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**Title:** A Hybrid Monte Carlo Method for Equilibrium Equation of State of Detonation Products

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**A HYBRID MONTE CARLO METHOD  
FOR EQUILIBRIUM EQUATION OF STATE  
OF OF DETONATION PRODUCTS**

**M. Sam Shaw, T-14  
Los Alamos National Laboratory**

**2001 TOPICAL CONFERENCE  
ON SHOCK COMPRESSION OF CONDENSED MATTER**

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## MONTE CARLO SIMULATION METHODS

Benchmark for perturbation theory methods

NPT, NVT - single species or fixed composition mixture

Gibbs ensemble - phase separation

$N_a$ PT - chemical equilibrium: fluid only

Hybrid Monte Carlo Method - extension of  $N_a$ PT

solid carbon + chemical equilibrium + phase separation

'Real' explosives - calculate final EOS directly

any potential is easy

equilibrium surface chemistry of carbon clusters

benchmark on whole, not just part

higher level quantum energy -

sample from lower level distribution

Partition function separated into three parts:

0=solid, 1=fluid1, and 2=fluid2.  $V=V_0 + V_1 + V_2$ .

Classical partition function - identical atoms

$$Q(N, V, T) = \left[ \frac{V^N}{N! \Lambda^{3N}} \right] \int e^{-\beta U} d\mathbf{s}_1 \cdots d\mathbf{s}_N \quad (1)$$

scaled coordinates,  $\mathbf{s}_i$ .

Isothermal-isobaric ensemble

$$\Delta(N, P, T) = \int_0^\infty e^{-\beta PV} Q(N, V, T) dV \quad (2)$$

$A_i$  the total number of atoms of type i - fixed

${}^k M_j$  the number of molecules of type j in box k.

$$\Delta(A_1, \dots, A_I, P, T) = \quad (3)$$

$$\int \int \int e^{-\beta P(V_0 + V_1 + V_2)} \sum' Q_0 Q_1 Q_2 dV_0 dV_1 dV_2$$

which can be rewritten as

$$\int \int \sum' e^{-\beta G_c(N_c, P, T)} e^{-\beta P(V_1 + V_2)} Q_1 Q_2 dV_1 dV_2$$

where  $e^{-\beta G_c(N_c, P, T)} = e^{W_0} = \Delta_c(N_c, P, T)$

approximate analytic scheme for solid carbon

e.g. cold curve plus Debye model

$Q_k$  denotes  $Q_k({}^k M_1, \dots, {}^k M_J, V_k, T)$

The prime on the summation -

only those sets of molecules with  $A_i$ 's conserved (over 3 boxes)

Include distinguishability, mass, internal degrees of freedom  $q_j$

$$Q_k({}^k M_1, \dots, {}^k M_J, V_k, T) = \quad (4)$$

$$\frac{V_k^{M_k} q_k^{{}^k M_1} \cdots q_k^{{}^k M_J}}{{}^k M_1! \cdots {}^k M_J! \Lambda_1^{3^k M_1} \cdots \Lambda_J^{3^k M_J}}$$

$$\times \int e^{-\beta U_k} d^k \mathbf{s}_1 \cdots d^k \mathbf{s}_{M_k}$$

$M_k$  - total number of molecules in box k

$U_k$  total potential energy of the molecules in box k

$$\Delta(P, T) = \int_0^\infty \int_0^\infty \int \int \sum' \left[ e^W dV_1 dV_2 \right]$$

$$d^1 \mathbf{s}_1 \cdots d^1 \mathbf{s}_{M_1} d^2 \mathbf{s}_1 \cdots d^2 \mathbf{s}_{M_2} \right] \quad (5)$$

$$W = -\beta G_c(N_c, P, T) - \beta[U_1 + PV_1] - \beta[U_2 + PV_2]$$

$$+ M_1 \ln V_1 + M_2 \ln V_2 \quad (6)$$

$$+ \sum_{j=1}^J \sum_{k=1}^2 \left[ {}^k M_j (\ln q_j - 3\Lambda_j) - \ln ({}^k M_j !) \right]$$

Monte Carlo simulation -

Markov chain with a limiting distribution proportional to  $e^W$

Accept trial move from r to s with probability:

$$P_{r \rightarrow s} = \text{Min}[1, \exp(W_s - W_r) p_{s \rightarrow r} / p_{r \rightarrow s}] \quad (7)$$

$p_{r \rightarrow s}$  is the unweighted probability of a move from r to s.

