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Title: PEAK DISPLACEMENTS AND INTERSTORY DRIFTS OF
NONLINEAR MULTI DEGREE OF FREEDOM SYSTEMS
USING PRINCIPAL COMPONENT ANALYSIS

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PEAK DISPLACEMENTS AND INTERSTORY DRIFTS OF NONLINEAR MDOF SYSTEMS USING PRINCIPAL COMPONENT ANALYSIS

I. Cuesta¹ and M. A. Aschheim²

ABSTRACT

Principal Components Analysis (PCA) is a method to extract the principal components (or modes) of response from recorded or computed response data, of systems exhibiting linear and/or nonlinear response. For linear systems, the PCA mode shapes coincide with the elastic mode shapes, if the nodal mass is uniformly distributed. For nonuniform mass distributions, the PCA modes are related to the elastic modes. The PCA technique is particularly valuable when applied to systems responding nonlinearly, because it identifies the "predominant mode" of response and the degree to which the response is in this mode.

This paper illustrates the use of the PCA technique for estimating floor and interstory drifts for a 12-story moment-resistant frame responding to earthquake ground motions. Linear and nonlinear responses are considered, and the observed mode shapes and the accuracy of drift estimates are discussed. The interaction of modal amplitudes in time is considered in detail. The peak roof drift and interstory drifts are expressed as linear combinations of the PCA modes, and are represented graphically, together with the observed interaction response. A technique is described to determine peak values of these quantities by maximizing the drift functions relative to the observed modal interactions.

Introduction

The large amounts of data generated in the nonlinear dynamic analysis of multi-degree-of-freedom (MDOF) systems has made it difficult to characterize the behavior of such systems. Investigators have used qualitative statements such as "responding predominantly in the first mode" while recognizing that the predominant mode of response may deviate from the elastic mode shape and without evaluating the degree to which the response is in a given mode. Principal Components Analysis (PCA) provides a simple tool to identify the relevant mode shapes and the degree to which the response is in each mode. The mode shapes obtained by PCA are eigenvectors that form an orthonormal basis for the response, although these modes may differ from the elastic mode shapes, and for nonlinear response, depend on the excitation. The

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PCA mode shapes are ordered in such a way that the first PCA mode shape is the shape vector that best represents the response, the second PCA mode shape is orthogonal to the first and best represents the response with the first mode removed, and so on. The corresponding eigenvalues identify the degree to which the variance in the response is represented by each mode.

PCA is used in the multivariate analysis of empirical data (see Mardia et al 1979, Duntzman 1989, and Venables and Ripley 1997). The use of PCA in earthquake-resistant structural engineering is relatively new. Wissapaisal and Aschheim (2000) and Inel (2001) established the properties of structural components based, in part, on agreement of the PCA modes determined for the structural model and those determined from the recorded responses of an instrumented bridge. Gutiérrez and Zaldivar (2000) independently applied PCA to data collected in pseudo-dynamic and shake table tests.

Theory of Principal Components Analysis

This section describes the basics of PCA and then introduces extensions required to relate the PCA modes to the elastic modes for systems having nonuniform nodal masses. While the technique is general and may be applied to any response quantity (e.g. velocities, accelerations, interstory drifts, etc), the derivation assumes displacement data will be used.

Let \mathbf{v} be an $n \times 1$ vector representing the displacements at n degrees of freedom at an instant of time. There are t observations of \mathbf{v} over the duration of response data. The deviation of \mathbf{v} from its mean over t observations, $\bar{\mathbf{v}}$, can be expressed in terms of a new orthonormal basis Φ :

$$\mathbf{v} - \bar{\mathbf{v}} = \Phi \mathbf{u} \quad (1)$$

Because Φ is orthonormal, $\Phi^T \Phi = I$, and therefore, $\Phi^T = \Phi^{-1}$. Premultiplying by Φ^T gives

$$\mathbf{u} = \Phi^T (\mathbf{v} - \bar{\mathbf{v}}) \quad (2)$$

with the mean of \mathbf{u} over the t observations being the $n \times 1$ vector $\mathbf{0}$. Let the covariance of \mathbf{v} be represented by the $n \times n$ covariance matrix \mathbf{C}_v . The (i, j) element of \mathbf{C}_v is the covariance between the i^{th} and j^{th} degrees of freedom, v_i and v_j , over the t observations, given by

$$\text{cov}(v_i, v_j) = \frac{1}{t} \sum_{k=1}^t (v_{i,k} - \bar{v}_i)(v_{j,k} - \bar{v}_j) \quad (3)$$

