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**Applied Technologies
Characterization and Analysis**

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ABSTRACT

An off-axis ultrasonic inspection technique using air-coupled ultrasonics has been developed to determine grain size in cast materials. The technique gives a uniform response across the volume of the component. This technique has been demonstrated to provide generalized trends of grain variation over the samples investigated.

Keywords: Ultrasonics, air-coupled, grain size, attenuation, non-destructive evaluation, chirp waveform

INTRODUCTION

Cast material has a grain structure that is relatively non-uniform. There is a desire to evaluate the grain structure of this material non-destructively. Traditionally, grain size measurement is a destructive process involving the sectioning and metallographic imaging of the material. Generally, this is performed on a representative sample on a periodic basis. Sampling is inefficient and costly. Furthermore, the resulting data may not provide an accurate description of the entire part's average grain size or grain size variation.

This project is designed to develop a non-destructive acoustic scanning technique, using Chirp waveforms, to quantify average grain size and grain size variation across the surface of a cast material. A Chirp is a signal in which the frequency increases or decreases over time (frequency modulation)ⁱ. As a Chirp passes through a material, the material's grains reduce the signal (attenuation) by absorbing the signal energy. Geophysics research has shown a direct correlation with Chirp wave attenuation and mean grain size in geological structures^{ii,iii}. The goal of this project is to demonstrate that Chirp waveform attenuation can be used to measure grain size and grain variation in cast metals (uranium and other materials of interest).

Traditionally, ultrasonics required the immersion of the test sample in a liquid or the application of gel to the test sample's surface in order to transmit the sound into and out of the sample. This fact greatly limited the scope of traditional ultrasonics to materials that were not susceptible to contamination issues. Air-coupled ultrasonics shows great promise in expanding the scope of inspection techniques for various materials. As the name suggests, air-coupled ultrasound uses air as the signal transmission medium.

There are several advantages for using an air-coupled ultrasonic system. Since no liquids or gels are needed for signal transmission, the inspection is non-contact and all contamination issues are eliminated. Since the inspection is non-destructive, the component can be returned to the customer intact after the inspection has been completed. Additionally, air-coupled inspections provide better component coverage and seamless scans compared to traditional ultrasonic inspections. Furthermore, the measurement of interest can be "mapped" over the surface of the component.

This report will discuss the theory of air-coupled ultrasonics, the inspection method developed to determine grain size and variation, and results obtained using the air-coupled ultrasonic inspection method.

ACOUSTIC ATTENUATION FROM GRAIN SCATTERING

Simply defined, attenuation is the decrease in signal strength as the sound propagates through the material. In terms of polycrystalline materials, grain boundaries and precipitates scatter the incident sound wave as the wave progresses through the medium^{iv}. Figure 1 illustrates this phenomenon.

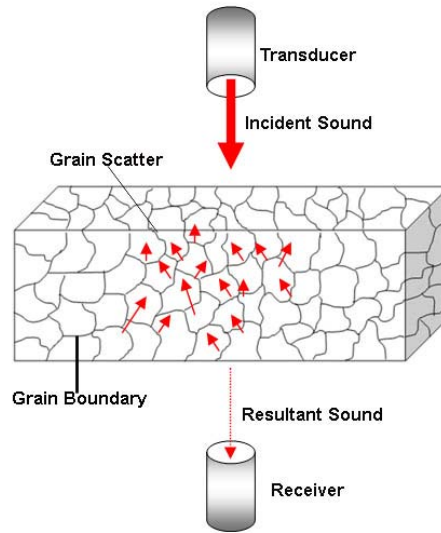


Figure 1 – Acoustic attenuation caused by sound scattered by the grain boundaries

Formulations on Grain Boundary Acoustic Scattering

As an acoustic wave propagates through a material, the attenuation of the amplitude of the acoustic wave can be mathematically represented by:

$$A = A_0 e^{-\alpha x} \quad (1)$$

where A_0 is the initial amplitude of the acoustic wave, x is the distance the wave has propagated through the material, and α is the material's attenuation coefficient^v.

Mason and McSkimin formulated (based on Rayleigh's findings) that each grain in a material contributes to the acoustic scatter within the material^{vi}. This formulation predicts that grain scattering attenuation depends upon the ratio of the grain size and the acoustic wavelength.

Grayeli *et al.* stated that there are three acoustic scatter regimes. At low frequencies, with the acoustic wavelength λ much larger than the grain diameter D , the wavelength to grain size ratio is $\frac{\lambda}{2\pi D} > 1$ and the acoustic wave will be attenuated by Rayleigh scattering represented by:

$$\alpha = C_4 D^3 f^4 \quad (2)$$

where D is the grain size diameter, f is the acoustic frequency, and C_4 is a material constant that depends on the material's single crystal elastic constants, mass density, and acoustic velocity. The value for C_4 is different for longitudinal and shear waves.

When the acoustic wavelength is on the order of the grain size, the wavelength to grain size ratio is $\frac{\lambda}{2\pi D} \approx 1$ and the acoustic signal is scattered stochastically. For stochastic acoustic scattering, the material's attenuation coefficient can be represented by:

$$\alpha = C_2 D f^2 \quad (3)$$

where C_2 is material constant dependent on the crystal's plastic constants.

