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Title: STATUS OF B_K FROM LATTICE QCD

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Submitted to: Proceedings of the CKM Unitary Triangle Workshop
held at CERN
Geneva, Switzerland
February 13 -16, 2002

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Status of B_K from Lattice QCD

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Abstract

A brief review of lattice calculations of the bag parameter B_K relevant for understanding indirect CP violation in the neutral kaon sector is given. A status report on current state-of-the-art calculations is presented as well as a discussion of the value of B_K exported to phenomenologists. This review was presented at the CKM Unitarity Triangle Workshop held at CERN during Feburary 13-16, 2002.

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I. INTRODUCTION

The most commonly used method to calculate the matrix element $\left\langle \overline{K^0} \mid Z\ (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}(\mu) \mid K^0 \right\rangle$ is to evaluate the three point correlation function shown in Fig. 1. This corresponds to creating a K^0 at some time t_1 using a zero-momentum source; allowing it to propagate for time $t_O - t_1$ to isolate the lowest state; inserting the four-fermion operator at time t_O to convert the K^0 to a $\overline{K^0}$; and finally allowing the $\overline{K^0}$ to propagate for long time $t_2 - t_O$. To cancel the K^0 ($\overline{K^0}$) source normalization at times t_1 and t_2 and the time evolution factors $e^{-E_K t}$ for times $t_2 - t_O$ and $t_O - t_1$ it is customary to divide this three-point function by the product of two 2-point functions as shown in Fig 1. If, in the 2-point functions, the bilinear operator used to annihilate (create) the K^0 ($\overline{K^0}$) at time t_O is the axial density $\bar{s}\gamma_4\gamma_5 d$, then the ratio of the 3-point correlation function to the two 2-point functions is $(8/3)B_K$.

A key prediction of chiral symmetry is that the matrix element should behave as

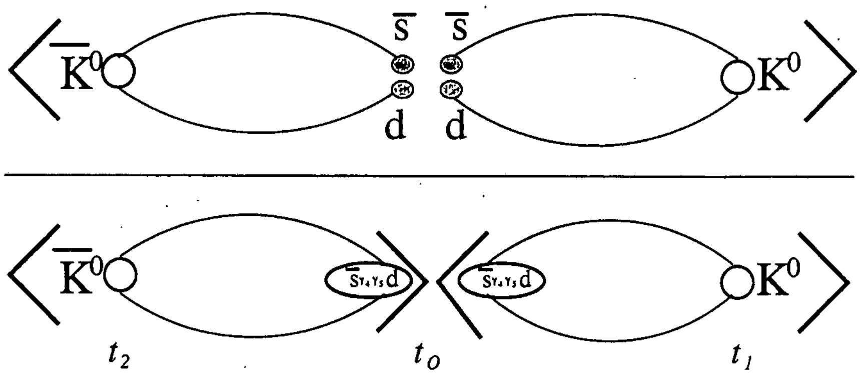
$$\left\langle K^0 \mid Z\ (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}(\mu) \mid \overline{K^0} \right\rangle \;=\; (8/3)B_K M_K^2 F_K^2 + O(M_K^4)\,.$$

Earliest calculations of B_K were done using Wilson fermions and showed significant deviations from this behavior. It was soon recognized that these lattice artifacts are due to the explicit breaking of chiral symmetry in the Wilson formulation [1–5]. Until 1998, the only formulation that preserved sufficient chiral symmetry to give the right chiral behavior was Staggered fermions. First calculations using this approach in 1989 gave the quenched estimate $B_K(NDR, 2\text{GeV}) = 0.70 \pm 0.01 \pm 0.03$. In hindsight, the error estimates were highly optimistic, however, the central value was only 10% off the current best estimate, and most of this difference was due to the unresolved $O(a^2)$ discretization errors.

