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PERFORMANCE OF DAMAGED SOIL-CONCRETE WRAPAROUND DAM SECTIONS UNDER DYNAMIC LOADING

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ABSTRACT

Predicting seismic or shock loading damage of the soil-concrete interface where an embankment wraparound dam provides support to the end monoliths in a concrete gravity dam is an inherently challenging three-dimensional coupled problem. We wish to predict formation and growth of a crack between the soil and concrete with a sustained flow of water. Further, we seek to better understand all critical phenomenology of this type of problem such as the potential mitigating and stabilizing role of upstream and downstream filter zone and shell materials. This collaborative research effort will ultimately determine whether advances in computational platforms, constitutive soil models (advances in representing particulates, tension, flow, and hydraulic erosion), and physical testing (advances in centrifuge and flume testing) can be applied successfully to solve this complex problem. Our focus is (1) to develop and validate high fidelity numerical models to investigate crack formation, soil erosion, transport of materials, and stability as part of the erosion process, and deposition within interface cracks; and (2) to investigate the performance of the filter zone materials if an extreme loading event such as an earthquake or shock damages the wraparound section. Our numerical tools include both continuum and discrete approaches. The continuum approach is based on the drift-flux multiphase model where a fluid and a solid are represented as interpenetrating continua and can account for turbulent flow characteristics, particle lift forces due to shear flow, particle collisions, and gravity settling. The discrete particle approach is also applied and is useful when deriving constitutive laws and parameterizations of soil behavior. Different experimental validation studies are under consideration for model validation and calibration. Several case studies for different crack sizes and orientations, particle sizes and environmental hydraulic conditions may be required to confirm the conditions necessary for self-healing or catastrophic growth of a crack. We will present both numerical and experimental findings to date on this effort in light of necessary considerations for further study.

PROBLEM DESCRIPTION

Dynamic displacement from a multihazard event, e.g. earthquakes or terrorist-placed explosives, may result in a crack between concrete sections of gravity dams and the embankment wraparound sections (Figure 1), damaging the filter or core materials and potentially initiating systematic failure of the zoned embankment. For example, substantial transverse abutment cracking were observed in Austrian Dam during Loma Prieta earthquake in 1989. Shin-Kang Dam was partially destroyed after Chi-Chi earthquake that caused the irregular ground movements and separation of most of the concrete blocks from the foundation rock, which consists of layers of mudstone, siltstone and sandstone (Bureau, 2003).

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Today's advances in physical testing, numerical analysis and computer hardware may allow a fundamental understanding of the mechanisms, physics, and processes and quantify and reduce risks associated with the problem.

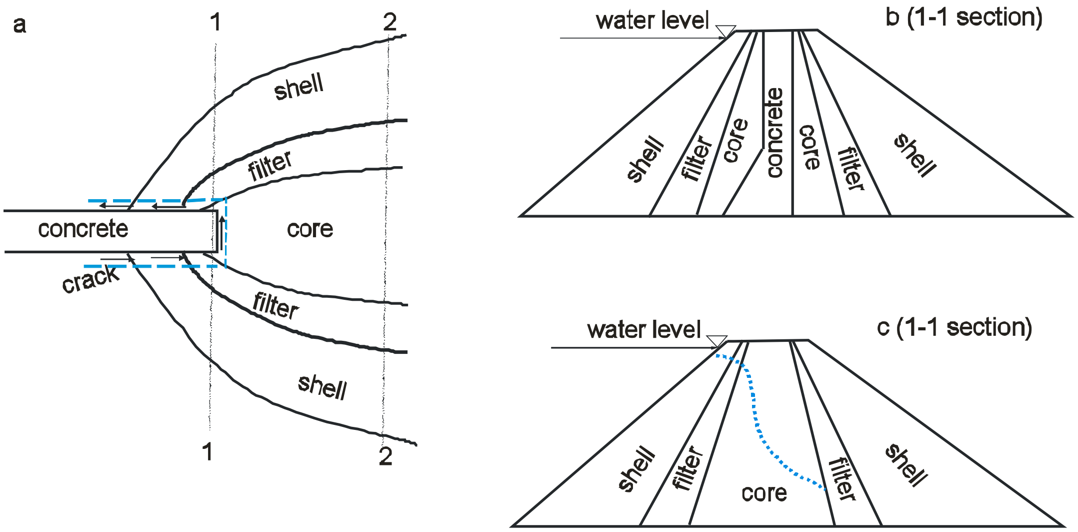


Figure 1. Schematic description of typical layers for concrete embankment dam. (a) - top slice view. (b), (c) - side slice views. Embankment layers, water level and phreatic line (that is, the line corresponding to the phreatic surface lying above the saturated zone when seen in a vertical plane) are shown