Since there is an inverse relationship between wavelength and frequency, at very high frequencies, the wavelength is very small. Therefore, the wavelength to grain size ratio becomes $\frac{\lambda}{2\pi D} \ll 1$. In this case, specular reflection and refraction occur at the grain boundaries and can be represented by:

$$\alpha = \frac{C_0}{D} \quad (4)$$

Botvina *et al.* named this the diffusion scattering regime^{vii}.

AIR COUPLED ULTRASONICS

Traditional ultrasonic inspections require water as the medium in which sound is transferred. This fact greatly limited the scope of materials in which ultrasonic inspections could be used. Air-coupled ultrasonics provides a new and potentially powerful method of inspecting materials. As the name suggests, air-coupled ultrasonics uses air as the medium in which sound is transmitted. The following section will give a brief overview of the theory behind air-coupled ultrasonics.

Air Coupled Ultrasonics Defined

As mentioned previously, air coupled ultrasonics uses air as the sound transmission medium. Air coupled ultrasonics inspections are non-contact and provide greater coverage of the part. The method in which sound propagates through materials is through reflection and transmission at material boundaries. Sound reflection and transmission is governed by the acoustic impedances of the materials. Acoustic impedance is a material property which quantifies the material's ability to restrict sound transmission. Figure 2 illustrates how the initial incident sound wave is reflected and transmitted into the next medium.

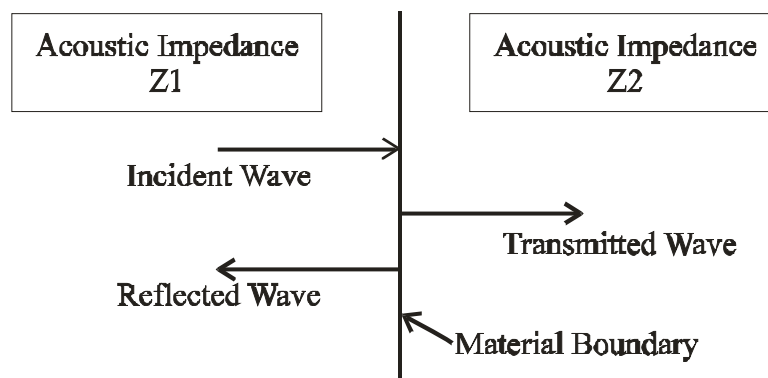


Figure 2-Sound propagation at material boundaries

The acoustic impedance can be represented mathematically as:

$$Z = \rho V \quad (5)$$

where ρ and V are the part density and velocity respectively. The amount of sound reflected and transmitted is governed by the reflection and transmission coefficients. Referring back to Figure 2, the reflection coefficient can be written mathematically as:

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (6)$$

The transmission coefficient can be written as:

$$T = \frac{2Z_2}{Z_2 + Z_1} \quad (7)$$

Therefore, if A is the amplitude of the incident wave, then it follows that the reflected wave, A_r , can be written as:

$$A_r = AR = A \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (8)$$

Similarly, the transmitted wave, A_t , can be expressed as:

$$A_t = AT = A \frac{2Z_2}{Z_2 + Z_1} \quad (9)$$

Assuming no losses of sound to other phenomenon, the following expression must hold:

$$A + A_r = A_t \quad (10)$$

Acoustic Impedance Mismatch

Referring back to Figure 2, if the acoustic impedances of the transmission mediums are drastically different, an impedance mismatch occurs. In this investigation, the initial transmission medium is air. The acoustic impedance of air (Z_{air}) is .000427 g/(cm²μs). The following example illustrates the significance of impedance mismatch in air-coupled ultrasonics.

Uranium Example

The following is an exercise to determine the reflection coefficient at the air/uranium interface. Empirically, the density of the uranium is 18.7 g/cc. The velocity of the material is .337 cm/μs. Therefore, the acoustic impedance is

$$\begin{aligned} z_{st} &= \rho V = \left(18.7 \frac{g}{cm^3} \right) \left(.337 \frac{cm}{\mu s} \right) \\ &= 6.3019 \frac{g}{cm^2 \mu s} \end{aligned}$$

At this stage, it is evident that the acoustic impedance of uranium is much greater than air – on the order of 15,000 times greater.

The reflection coefficient is

$$\begin{aligned} R &= \frac{z_{air} - z_{st}}{z_{air} + z_{st}} = \frac{.000427 \frac{g}{cm^2 \mu s} - 6.3019 \frac{g}{cm^2 \mu s}}{.000427 \frac{g}{cm^2 \mu s} + 6.3019 \frac{g}{cm^2 \mu s}} \\ &= -.99986 \end{aligned}$$

Therefore, nearly 99.986% of the incident wave is reflected. This poses a significant issue when using air coupled ultrasonics.

Acoustic mismatch is the main cause of signal loss in through transmission inspections. The following table shows the discrepancy in transmission energy and sound loss of air compared to water as a transmission medium through a sample of Uranium. Water's higher acoustic impedance of 0.1480 g/(cm²μs), yields higher transmission coefficient energy and lower sound loss compared to air. This phenomenon is due to lower impedance mismatch between water and Uranium.