Standard identities applied to Eq. 2 allow the covariance of \mathbf{u} , \mathbf{C}_u , to be expressed as

$$\mathbf{C}_u = \Phi^T \mathbf{C}_v \Phi \quad (4)$$

If the orthonormal basis Φ is selected to be the set of eigenvectors of \mathbf{C}_v , then \mathbf{C}_u is a diagonal matrix, with each term on the diagonal equal to the eigenvalue that corresponds to an eigenvector of \mathbf{C}_v .

By convention, the eigenvectors of Φ are arranged in sequence such that the corresponding eigenvalues are in descending order ($\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$). Because \mathbf{C}_u is diagonal, the displacements u_i and u_j (expressed in terms of the orthonormal basis Φ) are uncorrelated for $i \neq j$. One can show that: (1) The eigenvectors contained in Φ are the principal components of the displacement response \mathbf{v} (that is, ϕ_1 is oriented to maximize the variance; ϕ_2 maximizes the variance with ϕ_1 removed, and so on), and (2) the variance in the displacement response

represented by each eigenvector is proportional to its eigenvalue; that is, each component represents $\lambda_i / \text{tr}(\Lambda)$ of the total variance in displacement response. For linear elastic systems with uniform nodal masses, the application of PCA to displacement response data determines PCA mode shapes (the eigenvectors of Φ) that coincide with the elastic mode shapes, provided that the data samples are frequent enough and over a sufficiently long duration.

Because the eigenvectors are sequenced according to their eigenvalues, the most efficient representations of ν using k eigenvectors will be obtained by selecting the first k eigenvectors:

$$\nu(t) - \bar{\nu} \approx \sum_{i=1}^k \phi_i u_i(t) \quad (5)$$

The mean $\bar{\nu}$ can be expressed in terms of the mode shapes, as $\bar{\nu}_i = \phi_i \phi_i^T \bar{\nu}$, resulting Eq. 5 into

$$\nu(t) \approx \sum_{i=1}^k \phi_i u_i(t) = \sum_{i=1}^k \phi_i \phi_i^T \nu(t) \quad (6)$$

Because the principal components are orthogonal, the coefficients u_i are invariant with changes in k , and their values are determined by the simple dot product $u_i(t) = \phi_i^T \nu(t)$. The number of eigenvectors to be used, k , depends on the desired level of precision; inspection of Λ reveals the benefit of using additional components in Eq. 6.

Principal Components in the Response of a 12-story Building

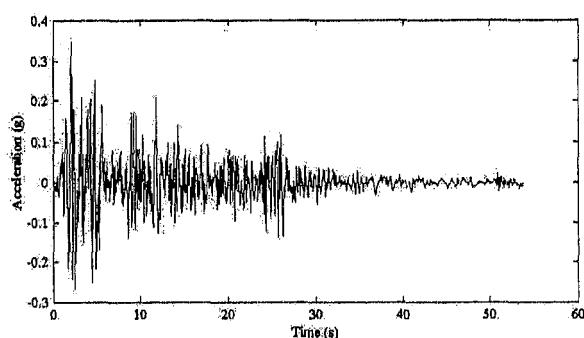


Figure 1. The El Centro record (1940).

The theory of principal components analysis was used to study the dynamic response of a 12-story moment-resistant frame building subjected to the North-South El Centro record (5/18/1940). The acceleration ground history of this record is plotted in Fig. 1. To obtain linear and nonlinear responses, the amplitude of the acceleration ground motion was scaled by factors of 1 and 4, respectively.

