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Development of an Autonomous Continuous Monitoring System for Mechanical Damage Detection

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ABSTRACT

The primary objective of damage identification is to ascertain the existence of damage within a mechanical system. This study applies the Sequential Probability Ratio Test (SPRT) to examine if damage is present or not. In the original formulation of the SPRT, the distribution of data is assumed Gaussian and thresholds for monitoring are set focusing on the center mass properties of the distribution. Decision-making for damage identification is, however, often sensitive to the tails of the distribution and the tails may not necessarily be governed by Gaussian characteristics. By modeling the tails using the technique of Extreme Value Statistics (EVS), the thresholds for the SPRT may be set more accurately avoiding the unnecessary normality assumption. The proposed combination of the SPRT and the EVS is demonstrated using experimental data collected from a three-story frame structure with bolted connections.

INTRODUCTION

The primary goal of structural health monitoring is simply to identify from measured data if a structure has deviated from a normal operational condition. Particularly, vibration-based damage detection techniques assume that changes of the structure's integrity affect characteristics of the measured vibration signals enabling one to detect damage. Many current approaches to this problem involve methods that leave much to the interpretation of analysts. These methods may enable a trained eye to discern and locate damage but are not easily automated or objective. In an attempt to automate the damage identification procedure, the SPRT is employed for the decision-making procedure. The original SPRT assumes that the extracted features have a Gaussian

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distribution. This normality assumption, however, may place misleading constraints on the tails of the distribution. As the problem of damage detection specifically focuses attention on the tails, the assumption of normality is likely to lead the analysis astray. To overcome this difficulty, the performance of the SPRT is improved by integrating the EVS, which specifically models behavior in the tails of the distribution of interest, into the SPRT.

METHODOLOGY

A unique combination of time series analysis, hypothesis testing, and extreme value statistics is presented to automate the damage identification procedure with special attention to statistical inference for decision-making. First, damage-sensitive features are extracted using the time series analysis of vibration signals. Then, a brief summary of the SPRT is presented and the EVS is incorporated with the SPRT.

Time Series Analysis

A linear prediction model combining auto-regressive (AR) and auto-regressive with exogenous inputs (ARX) models is employed to extract features for the subsequent SPRT analysis. First, all time signals are normalized prior to fitting an AR model by subtracting its mean and dividing by the standard deviation. Then, an AR(*r*) model with *r* auto-regressive terms is constructed for a given time signal $x(t)$ (Sohn and Farrar, 2001):

$$x(t) = \sum_{j=1}^{r} \phi_{xj} \; x(t-j) \; + e_x(t) \tag{1}$$

Assuming that the error between the measurement and the prediction obtained by the AR model [$e_x(t)$ in Equation (1)] is mainly caused by the unknown external input, an ARX model is employed to reconstruct the input/output relationship between $e_x(t)$ and $x(t)$:

$$x(t) = \sum_{i=1}^{p} \alpha_i \; x(t-i) \; + \sum_{j=1}^{q} \beta_j \; e_x(t-j) + \varepsilon_x(t) \tag{2}$$

where $\varepsilon_x(t)$ is the residual error after fitting the ARX(*p,q*) model to $e_x(t)$ and $x(t)$ pair. Ljung (1999) suggests keeping the sum of p and q smaller than r ($p+q \le r$). Although the p and q values of the ARX model are set rather arbitrarily, similar results are obtained for different combinations of p and q values as long as the sum of p and q is kept smaller than r.