Table 1. Comparison of transmission coefficient energy and signal loss of water and air in transmission media		
	Transmission Coefficient-Energy	Loss - db
Water - U	0.7225	-2.82
Water - U - Water	0.5220	-5.65
Air - U	3.589E-04	-68.90
Air - U - Air	1.288E-07	-137.80

Chirp Waveforms

It is evident from Table 1 that a traditional ultrasonic signal transmitted through a material using air as a medium will incur tremendous sound loss. In order to alleviate this problem, chirp waveforms will be used. The chirp waveform is a tone burst with a varying frequency that can be up to 600 μs long (Figure 3). The frequency range of the chirp defines the bandwidth of the system. This enables the transmitter to be excited by considerable power.

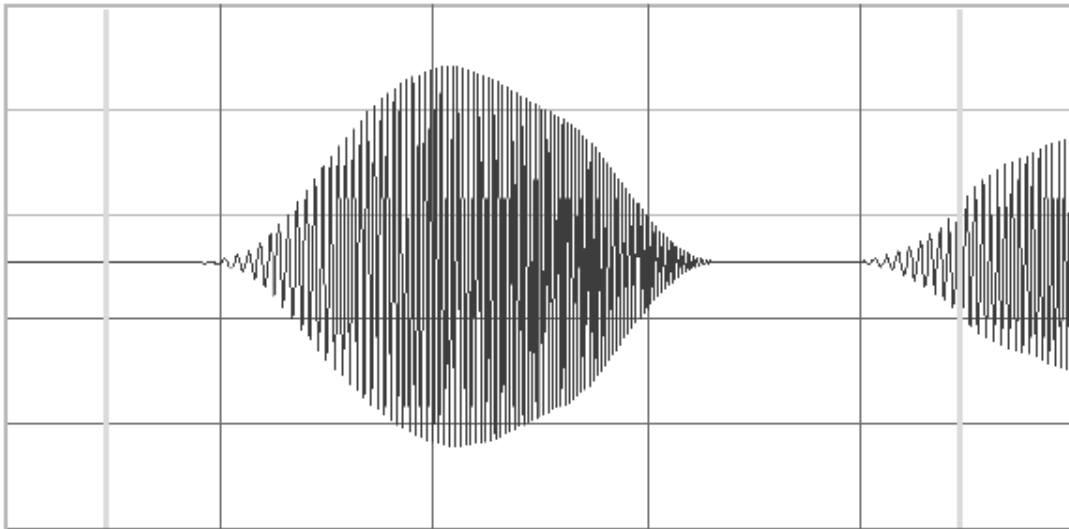


Figure 3-Example of a chirp waveform

When signal averaging is incorporated with the chirp waveforms, small signals can be extracted and resolved. The detected signal is then cross correlated with the excitation signal. The cross correlation enables microsecond resolution even though the excitation pulse may be many microseconds long. This combination can improve the signal to noise ratio up to 80dB. A schematic of the system is shown in Figure 4.

