INL/CON-10-17540
PREPRINT

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PSAM 10

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June 2010

Impact of Uncertainty on Calculations for Recovery from Loss of Offsite Power

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Abstract: Uncertainty, both aleatory and epistemic, can have a significant impact on estimated probabilities of recovering from loss of offsite power within a specified time window, and such probabilities are an input to risk-informed decisions regarding the significance of inspection findings in the U.S. Nuclear Regulatory Commission's Reactor Oversight Process. In particular, the choice of aleatory model for offsite power recovery time can have a significant impact on the estimated nonrecovery probability, especially if epistemic uncertainty regarding parameters in the aleatory model is accounted for properly. In past and current analyses, such uncertainty has largely been ignored. This paper examines the impact of both aleatory and epistemic uncertainty on the results, using modern open-source Bayesian inference software, which implements Markov chain Monte Carlo sampling. It includes examples of time-dependent convolution calculations to show the impact that uncertainty can have on this increasingly frequent type of calculation, also. The results show that the "point estimate" result, which is an input to risk-informed decisions, can easily be uncertain by a factor of 10 if both aleatory and epistemic uncertainties are considered. The paper also illustrates the use of Bayesian model selection criteria to aid in the choice of aleatory model.

Keywords: PRA, Uncertainty, Bayesian inference

1. INTRODUCTION

Uncertainty, both aleatory and epistemic, can have a significant impact on estimated probabilities of recovering from loss of offsite power (LOSP) within a specified time window, and such probabilities are an input to risk-informed decisions regarding the significance of inspection findings in the U.S. Nuclear Regulatory Commission's Reactor Oversight Process. In particular, the choice of aleatory model for offsite power recovery time can have a significant impact on the estimated nonrecovery probability, especially if epistemic uncertainty regarding parameters in the aleatory model is accounted for properly. In past and current analyses, such uncertainty has largely been ignored.

This paper examines the potential impact of both aleatory and epistemic uncertainty on the results of recovery calculations for LOSP sequences. Aleatory uncertainty refers to the variability in LOSP recovery time expressed by a probabilistic model (exponential, Weibull, etc.), and epistemic uncertainty refers to the choice of aleatory model for recovery time, and to state-of-knowledge uncertainty as to the parameter values in this aleatory model.

OpenBUGS, a modern, open-source Bayesian inference software package, which implements Markov chain Monte Carlo (MCMC) sampling, is used for the analyses presented herein. More details on OpenBUGS can be found in (1) and (2). Examples of modern Bayesian inference in PRA with OpenBUGS can be found in (3).

2. EXAMPLE

Consider the following times to recover offsite power following a failure of the offsite grid (a grid-related loss of offsite power). The units are hours: 0.1, 0.133, 0.183, 0.25, 0.3, 0.333, 0.333, 0.55, 0.667, 0.917, 1.5, 1.517, 2.083, 6.467.[2] The simplest aleatory model for these times is the exponential

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[2] The recovery times used in this example are for a U. S. region. They have not been rounded.

distribution, which has one parameter that must be estimated, namely the recovery rate, λ. Updating the Jeffreys prior for λ with these data, one finds the posterior mean of λ to be 0.91/hr, with a 90% credible interval of (0.55, 1.35). The probability of not recovering power by time t, which is conditional upon λ, is given by $e^{-\lambda t}$. This must be averaged over the posterior distribution for λ to account for the epistemic uncertainty in λ:

$$P(T \le t) = \int_0^\infty e^{-\lambda t} \pi_1(\lambda \mid t_1, t_2, \ldots, t_n) d\lambda.$$

This is implemented using the `cumulative()` function in OpenBUGS, giving a mean nonrecovery probability at 8 hours of 0.0028, with a 90% interval of (2.0E-5, 0.012). The point estimate nonrecovery probability at 8 hours, using the maximum likelihood estimate (MLE) of λ, is only 6.7E-4, about a factor of 4 less than the mean nonrecovery probability calculated by propagating epistemic uncertainty in λ through the exponential aleatory model for recovery time. The OpenBUGS script used for these calculations is shown in Script 1 in the Appendix.

