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INFORMATION UNCERTAINTY TO COMPARE QUALITATIVE REASONING SECURITY RISK ASSESSMENT RESULTS

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Abstract: The security risk associated with malevolent acts such as those of terrorism are often void of the historical data required for a traditional PRA. Most information available to conduct security risk assessments for these malevolent acts is obtained from subject matter experts as subjective judgements. Qualitative reasoning approaches such as approximate reasoning and evidential reasoning are useful for modelling the predicted risk from information provided by subject matter experts. Absent from these approaches is a consistent means to compare the security risk assessment results. Associated with each predicted risk reasoning result is a quantifiable amount of information uncertainty which can be measured and used to compare the results. This paper explores using entropy measures to quantify the information uncertainty associated with conflict and non-specificity in the predicted reasoning results. The measured quantities of conflict and non-specificity can ultimately be used to compare qualitative reasoning results which are important in triage studies and ultimately resource allocation. Straight forward extensions of previous entropy measures are presented here to quantify the non-specificity and conflict associated with security risk assessment results obtained from qualitative reasoning models.

1 INTRODUCTION

In security risk assessment from malevolent actions (SRAMA) such as those of terrorism, there is an absence of quantitative historical data necessary for a conventional probabilistic risk assessment. Much of the information for SRAMA is elicited from subject matter experts (SMEs) as subjective judgements and is often available as qualitative imprecise values. An Approximate Reasoning (AR) model is a useful alternative to a probabilistic model when drawing conclusions using imprecise knowledge provided by SMEs. AR has numerous applications in engineering and control (Ross 1995, Lewis 1997) and recently has been applied to security risk assessment for malevolent actions (Bott and Eisenhawer 2006).

An important factor differentiating AR results in control applications to that of to those of SRAMA applications is the type of information used to validate the results. That is, in control applications historical data can be used to validate the AR results; however, for particular terrorist attacks there is generally an absence of historical data. For example, prior to September 11, 2001, there was no historical data for successful attempts using airplanes to attack World Trade Center Towers in New York. In the absence of specific historical data, the AR results for SRAMA applications can be realistcally verified by the SMEs. Apart from this SME verification approach there is no consistent means to quantify the difference in competing results. A comparison of confidence among the competing results is also important. For example, triage studies of input values contributing to the security risk are often a necessary part of the security risk assessment model and a means to consistently measure the effect of this change for comparison of the results is critically important. Modifying a particular input value can alter the resulting AR risk value and this change may not be sufficiently or consistently quantified using only SME verification.

This study, therefore, proposes quantifying the information uncertainty associated with each predicted AR result and using the measured quantities to conduct comparisons of the results. The term entropy has been defined as a measured quantity of information uncertainty related to non-specificity and conflict (Klir and Wierman 1999). Existing measures for entropy were not developed for use with AR results. This study extends entropy to AR results and it is unique in that a similar approach has not been previously pursued in AR or applied in the area of SRAMA. It is a novel approach because it examines both conflict and non-specificity associated with the SRAMA AR result. Moreover, this approach is distinctly different than previous approaches involving information uncertainty and linguistic values. In previous approaches the entropy quantified involves all the possible states described by a particular fuzzy set (Pal et al. 1994, Klir and Wierman 1999, Klir 2005); whereas, in this application the entropy quantified is associated with only one state described linguistically using fuzzy sets. Fuzzy sets are discussed in Section 2.

Quantification of non-specificity and conflict can be also be applied to security risk assessment results obtained using Evidential Reasoning (ER). Like AR, ER is an approach used to draw conclusions from information. The major difference between the two approaches is the imprecision associated with describing the state is captured with AR while the lack of certainty associated with assigning a particular state to one of several linguistic values is captured with ER. There have been recent attempts to combine AR and ER for SRAMA applications which are fuzzy evidential reasoning (Yang et al 2009) and belief measures on fuzzy sets (Darby 2007). In this paper, AR and ER are treated separately and are collectively referred to as qualitative reasoning and the reader is referred to Ross (Ross 2004) for AR and Yang (Yang et al. 2006) for ER.