The complete computational space of this problem spans from accurate damage prediction from an extreme multi-hazard event that creates a crack or imperfection and the soil erosion processes within the imperfection, to stability and collapse of eroded slopes. Relevant phenomena are inherently multiscale both in space and time: seismic events have a second to minute time scale, shock dynamic loading has a millisecond time scale, while catastrophic failure can happen from within a few hours to weeks after a triggering event. There are also intermediate time scales due to sloped soil collapse (seconds), water seepage or flow (minutes), and soil/gravel erosion and deposition (hours to days). In terms of length-scale, the problem can be studied at the full dam scale, at the multiphase crack flow scale, or at the particle-resolved scale by either resolving only the bigger gravel particles, or by including the soil particles. Some of these approaches are extremely computationally intensive requiring a large investment in computer resources and computational validation. For this initial conceptualization discussed here, we focus on the existence of a specific imperfection between a concrete and embankment section and whether or not such an imperfection or crack can remain open for sustained water flow or initiate an internal erosion process. Internal erosion of embankment dams under normal conditions is well described in literature (see Fell et al., 2001). It is defined as a process of washout of fine-grained particles from a dam's filling material by water seepage as it flows through cracks, gaps at interfaces between embankment sections or their foundations, or other preferential flow paths. Piping is among the most important causes of dam failure under normal operational conditions. It occurs when internal erosion due to water flow through poorly compacted soil or a crack in the embankment dam get localized over time to form a ‘pipe’. If the pipe continues to grow, it may lead to the complete collapse of the dam and release a major flood. According to Foster et al. (2000) about 46% of all dam failures can be attributed to some form of piping, and about 30.5% of all dam failures were due to piping through the embankment. For example, both Teton Dam and Baldwin Hills Dam failed due to piping in approximately four hours (Solava and Delatte, 2003; Saxena and Sharma, 2005). Design of existed dams varies widely depending on loading prehistory, material properties, and compatibility of materials. While internal erosion in embankment dams under normal operational conditions can be accurately monitored and prevented (e.g. Fell et al., 2001), the genesis of our work is primarily to help evaluate consequences of dynamic loading due to earthquakes or explosives for existed gravity dams. In this case the internal erosion can be a relatively rapid process since dynamic loading can enhance crack formation and increase seepage flow rates. At this stage we develop and evaluate numerical tools and capabilities to be able to address different scenarios of the damage formation. We plan to taylor them for particular material properties and dam structure at later stages based on real field data.

FAILURE CLASSIFICATION

We can group damage severity to soil-concrete interfaces into three categories:

- **Structural damage to watertight core material with minimal damage to the concrete interface.** Inelastic deformation could cause cracking in the core material and development of leakage paths which may or may not lead to further erosion. The question to be answered is whether or not larger voids can develop leading to instability. The associated soil deformation behaviour and permeability change can be studied experimentally or with mesoscale simulations of a representative volume element (RVE) deformation of the core material.
- **Significant damage to the interface between the core material and concrete.** Here there is no initial damage to the filters, but there is seepage flow or very low Reynolds number flow within the core section. The possibility of subsequent dam failure depends significantly on the filter design specifications. The filter can prevent further erosion if the core material fills the voids between filter grains and stop the flow. But it is also known, that a “suffusion” process in coarsely graded filter material, i.e., when finer particles are eroded through the pore space between the larger particles, could lead to the filter failure. While the filter design criteria are in most cases empirically derived, numerical modelling can serve a complimentary approach for investigating filter stability.
- **Significant opening between the concrete and both filter and core sections.** Large water flow through the gap can lead to high Reynolds number flows, which easily carry large particles. This regime will lead to rapid erosion of both filter and core zones and systematic (catastrophic) failure. But even in this case large gravel will have a high settling velocity and may occlude the lower portion of the crack, ultimately stabilizing the process. The filter and shell materials can be investigated experimentally and numerically to see if they fill the crack and prevent the erosion process from leading to failure.

The failure of a crack to self-heal is a gradual transition from a damage regime to an altered but stable condition. At one extreme, there is a light damage associated with unconnected shear-induced voids. At the other extreme, there is degradation or healing of moderately to severely damaged regions that can be linked to fluid flow, sliding of materials into the cracks, and possible erosion and downstream deposition of clay, sand or gravel. Removal of relatively large volume of earthen materials can alter stress state in a dam and eventually may lead to its collapse due to slope instability. Also cracks forming through the core could result in the downstream zone becoming saturated which could also contribute to slope instability. This problem is usually studied by finite element techniques (e.g. White and Borja, 2008).

We numerically represent the flow physical processes using two fundamentally different approaches in this paper. The first uses an Eulerian-Eulerian multiphase formulation (described in the next section), where fluid and particulate phases with different particle sizes are represented by corresponding mass fractions. This approach is most suitable when considering an entire system, but requires computationally demanding closure laws for interphase momentum exchange. The second approach explicitly resolves the volume occupied by solid particles and is applicable for densely packed particle beds. This second approach limits the number of necessary parameterizations and is often used to help parameterize the more computationally intensive Eulerian-Eulerian approach. Due to the large variation in particle size (from microns to centimetres), both approaches have to be used in a single simulation to evaluate the ability of the filter zones (with resolved particles) to capture the unresolved finer particles.