2.1. Alternative Aleatory Models

Two commonly used alternatives to the exponential model are the Weibull and lognormal distributions. I examine each of these in turn, starting with the Weibull distribution. For Bayesian inference, it is common to use the following parameterization of the Weibull distribution:

$$F(t) = 1 - \exp\left(-\lambda t^{\beta}\right)$$

In this parameterization, β controls the shape of the distribution; for $\beta = 1$, the Weibull distribution reduces to the exponential distribution. Starting with independent, diffuse prior distributions for β and λ, we find the following marginal results, using the OpenBUGS script shown in Script 2 in the Appendix.

Table 1 Summaries of marginal posterior distributions for Weibull parameters, obtained by updating independent, diffuse prior distributions with example recovery times

	Posterior mean	**90% interval**
β	0.83	(0.58, 1.11)
λ	1.01	(0.59, 1.525)

As a side note, the Bayesian inference was carried out with *independent* prior distributions for β and λ; however, β and λ are *dependent* in the posterior distribution. One measure of this is the correlation coefficient between β and λ, which is approximately -0.32.

The probability of not recovering power by time t is given for any aleatory model by $1 - F(t)$, where F(t) is the cumulative distribution function for the recovery time. For the Weibull distribution, *F(t)* is $\exp\left(-\lambda t^{\beta}\right)$. Propagating the uncertainty encoded in the *joint* posterior distribution for β and λ (the joint distribution accounts for the dependence between β and λ) gives a mean nonrecovery probability at 8 hours of 0.012, with a 90% interval of (5.6E-5, 0.052). Note the significantly larger mean value, a factor of about 4 larger than with the exponential model. Conceptually, this is caused by two issues: the considerable uncertainty in λ, and the negative correlation between β and λ in the joint posterior distribution. The point estimate nonrecovery probability with the Weibull model, using the MLEs of β and λ, is 2.8E-3, again about a factor of 4 less than the posterior mean value.

Turning now to a lognormal aleatory model, the density function is

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma t}\exp\left[-\frac{(\ln t - \mu)^2}{2\sigma^2}\right]$$

OpenBUGS uses $\tau = \sigma^{-2}$ as the scale parameter. The script in Script 3 in the Appendix uses independent diffuse priors for μ and σ to give the posterior summaries shown in Table 2.

Table 2 Summaries of marginal posterior distributions for lognormal parameters, obtained by updating independent, diffuse prior distributions with example recovery times

	Posterior mean	**90% interval**
μ	-0.61	(-1.16, -0.05)
σ	1.24	(0.89, 1.74)

The mean nonrecovery probability at 8 hours under the lognormal aleatory model is 0.023, with a 90% interval of (8.9E-4, 0.078). The point estimate nonrecovery probability, using the MLEs of μ and σ, is 8.8E-3, about a factor of 3 less than the mean value above.

To summarize the results so far, the three aleatory models for LOSP recovery time give widely varying mean nonrecovery probabilities at 8 hours, when parameter uncertainty is properly accounted for in a fully Bayesian analysis. Note that using the maximum likelihood parameter estimates gives point estimate nonrecovery probabilities at 8 hours that are significantly less than the posterior means from the fully Bayesian analysis. This important result, which has been realized for some time, is summarized in Table 3.

Table 3: Comparison of point estimate nonrecovery probabilities at 8 hours with posterior mean values from fully Bayesian analysis

Model	**Point estimate nonrecovery probability at 8 hours using MLEs**	**Posterior mean nonrecovery probability at 8 hours**	**Ratio of posterior mean to point estimate**
Exponential	6.7E-4	2.8E-3	4.2
Weibull	2.8E-3	0.012	4.3
Lognormal	8.8E-3	0.023	2.6

2.2 Convolution Calculations

Traditional PRA calculations to quantify sequence minimal cut sets ignore the time dependence of emergency diesel generator (EDG) failure to run. A typical PRA cut set of interest might involve EDG failure to run and failure to recover offsite power before core uncovery. In quantifying such a cut set, traditional PRA calculates the probability of EDG failure to run separately from the probability of not recovering offsite power before core uncovery, and then multiplies the resulting probabilities together. The more accurate approach to quantifying such cut sets is to convolve (i.e., integrate over) the EDG failure time with the offsite power recovery time, treating these times as stochastically independent. The result of this more accurate calculation is often a smaller cut set risk metric, and so this calculation is becoming more frequent as analysts seek to avoid unnecessary conservatism in risk inputs to the NRC's Reactor Oversight Process.