Sequential Probability Ratio Test (SPRT-N)

In the previous section, the AR-ARX model is constructed using the time signal from the baseline structure. When a new time series $y(t)$ is obtained from unknown condition of the structure, the prediction or residual error becomes:

$$\varepsilon_y(t) = y(t) - \sum_{i=1}^{p} \alpha_i \; y(t-i) - \sum_{j=1}^{q} \beta_j \; e_y(t-j) \tag{3}$$

where $\varepsilon_y(t)$ is the residual error of $y(t)$ obtained using the previously estimated AR-ARX model. $e_y(t)$ is computed in a similar way to $e_x(t)$ in Equation (1). When a new time signal is obtained from a damaged structure and fed to the AR-ARX model, the time prediction model trained with the undamaged cases will not properly predict the new time series. Therefore, the standard deviation of the residual error $\varepsilon_y(t)$ is expected to increase compared to that of the baseline residual error $\varepsilon_x(t)$.

Based on this premise, a simple hypothesis test discriminating two hypotheses is constructed using the standard deviation of the residual errors as the parameter in question (Sohn et al, 2002):

$$H_o : \sigma(\varepsilon_y) \le \sigma_o, \quad H_1 : \sigma(\varepsilon_y) \ge \sigma_1, \quad 0 < \sigma_o < \sigma_1 < \infty \tag{4}$$

When the standard deviation of the residual error $\sigma(\varepsilon_y)$ is less than a user specified lower bound σ_o, the system in question is considered undamaged. On the other hand, when $\sigma(\varepsilon_y)$ becomes larger than the other user specified upper bound σ_1, the system is suspected damaged. Note that the selection of σ_o and σ_1 is structure-dependent and it might be necessary to use signals from both undamaged and damage cases to establish these two decision boundaries.

A SPRT starts with observing a sequence of the residual errors, $\{\varepsilon_y(i)\}$ $(i = 1, 2, \ldots)$. Define this accumulated data set at stage n is denoted as $E_n = [\varepsilon_y(1), \ldots \varepsilon_y(n)]$. The goal of a statistical inference is to reveal the probability model of E_n, which is assumed to be at least partially unknown. When the statistical inference is cast as a parametric problem, the functional form of E_n is assumed known and the statistical inference poses some questions regarding the parameters of the probability model. For instance, if $\{\varepsilon_y(i)\}$ are independent and identically distributed (i.i.d.) normal variables, one may pose some statistical test about the mean and/or the variance of this normal distribution.

A sequential test is one of the simplest tests for such a statistical inference where the number of samples required before reaching a decision is not determined in advance. An advantage of the sequential test is that, on average, a smaller number of observations are needed to make a decision compared to the conventional fixed-sample size test. For the well-established fixed-sampling tests, the sample size n is fixed, and an upper bound on the type I error is pre-specified. Then, an optimal fixed-sample test is selected by minimizing the probability of type II error. On the other hand, a sequential test specifies upper bounds on the probabilities of type I and II errors, and minimizes the sample number required to make a decsion. Among various valid sequential tests, it can be proven that the SPRT minimizes on average the sample size required to make a correction making it an optimal sequential test (Ghosh, 1970). Because of the sensitivity of the SPRT to signal disturbance, the SPRT has been applied for the surveillance of nuclear power plant components (Humenik and Gross, 1990).

For the hypothesis test in Equation (4), a SPRT, S(*b*,*a*), makes three distinctive decision at stage n (Ghosh, 1970):

$$\begin{aligned} &\text{Accept } H_o \text{ if } Z_n \le b \\ &\text{Reject } H_o \text{ if } Z_n \ge a \\ &\text{Continue observing data if } b \le Z_n \le a \end{aligned} \tag{5}$$

where the transformed random variable Z_n is the natural logarithm of the probability ratio at stage n:

$$Z_n = \ln \frac{f(E_n \mid H_1)}{f(E_n \mid H_o)} = \ln \frac{f(E_n \mid \sigma_1)}{f(E_n \mid \sigma_o)} \text{ for } n \ge 1 \tag{6}$$

