Non-Universality of Transverse Momentum Dependent Parton Distributions at Small-x

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Abstract

We study the universality of the transverse momentum dependent parton distributions at small-x, by comparing the initial/final state interaction effects in dijet-correlation in pA collisions with that in deep inelastic lepton nucleus scattering. We demonstrate the non-universality by an explicit calculation in a particular model where the multiple gauge boson exchange contributions are summed up to all orders. We further comment on the implications of our results on the theoretical interpretation of di-hadron correlation in dA collisions in terms of the saturation phenomena in deep inelastic lepton nucleus scattering.

I. INTRODUCTION

The Feynman parton distribution functions describe the internal structure of hadrons in terms of the momentum distributions of partons in the infinite momentum frame. These distributions depend only on the longitudinal momentum fractions of the target hadrons carried by the partons. The measurements of high energy hadronic processes depending on the Feynman parton distributions have been made possible by proving the associated factorization theorem, which guarantee the parton distributions studied in different processes are universal [1].

In recent years, hadronic physicists become more interested in semi-inclusive high energy processes, where one hopes to study the intrinsic transverse momentum of partons inside hadrons. The additional transverse momentum dependence will help to picture the parton distribution in a three-dimension fashion and build the hadron tomograph through the partonic structure [2]. A number of novel hadronic physics phenomena are also strongly associated with the transverse momentum dependent parton distributions. For example, the single transverse spin asymmetries [3–6] and small-x saturations phenomena [7, 8] are both related to the transverse momentum dependent parton distributions. In the last few years, great progress has been made in understanding the fundamental questions associated with these transverse momentum dependent parton distributions, such as the gauge invariance and the QCD factorization [4, 5, 9, 10]. In particular, the non-universality of these distribution functions due to the final/initial state interaction effects has attracted intensive investigations. It has been found that the difference between final state interaction in deep inelastic scattering and the initial state interaction in Drell-Yan lepton pair production in pp collisions leads to an opposite sign for the single spin asymmetries in these two processes [3, 4]. More complicated relation was further found for the single spin asymmetry in dijet-correlation in pp collisions as compared to those in DIS and Drell-Yan processes [11–16], and eventually a standard transverse momentum dependent factorization breaks down for this process [14].

In this paper, we will extend the universality discussions of the transverse momentum dependent parton distributions to the small-x domain, where the k_t-dependent distributions have been a common practice to describe the relevant physics phenomena [8]. We expect the non-universality for these objects as well. However, because of different approximation has been made in the small-x region, the general arguments of Refs. [14, 15] on the non-universality may not apply. As far as we know, there has been no discussion on this issue in the literature[1]. In this paper, we will study this. We will carry out an explicit calculation in a model where both small-x and low transverse momentum approximation are valid. Furthermore, we will resum the initial/final state interactions to all orders in perturbation to study the associated universality property.

In particular, we investigate the universality of the small-x transverse momentum dependent parton distributions probed in hadronic dijet-correlation in nucleon-nucleus collisions, as compared to that in the deep elastic lepton-nucleus (nucleon) scattering. There have been experimental results on di-hadron correlation in dA collisions at RHIC reported by the STAR collaboration, and interesting phenomena were found [18]. However, the theoretical interpretation is not yet clear at this moment [19–22]. As schematically shown in Fig. 1(a), two partons from the nucleon projectile and nucleus target collide with each other, and produce two jets in the final state,

$$p + A \to \mathrm{Jet1} + \mathrm{Jet2} + X \ , \tag{1}$$

where the transverse momenta of two jets are similar in size but opposite to each other in direction, $\vec{P}_{1\perp} + \vec{P}_{2\perp} \approx 0$. In ideal case, these two jets are produced back-to-back. However, the gluon radiation and intrinsic transverse momenta of the initial partons will induce the imbalance between them. We are particularly interested in the kinematic region that the imbalance $\vec{k}_\perp = \vec{P}_{1\perp} + \vec{P}_{2\perp}$ is much smaller than the transverse momentum of the individual jet, $|\vec{k}_\perp| \ll |\vec{P}_{1\perp}| \sim |\vec{P}_{2\perp}|$. Only in this region, the intrinsic transverse momentum can have significant effects. Since there are two incoming partons, both intrinsic transverse momenta can affect the imbalance between the two jets. For big nucleus and small-x, the dominant contribution shall come from the intrinsic transverse momentum of the parton from the nucleus, for which we labeled as $q_\perp$ in Fig. 1(a). In the following, we will focus on this contributions. Of course, we emphasize that both contributions shall be taken into account to describe the dijet-correlation in pA scattering.