All such calculations of which the author is aware have been point estimates, using the MLEs of the aleatory model parameters. So the question is what might the impact be of including epistemic uncertainty about the parameters of the aleatory model in such a calculation? To explore this question, assume that the EDG failure rate is lognormally distributed with a mean of 1E-3/hr and an error factor of 10. Assume that core uncovery will occur 1.5 hours after failure of the EDG.[3] Also assume that 12 minutes are required for the operators to restore offsite power to the safety loads once it becomes available on the grid. Therefore, the time available to avert core damage will be 1.32 hours.

In general, if t is the time at which the EDG fails, and t_c is the time available to avert core damage, the probability of the cut set will be given by the following convolution integral:

$$\int_0^{t_m} \lambda e^{-\lambda t}\left[1 - F\left(t + t_c\right)\right] dt \qquad (1)$$

In Eq. 1, t_m is the mission time of the EDG, whose failure time is assumed to be exponentially distributed with rate λ, and F(•) is the cumulative distribution function for the offsite power recovery time. It is common in PRA to adjust the traditional cut set probability by multiplying by a recovery factor, which is the value from Eq. 1 divided by the probability of EDG failure over the mission time, with 24 hours being a typical EDG mission time.

For the sample times, the point estimate nonrecovery probability for the exponential model is 3.3E-4, and 3.7E-4 for the Weibull and lognormal models (obtained using the parameter MLEs). The corresponding recovery factor point estimates are 0.014 and 0.015. OpenBUGS was used to propagate epistemic uncertainties through the convolution integral. This is done using the `integral()` function in OpenBUGS. OpenBUGS correctly reflects the dependence between the Weibull parameters in the joint posterior distribution, as well. Table 4 shows the posterior mean and 90% credible interval for the nonrecovery probability and recovery factor, for each of the three aleatory models under consideration. The means are within a factor of two of the point estimates, but the uncertainties are relatively large.

Table 4 Posterior means and 90% credible intervals for nonrecovery probabilities and recovery factors from convolution of EDG failure time with offsite power recovery time

Model	Posterior mean nonrecovery probability (90% credible interval)	Posterior mean recovery factor (90% credible interval)
Exponential	4.0E-4 (8.8E-6, 1.6E-3)	1.6E-2 (3.8E-3, 3.7E-2)
Weibull	5.3E-4 (1.1E-5, 2.1E-3)	2.2E-2 (4.8E-3, 5.5E-2)
Lognormal	6.0E-4 (1.0E-5, 2.4E-3)	2.5E-2 (3.9E-3, 7.1E-2)

3. MODEL CHECKING

So which aleatory model among the three is the best? A simple way to begin answering this question is via nonparametric exploratory data analysis. A cumulative hazard plot, as described in (4), can be used to quickly check for gross departures from the simplest aleatory model, the exponential distribution. Figure 1 shows the cumulative hazard plot for the sample recovery times. There is an obvious curvature to the plot, suggesting that the exponential model may not be adequate.

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[3] Actually, the time to core uncovery will be a function of the time of EDG failure, because of the decreasing decay heat load, but this complication is ignored.