where $f(E_n \mid H_o)$ or $f(E_n \mid \sigma_o)$ is the conditional probabilty of observing the accumuated data set E_n given the assumption that the null hypothese is true. $f(E_n \mid H_1)$ or $f(E_n \mid \sigma_1)$ is defined in a similar fashion. Without any loss of generality, Z_n is defined zero when $f(E_n \mid \theta_1) = f(E_n \mid \theta_o) = 0$. b and a are the two stopping bounds for accepting and rejecting H_o, respectively, and they can be estimated by the following Wald approximations (Wald, 1947):

$$b \cong \ln\frac{\beta}{1-\alpha} \text{ and } a \cong \ln\frac{1-\beta}{\alpha} \tag{7}$$

where α and β the predetermined upper limits for type I and II errors, respectively. When implementing the SPRT, a trade-off must be considered before assigning values for α and β. When there is a large penalty associated with false positive alarms (for example, alarms that shut down traffic over a bridge), it is desirable to keep α smaller than β. On the other hand, for safety critical systems such as nuclear power plants, one might be more willing to tolerate a false positive alarm to have a higher degree of safety assurance. In this case, β is often specified larger than α.

If $Z_n = \sum_{i=1}^{n} z_i$, the modified observations $\{z_i\}$ $(i = 1, 2, \ldots)$ are defined as follows:

$$Z_n = \sum_{i=1}^{n} z_i\ ,\ z_1 = \ln\frac{f(E_1 \mid \sigma_1)}{f(E_1 \mid \sigma_o)} \text{ and } z_i = \ln\frac{f(E_i \mid \sigma_1) f(E_{i-1} \mid \sigma_o)}{f(E_i \mid \sigma_o) f(E_{i-1} \mid \sigma_1)} \tag{8}$$

Assuming that E_n has a normal distribution with mean μ and standard deviation σ, z_i can be related to $\varepsilon_y(i)$:

$$z_i = \frac{1}{2}(\sigma_o^{-2} - \sigma_1^{-2})(\varepsilon_y(i) - \mu)^2 - \ln\frac{\sigma_1}{\sigma_o} \tag{9}$$

In a graphical representation of the SPRT S(*b,a*), Z_n, which is the cumulative sum of the transformed variable z_i, is continuously plotted against the two stopping bounds *b* and *a*. A Z_i value less than *a* is indicative of acceptance of the H_o hypothesis, while a Z_i value greater than *b* indicates an acceptance of the H_1 hypothesis. This SPRT using the normality assumption is referred to SPRT-N hereafter.

SPRT Combined with Extreme Value Statistics (SPRT-G)

Now, the SPRT is extended to the extreme values of the parent distribution, the distribution of the residual errors. In the previous section, the SPRT is formulated assuming that the residual errors have a normal distribution. However, slight errors in the normality assumption of the parent distribution can lead to larger errors for the extremes resulting in errneous false positive/negative indications of damage. To avoid this problem, the SPRT is reformulated using the probability distributions of extreme values (SPRT-G). There are only three possible choices for the distributions of the extremes regardless the parent distrubution type: Gumbel, Weibull, or Frechet. Particulary, because the maxima of a normal distribution are known to have a Gumbel distribution and the residual errors of the experimental study presented later are close to a normal distribution, the derivation presented here focuses on incorprating Gumbel distribution for maxima values into the SPRT. The cumulative distribution function for a Gumbel distribution is (Fisher and Tippet, 1928):

$$\text{Gumbel} \qquad F_M(x) = \exp\left[-\exp\left(-\frac{x-\lambda}{\delta}\right)\right] \qquad \text{for} \quad \begin{array}{c} -\infty < x < \infty \\ \delta > 0 \end{array} \tag{10}$$