SOIL EROSION IN A CONCRETE-WRAPAROUND CRACK

If the shell and filter zone materials do not effectively fill the cracks and prevent higher flow of water, then erosion of soil within the initial crack may potentially lead to dam failure necessitating an understanding of erosion characteristics of the soil within the dam core. Detailed validation experiments and numerical simulations of soil at the micro scale are needed to better understand and parameterize soil properties. It is well known that different soils erode at different rates depending on factors such as grain size, soil composition, plasticity, compaction, water content, stress-history, and soil cohesion. Several experimental methods have been developed to analyze soil erosion, the most common being the hole erosion test, or "HET" (e.g., Wan and Fell, 2004; Bonelli and Brivois, 2008), and the jet erosion test, or "JET" (e.g., Hanson and Cook, 2004). Such methods provide data for specific types of soils, though the general constitutive relationship between erosion parameters and geotechnical or chemical soil properties remain unknown (Wan and Fell, 2002).

An initial computational approach to the problem is based on sediment transport modeling. Most hydraulic sediment transport models represent both soil and fluid as a continuum (e.g., Brethour, 2001, Zhao et al., 2007) where the erosion domain is represented as two media. The first medium is a packed solid domain consisting of particles completely bound together and forming a solid-like structure; the second medium is a mixture of suspended particles and fluid. At the surface of the packed bed of sediments, the fluid shear stress acts to remove sediment (first medium). If the fluid stresses are high enough to break the frictional forces between the particles, they begin to migrate from the surface forming suspended particles (second medium).

The majority of sediment transport models assume that sediment motion occurs when the Shields parameter $\theta = \frac{\tau}{gd(\rho_s - \rho_s)}$ exceeds a critical value θ_{cr}. Here ρ_s and ρ_f are sediment grain and fluid density, correspondingly, and d is the grain diameter. The erosion process is then simulated by an erosive constitutive law that manifests either as the mass or momentum exchange terms at the eroded boundary. In this case the packed sediments are not simulated and the solid domain is represented as a moving boundary. Bonelli et al. (2006) proposed a phenomenological piping erosion model using asymptotic analysis assuming that: (1) sediment concentration is low and does not affect density, velocity and stresses, (2) the flow is quasi steady and (3) interface velocities are low and do not contribute to the inertia. Using these assumptions they showed that dimensionless hole radius can be estimated as an exponential function of dimensionless time and initial and critical stress, when the hole is progressively eroding as

$$\frac{R(t)}{R_0} = 1 + \left(1 - \frac{\tau_c}{\tau_0}\right)\left(e^{t/t_{er}} - 1\right), \tag{1}$$

where $R(t)$ is hole radius at time t, R_0 is the initial radius at time zero, τ_c is critical shear stress, τ_0 is shear stress at time zero. Characteristic erosion time scale is defined as $t_{er} = \frac{2L\rho_s}{k_{er}\Delta h}$, where k_{er} is erosion coefficient.

The dimensional flow rates are proposed to estimate as

$$Q^* = \frac{Q(t)}{Q_0} = \left(\frac{R(t)}{R_0}\right)^{5/2} = \left[1 + \left(1 - \frac{\tau_c}{\tau_0}\right)\left(e^{t/t_{er}} - 1\right)\right]^{5/2} \tag{2}$$

This model can be used for both interpreting HET experiment as well as predicting crack growth if soil properties are known from HET experiment. The significant advantage of this method for interpreting HET experiment that the final hole diameter does not need to be measured to find soil erosion coefficient k_{er}. Assuming that erosion coefficient k_{er} and critical stress τ_c are known from HET experiment for particular soil type, it is straightforward to estimate crack growth in time from equation (1). While this model was applied to numerous HETs experiments and was found to reproduce HET test results quite well it is not generally applicable for other erosion processes, e.g. JET experiment.

Another approach is to introduce a diffusion term (the lift force) for the solid phase (Brethour, 2001). We prefer to use this general approach since both solid domain and domain with a mixture of fluid and suspended sediments are simulated simultaneously as one system. The diffusion term (lift force) is introduced as $F = \nabla(k\nabla C)$, where the diffusion coefficient k depends upon the shear stress and critical shear stress difference $(\tau - \tau_c)$, and C is the particle phase concentration.