To understand the universality property of the transverse momentum dependent parton

[1] An important factorization breaking effect has been discussed in [17], which is however different from the non-universality issue we are investigating in this paper. See more detailed discussions below.

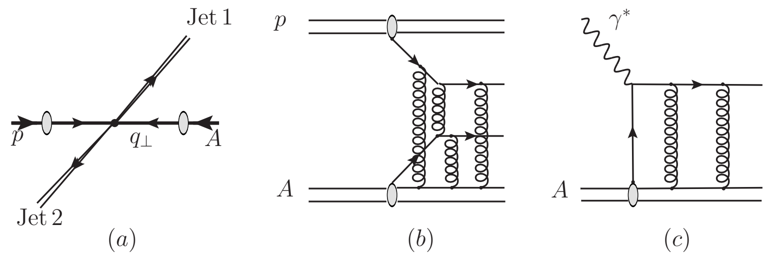

FIG. 1. (a) Schematic diagram showing that two partons from the nucleon projectile and the nucleus target collide and produce two jets in the final state, where the intrinsic transverse momentum $q_\perp$ from nucleus dominates the imbalance between the two jets; (b) initial/final state interactions may affect the transverse momentum dependent quark distribution from the nucleus in this process; (c) as a comparison, in deep inelastic lepton-nucleus (nucleon) scattering, there is only the final state interaction effect.

distribution from the nucleus, we have to study the multi-gluon exchange between the hard scattering part with the nucleus target [5, 7, 14, 15]. We show the generic diagrams of these interactions in Fig. 1(b), for the particular partonic channel $qq' \to qq'$. All other channels shall follow accordingly. Since the incoming and outgoing partons are all colored objects, there exist initial state interaction with the initial parton from the nucleon projectile, and final state interactions with the outgoing two partons. For comparison, we also plot in Fig. 1(c) the similar diagram for the deep inelastic lepton scattering on nucleus target, where there is only final state interaction on the struck quark. Clearly, if the initial/final state interactions affect the transverse momentum dependence, we will conclude that they are not universal between these processes. We emphasize, however, that our discussions will not affect the universality for the transverse momentum integrated parton distributions. In particular, for the inclusive observables the initial/final state interaction effects can be summarized into one gauge link associated with the integrated parton distributions, which are universal between different processes[1].

The rest of the paper is organized as follows. In Sec. II, we will construct a model to investigate the universality property for the small-x parton distributions, where all gauge boson exchange contributions can be summed up together, including all initial and final state interactions. In Sec. III, we will summarize our results, and discuss the phenomenological implications, in particular, on the theoretical interpretation of the di-hadron correlation in

dA collisions at RHIC recently observed by the STAR collaboration.

II. INITIAL AND FINAL STATE INTERACTION EFFECTS

We take the partonic channel $qq' \to qq'$ as an example to show the initial/final state interaction effects and the quark distribution in dijet correlation $pA \to \mathrm{Jet1} + \mathrm{Jet2} + X$, and compare to that in deep inelastic scattering process. At small-x, quark distribution is dominated by gluon splitting, and can be calculated from the relevant Feynman diagrams [23–25]. For the purpose of our calculation, we follow an Abelian model of Refs. [7, 14, 15]. It is a scalar QED model with Abelian massive gluons with a mass λ. We construct the model in such a way that the big nucleus is represented by a heavy scalar target with mass M_A. The scalar quarks are generated by the Abelian gluon splitting and are dominant source at small-x. The associated quark distribution in deep inelastic scattering process in this model has been calculated in [5, 7].

Since we are interested in studying the final state interaction effects on the parton distribution from the nucleus, for convenience, we choose the projectile as a single scalar quark with charge g_2, which differs from charge of the scalar quark from the target nucleus g_1. In addition, we assume that the Abelian gluons attached to the target nucleus has an effective coupling g. All the partons in this calculation is set to be scalars with a mass m. The coupling g_2 being different from g_1 is to show the dependence of the parton distribution on the initial/final state interactions associated with the incoming parton. If the dependence on g_2 remains for the parton distributions, they are not universal [14, 15].

