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Surface Interactions Involved in Flashover with High Density Electronegative Gases

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Abstract

This report examines the interactions involved with flashover along a surface in high density electronegative gases. The focus is on fast ionization processes rather than the later time ionic drift or thermalization of the discharge. A kinetic simulation of the gas and surface is used to examine electron multiplication and includes gas collision, excitation and ionization, and attachment processes, gas photoionization and surface photoemission processes, as well as surface attachment. These rates are then used in a 1.5D fluid ionization wave (streamer) model to study streamer propagation with and without the surface in air and in SF₆. The 1.5D model therefore includes rates for all these processes. To get a better estimate for the behavior of the radius we have studied radial expansion of the streamer in air and in SF₆. The focus of the modeling is on voltage and field level changes (with and without a surface) rather than secondary effects, such as, velocities or changes in discharge path. An experiment has been set up to carry out measurements of threshold voltages, streamer velocities, and other discharge characteristics. This setup includes both electrical and photographic diagnostics (streak and framing cameras). We have observed little change in critical field levels (where avalanche multiplication sets in) in the gas alone versus with the surface. Comparisons between model calculations and experimental measurements are in agreement with this. We have examined streamer sustaining fields (field which maintains ionization wave propagation) in the gas and on the surface. Agreement of the gas levels with available literature is good and agreement between experiment and calculation is good also. Model calculations do not indicate much difference between the gas alone versus the surface levels. Experiments have identified differences in velocity between streamers on the surface and in the gas alone (the surface values being larger).

Contents

1	INTRODUCTION	23
1.1	Description of Effect of Dielectric Photoemission	26
1.2	Applications Involving Dielectric Surfaces	28
1.2.1	Pulsed Power Switching	28
1.2.2	Explosive Safety	31
1.2.3	TTU Experiments	32
1.3	Past Work In Literature	32
2	STATIC BREAKDOWN CALCULATION ON TTU EXPERIMENT	35
3	ONE DIMENSIONAL FLUID MODEL	42
3.1	Photoionization Source Term	42
3.1.1	Photoionization Secondary Excitation Coefficient	42
3.2	Air Rates	43
3.2.1	Ionization And Attachment Coefficients	43
3.2.2	Mobililty	44
3.2.3	Diffusion Coefficient	44
3.2.4	Recombination Coefficient	44
3.2.5	Photoionization Model	44
3.2.6	Simplified Photoionization Model (Photoabsorption and Effective Secondary Coefficients)	47
3.3	SF6 Rates	47
3.3.1	Ionization Coefficient	47

3.3.2	Attachment Coefficient	47
3.3.3	Electron Drift Velocity	48
3.3.4	Electron Diffusion	48
3.3.5	Ionic Mobility	48
3.3.6	Recombination	49
3.3.7	Photon Absorption Coefficient	49
3.3.8	Photoionization Excitation Coefficient	49
4	KINETIC CALCULATIONS	50
4.1	Monte Carlo Calculation to Obtain Surface Parameters	50
4.2	Monte Carlo Code Modifications for SF6	50
4.3	Results	54
5	ONE-DIMENSIONAL FLUID AND 1.5D FIELD MODEL	57
5.1	Axial Current And Charge Density	58
5.2	Volume Electric Fields	58
5.2.1	Disc Charges	58
5.2.2	Tube Charges	62
5.3	Summary Of One Dimensional System	65
5.3.1	Photoionization Computation	65
5.3.2	Discretization	65
5.3.3	Flux Corrected Algorithm	65
5.3.4	Zalesak Modification	68
5.4	Surface Electric Fields	72
5.4.1	Half Cylinder Charge	72
5.4.2	Dielectric Images	74