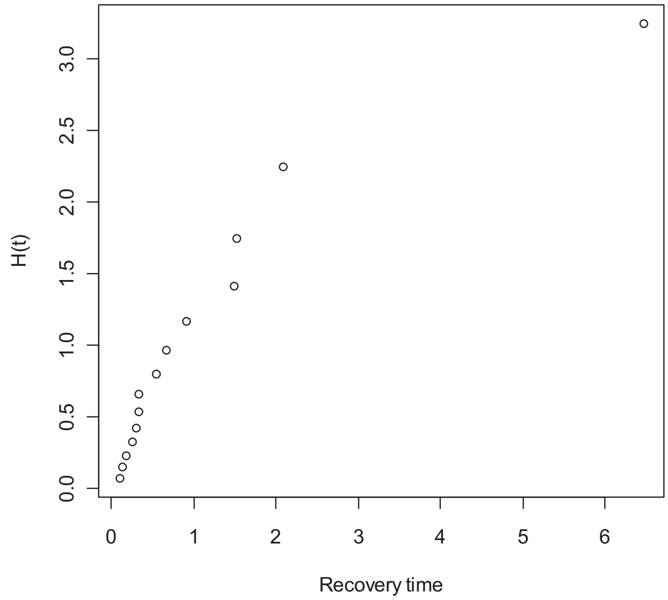

Figure 1 Cumulative hazard plot of sample recovery times. Curvature suggests exponential model may not be adequate.

A Bayesian answer to the question of adequacy of the aleatory model can be obtained by examining replicated recovery times from the posterior predictive distribution (3), (5), (6). Under the exponential model, the 97.5th percentile of the replicated recovery time is about 4.7 hours, so this model is not able to replicate the longest observed time of 6.467 hours. The Weibull model does better, with a 97.5th percentile replicated time approaching 6 hours. For the lognormal model, the 97.5th percentile replicated time is about 7.5 hours, so of the three models, only the lognormal model can replicate the observed data with significant probability. This is illustrated graphically in the figures below, which show 95% intervals from the posterior predictive distribution with the observed times indicated; the longest time falls outside the predictive interval for the exponential and Weibull models, but inside the interval for the lognormal model.

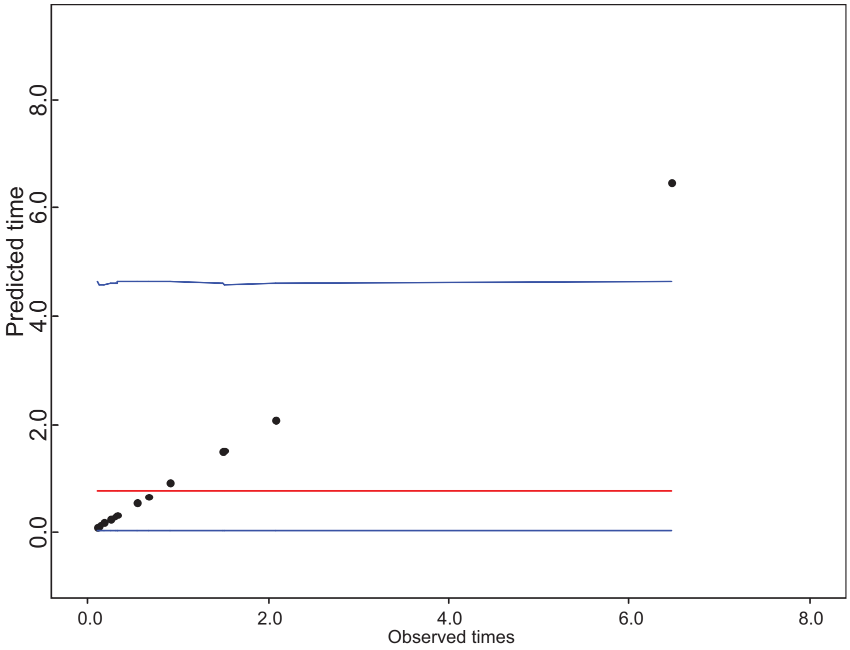

Figure 2 Plot of predicted vs. observed times with exponential aleatory model. Dots are the observed times. Red line is posterior mean predicted time, blue lines indicate 95% predicted interval. Exponential model cannot replicate observed variability in times.