Four types of moves:

Position moves -  $-\beta U$  terms and  $p_{r \rightarrow s} = p_{s \rightarrow r}$ .

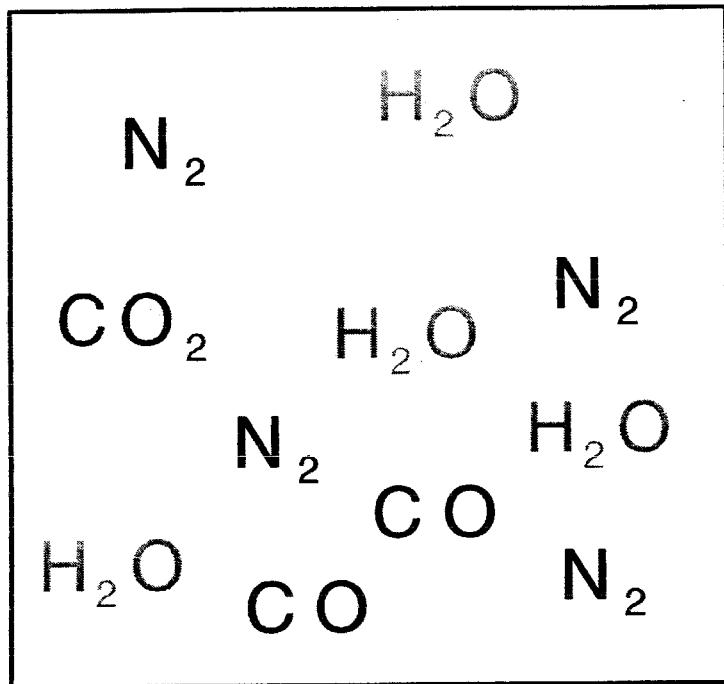
Volume moves - uniform scaling

$p_{r \rightarrow s} \neq p_{s \rightarrow r}$  - count the number of ways to choose particles

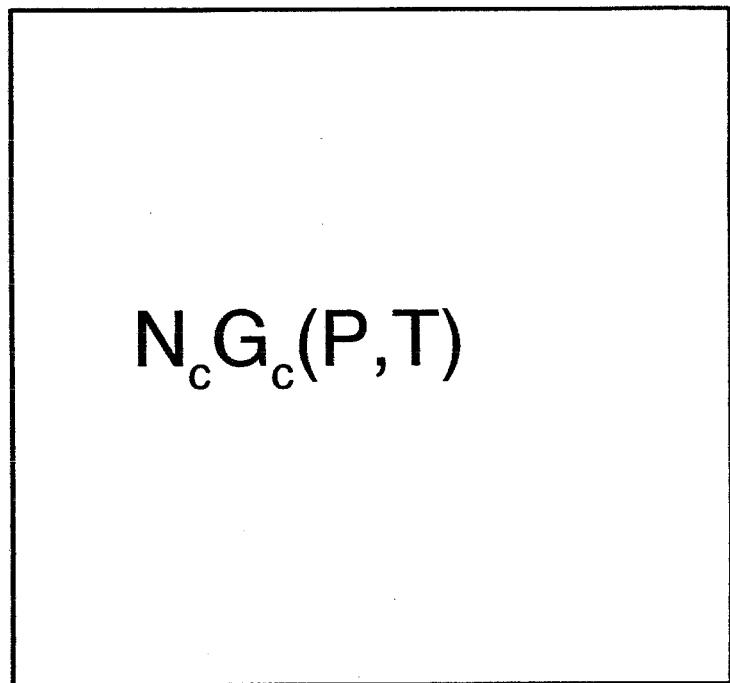
Between fluids - interchange particles or move a particle from one box to another

Chemistry moves - generalizations of the  $N_{atoms}PT$  ensemble to include solids.