Building Description and Modeling

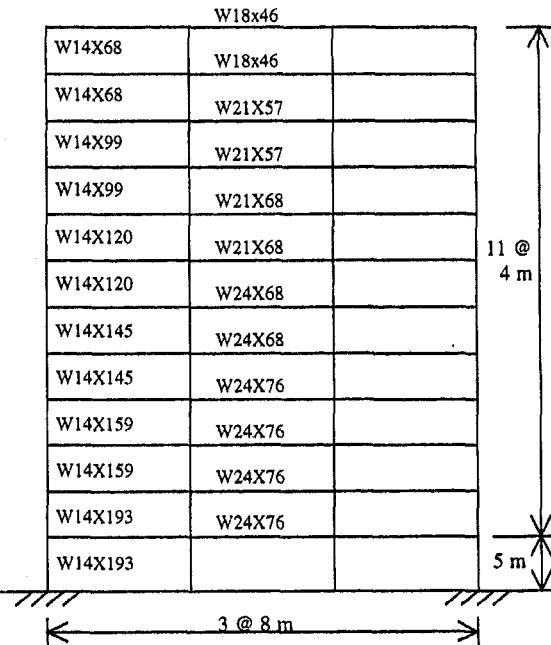
Figure 2 shows the 12-story moment-resistant steel frame building used in this study. The building was designed by Black (2000) to illustrate a design approach for limiting drift. The building has 3 bays, uniform floor masses, with a total weight of 6,612 kN. The steel is Grade A36. Non-linear pushover analysis was done by applying lateral forces in proportion to the mode shapes, and bilinear curves were fit to the capacity curves following Chopra and Goel procedure (Chopra and Goel, 2001). The bilinear curves have post-yield stiffness equal to 13.6% and 12.2% of the initial stiffness, yield displacements of 36.3 and 10.8 cm, and base shear yield coefficients of 0.178 and 0.141, for the 1st and 2nd modes, respectively.

Displacement Response

The building has uniform floor masses. PCA mode shapes were computed for the floor displacements (relative to the base), using 50 sec of response data at 0.01 s intervals. The first and second PCA mode shapes are compared with the first two elastic modes in Figure 3. While the mode shapes coincide for the 1x El Centro case, some deviation in the mode shapes appears for nonlinear response (4x El Centro). The variance of the displacement response explained by each mode shape, $\lambda_i / \text{tr}\Lambda$ and the cumulative variance are given in Table 1. For linear response, 96.79% of the variance in displacement response is represented by the 1st mode, while 93.4% of the variance is represented by the 1st PCA mode for the nonlinear case. The first two PCA modes account for more than 99% of the variance for both cases. The PCA mode shapes are oriented to best represent the response, but because the elastic mode shapes are similar in this example, they would be almost as good. Because such a large percentage of the variance in the displacement response is represented by the first mode, an equivalent SDOF system based on this mode can be expected to provide a reasonably accurate estimate of the displacement response, for both linear and nonlinear cases.

PCA can be applied to smaller intervals of data to better capture the variation of the mode shapes with time (although the reduced data sample affects their resolution). Figure 4 shows the PCA mode shapes obtained using a 5-sec window advanced over the displacement data in 1-sec increments, in dashed lines, with the elastic mode shapes shown in solid lines. For both cases, the PCA mode shapes appear as perturbations about the elastic modes. Associated with the moving window mode shapes are moving window eigenvalues; Fig. 5 plots the cumulative proportion of variance of the first mode, the first two modes, and the first three modes. Three regions may be clearly distinguished in both the linear and nonlinear cases. In the first region, for 5-sec windows that start between 0 and 25 sec, the 1st and 2nd modes are appreciable; between 25 and 38 sec, the first three modes contribute to the response, and beyond 38 sec, the response is nearly all in the 1st mode.

Figure 6 plots the drift profile for the two cases at the instant that the peak roof displacement is reached, using a solid line. Approximations of this drift using Eq. 5 with $k = 1$ and 2 are shown as "PCA 1" and "PCA 1+2" using dashed and dotted lines. Estimates of this drift using either the first or the first two elastic mode shapes according to the "Modal Pushover Analysis" method (Chopra and Goel 2001) were reasonable for the elastic case, but led to significant disparities in the nonlinear case. The MPA estimates were made using "equivalent" SDOF oscillators, having properties determined from the pushover curve according to the investigator's recommendations, subjected to the scaled accelerogram, and using the modal damping present in the nonlinear



Note: all columns and beams within a story are identical

Figure 2. Framing configuration.

Because such a large percentage of the variance in the displacement response is represented by the first mode, an equivalent SDOF system based on this mode can be expected to provide a reasonably accurate estimate of the displacement response, for both linear and nonlinear cases.

dynamic analyses. Values are given in Table 2.