5.4.3	Total Disc Charge and Field	75
5.4.4	Surface Strip Charge	75
5.5	Tube Charges	78
5.5.1	Semi-Circular Tube	78
5.5.2	Total Tube Charge and Field	80
5.5.3	Surface Line Charges	80
5.6	Summary Of Surface Equations	81
5.6.1	Air Photoemission	82
5.6.2	SF6 Photoemission	82
6	MODEL RESULTS	83
6.1	Critical Fields	83
6.2	Calculation of Sustaining Fields	83
6.3	Air	89
6.4	Air and Surface	97
6.5	SF6	99
6.6	SF6 and Surface	99
6.7	High Pressure SF6	103
6.8	Run Times	103
7	RADIAL EXPANSION OF HEAD	106
7.1	Axial Tube Field	106
7.2	Radial Cylindrical Shell Field	106
7.3	Selection Of New Radius	111

7.4	Radial Expansion With Photoionization	111
7.5	Continuity Equations In Radial Coordinates	112
8	TWO DIMENSIONAL RADIAL EXPANSION	120
8.1	The Two Dimensional Fluid Code	120
8.1.1	Convection algorithm	121
8.1.2	Diffusion algorithm	128
8.1.3	Source algorithm	128
8.1.4	Electric Field	129
8.1.5	Results	131
8.2	Radial Expansion On Surface	136
9	EXPERIMENTS WITH UNIFORM FIELD	140
9.1	Threshold Electric Field Levels	150
9.1.1	Volume Results	151
9.1.2	Teflon Surfaces	151
9.1.3	High Density Polyethylene	155
9.1.4	Summary Of Threshold Fields	155
9.1.5	Effect of Flashover History	155
9.2	Streak Photography	162
9.2.1	Velocity Results	168
10	EXPERIMENTS WITH NONUNIFORM FIELD	171
10.1	Description of Experiment	174
10.2	Air Results	180
10.3	SF6 Results	182

10.4 Rod-Plane Problem	182
10.4.1 Line Charge	186
10.4.2 Far Field	187
10.4.3 Line And Point Charges	188
10.4.4 On Axis Field	191
10.4.5 Integral Equation	191
10.4.6 Field At Plane Opposite From Plane With Rod	193
10.4.7 More Exact Approach	198
11 SUMMARY OF PAPERS ON LEADER BREAKDOWN IN ELECTRONEGA-	
 TIVE GASES	198
11.1 Chalmers 1972	198
11.1.1 Nitrogen	198
11.1.2 Oxygen	199
11.1.3 SF6	199
11.2 Malik 1978	199
11.2.1 Factors that Affect Discharge Voltage in SF6	200
11.3 Niemeyer 1983	200
11.3.1 Thermodynamic state of leader channel	202
11.3.2 Ionization zone and leader propagation	202
11.3.3 Interaction of leader and gap geometry	204
11.3.4 Leader propagation criterion	204
11.4 Pinnekamp 1983	204
11.4.1 E Field	205
11.4.2 Discharge mechanisms	205
11.5 Chalmers 1984	205

11.6 Chalmers 1987	206
11.7 Wiegart 1988	206
11.8 Niemeyer 1989	207
11.8.1 Phases of Leader Breakdown	207
11.8.2 Streamer Properties	208
11.8.3 Streamer Corona	208
11.8.4 Leader Inception	209
11.8.5 Summary of Formula	210
11.8.6 Leader Channel Development	210
11.8.7 Leader Propagation Through the Gap	210
11.8.8 Conclusions Of Paper	211
12 CONCLUSIONS	211
13 REFERENCES	213

Figures

1.	Breakdown voltage versus distance with and without dielectric spacer [7].	24
2.	Photoionization process.	26
3.	Dielectric photoemission process.	27
4.	A laser-triggered high-voltage gas breakdown switch. The laser triggered gap is on the left end. The cascade gap region on the right is the intended breakdown path after the trigger gap is closed. The insulator housing is shown as the green colored region.	28
5.	An open shutter photograph of an insulator housing flashover of the gas breakdown switch. The trigger gap on the left is clearly visible, along with the cascade gap discharges. However the tracks on the insulator clearly indicate abnormal behavior.	29
6.	A set of framing camera images showing the rapid progression of the insulator housing discharge.	29
7.	Damage to the dielectric housing caused by surface flashover.	30
8.	Breakdown across dielectrics in an explosive handling bay.	31
9.	Possible connector breakdown paths.	32
10.	TTU grooved experimental fixture and discharge paths for two different dielectric materials.	33
11.	View of electrodes buried in Lexan.	35
12.	Field lines of TTU experiment.	36
13.	Breakdown in nitrogen lifts off the surface.	36
14.	Breakdown in air hugs the surface.	36