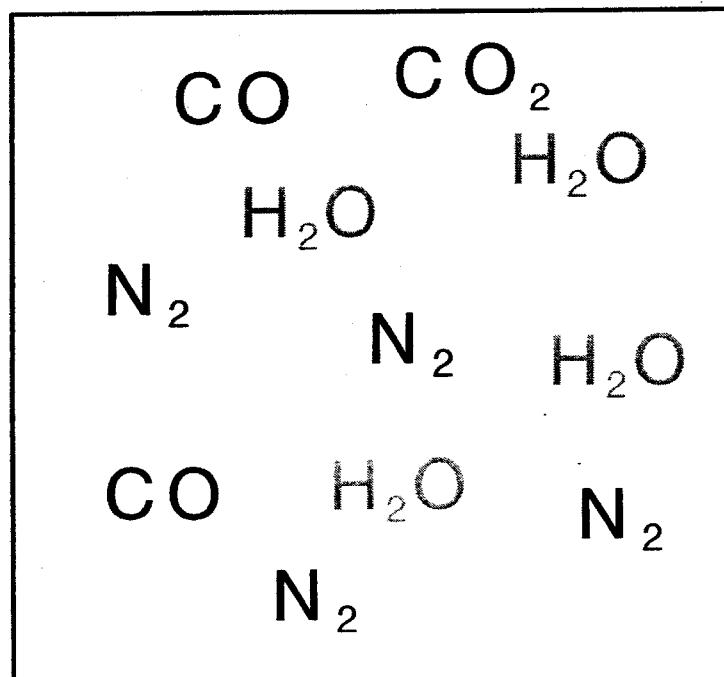
FLUID 1



VIRTUAL SOLID



FLUID 2



$N_a$  PT ensemble

move types:

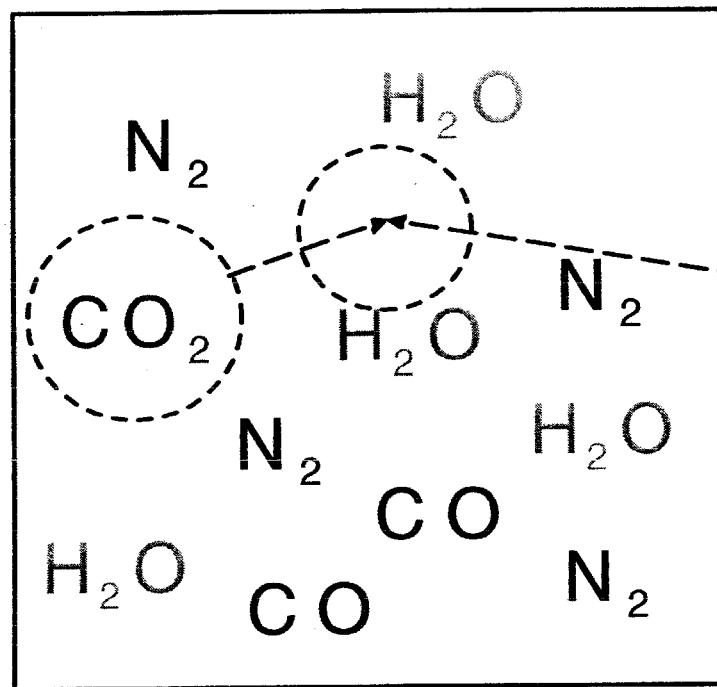
position

volume

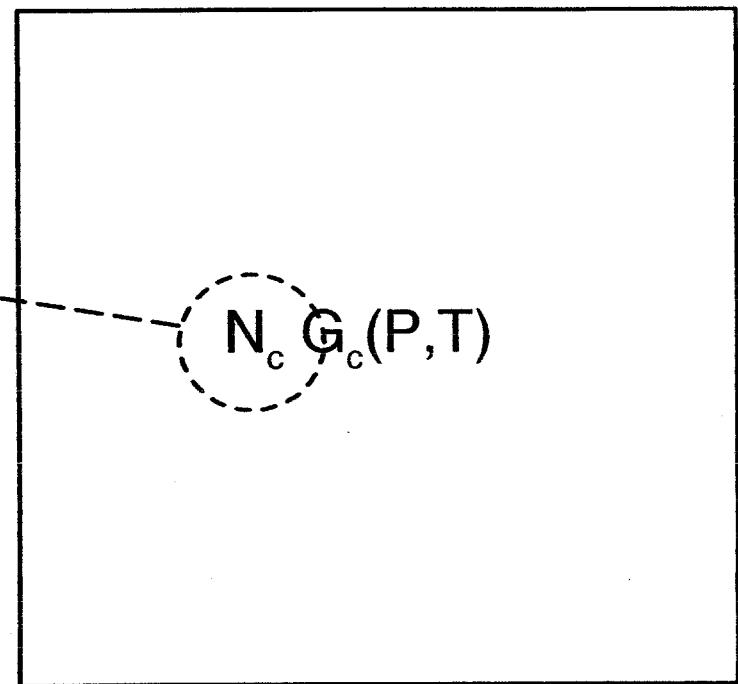
between  $F_1$  &  $F_2$

chemistry

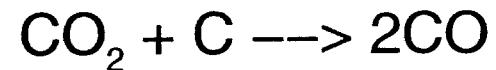
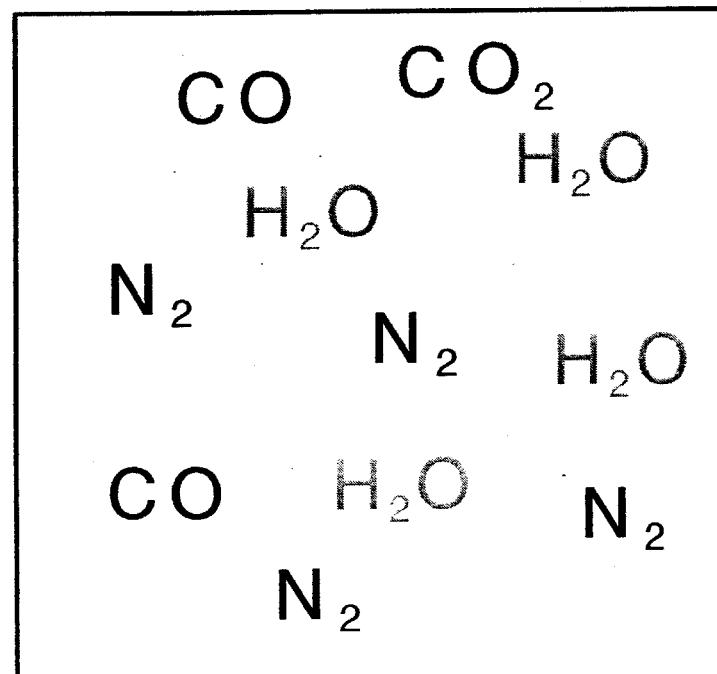
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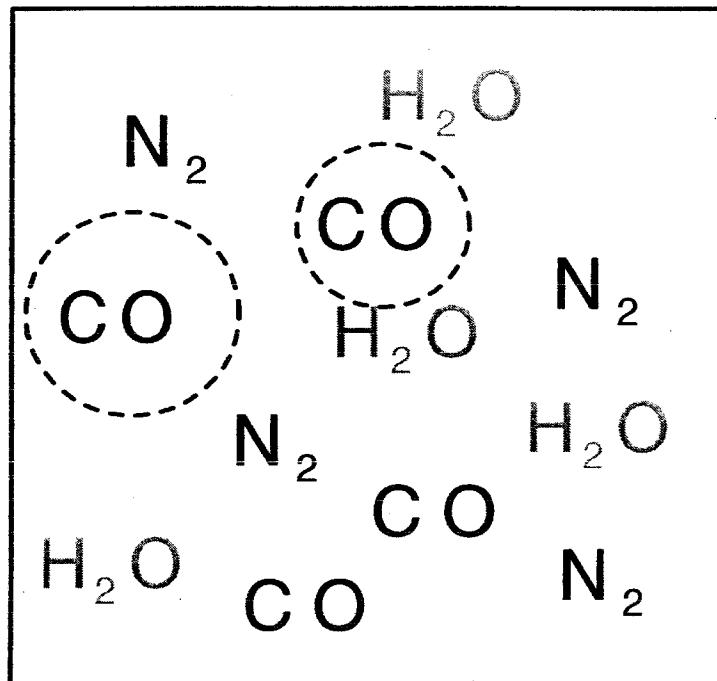
