot{J} + \Sigma \dot{B} \} \{ \Pi \dot{x} + \Sigma \epsilon \} - C - G . \quad (63)$$

Premultiplying Eq. (63) by the $(N \times N)$ nonsingular matrix $[D^{-1} [J^T, B^T]]^T$ and utilizing the matrix relations in Eq. (5) separates the model into two sets of equations:

$$\ddot{x} = J D^{-1} \{ \tau - C - G \} + \dot{J} \{ \Pi \dot{x} + \Sigma \epsilon \} , \quad (64)$$

$$\dot{\epsilon} = B D^{-1} \{ \tau - C - G \} + \dot{B} \{ \Pi \dot{x} + \Sigma \epsilon \} . \quad (65)$$

The M second order differential equations of motion governing the motion of the point, link, or end effector being controlled in Cartesian space are given by Eq. (64) as functions of the variables $\{\dot{x}, \ddot{x}, \epsilon, \dot{\epsilon}, q, \tau\}$. On the other hand, Eq. (65) is comprised of $(N - M)$ second order differential equations of motion which govern the behavior of the redundant DOF in terms of the variables $\{\dot{x}, \epsilon, \dot{\epsilon}, q, \tau\}$.

The problem now considered is to determine a control input τ to Eqs. (64) and (65) so that the Cartesian variables $\{\dot{x}, \ddot{x}\}$ and the redundant DOF variables $\{\epsilon, \dot{\epsilon}\}$ will be controlled independently. The proposed controller consists of the sum of the outputs of a $(N \times 1)$ primary controller (τ^p) that is designed for cancellation of nonlinear terms in the model and a $(N \times 1)$ secondary controller (τ^s) that performs closed-loop servoing. The composite control (τ) is specified as $\tau = \tau^p + \tau^s$. The primary and secondary controllers are defined by:

$$\tau^p = - \hat{D} \{ \hat{\Pi} \hat{J} + \hat{\Sigma} \hat{B} \} \{ \hat{\Pi} \hat{x} + \hat{\Sigma} \hat{\epsilon} \} + \hat{C} + \hat{G} , \quad (66)$$

$$\tau^s = \hat{D} \left[\hat{\Pi} \tau_{cart}^s + \hat{\Sigma} \tau_{redun}^s \right] \quad (67)$$

where the superscript $\hat{\cdot}$ denotes that the quantity is estimated as a function of the desired (reference) feedforward trajectory $(q^{ref}, \epsilon^{ref}, \dot{x}^{ref})$ and/or the feedback

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variables (q, ϵ, \dot{x}) . In Eq. (67), τ_{cart}^s and τ_{redun}^s are $(M \times 1)$ and $((N - M) \times 1)$ vectors, respectively, representing new control input variables to be determined.

Substituting the composite controller $\tau = \tau^p + \tau^s$ defined by Eqs. (66) and (67) into Eqs. (64) and (65), under the assumption that the following relations hold:

$$\begin{aligned} \hat{D} &= D & \hat{C} &= C & \hat{G} &= G & \hat{J} &= J & \hat{B} &= B & \hat{j} &= j, \\ \hat{\dot{B}} &= \dot{B} & [\hat{\Pi}, \hat{\Sigma}] &= \{[J^T, B^T]^T\}^{-1} \end{aligned} \quad (68)$$

leads to the closed-loop system:

$$\ddot{x} = \tau_{cart}^s, \quad (69)$$

$$\dot{\epsilon} = \tau_{redun}^s \quad (70)$$

in which Eq. (5) has been used. Suppose τ_{cart}^s and τ_{redun}^s are selected by the designer to control $\{\dot{x}, \ddot{x}\}$ and $\{\epsilon, \dot{\epsilon}\}$ to track reference or commanded trajectories, respectively. Since Eqs. (69) and (70) reveal that the (nonredundant) Cartesian- and redundant-controlled DOF have been completely decoupled, then τ_{cart}^s and τ_{redun}^s are noninteracting controllers. The structure of the proposed composite controller reveals that all N actuated manipulator joints contribute simultaneously to the control of the Cartesian- and the redundant-DOF during motion of the manipulator.