In 1997, the staggered collaboration refined its calculation and obtained 0.62(2)(2) [6], again the error estimate was optimistic as a number of systematic effects were not fully included. The state-of-the-art quenched calculation using Staggered fermions was done by the JLQCD collaboration in 1997 and gave $B_K(2\text{GeV}) = 0.63 \pm 0.04$ [7]. This estimate was obtained using six values of the lattice spacing between $0.15 - 0.04$ fermi, thus allowing much better control over the continuum extrapolation as shown in Fig. 2 along with other published results. This is still the benchmark against which all results are evaluated and is the value exported to phenomenologists. This result has three limitations: (i) It is in the quenched approximation. (ii) All quenched calculations use kaons composed of two quarks of roughly half the "strange" quark mass and the final value is obtained by interpolation to a kaon made up of $(m_s/2, m_s/2)$ instead of the physical point (m_s, m_d). Thus, SU(3) breaking effects ($m_s \neq m_d$) have not been incorporated. (iii) There are large $O(a^2)$ discretization artifacts, both for a given transcription of the $\Delta S = 2$ operator on the lattice and for different transcriptions at a given value of the lattice spacing, so extrapolation to the continuum limit is not as robust as one would like. A conservative estimate of the combined associated systematic error is 0.1 [8].

In the last four years a number of new methods have been developed and the corresponding results are summarized in Table 1.

- The Rome collaboration has shown that the correct chiral behavior can be obtained using $O(a)$ improved Wilson fermions provided non-perturbative renormalization constants are used. Their latest results, with two different "operators", are



Ratio of correlation functions needed to calculate B_K

FIG. 1:

Collaboration	year	B_K(2GeV)	Formulation	Renormalization	a^{-1} (GeV)
Staggered [6]	1997	0.62(2)(2)	staggered	1-loop	∞
JLQCD [7]	1997	0.63(4)	staggered	1-loop	∞
Rome [9]	2002	0.63(10)	Improved Wilson	NP	∞
Rome [9]	2002	0.70(12)	Improved Wilson	NP	∞
CP-PACS [10]	2001	0.58(1)	Domain Wall	1-loop	1.8 GeV
CP-PACS [10]	2001	0.57(1)	Domain Wall	1-loop	2.8 GeV
RBC [11]	2002	0.53(1)	Domain Wall	NP	1.9 GeV
DeGrand [12]	2002	0.66(3)	Overlap	1-loop	1.6 GeV
DeGrand [12]	2002	0.66(4)	Overlap	1-loop	2.2 GeV
GGHLR [13]	2002	0.61(7)	Overlap	NP	2.1 GeV

TABLE I: Quenched estimates for B_K evaluated in the NDR scheme at 2GeV. The fermion formulation used in the calculation, the method used for renormalizing the operators, and the lattice scale at which the calculation was done are also given. NP indicates non-perturbative renormalization using the RI/MOM scheme and $a^{-1} = \infty$ implies that the quoted result is after a continuum extrapolation.

$B_K(2\text{GeV}) = 0.63(10)$ and $0.70(12)$ [9]. These, while demonstrating the efficacy of this method, do not supplant the staggered result, as the continuum extrapolation is based on only three points and the data have larger errors. The discretization errors can be characterized as $B_K(a) = B_K(1 + a\Lambda)$ with $\Lambda \approx 400$ MeV and are similar in magnitude to those with staggered fermions at $1/a = 2$ GeV, as are the differences in estimates with using different operators. In the staggered formulation, the artifacts are, however, $O(a^2\Lambda^2)$ and $O(\alpha_s^2)$ and the data suggest an unexpectedly large $\Lambda \sim 900$ Mev.

- Four collaborations have new results using domain wall and overlap fermions as shown in Table I [10–14]. Both formulations have built in chiral symmetry at finite a and $O(a)$ improvement. Each of these collaborations have used slightly different methodology,