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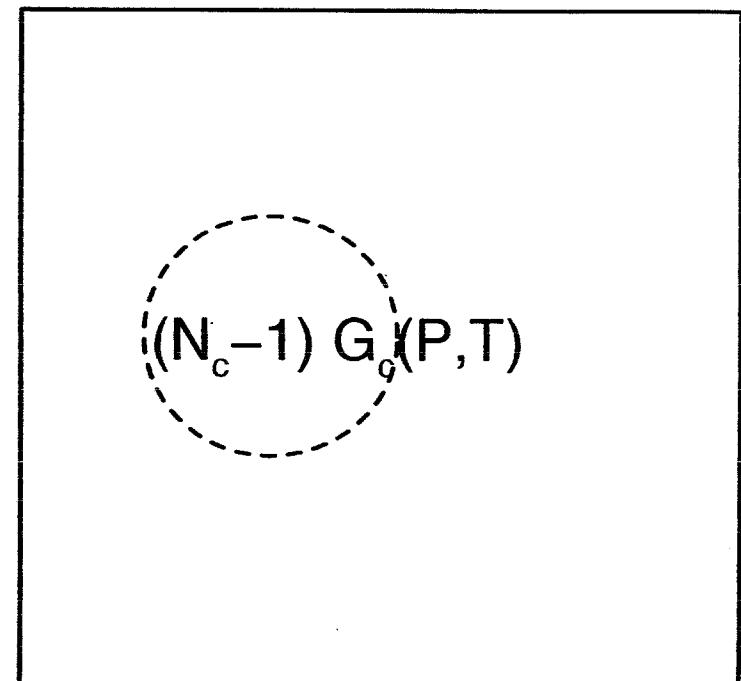
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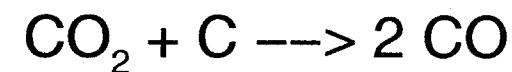
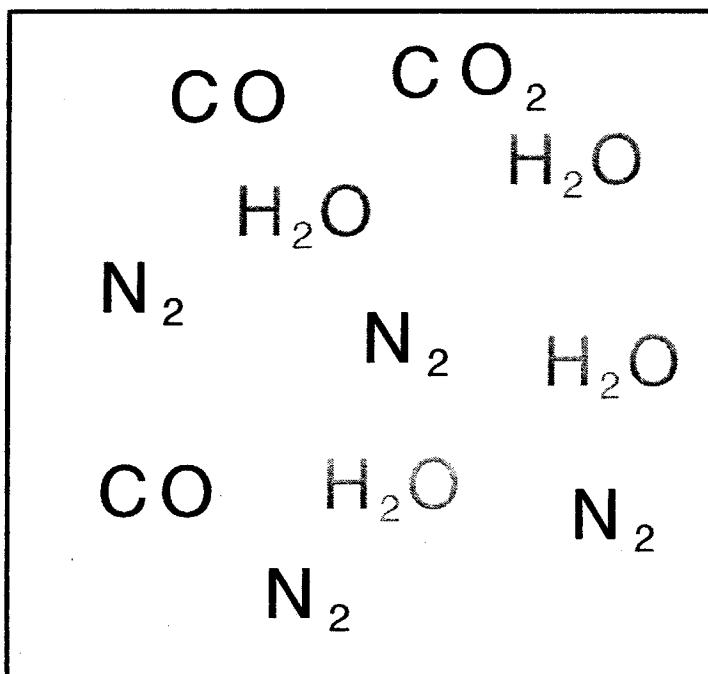
FLUID 1