If, as an approximation, just the first two modes are considered, then simple graphical techniques can be used to gain insight into the components that contribute to floor displacements. Figure 7 plots the interaction between the normalized amplitudes u_1/u_{1m} and u_2/u_{2m} where u_{im} are the peak displacements, $u_{im} = \max|u_i(t)|$, of the first and second modes, over time, for the linear and nonlinear cases. Using Eq. 6 with $k = 2$, the displacement of floor j is estimated $v_j(t) = \phi_{j1}u_1(t) + \phi_{j2}u_2(t)$. Therefore, the displacement of floor i , v_i , can be expressed as a line in the space u_1/u_{1m} vs u_2/u_{2m} , $c_j = \phi_{j1}u_1(t) + \phi_{j2}u_2(t)$, where c_j is a constant. The slope of this straight line is given by $\partial u_2 / \partial u_1 = -\phi_{j1} / \phi_{j2}$. The angle of constant drift is $\alpha_j = \arctan(-\phi_{j1} / \phi_{j2})$, which is only function of the PCA mode shape components. Rearranging coefficients, the line of constant drift in Fig. 7 is:

$$\frac{c_j}{\underbrace{\phi_{j1}u_{1m}}_{b_j}} = \frac{u_1(t)}{\underbrace{u_{1m}}_x} + \frac{u_2(t)}{\underbrace{u_{2m}}_y} \cdot \left(\frac{\phi_{j2}u_{2m}}{\phi_{j1}u_{1m}} \right) \quad (7)$$

Therefore, in a X-Y coordinate system, the straight lines with constant slopes are given by $\partial y / \partial x = -1/a_j$. The displacement has a maximum that intercepts this line (solid lines), as shown in Fig. 7. Different floors will have different coefficients c_j , and thus the slopes of the lines that represent v_j vary with j . It is thus possible to represent the peak displacements of many floors as the intersections of lines having different slopes with the interaction data shown in Fig. 7. Estimates of peak floor displacements in future earthquakes depend on (a) being able to identify appropriate mode shapes and (b) being able to estimate the interaction surface. Investigations to date indicate that the interaction surfaces can vary substantially from an ellipse (the shape assumed in SRSS approximations) and that estimates of modal peak responses are prone to significant scatter. Mathematical extensions to three or more modes may be developed, but the small contributions of the third and higher modes to the variance in displacement response suggest that there is limited benefit in doing so.

Interstory Drift Index Response

In this section, PCA is applied in two different ways to characterize interstory drift. In the first, interstory drift is calculated by superposition of the interstory drifts associated with the PCA mode shapes, using the PCA mode shapes that were determined for the floor displacement response data. In the second, PCA mode shapes are determined for the interstory drift data, with the interstory drifts being determined as differences in the floor displacement response data.

Interstory Drift as Differences of the PCA Mode Shapes

As discussed previously, the PCA mode shapes of the displacements, Φ , resemble or coincide the elastic mode shapes. Interstory drifts for the j^{th} story associated with each mode

shape can be computed as

$$\Delta_j(t) = \gamma_{j1}u_1(t) + \gamma_{j2}u_2(t) \quad (8)$$

where the drift mode shapes are determined with the PCA displacement mode shapes: $\gamma_{ij} = \phi_{ij} - \phi_{ij-1}$. The parallel structures of Eqs (6) and (8) indicate that the expression for constant interstory drift plots as a line having slope $\alpha_j = \arctan(-\gamma_{j1}/\gamma_{j2})$, on Figure 7 (dashed lines). Thus, the largest interstory drift that occurs for a given excitation can be found by maximizing the function over the domain of interaction between u_1 and u_2 , with the maximum occurring in the intersection between the corresponding straight line and the displacement curve given by the two modes. This illustrates the versatility of the plot of u_1 vs. u_2 , for estimating both floor displacements and interstory drifts, the dependence of the maxima of the quantities on the actual interaction, the fact that the maxima occur at different times (represented by different intersection points), and the difficulty of (a) anticipating the shape of the interaction surface *a priori* and (b) the need for accurate estimates of the individual modal peaks if estimates of these quantities are to be made.