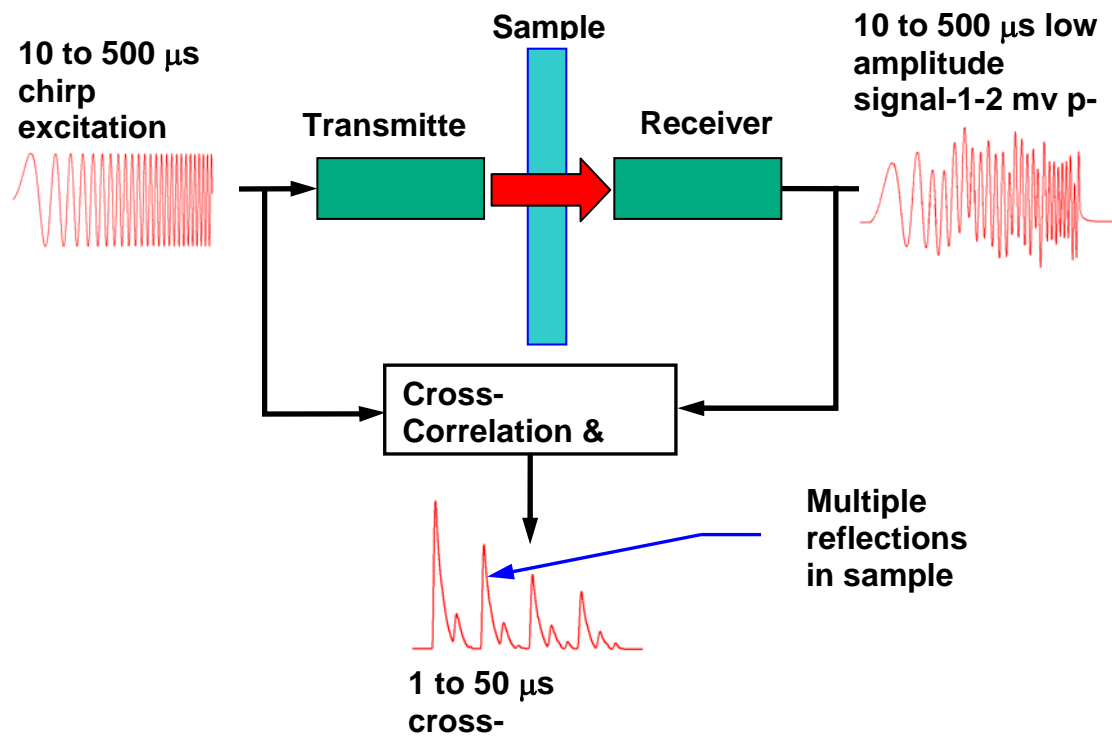


Figure 4 - Schematic of cross - correlation chirp system

NCA 1000 Air Coupled Ultrasonic System

The NCA 1000 air coupled ultrasonic inspection system (from VN Instruments) is a self-contained ultrasonics analysis system which uses chirp waveform technology and digital signal processing to investigate a multitude of material properties. As mentioned previously, the Chirp waveform allows for proper selection of a transmission frequency by using a tone burst signal.

The NCA 1000's digital signal processing feature allows for calculations to be made in real time. The power of the NCA 1000 is the system's capability of being operated in reflection and transmission modes simultaneously while using up to four transducers in the process^{viii}. Figure 5 illustrates transmission modes used by the NCA 1000.

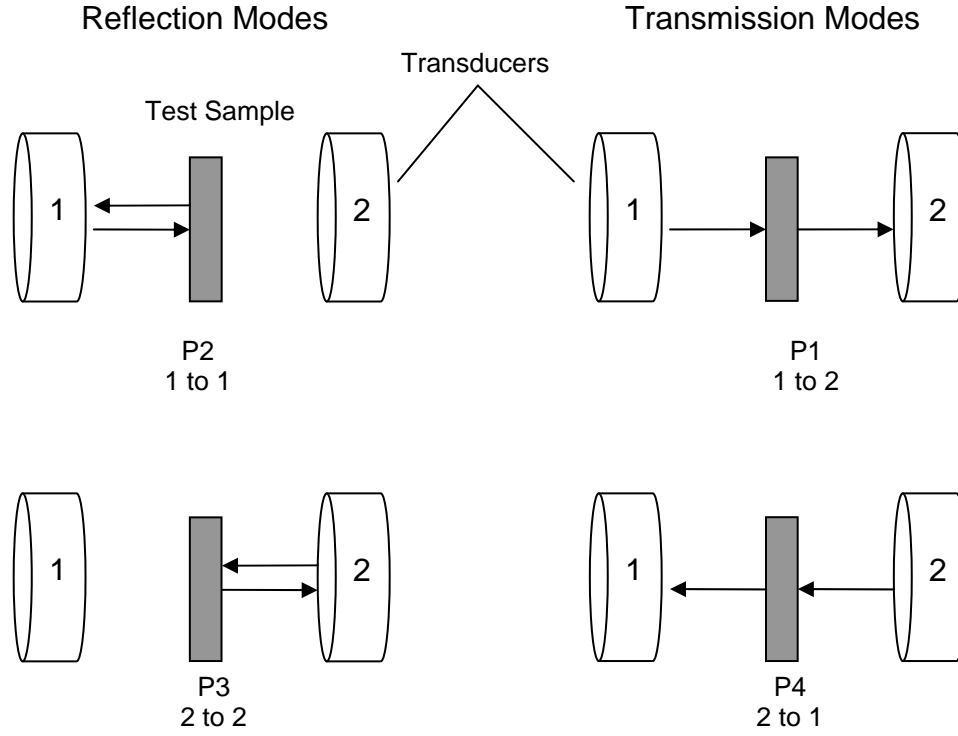


Figure 5 – NCA 1000 Transmission Modes

NCA 1000 Power Output

To calculate the power output by the system, the excitation signals were measured (Figure 6). The peak-to-peak voltage was measured to be 168 V. The RMS (root mean square) voltage can be calculated from:

$$V_{RMS} = \frac{V_{P-P}}{2\sqrt{2}} \quad (11)$$

where V_{P-P} is the peak-to-peak voltage. V_{RMS} was calculated as 59.4 V. Using the similar methodology, the peak-to-peak current and RMS current were 13.53mA and 4.78 mA respectively.

The peak power of the system is 0.57 Watts (W). The average power is calculated by:

$$\begin{aligned} P_{RMS} &= V_{RMS} \times I_{RMS} \\ &= 59.4V \times 4.78mA = 0.28W \end{aligned} \quad (12)$$

Using a pulse width of 300 μ s and a period of 14ms, the average power is:

$$P_{avg} = P_{RMS} \times \frac{0.3}{14.0} = 6.1mW \quad (13)$$

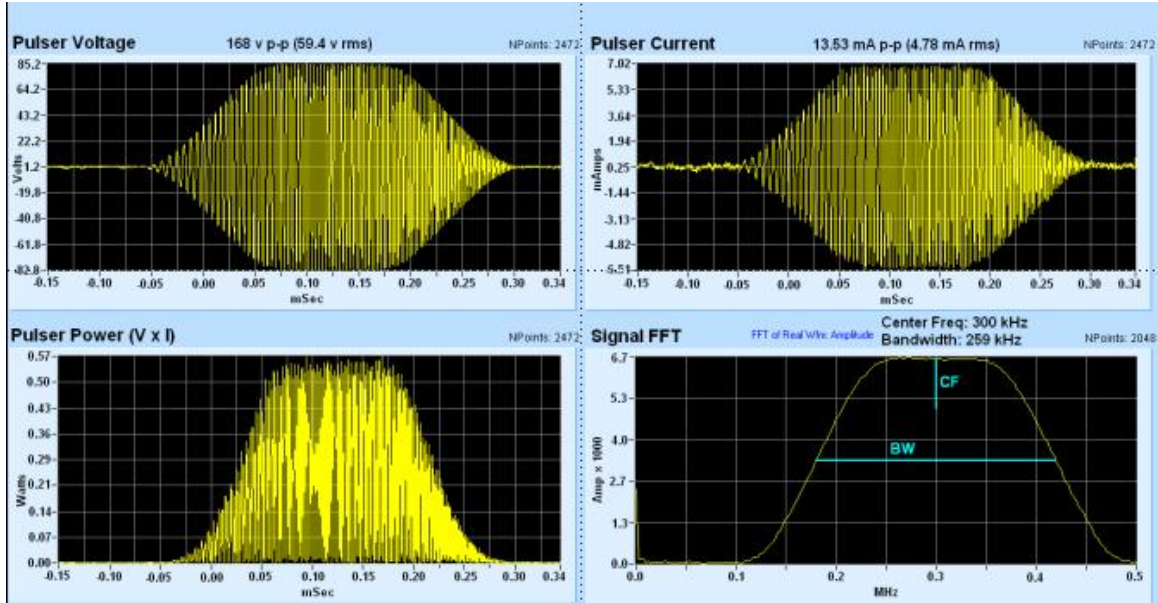


Figure 6 – Excitation signals measured on the NCA 1000 system

Capacitive Transducers

Due to high acoustic impedance mismatches and the highly attenuative transmission medium (air), transducers with broad bandwidth and good sensitivity were required. Capacitive transducers are an excellent choice for this particular application. Capacitive transducers offer superior bandwidth, good stability over time, and good sensitivity. Since these transducers have such a broad bandwidth, the NCA 1000 can “tune” the transducer for the application needed. The transducer used in this investigation is the “CAP3” (300 kHz capacitive) transducer. The transducer is shown in Figure 7 along with typical parameters listed in Table 2.

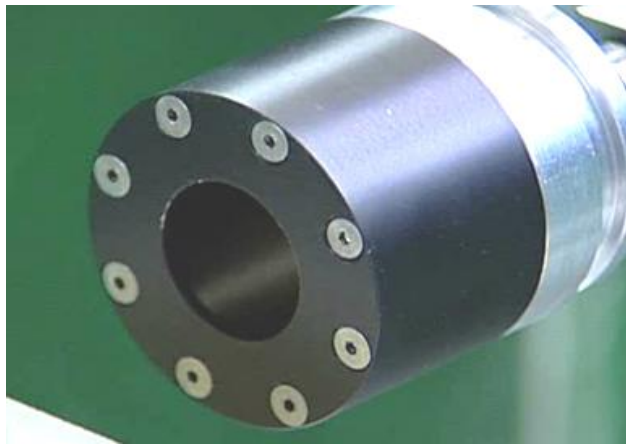


Figure 7 - CAP3 transducer

Table 2. Specifications for the remote module

Peak Frequency	272.6 KHz
-3dB Width	219.0 KHz
-6dB Width	344.5 KHz
Peak Voltage Gain	-24.06 dB
Mean Voltage Gain	-26.50 dB

SCANNING TECHNIQUE DETERMINATION

To non-destructively determine the grain size of a component, three inspection techniques were considered. The three proposed techniques were: pulse-echo inspection, through-transmission inspection, and off-axis scanning. An illustration of the proposed techniques is shown in Figure 8.

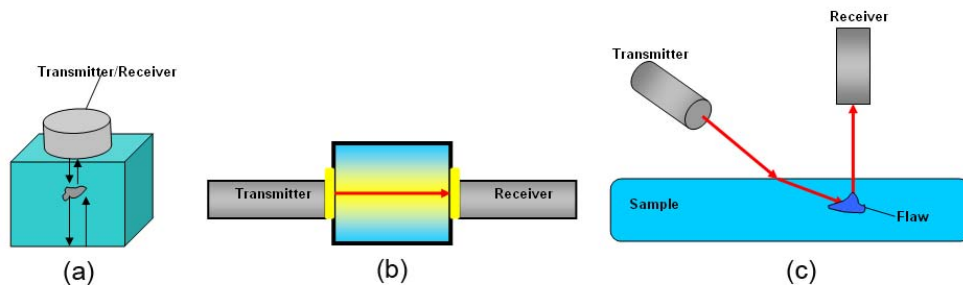


Figure 8 – Proposed scanning techniques to determine grain size: (a) pulse-echo, (b) through transmission, (c) off-axis scanning

Pulse-Echo Technique

As the name suggests, the pulse echo technique utilizes a single transducer to send and receive the acoustic wave^{ix}. Intuitively, from the uranium example presented earlier in this investigation, 99.986% of the transmitted wave is reflected back to the transducer. This fact would suggest that this technique would be a prime candidate for grain size measurement. The drawback to

this technique is that the acoustic sound wave will not travel across the material surface. The wave will reflect normal to the surface and thus will not interact with the grain boundaries. Additionally, as the transducer scans the material's surface, the reflection normal from the grain boundaries will be overwhelmed from the reflection of the bulk material itself.