5. CONCLUSION AND FUTURE WORK

The conceptual input relegation control philosophy [1] has been applied to the problem of resolving the kinematic redundancy of a serial-link manipulator as well as to dynamically modeling and controlling such a system. A new vector variable ϵ quantifying the redundant DOF was introduced and defined as linear combinations of the generalized velocities. The equations defining vector ϵ together with the kinematic velocity model yielded a well-specified solution for the generalized velocities. The general procedure was also used to resolve the redundancy at the acceleration level. A criteria for choosing ϵ and several approaches for selecting the $B(q)$ matrix were presented. In the first approach, B was picked to be a constant matrix such that ϵ is a subset of the generalized velocities. This resulted in determining the inverse of the composite matrix $(J^T(q), B^T)^T$ analytically thus avoiding a numerical inversion. Furthermore, analytical and numerical methods for determining $B(q)$ such that it is orthogonal to the rows of the Jacobian matrix $J(q)$ were suggested, some of which applied to manipulators with multiple degrees of redundancy. The advantages of this were demonstrated through relegating the redundant DOF to produce minimum Euclidean norm solutions for the generalized velocities and accelerations, respectively. Additionally, a rigid body dynamical model consisting of two distinct sets of equations of motion was derived. One set is the Cartesian space equations of motion governing the behavior of the nonredundant DOF. The other set governs the redundant DOF. A control architecture was suggested, which according to the separated form of the model, decouples the Cartesian- and redundant-DOF.

The research in this report has uncovered and identified a wealth of open research issues that warrant future attention. Vectors $\{\epsilon, \dot{\epsilon}\}$ were selected to minimize the Euclidean norm of the generalized velocities and accelerations, respectively. Clearly other criteria and analytical methods for selecting $\{\epsilon, \dot{\epsilon}\}$ need to be developed. The analytical methods for choosing $B(q)$ to be orthogonal to the rows of the Jacobian matrix in Section 3.3 need to be extended to allow for an arbitrary number of degrees of redundancy. The vector cross product method in Section 3.5 for deriving the orthogonal complement of the Jacobian allows for multiple degrees of redundancy. Unfortunately, the method is restricted to Jacobians having only two rows. Further research is needed to generalize this approach.

It has been shown in this report that the problems of resolving the kinematic redundancy of a serial link manipulator and of dynamically modeling a constrained mechanical system [7,8,9] are intimately related. In view of this, another suggested future work area is to investigate the uses of the redundant DOF during hard contact motion of a serial-link redundant manipulator. Additionally, it is felt to be very worthwhile to combine the results presented here with those on modeling the constrained motion of a nonholonomic omnidirectional wheeled mobile platform [15,20] to study the problems of kinematic redundancy and rigid body constraints in a combined mobility/manipulation system.

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APPENDIX A

THE JACOBIAN MATRIX FOR THE CESARm RESEARCH MANIPULATOR

In this appendix the (3×4) Jacobian matrix which transforms the joint velocities of the lower four joints of the CESARm research manipulator [6,10,11,12] to obtain the Cartesian translational velocities of the centerpoint of the wrist is specified. Let j_{lm} denote the element located at the intersection of the l th row and m th column of $J(q)$. Let $c_i = \cos(q_i)$ and $s_i = \sin(q_i)$, $i = 1, 2, 3, 4$. The elements are defined by:

$$\begin{aligned} j_{11} = & a_4 s_1 s_2 s_4 - a_4 (c_1 s_3 + s_1 c_2 c_3) c_4 - a_3 c_1 s_3 - a_3 s_1 c_2 c_3 \\ & - d_3 s_1 s_2 + d_2 c_1 , \end{aligned} \quad (A.1)$$

$$\begin{aligned} j_{12} = & - a_4 c_1 c_2 s_4 - a_4 c_1 s_2 c_3 c_4 - a_3 c_1 s_2 c_3 \\ & + d_3 c_1 c_2 , \end{aligned} \quad (A.2)$$

$$j_{13} = a_4 (-c_1 c_2 s_3 - s_1 c_3) c_4 - a_3 (c_1 c_2 s_3 + s_1 c_3) , \quad (A.3)$$

$$j_{14} = - a_4 (c_1 c_2 c_3 - s_1 s_3) s_4 - a_4 c_1 s_2 c_4 , \quad (A.4)$$

$$\begin{aligned} j_{21} = & - a_4 c_1 s_2 s_4 + a_4 (c_1 c_2 c_3 - s_1 s_3) c_4 - a_3 s_1 s_3 + a_3 c_1 c_2 c_3 \\ & + d_3 c_1 s_2 + d_2 s_1 , \end{aligned} \quad (A.5)$$

