LA-UR- 02-0863

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Title: Assessing Performance and Validating Finite Element Simulations Using Probabilistic Knowledge

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Submitted to: World Automation Conference
Orlando, FL
June 9-13, 2002

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ASSESSING PERFORMANCE AND VALIDATING FINITE ELEMENT SIMULATIONS USING PROBABILISTIC KNOWLEDGE

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ABSTRACT

Two probabilistic approaches for assessing performance are presented. The first approach assesses probability of failure by simultaneously modeling all likely events. The probability each event causes failure along with the event's likelihood of occurrence contribute to the overall probability of failure. The second assessment method is based on stochastic sampling using an influence diagram. Latin-hypercube sampling is used to stochastically assess events. The overall probability of failure is taken as the maximum probability of failure of all the events. The Likelihood of Occurrence simulation suggests failure does not occur while the Stochastic Sampling approach predicts failure. The Likelihood of Occurrence results are used to validate finite element predictions.

KEY WORDS: Likelihood-of-Occurrence, Stochastic Sampling, Model Validation, and Probability Theory.

PROBLEM DESCRIPTION

A metal encased explosive charge is placed inside a hollow containment vessel as shown in Figure 1. Following detonation, metal fragments travel outward impacting with the vessel wall. Fragment geometry, orientation, and velocity, and the minimum velocity necessary for a given fragment geometry to perforate the vessel thickness are the parameters dominating failure.[1] The two approaches used to assess the safety of the containment vessel follow the same logical framework. They first determine probable fragment geometries. Then for each independent fragment, the probability it is properly orientated is computed. Next, the probability properly orientated fragments travel with sufficient velocity to perforate the containment vessel is ascertained. From these calculations, the system's probability of failure is assessed. Each approach assesses failure differently.

Theory and empirical evidence is used to generate the distributions shown in Figures 2, 3, 4, and 5. Fragment thickness is assumed constant and diameter is the smaller of either non-thickness dimension. Mott [2] developed equations for predicting fragment mass and geometry. The probability a fragment has mass M is P(M). The probability a fragment has diameter D is P(D). Given a fragment has mass M and diameter D, the conditional probability it has a given length to diameter ratio is, P(L/D|D). The probability a given fragment occurs is expressed as

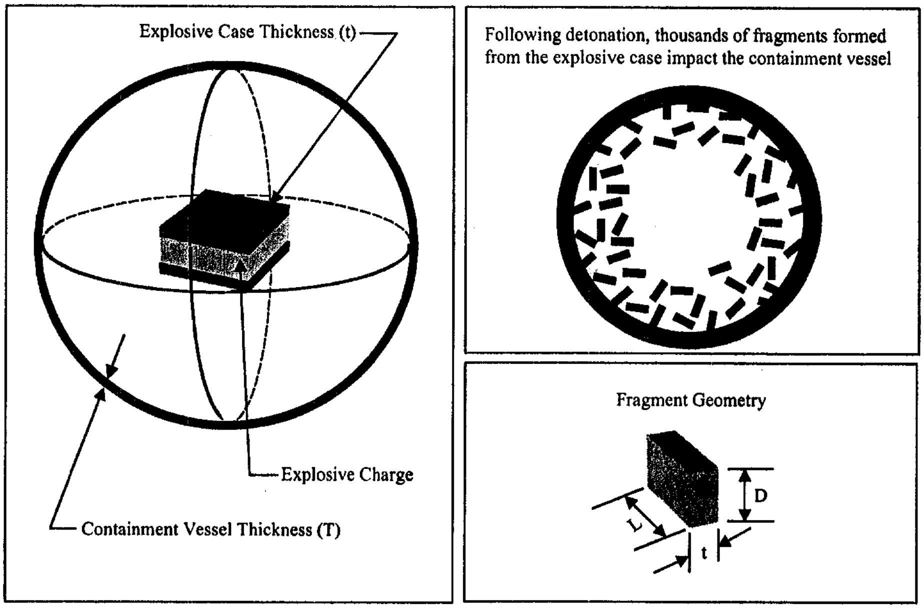

Figure 1: Schematic of experimental set up and fragment geometry.