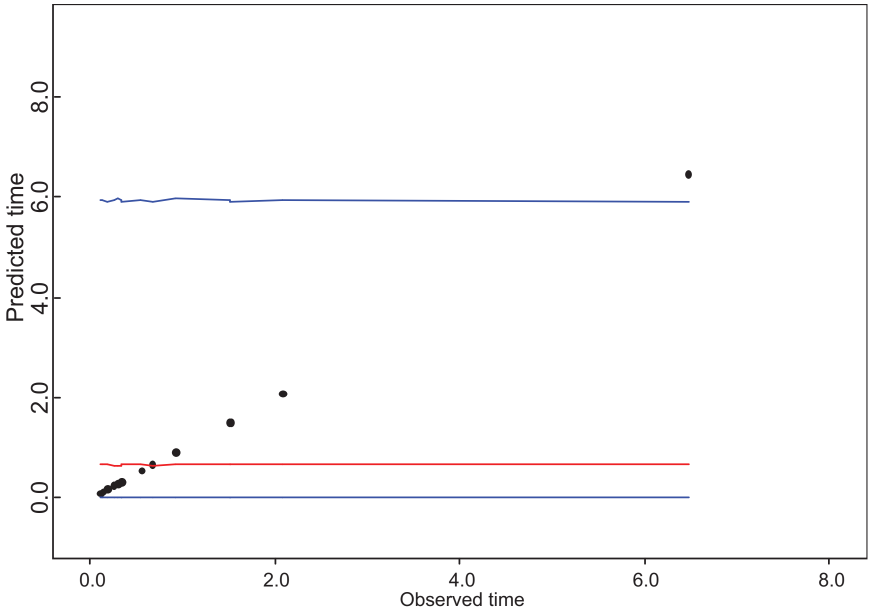

Figure 3 Plot of predicted vs. observed times with Weibull aleatory model. Dots are the observed times. Red line is posterior mean predicted time, blue lines indicate 95% predicted interval. Weibull model cannot replicate observed variability in times.

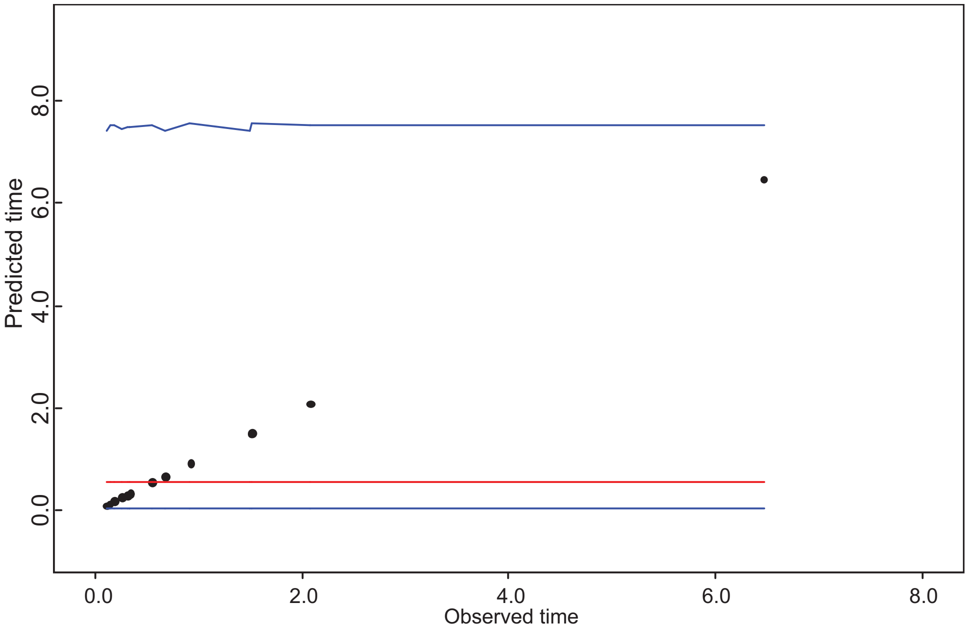

Figure 4 Plot of predicted vs. observed times with lognormal aleatory model. Dots are the observed times. Red line is posterior mean predicted time, blue lines indicate 95% predicted interval. Lognormal model can replicate observed variability in times (all observed times fall within predicted interval).