A challenge arises when simulating the flow of the solid laden fluid. Correct representation of the general rheological behavior of the solid and fluid mixture as well as transition from solid to fluid-like behavior is a very complicated problem. The common approach is to assume a shear stress-shear rate constitutive relationship:

$$T = -\left(\lambda - \frac{2}{3}\mu\right)(\nabla \cdot u)I + \mu S \tag{3}$$

where T is the shear stress, S is the shear rate, and μ and λ are shear and bulk viscosities, respectively. One approach to simulate fluid-particle suspensions (or mud flow) is to treat them as thixotropic yield stress fluids or Bingham fluids (Chanson et al., 2006). The granular bed deforms in the region in which the shear stress exceeds a certain value (yield stress). In these models the effective viscosity is determined by the shear stress and the shear rate as:

$$\mu_{eff} = \begin{cases} \mu_0 + T_0, T \geq T_0 \\ \infty, T \leq T_0 \end{cases} \tag{4}$$

where T_0 is the constant yield stress. In another approach (Zhao et al., 2007) the solid shear viscosity μ_s is represented as a sum of frictional μ_{fric}, collisional μ_{col} and kinetic viscosities μ_{kin}:

$$\mu_s = \mu_{fric} + \mu_{col} + \mu_{kin} \tag{5}$$

and an additional drag term dependent on the solid volume fractions is included in the momentum equations to represent solid-like behavior. Both approaches highlighted for Equations 4 and 5 have been incorporated into the numerical code used herein to investigate soil erodibility within the crack. To represent the settling of the sediment particles due to gravity we estimate the relative slip velocity between fluid and particles as

$$u_{slip} = \frac{d^2}{18\mu f_{drag}} \frac{\nabla P(\rho_s - \rho_f)}{\rho} \tag{6}$$

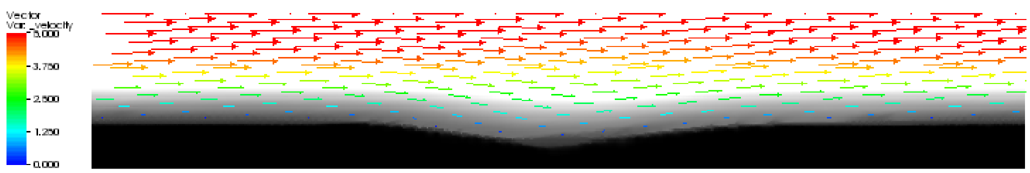
where drag term estimation (f_{drag}) is based on the Reynolds number

$$f_{drag} = \begin{cases} 1 + 0.15\,\mathrm{Re}^{0.687} & \text{if } \mathrm{Re} \leq 1000 \\ 0.0175\,\mathrm{Re} & \text{if } \mathrm{Re} > 1000 \end{cases} \tag{7}$$

For densely packed fluidized beds the drag equation (7) also takes into account volume fractions occupied by fluid and particles.

We apply numerical model described above to investigate erosion processes for HET and JET experiments. Figure 2 shows erosion simulations within a crack of maximum and minimum width 8 mm and 6 mm respectively. The flow is caused by a pressure gradient of 27 kPa/m (a hydraulic gradient of 2.75m/m). In this simulation we assume that critical shear stress was 8 Pa. This numerical configuration is close to FTHET010 experiment conditions considered in Bonelli et al., (2006). Figure 3 shows results of simulations of JET experiment. The 6.4 mm diameter submerged hydraulic jet was positioned initially 30 cm above the sample surface. The velocity at the jet nozzle was 6.6 m/s. As a submerged jet erosion test progresses with time, the scour surface in the zone of the impinging jet erodes away from the jet nozzle until an equilibrium depth is reached. The HET and JET methods yield significantly different estimates of the erosion coefficient and critical stress (Wahl et al., 2008). The numerical experiments described above are performed for the same type of soils. We estimate the factor of difference for erosion coefficient between the JET and HET experiments as 0.06 that agrees with experimental data (Regazzoni, 2008). It should be noted that JET method is more easily to perform and it is applicable to a wider range of soils. Therefore establishing connection in interpretation between these two tests is important task. Direct numerical simulation of soil erosion with continuum approach described above can be used to get more insight on the difference between JET and HET data. Also, the numerical approaches described above necessarily assume empirical constitutive laws for soil behavior that need further improvements and validation. In the scenario considered so far it is assumed that the hole is progressively eroding and eventually will lead to the dam failure if the filters do not prevent further erosion. The issues related to the filter design are considered in the next section.

a. Problem setup



b. Erosion of soil interface

Figure 2. Numerical simulation of multiphase flow in an interfacial gap between concrete and soil (HET experiment)

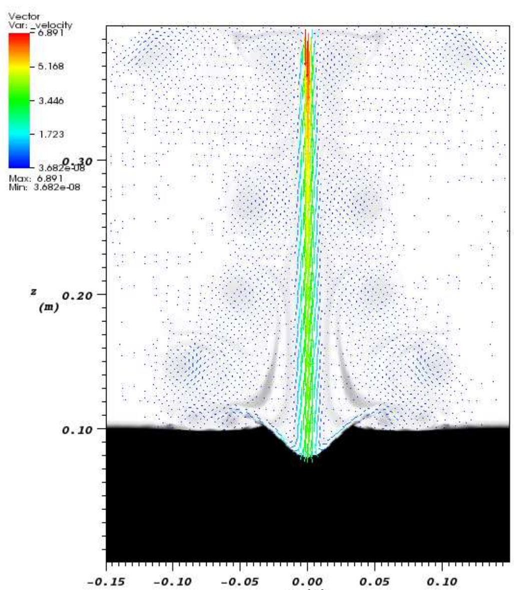


Figure 3. Numerical simulation of JET experiment.