15. Breakdown path for nitrogen lifts off the grooved surface.	37
16. Breakdown path for air hugs the grooved surface.	37
17. Breakdown path following the field line.	38
18. Breakdown path forced to hug surface.	39
19. Breakdown path following the field line.	40
20. Breakdown path forced to hug surface.	41
21. Function ψ as a function of pr (from Naidis [43]).	46
22. Processes for a streamer along a surface.	51
23. SF6 cross-sections used in kinetic simulations.	52
24. Avalanche in SF6.	53
25. Example of SF6 ionization and attachment.	53
26. SF6 absorption.	54
27. Photoemission for N2 and Teflon.	55
28. Photoemission for SF6 and Teflon.	56
29. SEE for Teflon.	56
30. Surface charge model consisting of semicylindrical charge in gas and surface strip charge.	72
31. Air streamer at 4 ns.	84
32. Air streamer at 10 ns.	85

33. Air streamer at 15 ns.	85
34. Air streamer at 20 ns.	86
35. E field peak position versus time 50 μm , 1e9.	87
36. E field peak magnitude vs. time 50 μm	88
37. Velocity of E field peak vs. time 50 μm	88
38. Anode-directed streamer peak magnitude vs. time for 52 kV sustaining field.	89
39. Anode-directed streamer peak magnitude vs. time for 53 kV sustaining field.	90
40. Anode-directed streamer peak magnitude vs. time for 54 kV sustaining field.	90
41. Applied field waveform.	91
42. 600 μm radius streamer with cathode-directed streamer suppressed at $x < 0$ mm.	93
43. 600 μm radius streamer with cathode-directed streamer suppressed at $x < -10$ mm.	93
44. 600 μm radius streamer with cathode-directed streamer suppressed at $x < 0$ mm.	94
45. 600 μm radius streamer with cathode-directed streamer suppressed at $x < -10$ mm.	94
46. Cathode-directed streamer below sustaining field level.	95
47. Cathode-directed streamer stops below sustaining field level.	95
48. Cathode-directed streamer above sustaining field.	96
49. Cathode-directed streamer propagates above sustaining field level.	96

50. Anode-directed streamer grows faster than cathode-directed streamer.	98
51. Cathode-directed streamer decays in a 52 kV/cm sustaining field.	100
52. Cathode-directed streamer is constant in a 54 kV/cm sustaining field.	100
53. Cathode-directed streamer continues to be constant in a 65 kV/cm sustaining field.	101
54. Cathode-directed streamer grows in a 66 kV/cm sustaining field.	101
55. Cathode-directed streamer propagates below sustaining field.	102
56. Tbird run times for surface and volume streamer codes.	105
57. Cathode-directed streamer with 20 μm cell size, 50 μm radius at 2.5 ns.	107
58. Cathode-directed streamer with 10 μm cell size, 50 μm radius at 2.5 ns.	107
59. Cathode-directed streamer with 5 μm cell size, 50 μm radius at 2.5 ns.	108
60. Cathode-directed streamer with 20 μm cell size, 300 μm radius at 2.5 ns.	108
61. Cathode-directed streamer with 10 μm cell size, 300 μm radius at 2.5 ns.	109
62. Radial expansion in air driven by various effective disc electron densities associated with the anode end of the streamer channel. Photoionization is present and the actual electron density is collapsed to a 2.5 μm shell at $r_0 = 150 \mu\text{m}$	114
63. Radial expansion in air driven by various effective disc positive ion densities associated with the cathode end of the streamer channel. Photoionization is present and the actual positive ion density is collapsed to a 2.5 μm shell at $r_0 = 150 \mu\text{m}$. Seed electrons with density 10^8 cm^{-3} are present in the space to initiate the avalanche.	115
64. Radial expansion in SF6 driven by various effective disc electron densities associated with the anode end of the streamer channel. Photoionization is present and the actual electron density is collapsed to a 2.5 μm shell at $r_0 = 150 \mu\text{m}$	116

