MEETING VIII BEVATRON RESEARCH CONFERENCE
December 8, 1953

Geometrical interpretation of energy-momentum conservation in collisions -- according to J. Blaton, Dansk-Vid. Selsk. 24, No. 20, (1950).

Symbols.	m	rest mass	Asterisks refer to c.m.
	k	momenta	No asterisks to lab. system
	E	total energies	I, II, before collision
			1,2, after collision

The units are such that c = 1; and if energies are expressed in Mev, momenta are in Mev/c.

Fundamental relations (conservation of energy and momentum)

$$E = \sqrt{m_I^2 + K_I^2} + \sqrt{m_{II}^2 + K_{II}^2} \qquad (1,1)$$

$$\vec{K} = \vec{k}_I + \vec{k}_{II} \qquad (1,2)$$

$$E = \sqrt{m_1^2 + K_1^2} + \sqrt{m_2^2 + K_2^2} \qquad (1,3)$$

$$\vec{K} = \vec{K}_1 + \vec{K}_2 \qquad (1,4)$$

The quantity

$$N = \sqrt{E^2 - K^2} = E^*$$

is an invariant (energy in c.m.)
Note:

$$\frac{E}{N} = \gamma \qquad \frac{K}{E} = \beta$$

where β and γ are the velocity and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ of the c.m.

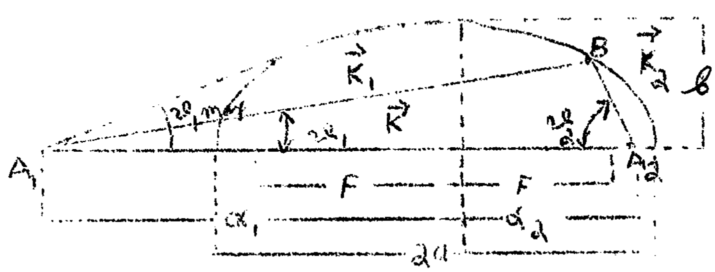


FIG 1

Parameters:

$$b = \frac{1}{2N}\sqrt{\{N^2 - (m_1+m_2)^2\}\{N^2 - (m_1-m_2)^2\}} \qquad (1,7)$$

b is the momentum of the particles after the collision, in the c.m. system.

$$a = \gamma b = \frac{b}{\sqrt{1-\beta^2}}$$

$$e_1^* = \sqrt{m_1^2 + b^2} \qquad\qquad e_2^* = \sqrt{m_2^2 + b^2}$$

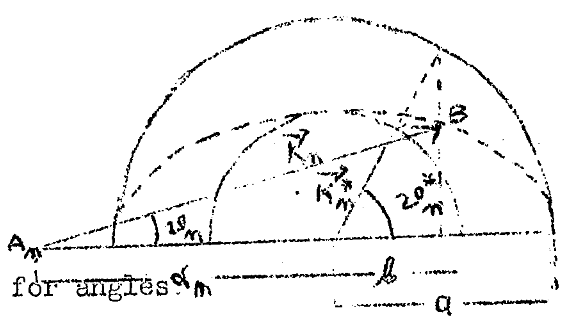
$$\alpha_1 = \beta\gamma e_1^*$$

$$\alpha_2 = \beta\gamma e_2^* = \underline{K} - \alpha_1$$

Distance between foci $= 2f = 2\sqrt{a^2 - b^2}$

Eccentricity $= \frac{f}{a} = \beta$

Transformation from c.m. to lab. system



Transformation for angles:

$$tg\,\vartheta_m = \frac{b \sin 2\vartheta_m^*}{\alpha_m + a\cos 2\vartheta_m^*} = \sqrt{1-\beta^2}\,\frac{\sin 2\vartheta_m^*}{\gamma_m + \cos 2\vartheta_m^*} \tag{2,1}$$

where $\gamma_m = \frac{\alpha_m}{a}$

Transformation for energies:

$$e_m = \frac{e_m^*}{\sqrt{1-\beta^2}} + f\cos 2\vartheta_m^* \tag{2,7}$$

Transformation for solid angles and cross sections.

$$\sigma_m(2\vartheta_m, \varphi) = \sigma_m^*(2\vartheta_m^*, \varphi)\,\frac{d\cos 2\vartheta_m^*}{d\cos 2\vartheta_m} \tag{2,11}$$

$$\sigma_m(\vartheta_m \varphi) = \pm\, \sigma^*(\vartheta_m^*, \varphi)\left(\frac{K_m}{b}\right)^2 \frac{1}{\sqrt{1 - \frac{\gamma_m^2 - \beta^2}{1-\beta^2}\sin^2\vartheta_m}} \qquad (2,12)$$

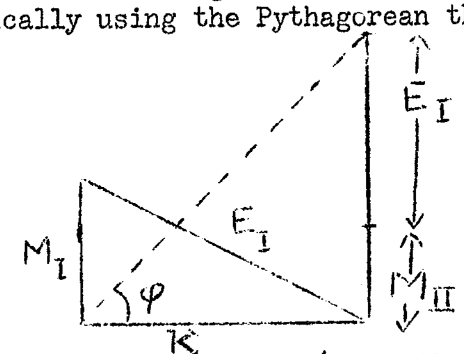
and for $\gamma_m = 1$ (elastic collision, particle at rest before collision).

$$\sigma_m = \sigma_m^* \frac{4(1-\beta^2)}{(1-\beta^2\cos^2\vartheta_m)^2} \cos\vartheta_m$$

Remarks:

K, b and the other quantities involved in Figure 1 may be easily calculated on a slide rule or constructed geometrically using the Pythagorean theorem.

e.g.



$$\cot\varphi = \frac{K}{E} = \beta^*$$

Threshold to create excess mass m by bombarding mass M_I with mass M_{II};
T_I = kinetic energy of M_I in lab system

$$T_I = \frac{m}{M_{II}}\left(M_I + M_{II} + \frac{m}{2}\right)$$