$$P(fragment) = P(L/D \mid D) \cdot P(D) = P(L/D \mid D) \cdot P(D \mid M) \cdot P(M) \quad (1)$$

Combining theory with engineering judgment, a velocity distribution is generated [3]. Each unique fragment has a critical velocity (v_{cr}) necessary to perforate. When a fragment travels faster than it's critical velocity it is a candidate for failure. Theory predicts the maximum probable velocity is 2 km/s and the minimum probable velocity is 1.4 km/s. Engineering judgment is that these values are three sigma events. A cumulative distribution is used to assess the likelihood a fragment travels with at least some specified velocity. This distribution does not take into account velocity's complex dependency on geometry. However, making velocity independent provides conservatism because it allows larger heavier fragments to be assessed with higher velocities than they actually achieve.

As a fragment travels toward the confinement vessel it spins and tumbles. Hydrodynamic calculations predict the critical angle of attack a fragment's leading edge must be within in order for perforation to occur is α=20 radians. Because this requirement exists for both pitch and yaw rotations the critical attack angle forms a cone. Since all fragment orientations are equally likely a uniform distribution is used allowing pitch and yaw rotations to be combined. The amount of vessel material the fragment has to penetrate for perforation to occur is minimal when α is perpendicular to the vessel wall, hence, treating all impacts within the α-cone as normal is conservative and can be represented deterministically [4]. The probability of being within the critical attack angle, P(α), is

$$P(\alpha) = \sum P(\Phi_i)\big|_{\alpha \geq \Phi \geq -\alpha} = P(\alpha \geq \Phi \geq 0) + P(0 \geq \Phi \geq -\alpha) = \frac{2\alpha}{\pi} \quad (2)$$

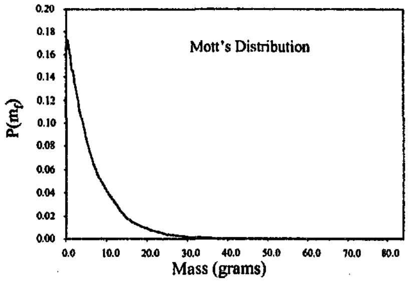

Figure 2. Probability Distribution for Mass.

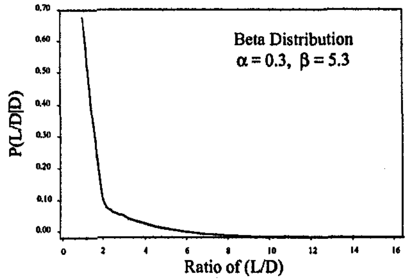

Figure 3. Conditional Probability Distribution for P(D/L | D).

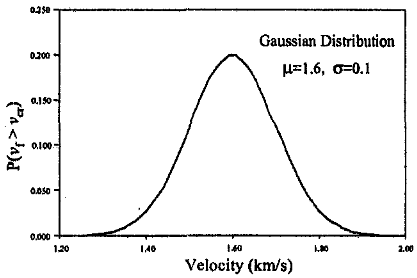

Figure 4. Probability Distribution for Velocity.

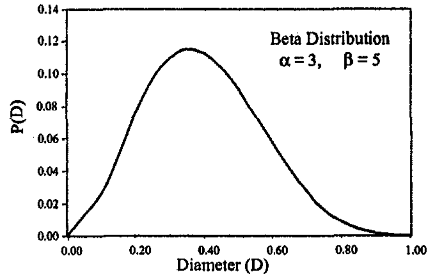

Figure 5. Probability Distribution for Diameter

LIKELIHOOD OF OCCURRENCE ASSESSMENT MODEL

Knowing the probability a given fragment perforates the vessel is of marginal utility without additionally knowing the likelihood the fragment occurs. Additionally, knowing the impact of a single fragment provides only limited insight into the probability the containment vessel survives an experiment. To determine a vessel's overall probability of failure the contributions of each fragment event is assessed. The Likelihood of Occurrence (LoO) model assesses all fragments simultaneously. The amount of explosive coupled with the mass and shape of the explosive casing account for the number of the fragments generated, their probable geometries, and their velocities. Many fragments possess similar geometry and can be grouped into discrete fragment configurations. The LoO model considers a 4x10 matrix of discrete Dx(L/D) fragment configurations. For each fragment configuration the number of fragments likely to be generated is the number of such fragments that could be formed from the case times the probability such fragments could exits. This can be expressed as