Through – Transmission Technique

In through-transmission inspection, an acoustic transmitter sends an acoustic wave through the medium to an acoustic receiver usually placed on the other side of the medium. This inspection technique is advantageous when inspecting materials for internal flaws. Perturbations in the through-transmission signal are indicators of defects within the transmission medium.

The drawback to using through-transmission in this investigation is two-fold. Problem 1: As mentioned previously, 99.986% of the incident acoustic wave is reflected at the air/uranium interface. This implies that only .014% of the incident wave is transmitted into the medium. Problem 2: As the transmitted acoustic wave propagates through the medium, the grain boundaries will scatter the transmitted wave. By the time the transmitted signal reaches the back surface of the material, the signal will be so weak that it will not be able to be resolved from ambient noise.

Off-Axis Scanning

In off-axis scanning, the transmitter sends an acoustic wave to the medium (usually oriented at an angle past the acoustic critical angle) while the receiving transducer monitors the spectra reflected from the medium. By transmitting the acoustic wave past the material's acoustic critical angle, the signal experiences mode conversion and becomes a surface wave and propagates along the surface of the material. This surface wave will interact with the grain boundaries and the reflected waves will be detected by the receiver. The spectra detected by the receiver can then be correlated to determine an average grain size. This technique was determined to be the most viable for the investigation.

COMPUTER SIMULATION

In order to evaluate parameters and to investigate acoustic effects on the microstructure of the cast material, an acoustic simulation was created. The simulation package selected for this investigation was Wave 2000. Wave 2000 is a stand alone software package exclusively designed for computational ultrasonics. The simulation package works by solving the two-dimensional acoustic elastic wave equation based on finite differences. The power of Wave 2000 lies in the software's ability to compute the full acoustic wave solution in an arbitrary two-dimensional object while allowing the user to select custom transmission and reception sources.

Grain Structure Implementation

In order for the simulation to effectively model the acoustic response of the grain structure, information on the grain structure must be implemented into the simulation routine. In order to accomplish this task, the cast component's microstructural image (obtained from metallography), was digitized and inserted into Wave 2000 as an object (Figure 9).

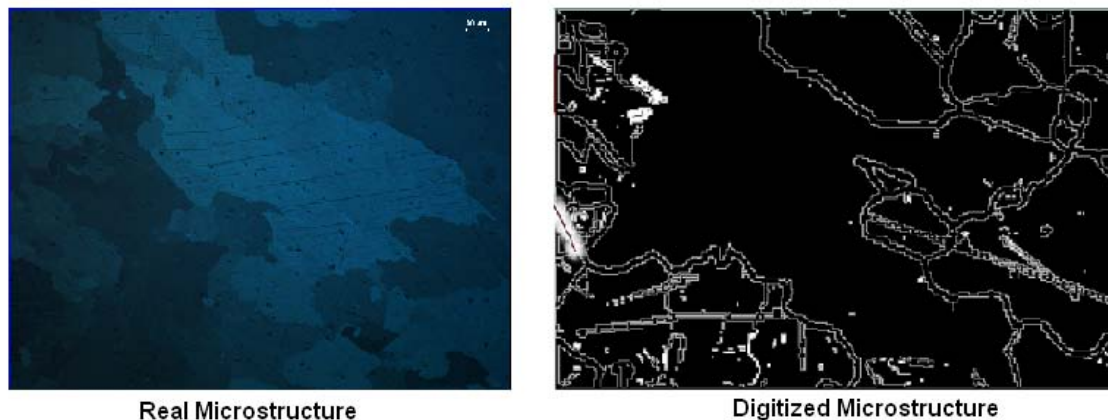


Figure 9 – Uranium microstructure inserted into Wave 2000

Once the image is inserted into Wave 2000, material properties, boundary conditions, placement of the transmitter and receivers can be specified. Figure 10 shows snapshots of the acoustic wave as it propagates along the surface of the microstructure. It is clearly evident that the grain boundaries scatter significant amounts of the acoustic wave.