BEHAVIOR OF GRAVEL IN ZONED FILTERS

Granular filters are one of the most important protective design elements of large, zoned embankment dams. The main role of filters is to prevent movement of the core materials past the filter zones and to lower the phreatic surface so that water safely seeps through the embankment at the downstream toe. Design of the size distribution of particles within the filter is based on a large body of empirical, physical experiments. Numerical rather than empirical determination of filter zone gradations is nontrivial. For example, the particle size distribution of the filter should contain small enough particles to have small void spaces and prevent erosion. However, it is well known that seepage flow within a coarsely graded filter material can erode finer particles thus generating a "suffusion" process that may over time compromise the performance of the filter. Also, clogging of the filter may occur causing loss of the filter's ability to remove water from protected soil. Existing filter design criteria are, in most cases, empirically derived (e.g. USACE Engineer Manual 1110-2-1901; Fell et al., 2005). Some of these criteria are listed below.

Clogging criterion: $$\frac{D_{15}^{filter}}{D_{85}^{soil}} \le 5 \tag{8}$$

Permeability criterion: $$\frac{D_{15}^{filter}}{D_{15}^{soil}} \ge 5, \tag{9}$$

where D_{15} and D_{85} are the diameters of a soil mixture which correspond to cumulative weight percentage of 15% and 85% in a particle size distribution.

Those criteria are derived for filters that are operated during normal conditions. During dynamic loading due to earthquakes or explosions the filter can be deformed homogeneously with associative dilatancy, as well as localized shear bands with even higher void ratios can be created. Numerical modeling holds promise as a complimentary approach for optimizing the selection of filter components. Numerical continuum formulations of flow through porous media are one way to approach such selection. Continuum approaches are often heavily dependent on a few unknown parameters such as soil permeability. The physical background of such parameters typically depends on the localized effects of the flow and pore geometry. Thus detailed simulations of the flow at the grain-scale can provide necessary physical parameters for the continuum formulations. An example of such direct simulations of fluid flow through the filter layer that consists of spherical particles is illustrated in Figure 4.

Another example of how local inhomogenities in the filter packing could affect its permeability is shown in Figure 5. Two different configurations of randomly packed spheres with the same porosity of 42% are considered (Figure 5 a, b). In the first configuration the anisotropy in packing in the vertical plane is introduced by removing few particles from initial randomly packed structure with porosity of 36%. In the second configuration the anisotropy is introduced in the horizontal plane in the same way. We estimate the maximum difference in pore pressure between these two cases as 22% (Figure 5c). Further studies to address the effect of local inhomogenities in filter structure on fine particle transport are underway.

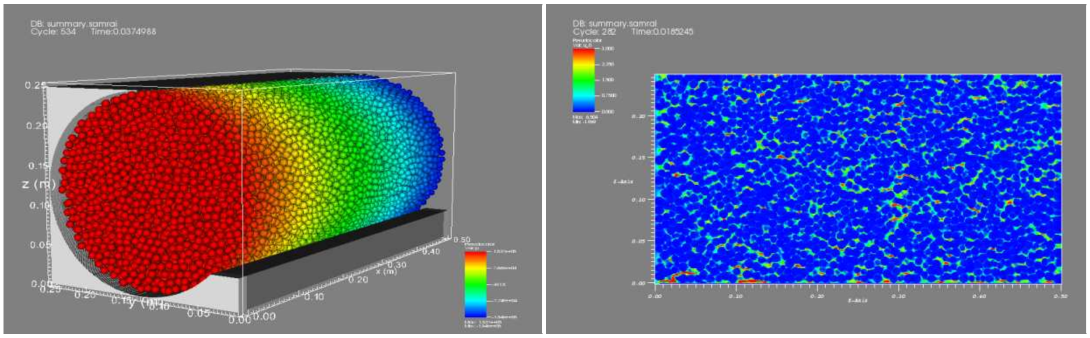


Figure 4. Simulation of the fluid migration through the gravel filter. Hydraulic pressure drop (left panel) and fluid velocity (right panel)