Section 3 provides a discussion on the quantities of non-specificity and conflict for AR and ER however, a general description of entropy is found in Klir (Klir 2005). The utility of a concept is measured by its applicability; therefore, simple AR and ER models are provided in Section 3. The quantification of conflict and non-specificity is illustrated on a simple AR and ER results in Section 3 and conclusions are provided in Section 4.

2 QUALITATIVE REASONING

A SME may indicate that the occurrence of a particular result is "highly likely", "somewhat likely", or "negligible" and the resulting consequences are "extremely costly", "moderately costly", or "insignificant". These expressions are called propositions and the kind of uncertainty associated with these propositions can be from vagueness, imprecision, and/or a lack of information regarding a specific state of the system. This type of uncertainty has collectively been called fuzzy uncertainty (Ross 2004). Fuzzy set theory provides a means for representing uncertainty contained in these propositions. Propositions of this type are commonly referred to as *fuzzy propositions* and express subjective ideas that can be interpreted slightly differently by various individuals. Reasoning using fuzzy propositions is referred to as approximate reasoning (Klir and Yuan 1995, Ross 1995). This section briefly describes fuzzy set theory for the purposes of this paper and the reader is referred to (Ross 2004) for an in depth description of each.

2.1 Fuzzy Set Theory

Natural language tends to be interpreted differently by various individuals. The linguistic values used by SMEs are no different and have a tendency to be vague and imprecise. For example, an SME may indicate that the process to construct a weapon device is "extremely difficult" or that it is "somewhat difficult". The precise meaning of these linguistic values may be interpreted slightly differently by various individuals; however, linguistic values may often be the values the SME is most confident in and comfortable providing. There is vagueness and imprecision associated with a linguistic value which has been termed fuzzy uncertainty. Fuzzy uncertainty is different from random uncertainty, where random uncertainty arises due to chance and deals with specific and well defined values such as the number on the top face of a die that is thrown. Random uncertainty is referred to as an aleatoric uncertainty and fuzzy uncertainty is referred to as an epistemic uncertainty. In some cases epistemic uncertainty may be reduced to aleatoric uncertainty but aleatoric uncertainty is non reducible uncertainty (Oberkampf et al. 2004, Zadeh 1995). Linguistic values such as "high", "medium", and "low" describe several specific states or conditions and are considered sets. The boundary that defines any one of these sets is unclear or fuzzy and thus these sets are called fuzzy sets.

A collection of objects having similar characteristics defines a universe of discourse, X. The individual elements, i.e. states, in X are denoted as x, with the same notations used for Y and y, and Z and, respectively. The elements can be grouped into various sets, such as $\tilde{A}$, $\tilde{B}$, or $\tilde{C}$. The set value of $\tilde{A}$, $\tilde{B}$, or $\tilde{C}$ may represent something like "high" which has a fuzzy boundary. The individual states of a fuzzy set can be mapped to a universe of membership values using a function theoretic form. If a state x is a member of the set $\tilde{A}$, then this mapping is given by Equation (1). A typical mapping of $\tilde{A}$ is shown in Figure 1.

$$\mu_{\tilde{A}}(x) \in [0, 1] \qquad (1)$$

The complement of $\tilde{A}$ is defined as:

$$\mu_{\bar{\tilde{A}}}(x_i) = 1 - \mu_{\tilde{A}}(x_i) \qquad (2)$$

The mapping for the complement is also shown in Figure 1. The mapping is known as a membership function and the membership of a specific state is x_i is referred to as the degree of membership. The degree of membership of x_i provides an indication of the fuzzy set's ability to describe the state.

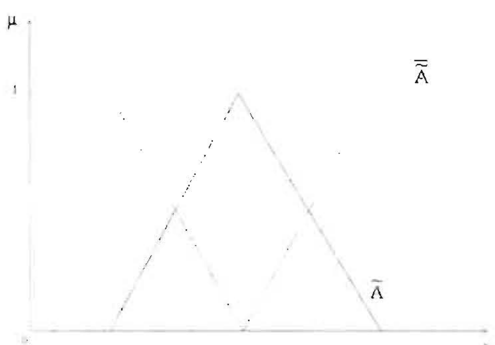

Figure 1: This caption has one line so it is centered.