$$\begin{aligned} j_{22} = & - a_4 s_1 c_2 s_4 - a_4 s_1 s_2 c_3 c_4 - a_3 s_1 s_2 c_3 \\ & + d_3 s_1 c_2 , \end{aligned} \quad (A.6)$$

$$j_{23} = a_4 (c_1 c_3 - s_1 c_2 s_3) c_4 - a_3 (s_1 c_2 s_3 - c_1 c_3) , \quad (A.7)$$

$$j_{24} = - a_4 (c_1 s_3 + s_1 c_2 c_3) s_4 - a_4 s_1 s_2 c_4 , \quad (A.8)$$

$$j_{31} = 0 , \quad (A.9)$$

$$\begin{aligned} j_{32} = & a_4 (-s_2 s_4 + c_2 c_3 c_4) + a_3 c_2 c_3 \\ & + d_3 s_2 , \end{aligned} \quad (A.10)$$

$$j_{33} = - a_4 s_2 s_3 c_4 - a_3 s_2 s_3 , \quad (A.11)$$

$$j_{34} = a_4 (c_2 c_4 - s_2 c_3 s_4) \quad (A.12)$$

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where $[a_3, a_4, d_2, d_3]$ are constant Denavit-Hartenberg link length parameters which are illustrated in Fig. 2 and whose values are:

$$[a_3, a_4, d_2, d_3] = [0.029, 0.508, 0.356, 0.635] (m) . \quad (A.13)$$

APPENDIX B

EXTENSION TO MANIPULATORS WITH TWO REDUNDANT DOF

The problem is to extend the analytical method for selecting B to specify a desired, reference value for the determinant of $(J^T, B^T)^T$ for manipulators containing two degrees of redundancy ($N - M = 2$). Let the $(2 \times N)$ matrix $B = (B_1^T, B_2^T)^T$, where $B_i = [b_{i1}, b_{i2}, \dots, b_{iN}]$, $i = 1, 2$.

Expanding along the last row of $(J^T(q), B^T(q))^T$ using the Laplace expansion [13], the determinant of $(J^T(q), B^T(q))^T$ is:

$$\det \begin{bmatrix} J(q) \\ B(q) \end{bmatrix} = B_2(q) \Delta \quad (B.1)$$

where, here again, $\det[]$ denotes the determinant of $[]$. In Eq. (B.1), $\Delta (= [\Delta_{N1}, \Delta_{N2}, \dots, \Delta_{NN}]^T)$ is the vector of cofactors, where Δ_{Ni} is the cofactor of the i th element of the N th row of $(J^T(q), B^T(q))^T$. Vector Δ is a function of the generalized coordinates and the elements of B_1 . The scalar cofactor Δ_{Ni} ($i = 1, 2, \dots, N$) is defined by:

$$\Delta_{Ni} = (-1)^{N+i} B_1^i \Delta_{N-1} \quad (B.2)$$

where Δ_{N-1} is a $(N \times 1)$ vector defined by:

$$\Delta_{N-1} = [\Delta_{N-1,1}, \Delta_{N-1,2}, \dots, \Delta_{N-1,i-1}, 0, \Delta_{N-1,i}, \Delta_{N-1,i+1}, \dots, \Delta_{N-1,N-1}]^T \quad (B.3)$$

where $\Delta_{N-1,k}$, ($k = 1, 2, \dots, N - 1$) is the cofactor of the k th element of the $(N - 1)$ th row of the $((N - 1) \times (N - 1))$ matrix contained within cofactor Δ_{N-1} . The i th element of Δ_{N-1} is zero.

Using the quantities defined above, the determinant of $(J^T(q), B^T(q))^T$ can be expressed in a concise notation:

$$\det \begin{bmatrix} J(q) \\ B(q) \end{bmatrix} = B_2 \Theta B_1^T \quad (B.4)$$

where Θ is an $(N \times N)$ matrix defined by:

$$\Theta = \begin{bmatrix} \Delta_{N-1}^T (-1)^{N+1} \\ \Delta_{N-1}^T (-1)^{N+2} \\ \vdots \\ \Delta_{N-1}^T (-1)^{N+N} \end{bmatrix} \quad (B.5)$$

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whose main diagonal elements are all zero. Θ is a function of the generalized coordinates and it will be shown by examples that it is a skew-symmetric matrix, i.e., $-\Theta = \Theta^T$.