65. Electron charge magnitude behavior in SF6 during anode radial expansion.	116
66. Negative ion charge magnitude behavior in SF6 during anode radial expansion.	117
67. Positive ion charge behavior in SF6 during anode radial expansion.	117
68. Radial expansion in SF6 driven by various effective disc positive ion densities associated with the cathode end of the streamer channel. Photoionization is present and the actual positive ion density is collapsed to a $2.5 \mu\text{m}$ shell at $r_0 = 150$ μm . Seed electrons with density 10^8 cm^{-3} are present in the space to initiate the avalanche.	118
69. Electron charge magnitude behavior in SF6 during cathode radial expansion.	118
70. Negative ion charge magnitude behavior in SF6 during cathode radial expansion.	119
71. Positive ion charge behavior in SF6 during cathode radial expansion.	119
72. Monte Carlo simulation of an avalanche near a surface.	120
73. Grid nomenclature.	123
74. Mass transfer across cell boundary.	125
75. Test case using SEPX4 to calculate potential.	130
76. E_x as a function of x obtained by finite-difference.	130
77. Initial conditions of the anode-directed streamer.	131
78. Anode-directed streamer at 0.01 ns.	132
79. Contour plot of anode-directed streamer at 0.01 ns.	133
80. Anode-directed streamer at 0.05 ns.	134
81. Contour plot of anode-directed streamer at 0.05 ns.	135

82. Anode-directed streamer at 0.05 ns calculated with 2D code.....	136
83. Radial expansion of anode-directed streamer at 0.05 ns calculated by 1D code.	137
84. Initial condition of cathode-directed streamer.	137
85. Cathode-directed streamer at 0.01 ns.	138
86. Cathode-directed streamer at 0.1 ns.	138
87. Cathode-directed streamer at 0.1 ns calculated with 2D code.	139
88. Radial expansion of cathode-directed streamer at 0.1 ns calculated by 1D code.	139
89. Contour plot of total charge density next to Teflon surface.	140
90. A representation of the flashover experiment showing the surface between the electrodes.	141
91. The physical layout of the surface flashover experiment.	142
92. The pulsed power is provided by a custom TG 125 trigger generator initiated by a L-3 40230 trigger. The resultant low jitter was necessary to capture the fast discharge with the streak camera.	143
93. The first pulse from the TG-125 trigger generator.	143
94. The voltage and current of the TG125 when the gap does not breakdown. The multiple pulses are because the TG 125 is a trigger generator and charges the transmission line that connects it to the chamber.	144
95. The load tank contains the resistive termination for the discharge chamber. The load is a solid resistor of the type which has been shown to maintain a pure resistance to a fraction of a nanosecond.	145
96. A crossectional view of the discharge chamber for high pressure surface flashover experiments.	146
97. The discharge chamber is capable of withstanding gas pressures up to 10	

atmospheres.	146
98. Along with the dielectric constant, the shape of the interface is used to maintain the electrical impedance throughout the chamber.	147
99. The impedance across the torlon/gas interface inside the discharge chamber.	148
100. An overlay of a pulse with a fast risetime measured at the input and at the load show the excellent impedance match when the test gap is shorted.	149
101. Details of the electrode design to insure good electrical contact between the inner conductor and the insulating test sample, which is not shown.	149
102. Open shutter pictures showing normal operation (upper) for multimegavolt voltages and a failure along the support rod in the output transmission line section.	152
103. The damage along the rod after a surface failure.	153
104. The voltage signal measured at the input of the gap when the gap breaks.	153
105. The input voltage and load current are normalized and overlaid to find the time to breakdown of the gap.	154
106. The electrical breakdown voltage of a HDPE surface in atmospheric air degrades with successive flashovers.	158
107. The threshold for electrical breakdown across a HDPE surface in atmospheric SF6 decreases for successive flashovers with significant scatter.	159
108. A open shutter photograph of a surface discharge in SF6 along a high density polyethylene surface where the discharge follows the damage path instead of the field enhancement.	160
109. Open shutter photography showing the discharge in SF6 prefers to follow the old discharge path rather than the field enhancement while air prefers the pin.	161
110. An overlay of the normalized electrical signals showing the relative timing of the applied voltage, the laser diode and the ramp of the streak camera. the ramp of the streak camera shows it is capturing the region of interest.	163