One can also use summary statistics derived from the posterior predictive distribution, as described in (3), (5), and (6). Using the Cramér-von Mises statistic, as described in (3), one can calculate a Bayesian p-value for each model, with a value near 0.5 being desirable. The script in Script 4 in the Appendix illustrates this calculation for the exponential model; the calculation is done analogously for the other models. The following Bayesian p-values were calculated for each of the three models: exponential, 0.42; Weibull, 0.44; lognormal, 0.58. By this criterion, all three models are reasonable choices.

The Bayesian information criterion (BIC), which penalizes models with more parameters to guard against over-fitting, can be used to select from among candidate models, as described in (5) and (6). The model with the smallest BIC is preferred. In this example, the estimated BICs are 34.2, 36.99, and 33.3 for the exponential, Weibull, and lognormal models, respectively. So based on BIC, we would select the lognormal model over the other two. Another criterion often used is the deviance information criterion (DIC), see (7). The DIC is calculated automatically by OpenBUGS, with no need for additional scripting. The lognormal model also has the smallest DIC, 30.16 versus 32.57 and 33.73 for the exponential and Weibull models, so it would be selected under this criterion, also.

In summary, among the three candidate aleatory models, the lognormal model would be chosen based on penalized likelihood criteria such as BIC or DIC. In terms of replicated recovery times, both the exponential and Weibull models tend to under-predict recovery times. The choice of aleatory model in this example makes a considerable difference in the estimated nonrecovery probability, especially when parameter uncertainty is accounted for in a fully Bayesian analysis.

4. BAYESIAN MODEL AVERAGING

There is uncertainty as to which of the three aleatory models is most appropriate for the LOSP recovery times under consideration. The Bayesian approach to addressing such model uncertainty, as described in (8), is to construct the posterior distribution for the quantity of interest, in this case the nonrecovery probability at 8 hours. Let p_{nonrec} denote this quantity. Let $\tilde{t}$ denote the vector of observed recovery times. Thus, we are interested in $\pi\left(p_{nonrec} \mid \tilde{t}\right)$, the marginal posterior distribution for the nonrecovery probability at 8 hours. This distribution is not conditional on any aleatory model.

To obtain this distribution, we must specify a discrete prior distribution, *p(M$_i$),* for each of the three aleatory models under consideration. $\pi\left(p_{nonrec} \mid \tilde{t}\right)$ is then given by

$$\pi\left(p_{nonrec} \mid \tilde{t}\right)=\sum_{i=1}^{3}\pi\left(p_{nonrec} \mid \tilde{t}, M_i\right)p(M_i \mid \tilde{t})$$

The posterior probability of each model, conditional upon the observed data, that appears in this equation, $p\left(M_i \mid \tilde{t}\right)$, is given by

$$p\left(M_i \mid \tilde{t}\right)=\frac{p(M_i)\int f\left(\tilde{t} \mid \theta_i\right)\pi\left(\theta_i \mid M_i\right)d\theta_i}{\sum_{j=1}^{3}p(M_i)\int f\left(\tilde{t} \mid \theta_i\right)\pi\left(\theta_i \mid M_i\right)d\theta_i} \tag{2}$$

Instead of basing inference on a posterior distribution that is conditional upon a single aleatory model, the Bayesian model-averaging approach uses a weighted-average posterior distribution, where the weights are the posterior probabilities of each model, conditional upon the observed data. Note that the prior parameter distributions must be proper in order for the integrals in the above equation to be defined. So far, the Bayesian inference has been done using extremely diffuse, barely proper priors. In order to do Bayesian model averaging, proper, informative priors are needed to ensure convergence of the integrals in Eq. 2. The analyst must also specify the prior probabilities for each model, which can be challenging. Finally, the integrals in Eq. 2 must be done numerically, often via MCMC.