Similar formulation of the SPRT can be easily derived for the other types of extreme value distribution and for minima values. Similar to Equation (4), the following hypothesis test is constructed using the standard deviation of the maxima as the parameter in question:

$$H_o : \sigma_M \le \sigma_{M,o}, \quad H_1 : \sigma_M \ge \sigma_{M,1}, \quad 0 < \sigma_{M,o} < \sigma_{M,1} < \infty \tag{11}$$

where σ_M is the standard deviation of the residual error maxima, and the subscript "*M*" denotes a quantity related to the maxima. $\sigma_{M,o}$ is a user specified lower limit of the standard deviation for the undamage contion, and $\sigma_{M,1}$ is the other user specified upper limit for the damage condition. It is observed that the change of the maxima distribution's standard deviation is monotomically related to the change of the parent distribution's standard deviation. Here, an indirect statitstical inference on the standard deviation of the parent distribution (the distribution of the residaul errors) is conducted by examining the standard deviation of the maximum values.

It can be shown that the model parameters, λ and σ, of the Gumble distribution are related to its mean μ_M and standard deviation σ_M (Castillo, 1987):

$$\delta = \frac{\sqrt{6}}{\pi}\sigma_M \text{ and } \lambda = \mu_M - 0.57772\,\delta \tag{12}$$

If the distribution of the maxima is preprocessed such that the mean value is zero, Equation (8) can be rewritten in terms of λ and σ:

$$z_1 = \ln\frac{f(E_1 \mid \lambda_1, \delta_1)}{f(E_1 \mid \lambda_o, \delta_o)} \text{ and } z_i = \ln\frac{f(E_i \mid \lambda_1, \delta_1) f(E_{i-1} \mid \lambda_o, \delta_o)}{f(E_i \mid \lambda_o, \delta_o) f(E_{i-1} \mid \lambda_1, \delta_1)} \tag{13}$$

For the original SPRT, E_n is defined as the accumulated sample points up to stage *n*. When the SPRT is applied to the maximum values like in Equation (13), $E_n = [\varepsilon_y(1), \ldots \varepsilon_y(n)]$ becomes a collection of the maximum values instead. That is, $\varepsilon_y(i)$ in E_n now becomes a maximum value obtained from samples. Aftere some manuplication, z_i for the maximum values is related to x_i:

$$z_i = -\ln\frac{\sigma_1}{\sigma_o} + \frac{\pi}{\sqrt{6}}\left(\sigma_o^{-1} - \sigma_1^{-1}\right)x_i + \exp\left(-\frac{x_i + 0.4504\,\sigma_o}{\sqrt{6}\ \sigma_o/\pi}\right) - \exp\left(-\frac{x_i + 0.4504\,\sigma_1}{\sqrt{6}\ \sigma_1/\pi}\right) \tag{14}$$

TEST STRUCTURE

The three-story frame structure shown in Figure 1 consists of Unistrut columns and aluminum floor plates. The floors are 1.3 cm-thick aluminum plates with two-bolt connections to brackets on the Unistrut. The base is a 3.8 cm-thick aluminum plate. Support brackets for the columns are bolted to this plate and hold the Unistrut columns. The floor layout from the top of the structure is shown in Figure 2. All bolted connections are tightened to a torque value of 0.7 Nm in the undamaged state. Four Firestone air mount isolators, which allow the structure to move freely in horizontal directions, are bolted to the bottom of the base plate.

The structure is instrumented with 24 single-axis accelerometers, two per joint (one on the plate and the other on the column) as shown in Figure 2. The accelerometers are numbered from the corner A to B, C, and D counterclockwise and from the top floor to the first floor. The nominal sensitivity of each accelerometer is 1 V/g. The shaker is connected to the base plate by a stringer as shown in Figure 1. The RMS voltage of the shake was fixed at 2 volts, and random signals were generated from the shaker. A 10-mV/lb-force transducer is also mounted between the stinger and the base plate. This force transducer is used to measure the input to the base of the structure. Using a commercial data acquisition system, the acceleration time histories are recorded. Each time signal gathered consisted of 8192 points and were sampled at 1600 Hz.