VIRTUAL SOLID



FLUID 2

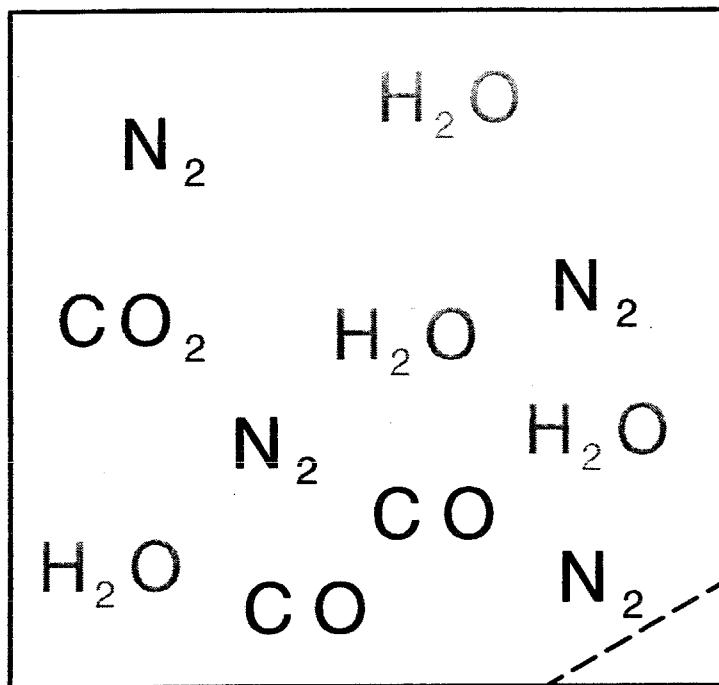


$\text{CO}$  replaced  $\text{CO}_2$

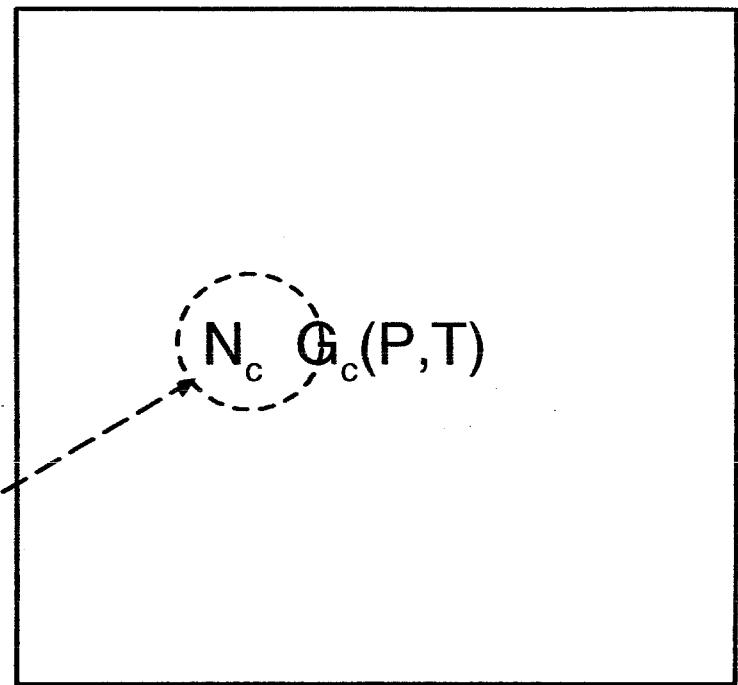
$\text{CO}$  inserted

$\text{N}_c$  replaced by  $\text{N}_c - 1$

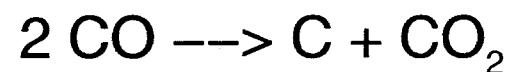
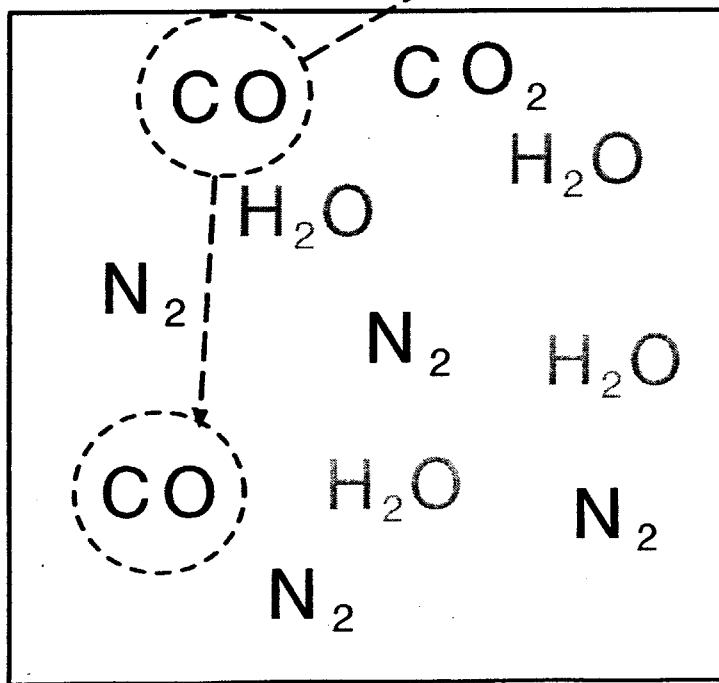