Principal Components of Interstory Drift

The second approach taken is to estimate the interstory drift, as $\Delta_j = \phi_{j1}z_1(t) + \phi_{j2}z_2(t)$, where $z_i(t) = \Delta(t)\varphi_i$, $\Delta(t)$ is the computed drift using Drain-2DX, and φ_i are the mode shapes obtained from the eigenvalue problem $C_\Delta\varphi = \Lambda\varphi$, where C_Δ is the covariance matrix of $\Delta(t)$. The slope of constant drift for each story j is given now by $\alpha_j = \arctan(-\phi_{j1}/\phi_{j2})$.

The PCA mode shapes, determined for 50 sec of response data at 0.01 sec intervals, are plotted in Fig. 8 and compared to the interstory drifts calculated from the elastic modes. The elastic and PCA mode shapes are different for both the linear (1 x El Centro ground motion) and the nonlinear (4 x El Centro) cases. The variance of the interstory drift index explained by each mode shape, $\lambda_i/tr\Lambda$ and the cumulative variance, $\sum \lambda_i/tr\Lambda$, are given in Table 1 for the first three mode shapes and for the two cases studied. The first mode contains 75.87% of the variance for the linear case, and 57.12% for the nonlinear case. The cumulative variance when the 1st and 2nd modes are added increases to more than 88%. The smaller variances obtained for interstory drifts, relative to the variances obtained for the displacement data, indicate that interstory drift estimates are prone to be less precise, particularly as inelasticity develops, for a given level of approximation (k).

Figure 9 shows the PCA mode shapes obtained with a 5-s window data in 1-s increment in dotted lines, and the elastic mode shapes in solid lines. In both cases, the PCA mode shapes are seen as variations around the elastic mode. This variation increases in the nonlinear case. Fig. 10 plots the cumulative proportion of variance, $\sum \lambda_i/tr\Lambda$, using the first mode, the first two modes, and the first three modes with largest variances, using 5-s window data in 1-s increments. For both linear and nonlinear cases the 3rd mode is relevant, and even more for the nonlinear case. The plots have two sections clearly distinguished. Between 0 and 38 s the 1st, 2nd, and 3rd modes are predominant. For times larger than 38 s, the 1st mode is only important.

Figure 11 plots the peak interstory drift indices (IDI) of each story for the linear and nonlinear cases. The peak displacements have been obtained using five procedures: 1) the "exact" IDI obtained with DRAIN-2DX program, 2) the estimates using the first principal component, 3) the estimates using the first and second principal components, 4) the estimates using the first model for the modal push-over analysis (MPA) method (Chopra and Goel 2001), and 5) the estimates using the 1st and 2nd modes for the modal push-over analysis method. While in the linear case the peak IDI are well-estimated using PCA or MPA, the peak IDI in the nonlinear case are overestimated using MPA.

Conclusions

The theory of Principal Components Analysis was described and applied to the calculation of floor displacements and interstory drifts. Based on the examples considered:

1. PCA is a useful technique for identifying the "predominant" mode of response of structures responding linearly and nonlinearly to earthquake ground motions. The mode shape and degree to which response is in this mode can be identified.
2. PCA can identify the elastic mode shapes of systems responding linearly.
3. Floor and roof displacements can be estimated based on the interaction of modal responses (Fig. 7) as the intersection of an interaction surface and a line. The shape of the interaction surface may vary significantly from an ellipse (assumed in SRSS combinations). Accurate estimates of displacement response require accurate estimates of the peak modal response amplitudes as well as presumptive assessments of the interaction surface.
4. While a single mode often is sufficient to represent over 90% of the variance in displacement response, two or three modes may be needed for similar accuracy in the estimates of interstory drift.
5. Superposition of modal responses, as described by Chopra and Goel (2001) led to reasonable estimates of displacement and interstory drift amplitudes for elastic response, but the accuracy of these estimates suffered for nonlinear response.

Acknowledgments

CAREER Award, any others?