The general class of $(N \times 2)$ orthogonal complements Σ of J is defined as the last two columns of $\{(J^T(q), B^T(q))^T\}^{-1}$:

$$\Sigma(q) = \frac{\Theta}{B_2 \Theta B_1^T} [-B_2^T, B_1^T] \quad (B.6)$$

where a specific Σ is obtained by the designer's choice of B_1 and B_2 .

Suppose we select B_2 as:

$$B_2 = \left\{ \Theta \begin{bmatrix} 1 \\ 0_{((N-1) \times 1)} \end{bmatrix} \right\}^T = -{}^1\Delta_{N-1}^T (-1)^{N+1} \quad (B.7)$$

If we select B_1 as:

$$B_1 = \left[\frac{1}{B_2 B_2^T} \det^* \begin{bmatrix} J(q) \\ B(q) \end{bmatrix}, 0_{(1 \times (N-1))} \right] \quad (B.8)$$

where B_2 is defined in Eq. (B.7) and where $\det^*[J^T(q), B^T(q)]^T$ is a constant, desired value of $\det[J^T(q), B^T(q)]^T$ selected by the designer, e.g., select $\det^*[J^T(q), B^T(q)]^T \neq 0$ to make $(J^T(q), B^T(q))^T$ an invertible matrix. Then substituting Eqs. (B.7) and (B.8) into Eq. (B.4) yields:

$$\det \begin{bmatrix} J(q) \\ B(q) \end{bmatrix} = \det^* \begin{bmatrix} J(q) \\ B(q) \end{bmatrix} \quad (B.9)$$

Given this choice of B_1 and B_2 , the determinant of $[J^T(q), B^T(q)]^T$ is non-negative and can only be zero when vector ${}^1\Delta_{N-1} = 0_{(N \times 1)}$. Should all the components of ${}^1\Delta_{N-1}$ be identically zero, B_2 may be selected as $-{}^i\Delta_{N-1}^T (-1)^{N+i}$ for any $i (= 2, 3, \dots, N)$ for which at least one of the components of ${}^i\Delta_{N-1}^T$ (other than the i th component) is nonzero. B_1 would then be selected as:

$$B_1 = \left[0_{(1 \times (i-1))}, \frac{1}{B_2 B_2^T} \det^* \begin{bmatrix} J(q) \\ B(q) \end{bmatrix}, 0_{(1 \times (N-i))} \right] \quad (B.10)$$

An alternate analytical approach to picking B so as to specify a desired reference value of $\det[J^T, B^T]^T$ is now discussed. It is assumed that the row vector B_2 has been selected to be linearly independent of the rows of J . Suppose B_1^T is selected as:

$$B_1^T = \frac{\Theta^T B_2^T}{B_2 \Theta \Theta^T B_2^T} \det^* \begin{bmatrix} J \\ B \end{bmatrix} = \frac{\Theta B_2^T}{B_2 \Theta^2 B_2^T} \det^* \begin{bmatrix} J \\ B \end{bmatrix} \quad (B.11)$$

where the skew-symmetry of Θ has been exploited. Θ^2 is a symmetric matrix whose main diagonal elements are negative. Thus, the denominator of the right-hand side of Eq. (B.11) is a quadratic form [14]. Substituting Eq. (B.11) into Eq. (B.4) yields the solution for the determinant given by Eq. (B.9). Since $J \Sigma = 0_{M \times 2}$, it is easy to see that $J B_1^T = 0_{M \times 1}$.

The former method for determining matrix B is now illustrated through two examples.

Example B1. Consider the problem of the determining matrix B for the case of controlling a serial-link planar redundant manipulator with $N = 4$ revolute joints. The configuration of the system is shown in Fig. 3. Only the Cartesian translational motion of the end effector is specified and controlled. Thus $M = 2$ and the system has 2 redundant DOF. The composite matrix $(J^T, B^T)^T$ can be expanded to reveal its component elements:

$$\begin{bmatrix} J \\ B \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} & j_{13} & j_{14} \\ j_{21} & j_{22} & j_{23} & j_{24} \\ b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix} . \quad (B.12)$$