Two damage cases as shown in Figure 1 are investigated in this experiment: Damage 1 and 2. For each damage case, four bolts at each joint are loosened until hand tight, allowing relative movement between the floor plate and column. After each damage case, all the bolts were tightened back to the initial torque of 0.7 Nm. Five time series are measured from the initial undamaged case, and these time series are used for constructing the baseline AR-ARX model. Five time series are recorded under each damage cases, and additional five time series are obtained after tightening all bolts to the initial torque values. That is, a total of 20 time series are used for this experiment.

RESULTS

The time series analysis begins with the assumption that a “pool” of signals is acquired from a known structural condition of the system. In this example, multiple time series are recorded from the undamaged structure. The collection of these time series is called the “reference database” in this study. The construction of this reference database is shown to be useful for normalizing data with respect to varying operational and environmental conditions. The details of this data normalization can be found in Sohn and Farrar (2001).

Instead of independently analyzing 24 time histories from each accelerometer, the point-by-point difference between time series from the two adjacent accelerometers at a joint is first computed. Then, the resulting 12 time series corresponding to each joint are used for the AR-ARX modeling. The order r in the AR model is set to 25, and the p and q orders for the ARX model are set to 20 and 5, respectively.

Next, the SPRT-N and SPRT-G are applied to the residual errors obtained from the AR-ARX modeling. The type I & II errors are set to 0.001. The formulation of the SPRT is based on the premise that, when a system being monitored undergoes a structural change such as damage, a signal measured under the new structural condition will be significantly different from the signal obtained from the initial undamage case. Therefore, when a time prediction model is constructed using the baseline undamaged time signal, the prediciton error of the newly obtained signal, which is again from the damaged case, will depart from that of the baseline signal. Particularly, the prediction error of the new singal is expected to increase. Based on this observation, the sequential hypothesis test shown in Equation (4) is conducted using SPRT-N. In this particular example, σ_o and σ_1 in Equation (4) are set to 0.40 and 0.42, respectively. Note that the establishment of the σ_o and σ_1 values is based on the observation of actually damaged cases. That is, changes of the standard deviation should be first monitored for the corresponding damage cases to select the appropriate σ_o and σ_1 values. In a similar fashion, the sequential hypothesis test in Equation (11) is cast for SPRT-G. $\sigma_{M,o}$ and $\sigma_{M,1}$ in Equation (11) are set to 0.24 and 0.26, respectively.

The results of damage classification are reported in Table 1. The results using SPRT-G is presented in parenthesis when the results from SPRT-N and SPRT-G are different. To briefly summarize the results, both methods illustrate comparable performance. SPRT-N and SPRT-G do not show any false-positive indications of damage for all five undamaged cases. For Damage 1, the damaged joint is located at the corner A on the first floor, and this joint is associated with sensor readings from channels 17 and 18. Using SPRT-N and SPRT-G, the correct damage location is correctly revealed for all five cases of Damage 1. For Damage 2, where the bolts at the corner C on the third floor is hand tight and this joint corresponds to channel 5 and 6 readings, SPRT-N indicates that the adjacent joint at the corner D on the same floor is most likely damaged. SPRT-G also suggests the existance of damage at the same adjacent joint but correclty identifies the actually damaged joint 3 times out of the five exampled time series.

CONCLUSIONS

In an effort to find an automated and objective method for damage identification, a unique integration of time series analysis, statistical inference, and extreme value theory is explored. Time series analysis techniques solely based on the measured vibration signals are first employed to extract damage-sensitive features from a structure for damage classification. While there has been increasing interest in the field of structural health monitoring, the decision as to whether a structure is damaged or not tends to be made on the basis of exceeding some heuristic threshold. In this study, the sequential probability ratio test (SPRT) is employed to provide a more principled statistical tool for this decision-making procedure, excluding unnecessary interpretation of the measured data by users. Finally, the performance and robustness of damage classification is improved by incorporating extreme values statistics of the extracted features into the SPRT. The applicability of the SPRT to structural health monitoring is demonstrated using measured time signals from a three-story frame structure tested in a laboratory environment. The framework of the proposed SPRT method is well suited for developing a continuous monitoring system, and can be easily implemented on digital signal processing (DSP) chips automating the damage classification process.