$$\#fragments = \left[\frac{m_{case}}{m_{fragment}}\right]\cdot\left[\frac{P(M)}{\sum_{i=1}^{\#dg} P_i(M)}\right]\cdot P(L/D\,|\,D)\cdot P(D) \tag{3}$$

The number of fragments likely to be within the α-cone orientation is

$$\#candidates = P(\alpha) * \#fragments = \frac{2\alpha}{\pi}\cdot\left[\frac{m_{case}}{m_{fragment}}\right]\cdot\left[\frac{P(M)}{\sum_{i=1}^{\#dg} P_i(M)}\right]\cdot P(L/D\,|\,D)\cdot P(D) \tag{4}$$

The P(M) is normalized w.r.t. the number of discrete fragment geometries. Perforation occurs if these candidate fragments move at or above the minimum speed a fragment of dimensions LxD has to travel to perforate a target of thickness T, [4]. The probability a fragment perforates the containment vessel is expressed as

$$P(perforation)_{L,\,L/D} = \#candidates \cdot P(v_{fragment} > v_{cr}) \tag{5}$$

The containment vessel's overall probability of failure is the sum of the discrete fragment configuration probabilities of perforation. For N=40 discrete fragment configurations the overall probability of failure becomes

$$P(failure) = \sum_{i=1}^{N} P_i(perforation) = 1.23x10^{-13} \tag{6}$$

This probability of failure represents a cumulative rather than maximum value. The LoO model treats all fragment events as happening simultaneously. This approach is akin to a Bayesian probability problem using the Law of Total Probability, [5]. The method of maximum likelihood was used to obtain a point estimate for the (L/D) (T/L) ratios most likely to be involved in a perforation, [6]. The fragment most likely to perforate the containment vessel has D =0.75cm and L=11.75cm. This fragment has a probability of perforation of $1.614x10^{-14}$.

STOCHASTIC SAMPLING ASSESSMENT MODEL

The Stochastic Sampling (SS) model assesses probability of failure by determining the frequency at which fragment penetrations occur. The primary difference between the SS model and the LoO model is the SS model defines an event as a single fragment response. The logic is modeled using an influence diagram. The stochastic sampling model uses Latin Hyper-Cube to randomly select different combinations of D and (L/D), [7]. The model assumes all fragment orientations are equally probable and fragments whose angle of attack is within the α-cone angle are assumed perpendicular to the containment vessel wall. The SS model's approach is analogous to reaching into a bin of randomly shaped fragments and throwing them one at a time with random velocity and then counting how many fragments perforate. As the number of samples increases the mean response approaches the true mean. However, what happens in the tail regions is more difficult to determine and it is the tail region that drives failure estimates.

TABLE 1. RESULTS OF FORTY UNIQUE STOCHASTIC SAMPLING SIMULATIONS MEASURED WITH RESPECT TO THEIR WORST PROBABLE POINTS (WPP).

Minimum WPP	Maximum WPP	Mean of WPP $\overline{x}$	Standard deviation s	WPP upper 95% confidence point
1.35E-09	2.96E-09	2.57E-09	3.42E-10	3.25E-09

One way to measure the probability of failure is to take the maximum probability of perforation for all the individual fragments. A shortcoming of this Worst Probable Point (WPP) measure is it does not take into account net contributions from other less probable fragment events. Nonetheless, the WWP, along with the 95% confidence value were used. The stochastic process converged at thirty thousand events using a Minimal Standard Randomizer method and a fixed seed. Table 1 shows the results of forty simulations using different sampling seeds. Each simulation used 30,000 stochastic samples, [4]. For the forty simulations 95% of the maximum WPPs are less than or equal to 3.25E-9.