111. An overlay of a streak image with the test gap shows the relation between the time evolution of light emission within the gap for a volume discharge. The field enhancement pin which initiates the discharge is located on the electrode on the right hand side.	164
112. A streak image of a volumetric discharge in laboratory air.	164
113. The view of the surface discharge gap as as seen from the streak camera.	165
114. The streak image obtained with the EG&G camera of a discharge across a HDPE surface in air.	165
115. The streak image of an atmospheric air discharge across a HDPE surface was hand digitized to get distance versus time data. The shape of the curve is radially different than that measured in volumetric air and much faster.	166
116. A screen shot from the Hammamastu fast streak camera showing the timing mark as well as its relation to the gap geometry.	167
117. In processing the data of a streak image, the light intensity levels are extracted and then overlaid with the image.	168
118. The streak image of the previous figure with the background radiation removed.	169
119. The extracted light intensity levels for a surface flashover shot with and without the background removed.	169
120. A schematic describing the effect of timing jitter in the system on the time resolution of the streak camera.	170
121. The streak image of a volume breakdown in SF6 at a pressure of 1 atmosphere (absolute).	171
122. The streak image of a discharge across a HDPE surface immersed in 1 atmosphere (absolute) of SF6.	172
123. A comparison between intensity levels for the streak photographs of the two previous figures. The introduction of the surface speeds up the discharge by an order of magnitude.	173

124. Illustration of the symbols used in calculating the minimum field. The diameter of the pin, $2a$, is 1 mm and the height is 10 mm in this experiment.	174
125. The sharp pin on the electrode initiates the discharge. The streamer must travel through a minimum field, the streamer sustaining field, to cross the gap and cause current to flow.	175
126. Another view of the electrode with the pin.	176
127. The test gap. The electrodes are identical with the exception of the pin.	176
128. The layout of the experiment for studying sustaining fields.	177
129. The excitation waveform for the sustaining field experiments.	178
130. The chamber for studying gas discharges is suitable for pulses of nanosecond duration because it is well matched in impedance.	178
131. The experimental layout.	179
132. Overlays of the normalized waveforms of the input voltage and load current showing their relative timing. The current waveform is shift 4.5 ns to account for physical distance between the monitors. The first current bump is the capacitance of the gap.	183
133. The negative voltage waveforms for SF6 at 1 atm during the sustaining field calculations.	184
134. The gap does not breakdown until the value at the tip of the pin exceeds the critical value. The experiment shows very repeatable results.	184
135. A blowup of three overlaid voltage signals showing the repeatability of the measurement.	185
136. Rod-plane geometry excited by uniform field.	186
137. Comparison of axial field in a rod plane geometry. In this case the rod connects to the plane and the excitation is a uniform field in the z direction. The numerical solution of the integral equation and the approximate formula are compared. The rod height is $h = 1$ cm.	194

138. Comparison of axial field in a rod plane geometry. In this case the rod connects to the plane and the excitation is a uniform field in the z direction. The numerical solution of the integral equation and the approximate formula are compared. The rod height is $h = 4$ cm.	194
139. Comparison of line charge and line-point charge models for rod in iniform field with numerical solution.	196
140. Structure of leader breakdown.	201
141. Extent of ionization zone.	203