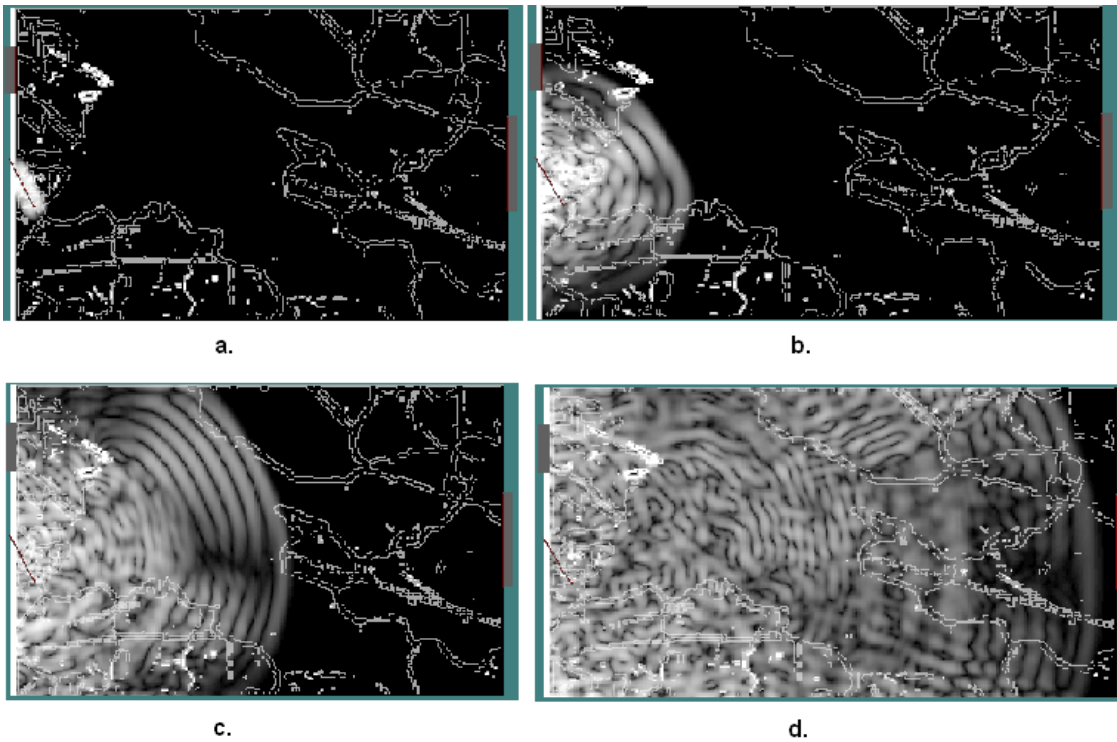


Figure 10 – Acoustic wave propagation along the component's microstructure

GRAIN SIZE DETERMINATION

Traditional Determination Using the Mean Lineal Intercept Method

The techniques used to measure the grain size varied tremendously and were difficult to reproduce. The most widely used and established method is the mean lineal intercept method (also known as Heyn's, technique). Meier defines the mean lineal intercept length as the average length of a line segment that crosses a sufficiently large number of grains^x. For spherical grains, the grain's diameter should be proportional to the equivalent diameter of a spherical grain. The mean lineal intercept length is determined by laying a number of randomly placed test lines on the image and counting the number of times that grain boundaries are intercepted^{xi}.

Mathematically, the mean lineal path is defined as:

$$L_L = \frac{1}{N_L} = \frac{L_T}{PM} \quad (14)$$

where N_L is the number of intercepts per total length of the test lines L_T , P is the total number of grain boundary intersections and M is the magnification. The following example uses this method to calculate the grain size in a depleted uranium microstructure.

Uranium Example

Figure 11 shows the microstructure of depleted uranium in which five test lines have been used to subdivide the image.



Figure 11 – Lineal Path Determination of Grain Size

The image's magnification, M , is 100X. The length of each test line is 4.8 inches. Therefore, since there are five test lines, the total test line length, L_T , is 24 inches or 60,9600 μm . Each test line's intersection with a grain boundary is marked with a yellow line. The total number of intersections, P , is 30. Therefore, the mean lineal path (average grain size) is:

$$\bar{L}_L = \frac{609600 \mu m}{(30)(100)} = 203.2 \mu m$$

It is clear that this value will change based upon the number of vertical test lines used. Additionally, if the grain boundaries are not equiaxed, i.e. symmetric in orthogonal directions, then there will be a significant error in the grain size measurement. In that case, the lineal path must be done using horizontal test lines as well.

The uranium coupons used in this investigation had average grain sizes ranging from 50 μm to 300 μm . The coupons were subdivided into four groups. Table 3 lists the groups and the grain size range within each group.

Table 3. Grain size range used in the investigation

Coupon Group	Grain Size Range (μm)
A	50 - 110
B	110 - 170
C	170 - 230
D	230 - 300

Acoustic Attenuation in Cast Uranium

It has been theorized that acoustic attenuation measurements can be used to effectively predict the grain size of polycrystalline materials. Therefore, the acoustic attenuation in cast uranium must be done in order to effectively predict grain size. Attenuation measurements were made on the uranium specimens with different grain sizes using the off-axis scanning method. Grayeli *et al.* postulated that the attenuation of longitudinal waves in each specimen can be plotted as a function of ultrasonic frequency and fitted to an equation of the form:

$$\alpha = Af^n(x) \quad (15)$$

such that n is the slope of a straight line on a log-log plot. The attenuation factor A and the exponent n varied with the selected sample. It was observed that the magnitude of A increased with increasing grain size and the change in the exponent n may have been caused by the nonhomogeneity of grain size in the samples. The exponent n ranged from 1.43 to 2.3. Figure 12 shows attenuation vs. frequency for the uranium samples. As expected, attenuation increases with increasing grain size.