2.2 Fuzzy Set Theory and Approximate Reasoning

An AR model uses the degrees of membership of elements in fuzzy sets to draw conclusions about a system such as risk of attack on a facility. The AR result is comprised of a vector of various fuzzy sets used to describe a specific state of risk and a respective degree of membership in each fuzzy set. Now suppose that an SME indicates that values $\tilde{A}$ and $\tilde{B}$ for states x_i and y_j, respectively, infers a particular value $\tilde{E}$ for z_k. The information provided is considered a rule governing the outcome z_k and can be represented as follows:

Rule 1: IF x_i is $\tilde{A}$ and y_j is $\tilde{B}$ THEN z_k is $\tilde{E}$

All the rules governing the particular outcome z_k involving values for x_i and y_j can be grouped together into a rule base, see Table 1. Now consider the situation when both x_i and y_j can be described by more than one value. In such a situation x_i and y_j have a degree of membership in each value that describes them. The values of x_i and y_j are used to identify the governing rule and infer the value of z_k. The inferred value of z_k will have an associated degree of membership which results from the conjunction ∧, i.e. taking the minimum value. of the degree of membership for x_i *AND* y_j in the values included in the governing rule. Take for example the rule specified above with $\mu_{\tilde{A}}(x_i) = 0.3$ and $\mu_{\tilde{B}}(y_j) = 0.6$, which results in a $\mu_{\tilde{E}}(z_k) = 0.3$. Another applicable governing rule may be:

Rule 2: IF x_i is $\tilde{B}$ and y_j is $\tilde{B}$ THEN z_k is $\tilde{E}$

with $\mu_{\tilde{B}}(x_i) = 0.7$ and $\mu_{\tilde{B}}(y_j) = 0.6$, which results in $\mu_{\tilde{E}}(z_k) = 0.6$. Both Rule 1 and Rule 2 result in the value $\tilde{E}$ for z_k but there are now two different values for the degree of membership. That is, either Rule 1 *OR* Rule 2 is applicable and the disjunction (∨), i.e. taking the maximum value. of $\mu_{\tilde{E}}(z_k) = 0.3$ and $\mu_{\tilde{E}}(z_k) = 0.6$, results in $\mu_{\tilde{E}}(z_k) = 0.6$. The conjunction and disjunction operations are used when the logical *AND* and *OR* are encountered, respectively. In each of the rules the logical *and* is encountered and the conjunction operation is used to determine the resulting degree of membership. The logical *OR* is encountered in the example because either Rule 1 *or* Rule 2 in result $\tilde{E}$. Additional logical operations can be found in (Ross 1995) as well as the axioms involved in fuzzy sets. It is important to note that the excluded middle axiom is not required for fuzzy sets; therefore, the resulting degree of membership for AR need not sum to 1.

Rule Base		Universe of Discourse X		
		$\tilde{A}$	$\tilde{B}$	$\tilde{C}$
Universe of Discourse Y	$\tilde{A}$	$\tilde{F}$	$\tilde{E}$	$\tilde{G}$
	$\tilde{B}$	$\tilde{F}$	$\tilde{E}$	$\tilde{E}$
	$\tilde{C}$	$\tilde{E}$	$\tilde{G}$	$\tilde{G}$

Table 1: Rule Base.

2.2.1 Application of AR in Risk.

This section illustrates the use of AR in SRAMA using a simple example to determine the risk of attack from success likelihood and the economic consequences of the attack. Table 2 provides the rule base used to infer the risk given the success likelihood and the consequences.