Applying Eq. (B.1) with $N = 4$, the vector of cofactors $\Delta (= [\Delta_{41}, \Delta_{42}, \Delta_{43}, \Delta_{44}]^T)$ is defined by:

$$\begin{aligned} \Delta = & \left[- \begin{vmatrix} j_{12} & j_{13} & j_{14} \\ j_{22} & j_{23} & j_{24} \\ b_{12} & b_{13} & b_{14} \end{vmatrix}, \begin{vmatrix} j_{11} & j_{13} & j_{14} \\ j_{21} & j_{23} & j_{24} \\ b_{11} & b_{13} & b_{14} \end{vmatrix}, \right. \\ & \left. - \begin{vmatrix} j_{11} & j_{12} & j_{14} \\ j_{21} & j_{22} & j_{24} \\ b_{11} & b_{12} & b_{14} \end{vmatrix}, \begin{vmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ b_{11} & b_{12} & b_{13} \end{vmatrix}^T \right] \end{aligned} \quad (B.13)$$

where $|\cdot|$ denotes the determinant of $\{\cdot\}$.

Applying the definition of the $^i\Delta_{N-1}$ given by Eq. (B.3), we have:

$$^1\Delta_3 = \begin{bmatrix} 0 \\ j_{13}j_{24} - j_{14}j_{23} \\ j_{14}j_{22} - j_{12}j_{24} \\ j_{12}j_{23} - j_{13}j_{22} \end{bmatrix} , \quad (B.14)$$

$$^2\Delta_3 = \begin{bmatrix} j_{13}j_{24} - j_{14}j_{23} \\ 0 \\ j_{14}j_{21} - j_{11}j_{24} \\ j_{11}j_{23} - j_{13}j_{21} \end{bmatrix} , \quad (B.15)$$

$$^3\Delta_3 = \begin{bmatrix} j_{12}j_{24} - j_{14}j_{22} \\ j_{14}j_{21} - j_{11}j_{24} \\ 0 \\ j_{11}j_{22} - j_{12}j_{21} \end{bmatrix} , \quad (B.16)$$

$${}^4\Delta_3 = \begin{bmatrix} j_{12}j_{23} & - & j_{13}j_{22} \\ j_{13}j_{21} & - & j_{11}j_{23} \\ j_{11}j_{22} & - & j_{12}j_{21} \\ 0 & & \end{bmatrix} . \quad (B.17)$$

Substituting Eqs. (B.14)–(B.17) into the right-hand side of Eq. (B.5) with $N = 4$ gives the solution for Θ :

$$\Theta = \begin{bmatrix} 0 & -j_{13}j_{24} + j_{14}j_{23} & -j_{14}j_{22} + j_{12}j_{24} & -j_{12}j_{23} + j_{13}j_{22} \\ j_{13}j_{24} - j_{14}j_{23} & 0 & j_{14}j_{21} - j_{11}j_{24} & j_{11}j_{23} - j_{13}j_{21} \\ -j_{12}j_{24} + j_{14}j_{22} & -j_{14}j_{21} + j_{11}j_{24} & 0 & -j_{11}j_{22} + j_{12}j_{21} \\ j_{12}j_{23} - j_{13}j_{22} & j_{13}j_{21} - j_{11}j_{23} & j_{11}j_{22} - j_{12}j_{21} & 0 \end{bmatrix} . \quad (B.18)$$

Equation (B.18) reveals that Θ is skew-symmetric. B_2 is given by:

$$B_2 = [0, j_{13}j_{24} - j_{14}j_{23}, j_{14}j_{22} - j_{12}j_{24}, j_{12}j_{23} - j_{13}j_{22}] \quad (B.19)$$

in which Eq. (B.7) has been invoked with $N = 4$. Given Θ and B_2 as defined by Eqs. (B.18) and (B.19), the quantities B_1 , $\det [J^T, B^T]^T$, and Σ are obtained by using Eqs. (B.8), (B.4), and (B.6), respectively.

Example B2. Consider the problem of determining matrix B for the case of controlling a serial-link planar redundant manipulator with $N = 5$ revolute joints. Both the Cartesian translational and rotational motions of the end effector are specified and controlled. Thus $M = 3$ and the system has 2 redundant DOF. The composite matrix $(J^T, B^T)^T$ can be expanded to reveal its component elements:

$$\begin{bmatrix} J \\ B \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} & j_{13} & j_{14} & j_{15} \\ j_{21} & j_{22} & j_{23} & j_{24} & j_{25} \\ j_{31} & j_{32} & j_{33} & j_{34} & j_{35} \\ b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \end{bmatrix} . \quad (B.20)$$