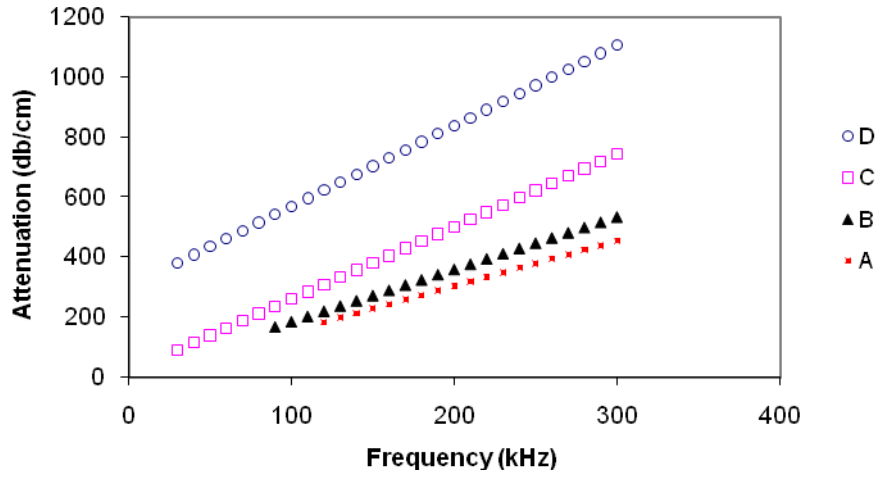


Figure 12 – Attenuation of longitudinal acoustic waves in uranium with different grain sizes

Rayleigh Scattering Attenuation

Since the wavelengths used in this investigation are much longer than the grain size, Rayleigh's theory of scattering attenuation of longitudinal waves in polycrystalline material should hold for cast uranium. In this theory, the acoustic scattering factor can be calculated by:

$$A_4 = \frac{0.55\pi^4 \bar{D}^3}{v_L^4} \left(\frac{2(c_{12} - c_{11}) + c_{44}}{5c_{11} + 2(c_{12} - c_{11}) + 4c_{44}} \right)^2 \quad (16)$$

Where D is the average grain size, V_L is the material's longitudinal velocity, and c_{12} , c_{11} , and c_{44} are the material's elastic constants. Figure 13 shows the values calculated from the actual measurement versus the predicted values from the Wave 2000 simulation.

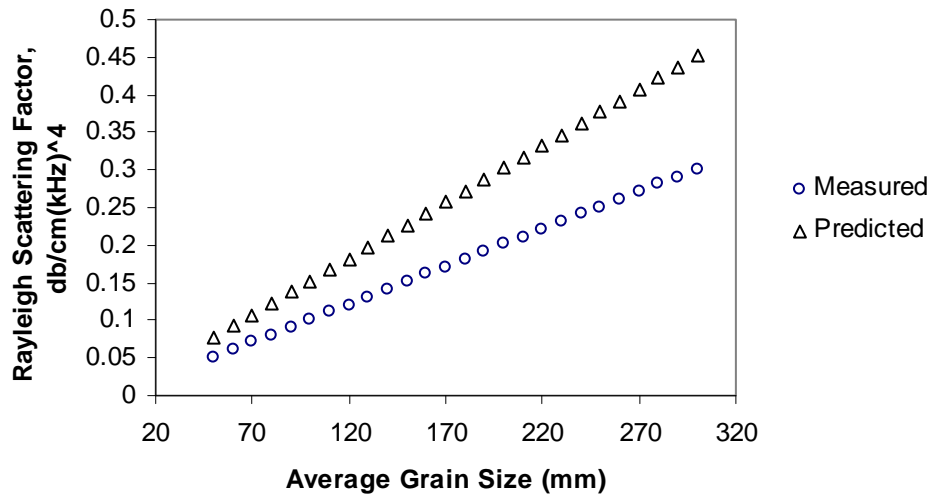


Figure 13 – Measured vs. Predicted Rayleigh Scattering Values for Uranium coupons

Grain Size Prediction

The spectral analysis of broadband acoustic signals attenuated in uranium shows that Rayleigh scattering is dominant in the acoustic response of uranium's grain structure. In general when predicting the grain structure, the following equation holds:

$$\overline{D} = w_r \overline{D}_r \quad (17)$$

Where D is the actual average grain size, D_r is the grain size predicted theoretically using Rayleigh scattering values and w_r is a weighting factor. Table 4 compares the average grain size measured using the lineal path method vs. the grain size predicted from the air coupled ultrasonic system and the Wave 2000 simulation for selected uranium samples.

Table 4. Prediction of Grain Size vs. Lineal Path Measurement

Coupon Group	Lineal Path (μm)	Air-Coupled (μm)	Wave 2000 (μm)
A	54.5	58.8	62.5
B	122.7	124.3	130.5
C	195.4	197.3	205.2
D	221.3	235.4	254.6

Even though the predicted values over-estimate the lineal path results, it is clearly evident that the selection of the off-axis scanning method provides data that shows the same general trend as the accepted measurement method.

CONCLUSION

The experiments have shown that air-coupled ultrasonics combined with the off-axis scattering technique can be utilized to determine grain-size variation in cast materials. Although Rayleigh's scattering theory slightly over estimates the grain sizes in the materials investigated, the theory provides a concrete basis for the analysis of grain scattering attenuation measurements. Furthermore, this technique successfully correlates acoustic attenuation with grain size. This technique provides a more efficient and safer inspection technique to inspect cast materials.

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