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Table 1. Variance and cumulative variance for the first three principal components of the displacement and the interstory drift index response.

x El Centro ground motion	PCA Mode	Displacement		Interstory drift index	
		Variance (%)	Cumulative Variance (%)	Variance (%)	Cumulative Variance (%)
1 (linear)	1	96.79	96.79	75.87	75.87
	2	2.92	99.71	18.77	94.64
	3	0.27	99.98	4.73	99.37
4 (nonlinear)	1	93.40	93.40	57.12	57.12
	2	5.94	99.34	30.97	88.09
	3	0.57	99.91	8.61	96.70

Table 2. Peak roof displacements (in m) obtained with DRAIN-2DX, for the first two principal components, and using the MPA method.

x El Centro ground motion	DRAIN-2DX	PCA 1 st mode	PCA 1 st + 2 nd modes	MPA 1 st mode	MPA 1 st + 2 nd modes
1 (linear)	0.336	0.298	0.326	0.297	0.303
4 (nonlinear)	0.575	0.630	0.546	0.767	0.787

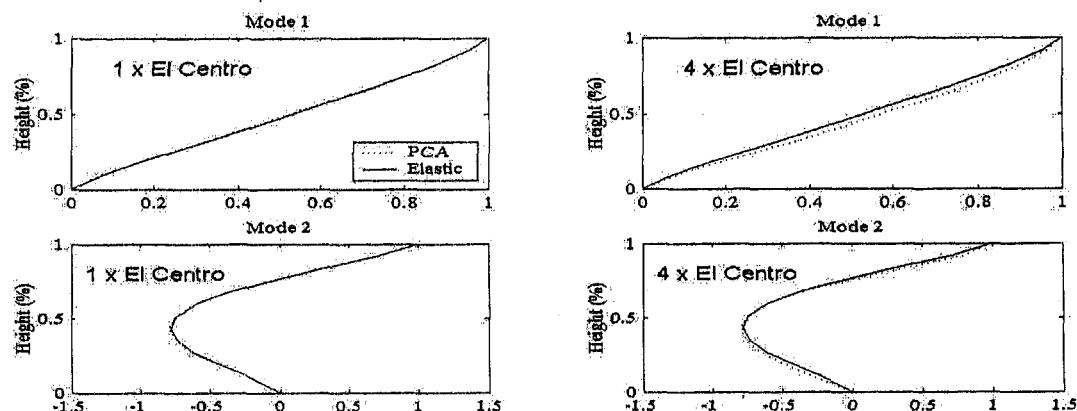


Figure 3. First and second mode shapes computed from elastic and PCA for all the data set.

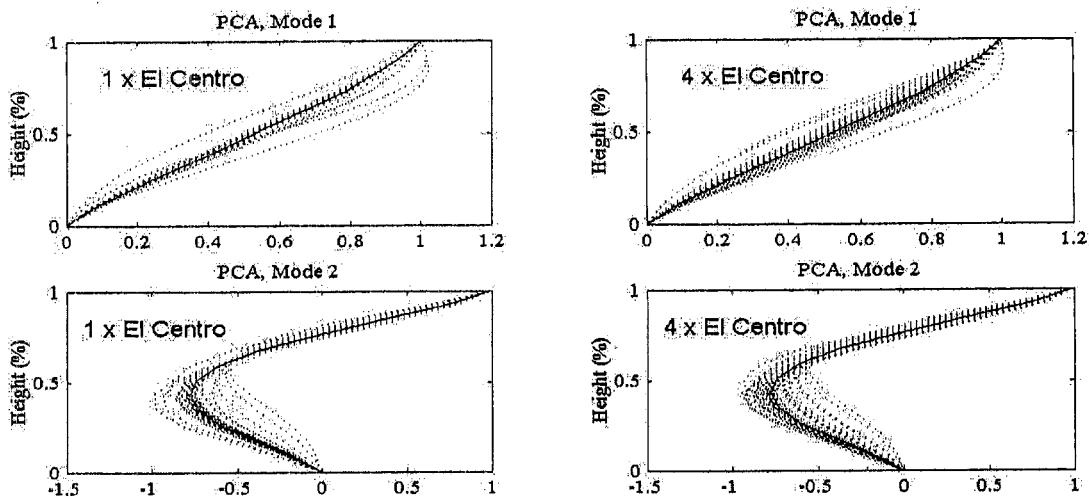


Figure 4. 1st and 2nd mode shapes computed from elastic and PCA for data sets taken in 5-s intervals.