The (5×1) vectors ${}^i\Delta_4 (i = 1, 2, \dots, 5)$ are obtained by applying Eq. (B.3) with $N = 5$:

$$^1\Delta_4 = \begin{bmatrix} 0 \\ - \begin{vmatrix} j_{13}, & j_{14}, & j_{15} \\ j_{23}, & j_{24}, & j_{25} \\ j_{33}, & j_{34}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{12}, & j_{14}, & j_{15} \\ j_{22}, & j_{24}, & j_{25} \\ j_{32}, & j_{34}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{12}, & j_{13}, & j_{15} \\ j_{22}, & j_{23}, & j_{25} \\ j_{32}, & j_{33}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{12}, & j_{13}, & j_{14} \\ j_{22}, & j_{23}, & j_{24} \\ j_{32}, & j_{33}, & j_{34} \end{vmatrix} \end{bmatrix}, \quad (B.21)$$

$$^2\Delta_4 = \begin{bmatrix} - \begin{vmatrix} j_{13}, & j_{14}, & j_{15} \\ j_{23}, & j_{24}, & j_{25} \\ j_{33}, & j_{34}, & j_{35} \end{vmatrix} \\ 0 \\ - \begin{vmatrix} j_{11}, & j_{14}, & j_{15} \\ j_{21}, & j_{24}, & j_{25} \\ j_{31}, & j_{34}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{13}, & j_{15} \\ j_{21}, & j_{23}, & j_{25} \\ j_{31}, & j_{33}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{13}, & j_{14} \\ j_{21}, & j_{23}, & j_{24} \\ j_{31}, & j_{33}, & j_{34} \end{vmatrix} \end{bmatrix}, \quad (B.22)$$

$${}^3\Delta_4 = \begin{bmatrix} 0 \\ - \begin{vmatrix} j_{12}, & j_{14}, & j_{15} \\ j_{22}, & j_{24}, & j_{25} \\ j_{32}, & j_{34}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{14}, & j_{15} \\ j_{21}, & j_{24}, & j_{25} \\ j_{31}, & j_{34}, & j_{35} \end{vmatrix} \end{bmatrix}, \quad (B.23)$$

$${}^4\Delta_4 = \begin{bmatrix} - \begin{vmatrix} j_{12}, & j_{13}, & j_{15} \\ j_{22}, & j_{23}, & j_{25} \\ j_{32}, & j_{33}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{13}, & j_{15} \\ j_{21}, & j_{23}, & j_{25} \\ j_{31}, & j_{33}, & j_{35} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{12}, & j_{15} \\ j_{21}, & j_{22}, & j_{25} \\ j_{31}, & j_{32}, & j_{35} \end{vmatrix} \\ 0 \\ \begin{vmatrix} j_{11}, & j_{12}, & j_{13} \\ j_{21}, & j_{22}, & j_{23} \\ j_{31}, & j_{32}, & j_{33} \end{vmatrix} \end{bmatrix}, \quad (B.24)$$

$${}^5\Delta_4 = \begin{bmatrix} - \begin{vmatrix} j_{12}, & j_{13}, & j_{14} \\ j_{22}, & j_{23}, & j_{24} \\ j_{32}, & j_{33}, & j_{34} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{13}, & j_{14} \\ j_{21}, & j_{23}, & j_{24} \\ j_{31}, & j_{33}, & j_{34} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{12}, & j_{14} \\ j_{21}, & j_{22}, & j_{24} \\ j_{31}, & j_{32}, & j_{34} \end{vmatrix} \\ - \begin{vmatrix} j_{11}, & j_{12}, & j_{13} \\ j_{21}, & j_{22}, & j_{23} \\ j_{31}, & j_{32}, & j_{33} \end{vmatrix} \\ 0 \end{bmatrix} \quad (B.25)$$

Substituting Eqs. (B.21)–(B.25) into the right-hand side of Eq. (B.5) with $N = 5$, it is easy to see that Θ is skew-symmetric. B_2 is given by:

$$B_2 = \begin{bmatrix} 0 \\ j_{33} (j_{14} j_{25} - j_{15} j_{24}) + j_{34} (j_{15} j_{23} - j_{13} j_{25}) + j_{35} (j_{13} j_{24} - j_{14} j_{23}) \\ j_{32} (j_{15} j_{24} - j_{14} j_{25}) + j_{34} (j_{12} j_{25} - j_{15} j_{22}) + j_{35} (j_{14} j_{22} - j_{12} j_{24}) \\ j_{32} (j_{13} j_{25} - j_{15} j_{23}) + j_{33} (j_{15} j_{22} - j_{12} j_{25}) + j_{35} (j_{12} j_{23} - j_{13} j_{22}) \\ j_{32} (j_{14} j_{23} - j_{13} j_{24}) + j_{33} (j_{12} j_{24} - j_{14} j_{22}) + j_{34} (j_{13} j_{22} - j_{12} j_{23}) \end{bmatrix}^T \quad (B.26)$$

in which Eq. (B.7) has been invoked. Vector B_1 can be determined from Eq. (B.8).