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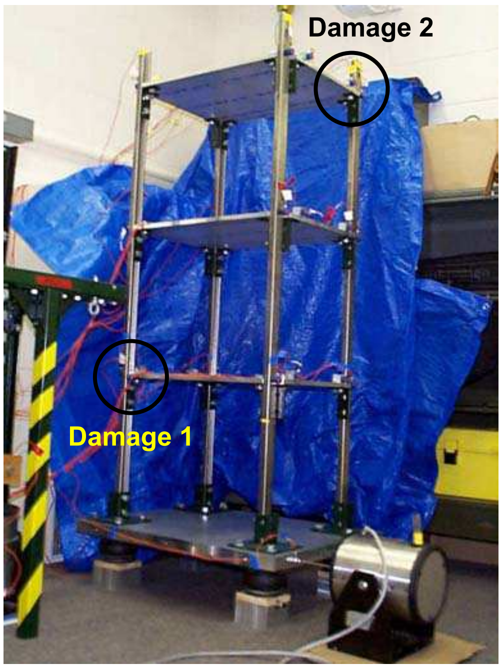

Figure 1: a three-story frame structure with dimension and damage locations

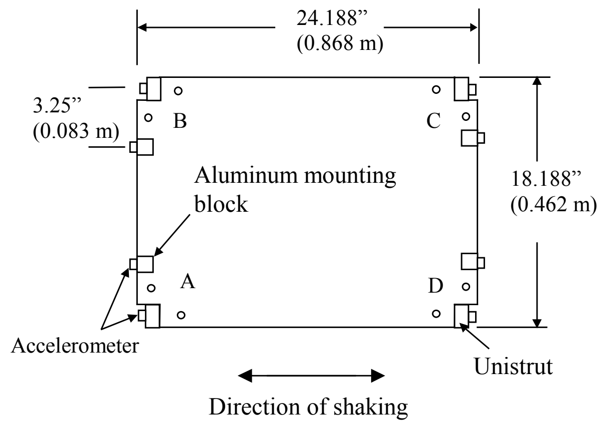

Figure 2: floor layout as viewed from above

Table 1: Damage classification results using SPRT-N and SPRT-G (in parenthesis)

Test Case	Ch1-2	Ch3-Ch4	Ch5-Ch6	Ch7-Ch8	Ch9-Ch10	Ch11-Ch12	Ch13-Ch14	Ch15-Ch6	Ch17-Ch18	Ch19-Ch20	Ch21-Ch22	Ch23-Ch24
Damage 1	0*	0	0	0	0	0	0	0	1(1)	0	0	0
	0	0	0	0	0	0	0	0	1(1)	0	0	0
	0	0	0	0	0	0	0	0	1(1)	0	0	0
	0	0	0	0	0	0	0	0	1(1)	0	0	0
	0	0	0	0	0	0	0	0	1(1)	0	0	0
Damage 2	0	0	0(1)	1	0	0	0	0	0	0	0	0
	0	0	0(0)	1	0	0	0	0	0	0	0	0
	0	0	0(0)	1	0	0	0	0	0	0	0	0
	0	0	0(1)	1	0	0	0	0	0	0	0	0
	0	0	0(1)	1	0	0	0	0	0	0	0	0

*The zero '0' denotes that the null hypothesis is accepted indicationg no damage is present at that joint, and the unity '1' denotes that the null hypothesis is rejected and the corresponding joint is damaged. The shaded areas represent the locations of the acutally damaged joints, and the hypothesis results in these shadded ares should ideally correspond to 1. The hypothesis results should be zero otherwise. For each undamaged and damage cases, five time series are recorded, and the corresponding damage classification results are shown.