Risk		Economic Consequence				
		Very Low	Low	Medium	High	Very High
Success Likelihood	Negligible	Very Low	Very Low	Very Low	Very Low	Very Low
	Extremely Unlikely	Very Low	Very Low	Very Low	Very Low	Low
	Very Unlikely	Very Low	Very Low	Very Low	Low	Medium
	Unlikely	Very Low	Low	Low	Medium	**Medium**
	Somewhat Likely	Very Low	Low	Low	Medium	**Medium**
	Likely	Low	Low	Medium	High	Very High
	Nearly Certain	Low	Low	Medium	High	Very High

Table 2. AR Risk Rule Base

An attack scenario S1 has the following vector of membership values for success likelihood and economic consequences:

S1(success likelihood): [0, 0, 0, 0.57, 0.43, 0, 0]

S1(economic consequences): [0, 0, 0, 0, 1]

The leftmost entry for degree of membership in the vector of success likelihood corresponds to "negligible", followed by "extremely unlikely", "very unlikely", "unlikely", somewhat likely", "likely" and the rightmost entry corresponds to "nearly certain". The leftmost entry for degree of membership in the vector of economic consequences corresponds to "very low" and so on to the rightmost entry corresponding to "very high". Using the rule base of Table 2 and AR operations of Section 2.2, "very high" economic consequences AND an "unlikely" success likelihood results in a "medium" risk with a degree of membership of 0.57. While a "very high" economic consequences AND a "likely" success likelihood results in a "medium" risk with a degree of membership of 0.43. Since either of these two rules, shown in bold in Table 2, result in "medium" risk the maximum of the resulting degree of membership values is used to determine the final degree of membership for a "medium" risk. The resulting vector of membership values for risk in scenario 1 are:

S1(risk): [0, 0, 0.57, 0, 0]

Corresponding to linguistic risk values of "very low", "low", "medium", "high", and "very high" from left to right.

2.3 Evidential Reasoning

Like AR, rules are used to draw conclusions about a particular outcome from a set of inputs using IF-THEN rules in ER. These IF-THEN rules consist of an antecedent and a consequence portion. The conditional portion of the rule, i.e. the IF x_i is $\tilde{A}$ and y_j is $\tilde{B}$ of Rule 1, forms the antecedent and the consequence of the antecedent includes THEN z_k is $\tilde{E}$. The main difference between an AR and ER model is the uncertainty involved in the reasoning. AR involves the uncertainty associated with imprecisely describing x_i using $\tilde{A}$; whereas, ER involves the uncertainty associated with assigning x_i to a particular crisp value A. A crisp set value has a precise well defined boundary and precisely describes x_i. The ER model uses the degrees of belief in the antecedent, instead of degree of membership as used in AR, to determine the degree of belief for the consequence. The SMEs degree of belief quantifies the evidence supporting a particular claim, i.e. x_i is A, and is similar to the basic evidence assignment (see Klir 2005) used to form other belief plausibility, and probability measures. The *bea* does not account for the uncertainty associated with imprecisely describing x_i. The degree of membership is used to assess the uncertainty involved in describing a specific state using an imprecise linguistic value.

The focus of this paper is on the results of the ER and AR methods in SRAMA. An ER result is comprised of a vector of various linguistic values representing a specific state and the *bea* for each linguistic value. One simple method of determining the *bea* associated with the inferred linguistic value in the consequence is to take the product of the *bea* values involved in the antecedent of the rule. This process is performed with all the pertinent rules in the rule base. Two or more rules in the rule base may result in the same linguistic value, in such a case these resulting *bea* values are summed to determine the resulting *bea* value for the linguistic value. It is important to note that the *bea* (*m*) must satisfy the following boundary conditions:

$$m(\emptyset) = 0 \qquad (3)$$

$$m(A) = - \sum_{A \in P(X)}^{j=1,2,3,\ldots,n} m(A_j) \qquad (4)$$

Equation 3 indicates that a *bea* value cannot be assigned to the proposition that x_i is defined by the null set, Ø, because the null set defines no states. Equation 4 indicates theat the sum of the *bea* values for x_i is A_j is equal to 1 where, A_j are crisp subsets of the power set *P(X)* with j =1, 2, 3, ... n must be equal to 1. The power set *P(X)* is the set if all subsets of *X*.