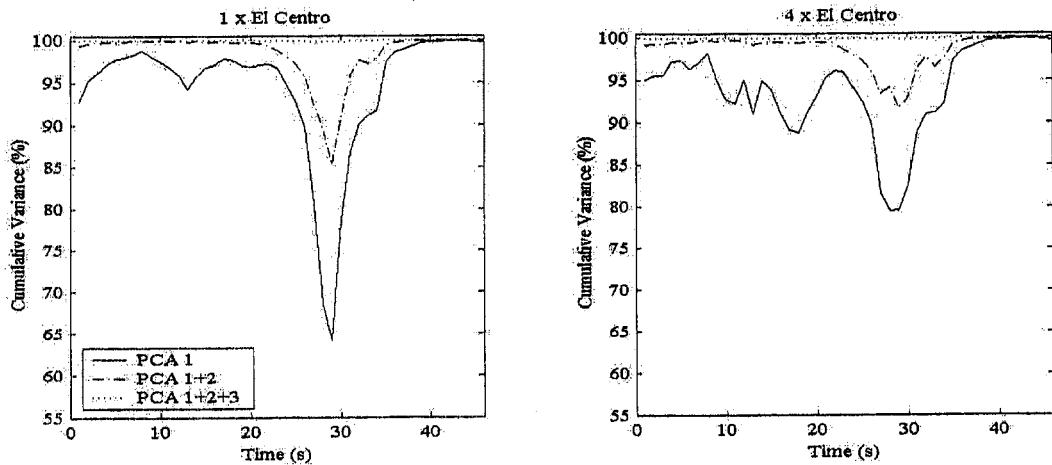


Figure 5. Cumulative proportion of variance in the displacement response represented by the first three principal components.

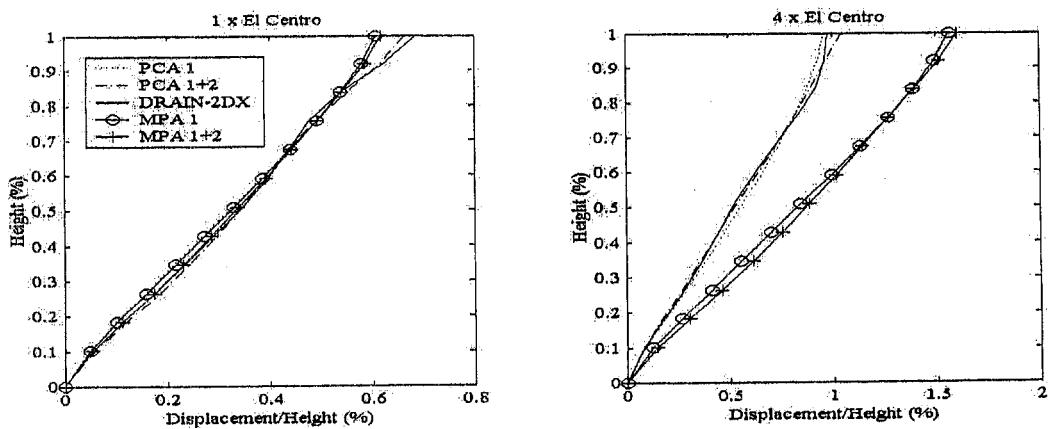


Figure 6. Comparison of the normalized peak displacement to the height of the building.

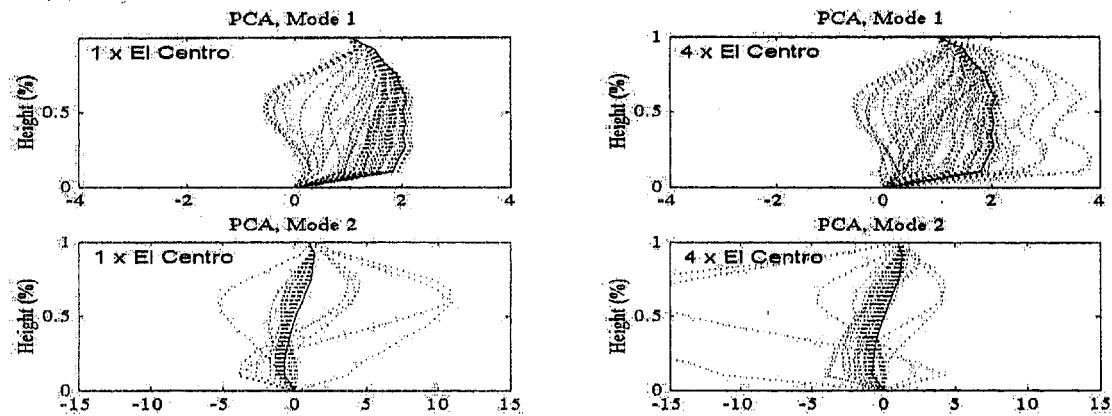


Figure 9. 1st and 2nd mode shapes computed from elastic and PCA for the interstory drift data set taken in 5-s intervals.