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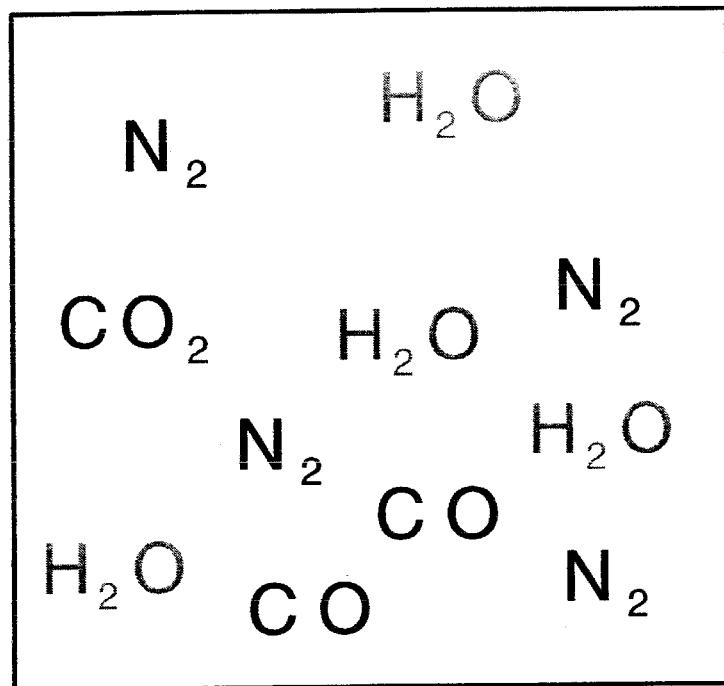
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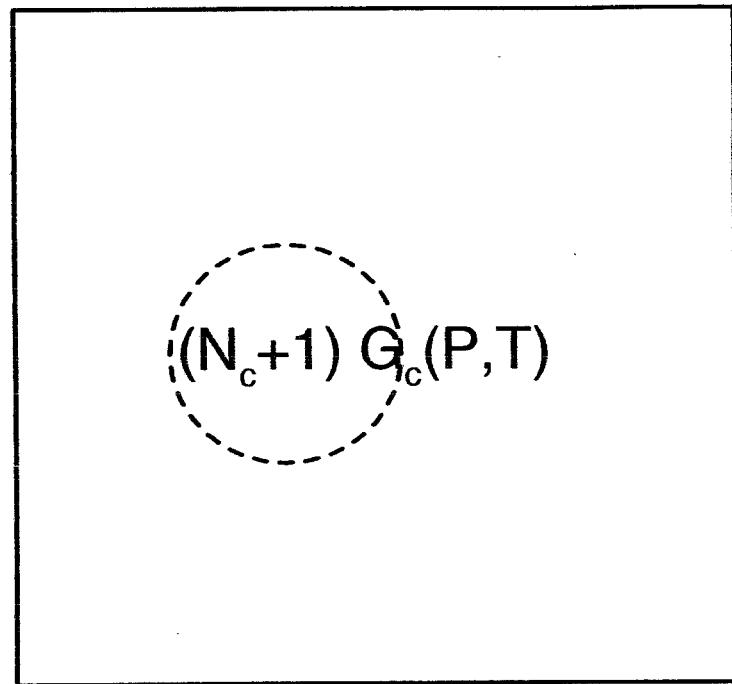
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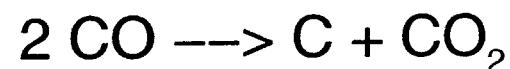
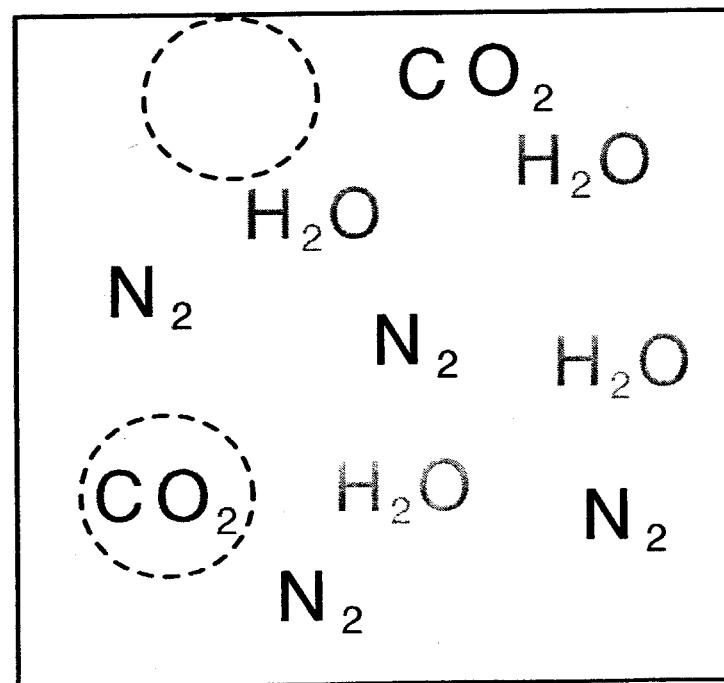
FLUID 1