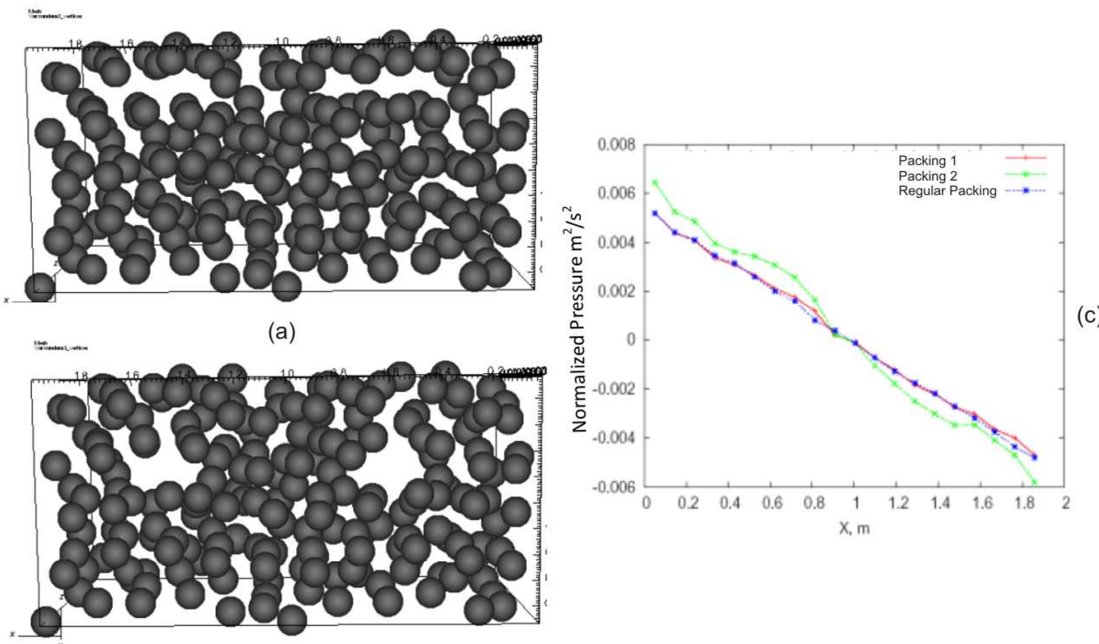
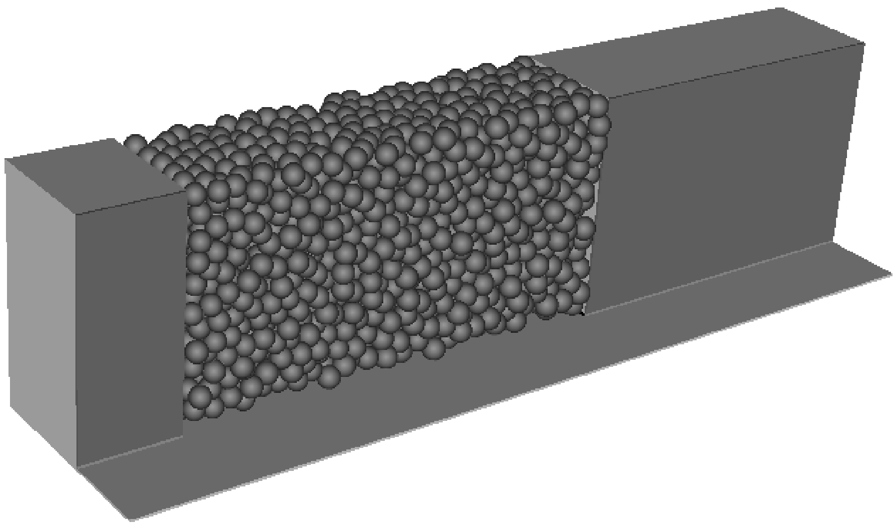
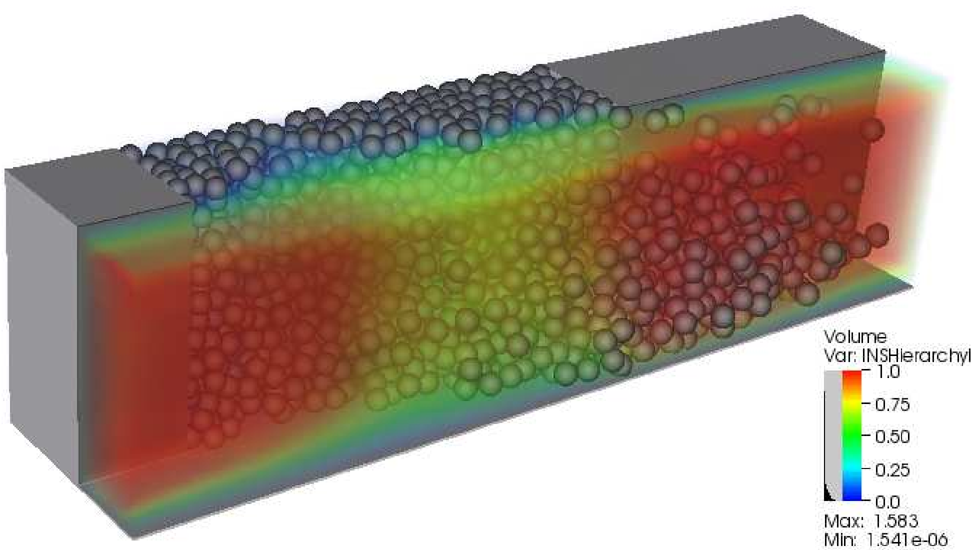