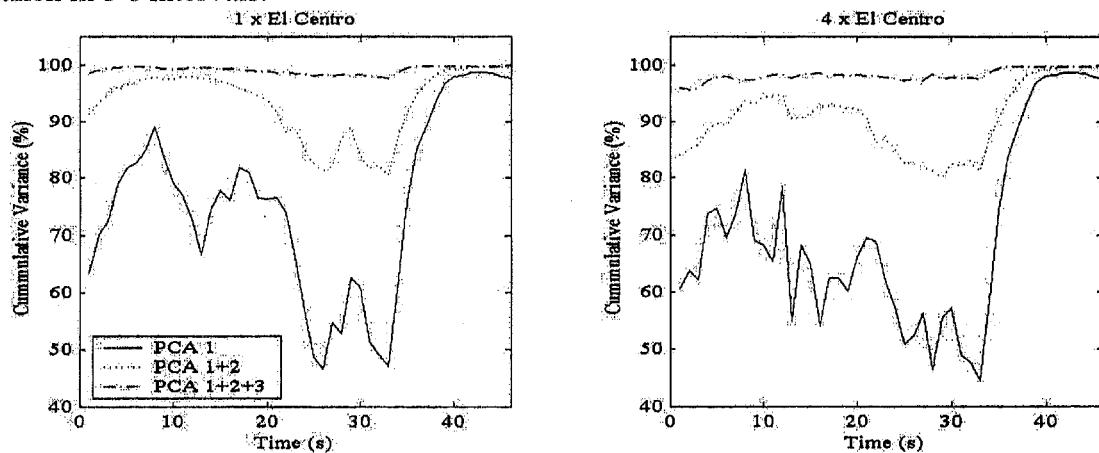


Figure 10. Cumulative proportion of variance in the interstory drift response represented by the first three principal components.

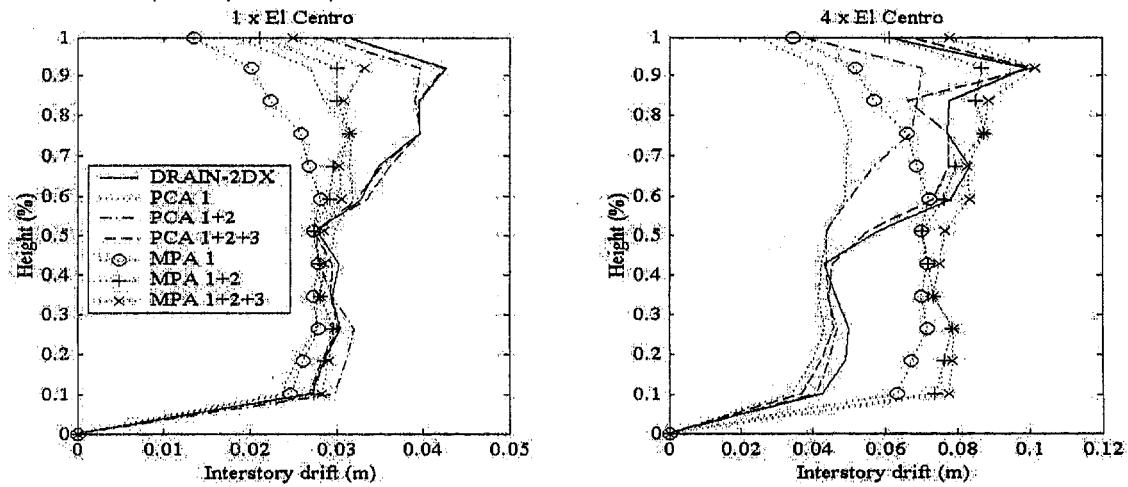


Figure 11. Comparison of peak interstory drift indices.