Tables

1	Air ionization coefficients (from “High Voltage Engineering: Fundamentals,” E. Kuffel, et. al. [36]).	43
2	Time for 300 kV/cm field.	92
3	Air sustaining field.	97
4	Time for 300 kV/cm field.	99
5	Air and surface sustaining field.	103
6	Time for 450 kV/cm field.	104
7	SF6 sustaining field.	106
8	Times and initial field for SF6 and surface.	110
9	SF6 and surface sustaining field.	111
10	Commercial Dry Air.	151
11	Sulfur Hexafluoride (SF6).	152
12	Nitrogen.	152
13	Teflon Surface immersed in Commercial Dry Air.	155
14	Threshold Electrical Breakdown Field for air.	156
15	Teflon Surface immersed in SF ₆	156
16	Threshold Electrical Breakdown Field for SF ₆	156

17 HDPE Surfaces immersed in Commercial Dry Air.....	156
18 Threshold Electrical Breakdown Field for air.	156
19 HDPE surface immersed in SF ₆	156
20 Threshold Electrical Breakdown Field for HDPE/SF ₆	156
21 HDPE Surfaces immersed in Commercial Dry Nitrogen.	157
22 Threshold Electrical Breakdown Field for nitrogen.	157
23 Summary of threshold values in Air	157
24 Summary of threshold values for SF ₆	157
25 Summary of threshold values in N ₂	157
26 Cathode-directed (Positive Charging Voltage) in air.	181
27 Anode-directed Streamers (Negative Charging Voltage) in air.....	181
28 Cathode-directed (Positive Charging Voltage) in SF ₆	182
29 Anode-directed Streamers (Negative Charging Voltage) in SF ₆	183

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1 INTRODUCTION

This report summarizes theoretical and experimental investigations and development of a simple streamer model for surface flashover in high pressure gases. Both highly electronegative (sulphur hexafluoride SF₆) and moderately electronegative (air) gases are treated. Motivation for this investigation arose from several sources: 1) flashovers of the insulating housings of high-voltage pulsed-power gas breakdown switches; 2) explosive safety concerns, where breakdown paths involving insulating surfaces are often involved; 3) a series of experiments where discharge paths in electronegative gases tracked dielectric surfaces; and 4) a number of investigations in the literature, where surface effects have been observed. Unlike vacuum, where insulating surfaces are known to be the weak point in electrical strength, surface effects in higher pressure gases are somewhat controversial.

These investigations are focused primarily on fast streamer behavior [1], [2], [3], [4], [5], [6]; a section at the end of the report has a brief summary of the literature on slower leader effects, some including the presence of insulating surfaces. In addition, surface effects can be thought of as including both first-order effects, such as, reductions in breakdown level, as well as secondary effects, such as changes in velocity or modifications of discharge path (where competing voltage levels are nearly the same). The modeling efforts are directed at the first-order issues associated with breakdown level, but we will touch on some of the secondary effects in the experiments and in the theoretical discussions. The first-order effects will include examinations of both critical field levels (required for avalanche growth) as well as sustaining field levels (required to maintain ionization wave or streamer propagation). The next four subsections discuss some of the motivations for this problem.

The examples that follow in this section are just a few of the many practical applications where knowledge of breakdown across a dielectric surface at atmospheric pressure is important. In 1998 we began to study this problem and found that the literature was empirical in nature and there was disagreement about whether the dielectric raised or lowered the breakdown voltage. Figure 1, for example, which is widely quoted in breakdown literature [7],[8], shows the breakdown field as a function of gap distance between two parallel-plate electrodes, with and without a dielectric spacer bridging the gap. The figure indicates that the dielectric surface reduces the breakdown voltage by as much as a factor of two. Later we found that the original paper giving these results was focused on the effect of humidity on breakdown and the indicated drop in breakdown voltage was explained by humidity on the surface of the dielectric [9]. Humidity effects are not a subject of investigation in this report (some recent measurements at Texas Tech University indicate that these effects diminish for times less than a microsecond).