We perform our calculations in covariant gauge, and the final results does not depend on the gauge we choose. We carry out the calculations in order of the coupling g. At each order, a gluon attaches the scalar quarks in partonic scattering part from the nucleus target [7, 14, 15]. As shown in Fig. 2, the lowest-order graphs contain one soft gluon exchange with a momentum k and $k^+ \ll P_A^+$ where the plus component of a momentum p is defined as $p^+ = (p^0 + p^z)/\sqrt{2}$ and the nucleus is moving in $+\hat{z}$ direction. We calculate the scattering amplitude in the infinite momentum frame of the nucleus, i.e., $P_A^+ \to \infty$. We can also perform the calculations in the target rest frame and take the limit of $M_A \to \infty$, which will lead to the same result [7]. The small-x approximation ($q^+ \sim k^+ \ll P_A^+$) will be taken throughout the following calculations. We will only keep the leading contributions

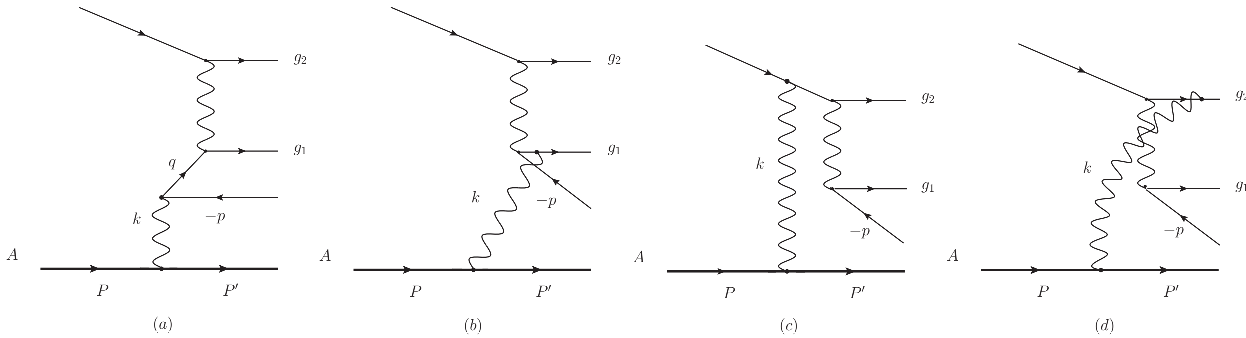

FIG. 2. Lowest-order graphs for di-jet production in hadron-hadron collision at small-x limit. In these graphs, there is one soft gluon exchange with momentum k in addition to the hard gluon exchange.

in this limit. In additional, we also follow the low transverse momentum approximation in terms of $q_\perp/P_{1\perp}$ $(q_\perp/P_{1\perp})$ by applying the power counting method [13]. An important simplification is the eikonal approximation, which replaces the gluon attachment to the initial and final state partons with eikonal propagator and vertex. After taking the leading order contributions, we find that the $q_\perp$ dependence of these diagrams can be included into an effective quark distribution [13], which takes the following form,

$$\tilde{q}\left(x, q_\perp\right) = \frac{x}{32\pi^2} \int \frac{dp^-}{p^-} \frac{d^2k_\perp}{\left(2\pi\right)^2} (4P^+p^-)^2 \left|A^{(tot)}\left(k,p\right)\right|^2 \ , \tag{2}$$

where $p_\perp = k_\perp - q_\perp$. The leading order contributions from Fig. 2 can be written as,

$$A^{(1)}\left(k,p\right) = gg_1 \frac{1}{k_\perp^2 + \lambda^2}\left[\frac{1}{D_1} - \frac{1}{D_2}\right] \ , \tag{3}$$

where we have defined $D\left(p_\perp\right) = 2xP^+p^- + p_\perp^2 + m^2$ and $D_1 = D(q_\perp)$ and $D_2 = D(p_\perp)$. In Eq. (3), the first term and second term in the square bracket correspond to Fig. 2 (a) and Fig. 2 (b), respectively. The contribution from Fig. 2 (c) and Fig. 2 (d) simply cancels. That means at the leading order in the coupling constant, the dependence on g_2 drops out, which however will change at higher orders.

At the next-to-leading order, there are 20 graphs in total in covariant gauge. We plot one of these graphs as an example in Fig. (3) (a), and additional diagrams can be obtained by attaching the gluons to all incoming and outgoing scalar quarks. The total contributions from these diagrams are

$$A^{(2)}\left(k,p\right) = \frac{i}{2}g^2 \int d[1]d[2] \left\{g_1^2\left[\frac{1}{D_1} + \frac{1}{D_2} - \frac{1}{D_{21}} - \frac{1}{D_{22}}\right] + g_1g_2\left[\frac{2}{D_2} - \frac{2}{D_{21}}\right]\right\} \ , \tag{4}$$