Given these difficulties, it is simpler to take an approximate route to Bayesian model averaging. The posterior model probabilities can be approximated using the BIC under the (probably reasonable) prior assumption that all aleatory models are equally likely *a priori* (8). The approximate posterior model weights under this approximation are given by

$$p\left(M_i \mid \tilde{t}\right)\approx\frac{\exp\left(-\Delta BIC_i / 2\right)}{\sum_{i=1}^{n}\exp\left(-\Delta BIC_i / 2\right)}.$$

In this equation, $\Delta\text{BIC}_i = (\text{BIC}_i - \min(\text{BIC}))$. Using this approximation for the posterior model probabilities gives a posterior mean nonrecovery probability at 8 hours of 0.015, with a 90% credible interval of (6.7E-5, 0.06).

5. CONCLUSIONS

LOSP sequences are frequently significant contributors to risk metrics used to evaluate findings in the U. S. Nuclear Regulatory Commission's Reactor Oversight Process. A principal contributor to risk in these sequences is the probability of not recovering offsite power in time to avert core damage, and this probability is determined by the aleatory model assumed for offsite power recovery time. Typical choices for this model are the exponential, Weibull, and lognormal distributions. Each of these distributions contains one or two parameters, which must be estimated, and these parameters are therefore subject to epistemic uncertainty. Whether or not the epistemic parameter uncertainty is propagated through the aleatory model can, as shown in this paper, have a significant effect on the estimated nonrecovery probability. This compounds the effect of the aleatory model.

This paper has illustrated tools that can be used to guide the choice of aleatory model, ranging from simple nonparametric plots suitable for exploratory data analysis, to plots and quantitative measures based on the posterior predictive distribution from a full Bayesian analysis, calculated utilizing the open-source MCMC package OpenBUGS. The Bayesian approach can also be used to propagate epistemic parameter uncertainties, including dependence among parameters, through convolution calculations used to correct for errors introduced by the approximate treatment of time-dependent EDG failure in traditional PRA cut set quantification approaches.

Finally, the paper touched upon the topic of Bayesian model averaging, via which aleatory and epistemic uncertainties (related to both parameter values and model choice) can be combined to yield overall results that include both uncertainty contributions.

Acknowledgements

This paper was prepared as an account of work sponsored by an agency of the U.S. Government. Neither the U.S. Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for any third party's use, or the results of such use, of any information, apparatus, product or process disclosed in this report, or represents that its use by such third party would not infringe privately owned rights.

This manuscript has been authored by Battelle Energy Alliance, LLC under Contract No. DE-AC07-05ID14517 with the U.S. Department of Energy. The United States Government retains and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a nonexclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

References

1. Lunn, D. J., et al., "WinBUGS - A Bayesian Modelling Framework: Concepts, Structure, and Extensibility," 2000, *Statistics and Computing*, pp. 325-337.
2. Lunn, David, et al., "The BUGS Project: Evolution, Critique, and Future Directions," 2009, *Statistics in Medicine*, Vol. 28, pp. 3049-3067.
3. Kelly, Dana L. and Smith, Curtis L., "Bayesian Inference in Probabilistic Risk Assessment - The Current State of the Art," 2009, *Reliability Engineering and System Safety*, Vol. 94, pp. 628-643.
4. Kelly, Dana L., Rodionov, Andrei and Uwe-Kluegel, Jens., "Practical Issues in Component Aging Analysis," Knoxville, TN : American Nuclear Society, 2008. ANS PSA Topical Meeting - Challenges to PSA During the Nuclear Renaissance.
5. Gelman, Andrew, Meng, X. L. and Stern, H., "Posterior Predictive Assessment of of Model Fitness via Realized Discrepancies," 1996, *Statistica Sinica*, pp. 733-807.
6. Gelman, Andrew et al., *Bayesian Data Analysis, Second Edition.* Chapman & Hall/CRC, 2004.
7. Schwarz, G., "Estimating the Dimension of a Model," 1978, *The Annals of Statistics*, Vol. 6, pp. 461-464.
8. Hamada, Michael S., et al. *Bayesian Reliability.* Springer, 2008.
9. Spiegelhalter, D., et al., "Bayesian Meaures of Model Complexity and Fit," 2002, *Journal of the Royal Statistical Society B*, 64, pp. 583-640.
10. Claeskens, Gerda and Hjort, Nils. *Model Selection and Model Averaging.* Cambridge University Press, 2008.