Figure 5. Simulation of the fluid migration through the gravel filter with local inhomogenities in packing in the vertical plane (a) and horizontal plane (b). (c) Difference in hydraulic pressure drop.

When there is a significant opening between concrete and both filter and core sections, the coarse grains and gravel become detached from the filter and slide into the crack, or are transported away by the flowing water. An example of gravel migration and sedimentation through a crack is illustrated in Figure 6 showing a computational domain of 0.55m x 0.13m x 0.13m. Here an initial layer of spherical gravel particles of diameter 1 cm is placed between two water resistant layers and an initial hydraulic pressure drop through a crack of 5 kPa/m is applied. As the fluid moves through a crack, gravel particles become detached from the filter layer and are transported downstream. Under certain conditions the gravel could fill the crack and prevent further erosion.



a. Initial setup



b. Gravel particles and flow field at time=0.5 s

Figure 6. Gravel motion through a crack. The magnitude of a velocity field is shown by color. An initial gravel layer is setup between two water resistent layers. As a fluid moves through the crack gravel particles become detached from the filter layer and transported downstream. Under certain conditions the gravel may fill the crack and prevent further erosion.

These simulations employ the Lagrange multiplier technique (Glowinski et al., 1999, Sharma and Patankar et al. 2005) and a stationary Eulerian grid with adaptive mesh refinement (AMR) to model fluid flow. Each particle is explicitly resolved on the Eulerian grid as a separate domain, using solid volume fractions. The fluid equations are solved throughout the entire calculation domain; however, Lagrange multiplier constraints are applied inside the particle domain such that the fluid within any volume associated with a solid particle moves as an incompressible rigid body. Mutual forces for the fluid-particle interactions are internal for the system. The particles interact with the fluid via fluid dynamics equations, resulting in implicit fluid-rigid-body coupling relations that produce realistic fluid flow around the particles (i.e., no-slip boundary conditions). In addition, particle-particle interaction is implemented using the DEM method (Cundall and Strack, 1979) with frictional, inelastic contact forces. The method is flexible enough to handle arbitrary particle shapes and size distributions and doesn't require extra parameterizations for fluid-particle interactions. This approach is suitable for developing parameterizations and constitutive lows for the continuum (or multiphase) approach described in previous section.

Additional computational complications arise when predicting erosion within a solid matrix structure consisting of gravel, sand, and fine particles (or clay). For example, the suffusion process, when finer soil particles are detached by seepage flow from the solid matrix, may ultimately lead to the filter failure. To our knowledge, the general erosion laws for this process have yet to be derived, and only a few attempts have been made toward a better understanding of numerical and experimental modeling of the suffusion process (Bonelli and Marot, 2008). The problem has similarities with the dialysis process used for blood filtration (e.g., Kleinstreur et al., 1983) where molecules diffuse from high concentration to low concentration through a semi-permeable membrane. However, unlike dialysis, during erosion the particles can be detached from the solid matrix (contrasting to a fixed membrane of known permeability in dialysis). It is clear, that the continuum formulation cannot model this class of problem. At the same time, the discrete representation of each grain ranging from finer particles (clay) to gravel-scale is computationally intensive requiring vast computational resources. We anticipate that carefully experimentally validated discrete simulations of the gravel together with continuum representation of finer particles described in the previous section are needed to study the internal erosion process in a gravel/sand/clay mixture.

CONCLUSIONS AND FUTURE WORK

In this paper, we have outlined a preliminary computational methodology to address a difficult multidisciplinary problem covering a wide range of length and timescales. The approach focuses on evaluating the consequences of dynamic loading of an interface between the end monoliths of a concrete gravity dam and where they meet up with and are supported by wraparound sections embankment dams. We consider aspects of the problem which involve significant, possibly non-laminar, fluid flow in opened cracks. The initial approach to the problem involves Eulerian-Eulerian multiphase methods, a phenomenological approach that requires multiple closure models. We plan to use functional parameterizations of these models available in the literature, while further constraining material parameters with HET and JET experiments for particular soil types.