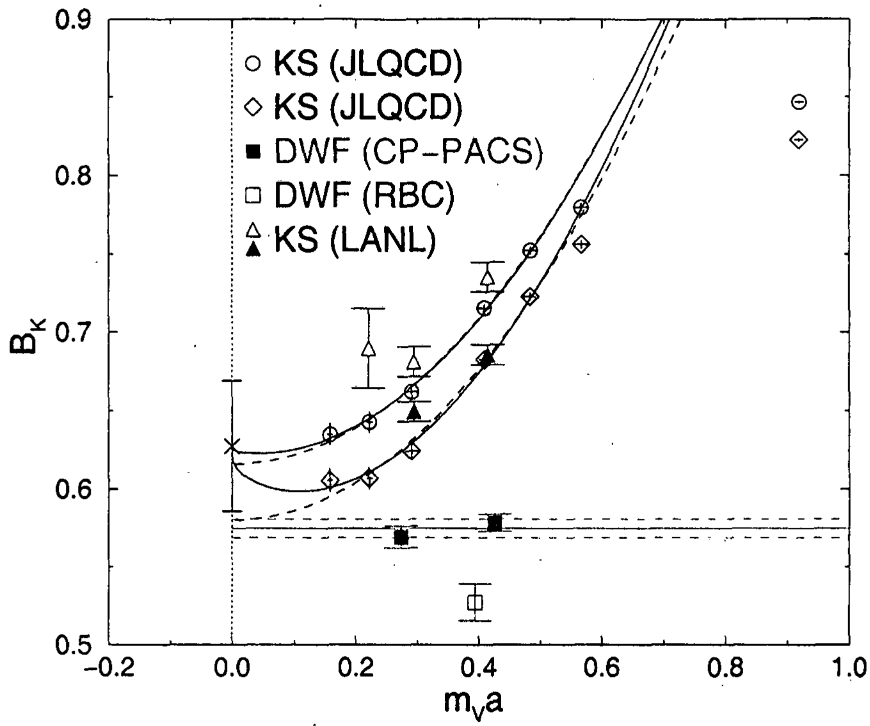


FIG. 2: Published estimates of B_K with fermion formulations that respect chiral symmetry. All results are in the quenched approximation.

so they cannot be compared head on, or combined to do a continuum extrapolation. Thus, the results are quoted with reference to the lattice spacing at which the calculation was done. The differences reflect $O(a^2)$ (and $O(\alpha_s^2)$ in cases where perturbative renormalization constants have been used) artifacts.

- Calculations using another method with good chiral behavior, twisted mass QCD, are in progress [15].

Starting with the current best lattice estimate, the JLQCD staggered result $B_K(2\text{GeV}) = 0.63(4)(10)$, conversion to $\widehat{B}_K$ requires making choices for α_s and the number of flavors in the perturbative expressions. Fortunately, it turns out that the result is quite insensitive to the choice one makes and the final estimate is

$$\widehat{B}_K = 0.86 \pm 0.06 \pm 0.14 \tag{1}$$

It is important to decide how to fold these errors in phenomenological analyses, especially since the estimate of the systematic error is not robust. ChPT estimates of quenching and SU(3) breaking effects both suggest a larger value. Taking into account this probably one sided systematic shift and the current data, a very conservative recommendation is to assume a flat distribution between 0.72 − 1.00 to account for the systematic uncertainty and convolute it with a gaussian distribution with $\sigma = 0.06$ on either end. Since this is a very conservative estimate, the suggested confidence level to associate with this broad distribution is 2σ. Given this uncertainty, current calculations are focused on reducing the quenching uncertainty and it is anticipated that the systematic error 0.14 will be reduced by at least a factor of two by year 2005.

Finally, the reasons why the quenched lattice estimate of B_K has been stable over time and considered reliable within the error estimates quoted above are worth mentioning:

- The numerical signal is clean and very accurate results are obtained with a statistical sample of even 50 decorrelated lattices.
- Finite size effects for quark masses $\geq m_s/2$ are insignificant compared to statistical errors once the quenched lattices are larger the 2 fermi.
- In lattice formulations with chiral symmetry, the renormalization constant connecting the lattice and continuum schemes is small ($< 15\%$), and reasonably well estimated by one-loop perturbation theory.
- The chiral expansion for the matrix element has no singular quenched logarithms (they cancel between the AA and VV terms) that produce large artifacts at small quark masses in observables like M_π^2, f_π etc. Also, for degenerate quarks, the chiral expansion between the quenched and full theories have the same form [16–19].
- ChPT estimates of quenching and SU(3) breaking systematic errors are at the $5-10\%$ level [16, 20, 21].

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