2.3.1 Application of ER

This section demonstrates the use of ER using a simple example to determine the effectiveness of physical inventory from the material inventory frequency and effectiveness of inventory verification. Table 3 provides the rule base used to infer the effectiveness of physical inventory from the material inventory frequency and effectiveness inventory verification. A processing facility F1 has the following vector of *bea* values for a specific material inventory frequency and specific effectiveness of inventory verification:
F1(material inventory frequency): [0, .1, .9, 0]
F1(effectiveness of inventory verification): [0, 0, 1, 0]

The leftmost entry for *bea* in the vector of success likelihood corresponds to "not applicable", followed by "occasionally", "regularly", and the rightmost entry corresponds to "continuously". The leftmost entry for the *bea* in the vector of effectiveness of inventory verification corresponds to "not applicable", followed by "low", "moderate", and the rightmost entry corresponding to "excellent". Using the rule base of Table 3 and ER operations of Section 2.3, a *bea* value of 0.1 in "occasionally" for physical inventory frequency AND a *bea* value of 1.0 in "moderate" for effectiveness of inventory verification results in a *bea* value of 0.1 in "low" for effectiveness of physical inventory. While a *bea* value of 0.9 in "regularly" for physical inventory frequency AND a *bea* value of 1.0 in "moderate" for effectiveness of inventory verification results in a *bea* value of 0.9 in "moderate" for effectiveness of physical inventory.

F1(effectiveness of physical inventory):
[0, 0.1, 0.9, 0],

and corresponding to values of linguistic effectiveness of physical inventory of: "not applicable", "low", "moderate", and "excellent" from left to right.

Effectiveness of Physical Inventory		Effectiveness of Inventory Verification			
		NA	Low	Moderate	Excellent
Physical Inventory Frequency	NA	NA	NA	NA	NA
	Occasionally	NA	Low	Low	Low
	Regularly	NA	Low	Moderate	Moderate
	Continuously	NA	Low	Moderate	Excellent

Table 3. Effectiveness of Physical Inventory ER Rule Base

3 QUANTIFICATION OF INFORMATION UNCERTAINTY IN QUALITATIVE REASONING

Decision makers are interested in the confidence associated with each of the competing alternatives. The quantity of uncertainty present in a result is related to the confidence (Devore 1999). That is, the less uncertainty present in the resulting alternative the more confidence one can have in the result. By measuring the information uncertainty present in each resulting alternative, the possible alternatives can be ranked ordered and the most credible alternatives can be determined based on the amount of information uncertainty. The quantification of Entropy for random uncertainty was addressed by Shannon (Shannon 1948). Klir (Klir 2005) elaborates on Shannon's measure of entropy and identifies *conflict* as the basis for the entropy measured by Shannon.

The measure of entropy proposed by Shannon works as follows: there exists a regular die with six faces all of which are equally likely to be thrown and there exists a six sided trick die with one side being twice as likely to be thrown as the remaining sides. The regular die has more entropy than the trick die because all sides are equally likely to occur in the regular die. The trick die is less uncertain because one side is twice as likely to be thrown as each of the remaining five; thus, one can have more confidence in the resulting trick die. De Luca and Termini (Deluca 1972) extended Shannon's measure of entropy to fuzzy uncertainty in a fuzzy set while others also presented alternative measures, see Yager (Yager 1979), and Higashi and Klir (Higashi and Klir 1982). Pal and Bezdek (Pal and Bezdek 1994) provide a good summary of many of the approaches used to measure entropy associated with a fuzzy set. Previous approaches quantified the entropy involved in an entire fuzzy set, whereas the current study examines quantifying the entropy involved in one state described using several fuzzy sets, i.e. the entropy associated with an AR result.