Appendix

This appendix lists the OpenBUGS scripts used in the analyses in the main body of the paper.

Script 1 OpenBUGS script for exponential aleatory model

```
model {
for(i in 1:N) {
      time[i] ~ dexp(lambda) #Exponential aleatory model
      }
#Jeffreys prior distribution for lambda
lambda ~ dgamma(0.0001, 0.0001)
#Predicted recovery time
time.pred ~ dexp(lambda)
#Calculate nonrecovery probability at 8 hours
prob.nonrec <- 1 - cumulative(time.pred, time.crit)
}
data
list(time=c(0.1,0.133,0.183,0.25,0.3,0.333,0.333,0.55,0.667,0.917,1.5,1.517
,2.083,6.467), time.crit=8, N=14)
inits
One chain
list(lambda=0.1)
```

Script 2 OpenBUGS script for Weibull aleatory model

```
model {
for(i in 1:N) {
      time[i] ~ dweib(beta, lambda) #Weibull aleatory model
      }
#Diffuse prior distributions
lambda ~ dgamma(0.0001, 0.0001)
beta ~ dgamma(0.0001, 0.0001)
#Predicted recovery time
time.pred ~ dweib(beta, lambda) #Weibull model
#Calculate nonrecovery probability at 8 hours
prob.nonrec <- 1 - cumulative(time.pred, time.crit)
}
data
list(time=c(0.1,0.133,0.183,0.25,0.3,0.333,0.333,0.55,0.667,0.917,1.5,1.517
,2.083,6.467), time.crit=8, N=14)
inits
#Two chains
list(beta=0.5, lambda=1)
list(beta=1, lambda=0.1)
```

Script 3 OpenBUGS script for lognormal aleatory model

```
model {
for(i in 1:N) {
    time[i] ~ dlnorm(mu, tau) #Lognormal aleatory model
    }
#Diffuse prior distributions
mu ~ dflat()
sigma ~ dgamma(0.0001, 0.0001)
tau <- pow(sigma, -2)
#Predicted recovery time
time.pred ~ dlnorm(mu, tau)
#Calculate nonrecovery probability at 8 hours
prob.nonrec <- 1 - cumulative(time.pred, time.crit)
}
data
list(time=c(0.1,0.133,0.183,0.25,0.3,0.333,0.333,0.55,0.667,0.917,1.5,1.517
,2.083,6.467), time.crit=8, N=14)
inits
#Two chains
list(mu=-0.5, sigma=1.1)
list(mu=-1, sigma=1.4)
```

Script 4 OpenBUGS script excerpt for calculating Bayesian p-value

```
model {
for(i in 1:N) {
      time[i] ~ dexp(lambda) #Exponential aleatory model
      time.ranked[i] <- ranked(time[], i)
#Replicate times from posterior predictive distribution
      time.rep[i] ~ dexp(lambda)
      #Rank replicate times
      time.rep.ranked[i] <- ranked(time.rep[], i)
#Calculate components of Cramer-von Mises statistic for observed and
replicate data
      F.obs[i] <- cumulative(time[i], time.ranked[i])
      F.rep[i] <- cumulative(time.rep[i], time.rep.ranked[i])
      diff.obs[i] <- pow(F.obs[i] - (2*i-1)/(2*N), 2)
      diff.rep[i] <- pow(F.rep[i] - (2*i-1)/(2*N), 2)
      }
#Calculate distribution of Cramer-von Mises statistic for observed and
replicate data
CVM.obs <- sum(diff.obs[])
CVM.rep <- sum(diff.rep[])
p.value <- step(CVM.rep - CVM.obs) #Mean value should be near 0.5
}
```