Dam embankments have complicated zoning, with an impermeable core, sand and gravel filters. Different particle sizes are necessary for effective design, but create additional computational challenges. Interphase momentum exchange terms are not well characterized for flows with wide particle size distributions. We plan to overcome these difficulties with high fidelity simulations resolving flow around the individual particles. To evaluate internal erosion processes, both approaches can be combined in a single simulation when bigger particles are directly resolved and smaller particles are represented by an Eulerian phase. Validation of simulations with scaled experiments of realistically zoned dams can help to build confidence in our computational tools, so they can be used to better assess the performance of these interface sections when subjected to deformations caused by dynamic loads.

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REFERENCES

1. Brethour, J.M., (2001) Transient 3D Model for Lifting, Transporting, and Depositing Solid material, Proc. 3rd Intrn. Environmental Hydraulics, Dec. 5-8, 2001, Tempe, AZ
2. Bonelli S, Benahmed N., Brivois O. (2006) On modelling of the hole erosion test. ICSE, 3rd International conference on scour and erosion, Amsterdam, 2006.
3. Bonelli S., and Brivois O. (2007) The scaling law in the hole erosion test with a constant pressure drop. Int. J. for Num. and Anal. Methods. in Geomech., 32(13), 1573-1595.
4. Bonelli S., Marot D. (2008) On the modelling of internal soil erosion. Int. Association for Computer Methods and Advances in Geomechanics, 2008.
5. Bradley J., et al. Wolf Creek Dam Consensus Report, 2007.
6. Bureau G. J. (2003) Dams and Appurtenant Facilities. Earthquake Engineering Handbook edited by Chenh, W.F. and Scawthorn, C. CRS press, 26.1 – 26.47.
7. Cundall P.A. and Strack O.D.L. (1979) A discrete numerical model for granular assemblies. Geotechnique, 29, 47-65.
8. Chanson, H., Jarny S., Coussot P. (2006) Dam break wave of thixotropic fluid. J Hydr Eng., ACSE, 2006, 132 (3), 280-293.
9. Fell R., MacGregor P., Stapledon D., Bell G. (2005) Geotechnical engineering of dams. Balkema, Leiden, ISBN 041536440.
10. Fell R., Wan C.F., Cyganiewicz J., Foster M. (2001) The time for development and detectability of internal erosion and piping in embankment dams and their foundations. UNICIV report R-399.

11. Foster M., Fell R., Spannagle M. (2000) The statistics of embankment dam failures and accidents. Can. Geotech. J., 37(5): 1000-1024.
12. Glowinski, R., Pan T.W., Hesla T.I., Joseph D.D. (1999). A distributed Lagrange multiplier/fictitiouss domain method for particulate flow. Int J Multiphase Flow, 25, 755-794.
13. Hanson G. J., K. R. Cook. (2004) Apparatus, test procedures, and analytical methods to measure soil erodibility in situ. Applied Engineering in Agriculture, 20 (4), 455-462.
14. Kleinstreuer C., Paller M., Ponzio R. J. (1983) Computer simulation of transport phenomena during dialysis deglycerolization of red blood cells. Computers and Biomedical Research, 1983, 16(1), 29-39.
15. Lee J.Y., Choi Y.K., Kim H.S., Yun S.T. (2005) Hydrologic characteristics of a large rockfill dam: Implications for water. Eng. Geol. 80 (2005), 4359.
16. Regazzoni P.-L. (2008) Report on the work concerning the tests of soils with the JET and the HET realized on E.R.D.C. and comparison with the tests realized on USBR and ARS. p 30.
17. Saxena K.R. and Sharma V.M., DAMS: Incidents and Accidents, A.A. Balkema Publishers, 2005.
18. Sharma N., Patankar N.A. (2005) A fast computation technique for the direct numerical simulation of rigid particle flows. J. Comp. Phys., 2005, 205, 439-457.
19. Solava S. and Delatte N. (2003) Lessons from the Failure of the Teton Dam, Proceedings of the 3rd ASCE Forensics Congress, October 19 - 21, 2003, San Diego, California.
20. Zhao Z., Fernando H.J.S. (2007) Numerical simulation of scour around pipelines using an Euler-Euler coupled two phase model. Env. Fluid Dyn., 2007, 7 (2), 121-142.
21. Wan, C. F., and Fell, R. (2002) Investigation of rate of erosion of soils in embankment dams. J. of Geotech. and Geoenv. Eng., ACSE, 2002, 130 (4), 373-380.
22. Whal T. L., Ragazzoni P.L., Erdoan Z. (2008). Determing erosion indices of cohesive soils with the hole erosion test and jet erosion test. USBR Report DSO-08-05.
23. White J. A., Borja R. I. (2008) Stabilized low-order finite elements for coupled solid-deformation/fluid-diffusion and their application to fault zone transients. Comput. Methods Appl. Mech. Engrg., 2008, 197, 4353-4366.
24. Engineer manual No. 1110-2-1901. Department of the Army, U.S. Army Corps of Engineers, Washington, DC, April, 1993.