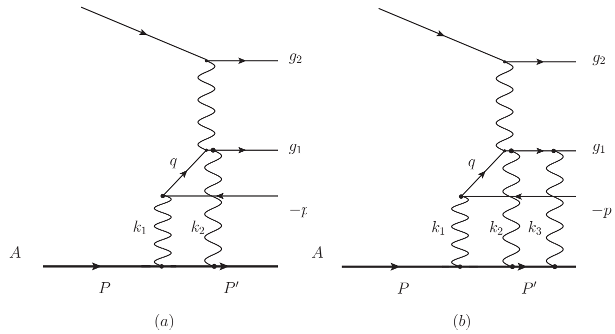

FIG. 3. Example diagrams for two (a) and three (b) gluons exchanges, where the gluons shall attach all charge particles in the upper part of the diagrams from the nucleus target.

where $\int d[1]d[2]$ stands for $\int \frac{d^2k_{1\perp}d^2k_{2\perp}}{(2\pi)^4}\frac{1}{k_{1\perp}^2+\lambda^2}\frac{1}{k_{2\perp}^2+\lambda^2}(2\pi)^2\delta^{(2)}(k_\perp - k_{1\perp} - k_{2\perp})$, and we further defined $D_{1i} = D(q_\perp - k_{i\perp})$ and $D_{2i} = D(p_\perp - k_{i\perp})$. Clearly, this result shows the dependence on g_2. In order to check this residue dependence in the amplitude squared for the quark distribution in Eq. (2), we will have to carry out the calculation of the amplitude up to order g^3.

At g^3 order, there are 120 diagrams in total with three soft gluon-exchange (see e.g., Fig. 3 (b)), including all possible permutation of the attachments of these three gluons on the target nucleus. Summing up all these graphs, we obtain the three gluon exchange amplitude,

$$A^{(3)}(k,p)=\frac{1}{3!}g^3\int d[1]d[2]d[3]\left\{g_1^3\left[\frac{1}{D_2}-\frac{1}{D_1}+\frac{3}{D_{13}}-\frac{3}{D_{21}}\right]\right.$$
$$\left.+g_1^2g_2\left[\frac{3}{D_2}+\frac{3}{D_{13}}-\frac{3}{D_{21}}-\frac{3}{D_{22}}\right]+g_1g_2^2\left[\frac{3}{D_2}-\frac{3}{D_{21}}\right]\right\}\ , \tag{5}$$

where $\int d[1]d[2]d[3]$ follows similar definition as in Eq. (4). Again, we see the dependence on g_2 coupling in the second and third terms. An important cross check of these results is that if we set $g_2 = -g_1$, effectively there will be no charge flow in the final state, and the quark distribution will be identical to that in the Drell-Yan process in the same model. Applying $g_2 = -g_1$, we can easily see that indeed Eqs. (4,5) reproduce those calculated in Ref. [26].

With the amplitude calculated up to g^3, we will be able to check the dependence on g_2 for the parton distribution in Eq. (2). Substituting the results in Eqs. (3-5) into Eq. (2), we find that the g_2 dependence still remains up to order g^4. If we drop all g_2 terms in these results, we will obtain the quark distribution in deep inelastic scattering process in the same model [5, 7]. This clearly shows that the transverse momentum dependent quark distribution $\tilde{q}(x, q_\perp)$ is not universal.

This non-universality is better illustrated when we sum up all order multi-gluon exchange contributions. To do that, we introduce the following Fourier transform [7],

$$A(R, r) = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 p_\perp}{(2\pi)^2} e^{-ik_\perp \cdot R_\perp - ip_\perp \cdot r_\perp} A(k, p) \ . \tag{6}$$

From the Fourier transforms of $A^{(1,2,3)}(k, p)$, we can easily see that they follow the expansion of an exponential form,

$$A^{(tot)}(R, r) = \sum_{n=1}^{\infty} A^{(n)}(R, r) = iV(r_\perp) \left\{1 - e^{igg_1[G(R_\perp + r_\perp) - G(R_\perp)]}\right\} e^{-igg_2 G(R_\perp)} \ , \tag{7}$$

where $G(R_\perp) = K_0(\lambda R_\perp)$ and $V(r_\perp) = K_0(Mr_\perp)/2\pi$ with $M^2 = 2xP^+p^- + m^2$. In the above result, the g_2-dependence seems only appear in a global phase and may not lead to a physics consequence. However, because the transverse momentum $q_\perp$ is conjugate to the coordinate variable difference $R_\perp$-$r_\perp$, This phase will lead to a non-universality contribution to the quark distribution defined in Eq. (2), for which the all order result reads as,