Shannon's measure of conflict has the form

$$S(p) = -\sum_{x\in X} p(x)\log_2 p(x), \qquad (5)$$

Equation 5 can be used with *bea* values on sets *A* instead of probability values *p* on *x* to determine the conflict in the ER result .

$$C = -\sum_{A\in X} m(A)\log_2 m(A), \qquad (6)$$

Klir and Wierman extended Equation 5 to use with Belief and Plausibility measures on various types of families of sets. De Luca and Termini's (Deluca and Termini 1972) measure for the entropy of a fuzzy set is similar to Shannon's but conceptually different. Shannon measures the conflict due to random uncertainty while De Luca and Termini measure the conflict due to the fuzzy uncertainty associate with a membership function for a fuzzy set. Deluca and Termini proposed quantifying the conflict of a fuzzy set from its membership function and the complement of the its membership function; as they proposed in the following equation to measure entropy in a fuzzy set (Deluca and Termini 1972):

$$D(\tilde{A}) = -\sum_{i=1}^{n} \mu_{\tilde{A}}(x_i)\log_2\mu_{\tilde{A}}(x_i) + \mu_{\bar{\tilde{A}}}(x_i)\log_2\mu_{\bar{\tilde{A}}}(x_i) \qquad (7)$$

Pal and Bezdek (Pal and Bezdek 1994) present several previously proposed alternative approaches to measure fuzzy uncertainty in a fuzzy set.

As discussed in the previous section, AR uses the degree of membership in linguistic values to predict the outcome of a system. The outcome resulting from the AR is expressed as a vector of linguistic values, e.g. fuzzy sets, and a respective degree of membership in each fuzzy set. That is, in an AR result there is only one membership value for each fuzzy set. The conflict due to fuzzy uncertainty as quantified from methods such as De Luca and Termini (Deluca and Termini 1972) and those summarized by Pal and Bezdek (Pal and Bezdek 1999) rely on the degree of membership for all the elements within the fuzzy set. In an AR model the conflict is not among one fuzzy set but several, that is, there is conflict among all the fuzzy set alternatives having a degree of membership greater than 0. This study involves the quantification of conflict present in the AR results rather than the conflict between elements in one particular fuzzy set; thus, there exists a fundamental difference between the work done previously and that of the current study. Equation 7 can be modified to account for the conflict involved in imprecisely describing a specific state *x* with the various fuzzy sets $\tilde{R}_i$ in the resulting vector $\vec{R}$ and the modified equation is presented in Equation 8. The difference between Equation 7 and 8 is that Equation 8 involves one state *x* potentially described using *n* fuzzy sets; whereas, Equation 7 involves one fuzzy set describing *n* different states x_i.

$$C(\vec{R}) = -\sum_{i=1}^{n} \mu_{\tilde{R}_i}(x)\log_2\mu_{\tilde{R}_i}(x) + \mu_{\bar{\tilde{R}}_i}(x)\log_2\mu_{\bar{\tilde{R}}_i}(x) \qquad (8)$$

$\vec{R}$ is the vector consisting of the degree of membership for each fuzzy set in the AR result for one scenario, and C is the conflict. $\tilde{R}_i(x)$ is the degree of membership of state x in the fuzzy set $\tilde{R}_i$

Another type of entropy, known as nonspecificity, reflects the ambiguity in specifying the exact solution (Klir 2005). Hartley (Hartley) first proposed measuring the lack of specificity which is simply related to the number of alternatives present. Klir (Klir 2005) simply defines the Hartley measure of uncertainty as:

$$H(f_E) = log_2|E|, \quad (9)$$

where f_E is any function of the subset E. The nonspecificity of an AR result can be determined through a straight forward extension of Equation 9. Considering that f_E instead represents a vector consisting of membership values for a specific state in several fuzzy sets. The Hartley measure has also been extended to probability distribution functions and membership function which are not discussed here and the reader is referred to (Klir 2005, Klir and Wierman 1999) for an in depth discussion.