APPENDIX C

DETERMINING SINGULARITY AND RANK OF MATRIX ($J^T J$)

In this appendix it will be proven by analytical techniques that matrix ($J^T J$) has less than full rank and thus is singular. The matrix relations given by Eq. (5) will be used in evaluating the rank of ($J^T J$). Additionally, the following matrix property will be used. Two ($n \times n$) real matrices X and Y are given. Matrix X is assumed to possess full rank n and thus is nonsingular. Then the following mathematical relation applies [13]:

$$\text{rank}(Y) = \text{rank}(XY) = \text{rank}(YX) = \text{rank}(X^T Y) = \text{rank}(YX^T) . \quad (C.1)$$

The rank of ($J^T J$) may now be determined. By the property of Eq. (C.1), the following relation holds:

$$\text{rank}(J^T J) = \text{rank} \left\{ \left[\begin{smallmatrix} \Pi^T \\ \Sigma^T \end{smallmatrix} \right] J^T J \left[\begin{smallmatrix} J^T, B^T \end{smallmatrix} \right] \right\} . \quad (C.2)$$

The rank of the ($N \times N$) matrix within the braces on the right-hand side of Eq. (C.2) is now analyzed by first carrying out the multiplications:

$$\text{rank}(J^T J) = \text{rank} \left\{ \left[\begin{smallmatrix} \Pi^T J^T J J^T, & \Pi^T J^T J B^T \\ \Sigma^T J^T J J^T, & \Sigma^T J^T J B^T \end{smallmatrix} \right] \right\} . \quad (C.3)$$

The right-hand side of Eq. (C.3) is simplified by invoking the matrix relations in Eq. (5):

$$\text{rank}(J^T J) = \text{rank} \left\{ \left[\begin{smallmatrix} J J^T & J B^T \\ 0_{((N-M) \times M)} & 0_{((N-M) \times (N-M))} \end{smallmatrix} \right] \right\} . \quad (C.4)$$

Equation (C.4) reveals that the last ($N - M$) rows of matrix ($[\Pi, \Sigma]^T J^T J [J^T, B^T]$) contain all zeros. Thus matrix ($[\Pi, \Sigma]^T J^T J [J^T, B^T]$) is singular. Furthermore, the $\text{rank}([\Pi, \Sigma]^T J^T J [J^T, B^T]) = M$. It should be mentioned that the upper M rows on the right-hand side of Eq. (C.4) form the matrix ($J [J^T, B^T]$), which has rank M because the rank of $J (= M)$ is unchanged by postmultiplying it by the ($N \times N$) nonsingular matrix $[J^T, B^T]$ [13].

Since matrix ($[\Pi, \Sigma]^T J^T J [J^T, B^T]$) is singular, then matrix ($J^T J$) is also singular with $\text{rank}(J^T J) = M$ because the ranks of these two matrices are the same as shown in Eq. (C.2). It should be mentioned that matrices ($J J^T$) and ($J^T J$) have the same eigenvalues, except that ($J^T J$) has ($N - M$) extra zero valued eigenvalues [14], which is readily confirmed by Eq. (C.4).

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48. Dr. James E. Leiss, 13013 Chestnut Oak Dr., Gaithersburg, MD 20878
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50. Professor Neville Moray, Department of Mechanical and Industrial Engineering, University of Illinois, 1206 West Green St., Urbana, IL 61801
51. Dr. Wes Snyder, Department of Radiology, Bowman Gray School of Medicine, 3005 Hawthorne Dr., Winston-Salem, NC 27103
52. D. Tesar, Department of Mechanical Engineering, University of Texas, 26 San Jacinto, Austin, TX 78712
53. Professor Mary F. Wheeler, Rice University, Department of Mathematical Sciences, P.O. Box 1892, Houston, TX 77251
- 54-63. Office of Scientific and Technical Information, P.O. Box 62, Oak Ridge, TN 37831

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