COMPARING RESULTS OF THE TWO MATHEMATICAL MODELS

While knowing the probability of failure is paramount to decision-making, having a measure of the overall quality of the containment vessel relative to performance is also useful. Margin and Safety are industry measures for the "goodness" of an engineered product relative to performance requirements. Generally based in statistical process control, margin is used to convey how near a product is to having a significant decrement in performance, [8]. Safety assessments invoke margin by measuring performance relative to acceptable risk, [9]. Margin and Safety provide a way to compare results of the two assessment methods. The capability ratio (Cp), is a ratio of the allowable performance spread to the actual performance spread relative to the statistical mean and standard deviation of the assessed fragment performance

$$C_p = \frac{\text{allowable performance spread}}{\text{actual performance spread}} = \frac{USL - \mu}{3\sigma} = \frac{USL - \bar{x}}{3s} \tag{7}$$

where USL is the upper statistical limit. For the LoO assessment, Cp_{LoC}=19,963, indicating a lot of margin in the vessel's performance. For the SS assessment, Cp_{SS}=7.823 which means while the bulk of the fragment events perform successfully as we approach the tail region the margin diminishes.

A product's performance can also be measured as a function of risk where risk is as a probability of failure times some perceived consequence. Determining appropriate values for failure and consequence is a matter of subjective interpretation often referred to as establishing risk criteria, Wirsching (1992). Since failure of the containment vessel is defined as a single fragment perforation, the desired probability of failure is the same for a single fragment event as it is for the entire system, i.e., $p_0 = p_{0LoO} = p_{0SS} = 1\text{x}10^{-9}$. This results in a desired safety index of 5.998. The LoO assessment has a computed safety index of 7.32, indicating the containment vessel is safe. The SS assessment has a computed safety index of 5.85, which is slightly less than the desired safety index indicating the containment vessel is slightly unsafe.

USING RESULTS TO VALIDATE FINITE ELEMENT PREDICTIONS

The probabilistic assessments can be used to validate finite element (FE) predictions. While the probabilistic models assess performance within domains of empirical knowledge and expert judgment, there remain other domains of interest. The goal is to use probabilistic models to validate FE predictions within domains of existing knowledge and once validated, use FE predictions to assess performance outside the domain of existing knowledge. With this strategy in mind, several issues immerge. For instance, how are the results of a numerical prediction to be compared with probabilistic predictions? How accurate are the predictions made by a FE tool outside the domain for which it was validated? While many other issues remain, only these two are considered here.

The LoO assessments are based on physical test information and can be used to validate the accuracy of the numerical tool for discrete parameter scenarios. The FE analysis predicts if a given fragment geometry traveling with a specific velocity and orientation perforates the vessel. The results are deterministic – the fragment either does or doesn't perforate. The FE predictions were in good alignment with the probabilistic assessments. However, the FE analysis cannot be used to say if the vessel is safe for operation without knowing the likelihood of occurrence for any failure prediction. The fact that a perforation may be predicted in an FE analysis is really only one piece of information necessary to make a performance assessment. For example, a fragment having dimensions D=0.5cm and L=8cm is predicted to perforate when traveling at a velocity of 1.5km/s. However, this fragment has only a 2.58×10^{-9} probability of occurring and when it does occur it needs to travel at 10km/s to perforate. To conclude from the numerical analysis that the vessel is unsafe would be an overstatement, highlighting the need to think beyond simply a numerical result.

CONCLUSIONS

Two different approaches were presented for probabilistically assessing a product's performance. The LoO method considered all probable fragments simultaneously assessing their combined contributions to failure. The SS method found the fragment event having the highest probability of failure. The LoO model assessed the vessel to be safe for use while the SS model assessed the vessel to be slightly unsafe. Results from the LoO assessment were used to validate FE predications. While the responses were consistent more comparisons are necessary before the FE code can be declared validated.

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