The nonspecificity in an AR result can be quantified using Equation 10

$$N(\vec{R}) = log_2|R|, \quad (10)$$

where R is the number of fuzzy sets with a non-zero degree of membership. Random uncertainty may be present in available information elicited from an SME but it is at an epistemic level and captured in the linguistic values provided by the SME. As a result the conflict due to random uncertainty is captured by 8. Equation 10 can also be used to quantify the non-specificity involved in an ER result and R is simply interpreted as the number of crisp linguistic sets. Both Equation 8 and 10 have units of bits of information from the use of the logarithm base 2 (Klir 2005).

3.1 Entropy in AR and ER results

The quantification of conflict and non-specificity in AR and ER results are demonstrated here using the examples provided in Section 2. Using Equation 6 the conflict involved in the ER result F1(effectiveness of physical inventory):[0, 0.1, 0.9, 0], is calculated as.

$$C = -[0.1\log_2(0.1) + 0.9\log_2(0.9)] = 0.469$$

The non-specificity involved in the ER result is calculated using Equation 10.

$$N(\vec{R}) = log_2|2| = 1$$

Using Equation 9 the conflict involved in the AR result S1(risk): [0, 0, 0.57, 0, 0], is calculated as follows. Recall that the membership of the complement is determined from Equation 2.

$$C(\vec{R}) = -[(0.57\log_2 0.57) + 0.43\log_2 0.43 = 0.986$$

The non-specificity involved in the AR result is calculated using Equation 10.

$$N(\vec{R}) = log_2|1| = 0$$

In addition to the ER and AR example provided previously two additional ER results and AR results are provided. The ER and AR results and their quantities of information uncertainty are presented in Tables 4 and 5, respectively.

ER result	Conflict	Nonspecificity
F1[0, 0.1, 0.9, 0]	0.469	1
F2[0, 0.2, 0.8, 0]	0.722	1
F3[0, 0.15, 0.75, 0.1]	1.054	1.585

Table 4. Entropy for ER results

AR result	Conflict	Nonspecificity
S1[0, 0, 0.57, 0, 0]	0.9858	0
S2[0, 0.3, 0.7, 0.2, 0]	2.883	1
S3[0, 0.2, 0.6, 0.2, 0.1]	2.484	1.585

Table 5. Entropy for AR results

The results demonstrate the utility of quantifying information uncertainty to compare the results. There is a recognizable and comparable difference in the conflict associated with all three ER results yet the resulting linguistic values are very similar, i.e. effectiveness of inventory verification is "low to mostly moderate". In the case of the AR results, there is also a recognizable difference in the conflict and the non-specificity. The non-specificity reflects a difference that can be discerned visually, the greater number of alternatives the greater the non-specificity. Alternatively, measuring the conflict provides comparative information that is not as easily discerned visually.

Tables 4 and 5 illustrate the quantification of the conflict and non-specificity using simplified AR and ER models. Actual AR and ER SRAMA models consist of several these inferences connected in series to ultimately infer such things as vulnerability and threat to a facility which are used to infer risk to the facility. Changes in the inputs to these inferences may affect the result. The measurement and comparisons of conflict and non-specificity can be used comparisons of the results and ultimately resource allocation to reduce the risk.

4 CONCLUSIONS

The implications of this study are pertinent to both AR and ER in SRAMA and information theory. ER and AR results for SRAMA have quantifiable amounts of information uncertainty. This study extends information theory to AR and ER SRAMA models. Straight-forward extensions of previous approaches used to measure the fuzzy uncertainty associated with a membership function are presented in this paper and used to quantify the information uncertainty in AR results. The measurement of conflict and non-specificity associated with AR and ER results is illustrated and used to compare the results to one another. In addition, the measured conflict and non-specificity in each result may be used to identify which result provided the most confidence by recognizing that a greater amount of confidence can be placed in the results with a lower value of uncertainty. The entropy measures of this study can be further extended to fuzzy evidential reasoning; however, in the current literature the fuzzy uncertainty associated with fuzzy evidential reasoning is not specifically identified with the degree of membership in a fuzzy set. The current paper would benefit from a discussion on the suggested requirements for entropy measures involving fuzzy sets and their relation to AR and ER results.

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