VIRTUAL SOLID



FLUID 2

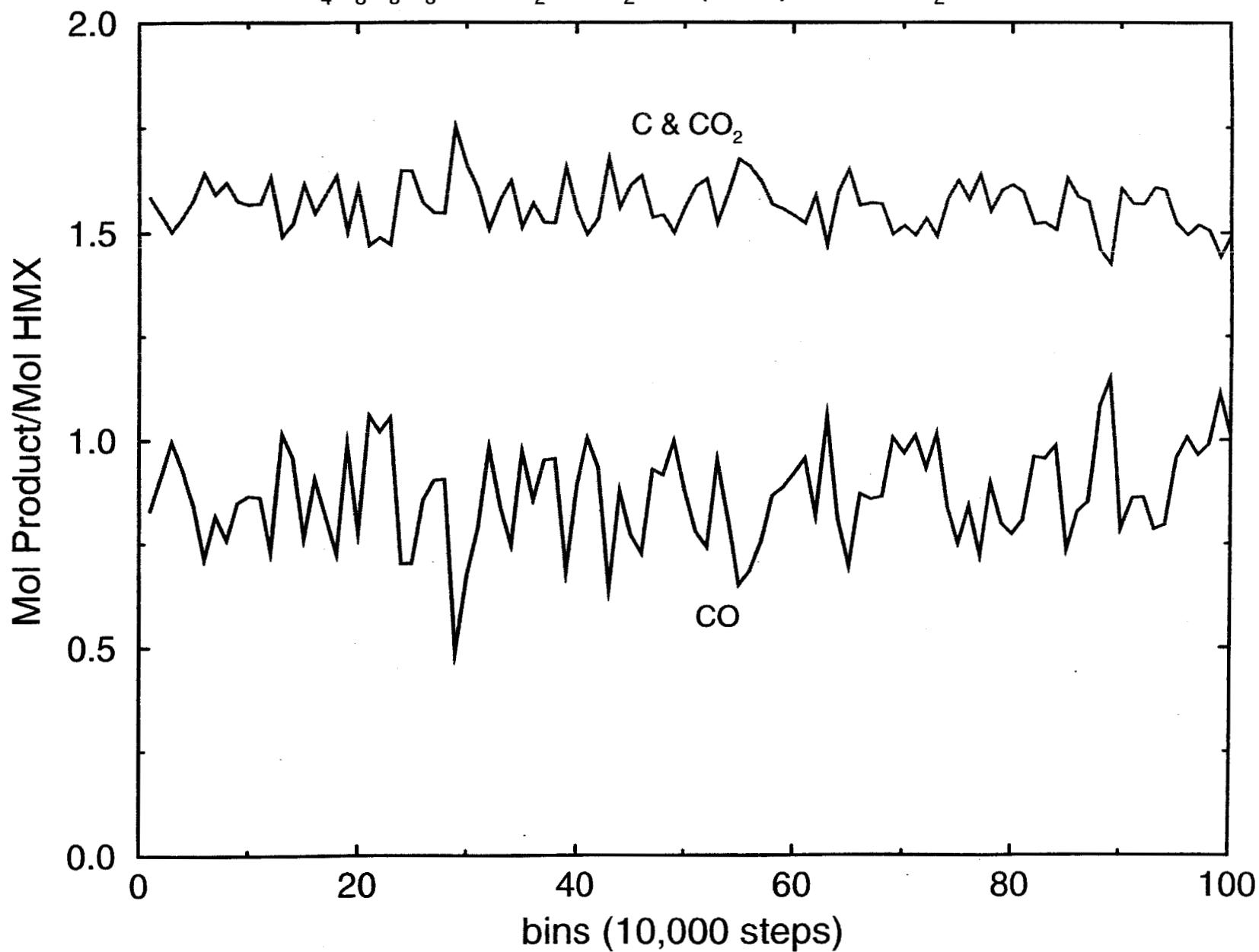
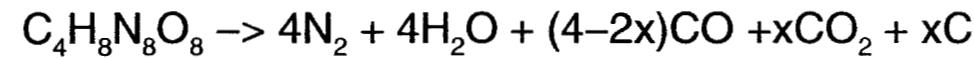


$\text{CO}_2$  replaced CO

void replaced CO

$N_c + 1$  replaced  $N_c$

# Hybrid MC – HMX Products 7.5 GPa 3000K



# HMX Products T=3000 K

blue: Lorentz-Berthelot cross potentials red:3% variations in  $r^*$