$$\begin{aligned}\tilde{q}(x, q_\perp) = & \frac{xP^{+2}}{2\pi^2} \int dp^- p^- \int d^2 R_\perp d^2 R'_\perp d^2 r_\perp e^{iq_\perp \cdot \left(R_\perp - R'_\perp\right)} e^{-igg_2\left(G(R_\perp) - G(R'_\perp)\right)} V(r_\perp) V(r'_\perp) \\ & \times \left\{1 - e^{igg_1\left[G\left(R_\perp + r_\perp\right) - G(R_\perp)\right]}\right\} \left\{1 - e^{-igg_1\left[G\left(R'_\perp + r'_\perp\right) - G\left(R'_\perp\right)\right]}\right\} \ ,\end{aligned} \tag{8}$$

where $r'_\perp = R_\perp + r_\perp - R'_\perp$. This transverse momentum dependent quark distribution is clearly different from that calculated in deep inelastic scattering in the same model [5, 7]. In other words, this distribution is not universal and the standard k_T-factorization breaks down. It is interesting to notice that the g_2 dependence disappears after the integration over the transverse momentum. This is consistent with the universality for the integrated parton distributions [5, 14, 15].

III. SUMMARY AND DISCUSSIONS

In this paper, we have demonstrated the non-universality for the small-x parton distributions in dijet correlation in pA collision, by an explicit calculation of the initial/final state interaction effects, and compare to that in deep inelastic scattering on the nucleus target. After summing up to all orders, we find that the net effects are summarized into a phase which leads to a non-vanishing contribution to the quark distribution and breaks the universality.

It has been argued that the light-cone gauge may simplify the factorization property for the hard scattering processes. For example, if we choose the advanced boundary condition for the gauge potential in light-cone gauge, the wave function of hadrons contain the final state interaction effects [5, 27]. However, as we showed in the above calculations, this does not help to solve the g_2-dependence in the quark distribution in the dijet correlation in hadronic process. In other words, the quark distribution from the nucleus target has to contain the interaction with the incoming (outgoing) quark with charge g_2, which can not be solely included into the wave function of the nucleus target.

The non-universality effect found in this paper is different from the factorization breaking effect discussed in Ref. [17], where the breaking effect decreases as the heavy quark mass (equivalent to our jet transverse momentum $P_{1\perp}$) increases. However, in this paper, we are discussing the non-universality effects is in the leading power, and does not vanish with large transverse momentum of the jet.

Our results show that there is no universality for the transverse momentum dependent parton distributions at small-x. This will impose a challenge to explain the dijet-correlation data in dA collisions at RHIC with the saturation phenomena observed in DIS experiments at small-x. The non-universality, on the other hand, provides an opportunity to study the QCD dynamics associated with the final/initial state interaction effects. For example, the STAR data indicate that the intrinsic transverse momentum of partons from the nucleus target are in order of 2-2.5GeV to explain the di-hadron azimuthal correlation disappearance at transverse momentum $P_{h\perp} \sim 2$GeV and forward rapidity region $\eta \sim 3.2$ [18], which corresponds to a saturation scale about 4-6GeV^2 [28]. This is much larger than a typical estimate of the saturation scale at the same value of $x \sim 10^{-3}$ from the analysis of the deep inelastic scattering data [29] which is about 2GeV^2. From our study, this is understandable. The saturation scales for the parton distributions in these two processes are not necessary the same because there is no universality between them. Besides, the additional initial/final state interaction effects (the g_2-dependence) may explain the saturation scale for dijet-correlation in pA collision is larger than that in deep inelastic scattering [19]. Since the quark distribution in Drell-Yan lepton pair production is the same as that in deep inelastic scattering process, the non-universality property shall manifest by comparing the dijet correlation and Drell-Yan measurements at the same kinematic region in pA collisions. We hope these can be carried out at RHIC in the near future.

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[28] This is a rough estimate. The disappearance of the azimuthal correlation between two hadrons happens when the saturation scale is the same order as the jet transverse momentum $P_\perp^{\rm jet}$, which is about 2.5GeV if we take the average fragmentation $\langle z\rangle \sim 0.8$ with $P_{h\perp}$ for hadron is around 2GeV. Taking into account the uncertainties from the fragmentation smearing effects, we estimate the saturation scale for the parton distribution from the nucleus target around 4-6GeV^2. Similar saturation scale has been used in [21].

[29] see, for example, H. Kowalski, T. Lappi and R. Venugopalan, Phys. Rev. Lett. **100**, 022303 (2008).