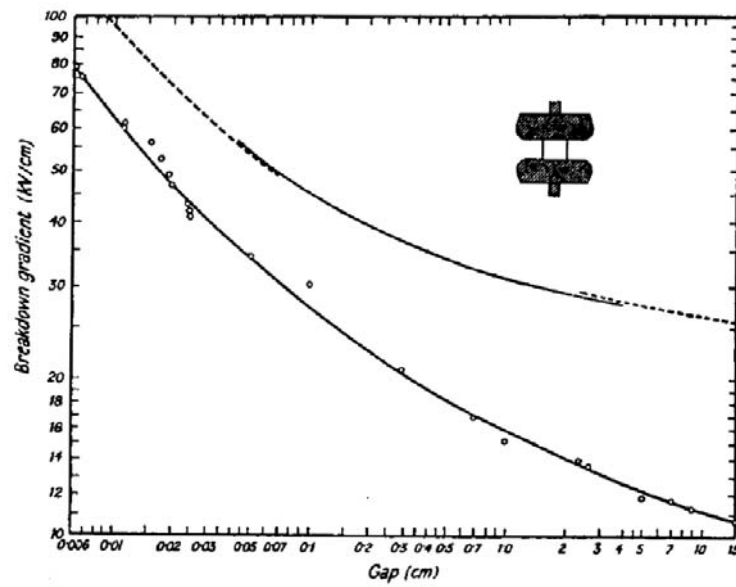


FIG. 7.35. 60 c./s. breakdown voltage characteristic for the sparkover across the surface of a glass or porcelain cylinder in air. The upper curve gives the breakdown characteristic of a uniform field in the absence of the cylinder.

Figure 1. Breakdown voltage versus distance with and without dielectric spacer [7].

In order to study the breakdown process we simulated the growth of an avalanche across a dielectric surface using 3D Monte-Carlo kinetic simulations [10]. During the course of that work, it became obvious that we had missed an effect that could play a major role in the propagation of an avalanche across a dielectric surface – that of photoemission from the surface. A follow-up report [11] investigated avalanche growth in nitrogen along dielectric surfaces, accounting for dielectric photoemission. The present report documents extensions made to the Monte Carlo code to account for attachment processes required to model electronegative gases. In addition, to address sustaining field levels with and without a surface, 1.5D and 2D fluid models are used, which allow later time development of the ionization wave (streamer). Interactions with the surface are included through rate equations derived from the 3D kinetic simulations. Radial expansion of the streamer body in air and SF6 are also examined to give an indication of the best choice [12] of streamer radius [13] in the 1.5D fluid model.

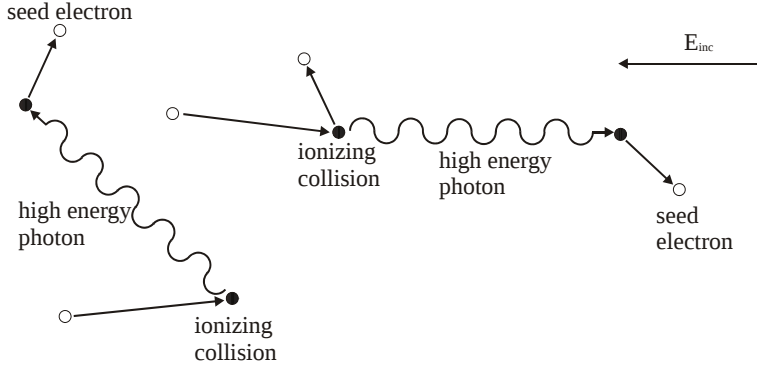


Figure 2. Photoionization process.

Because dielectric photoemission is a fundamentally new process associated with the presence of a surface we contrast it to the gas alone in the next subsection.

1.1 Description of Effect of Dielectric Photoemission

As the electrons are released, they are accelerated by the electric field until they collide with one of the neutral gas molecules. Initially, the electron energy is low enough that most of the collisions are either elastic or result in electron attachment. As the electron energy increases, the collisions become inelastic, transferring energy to and exciting the neutral gas molecule. Eventually, the electron energy becomes large enough that the collision ionizes the neutral, which adds to the number of electrons in the problem. The ionizing collisions lead to an exponential growth in the number of electrons, which is called an electron avalanche.

When a high-energy collision occurs, a photon may be released which has enough energy to ionize a second neutral at some distance from the collision site. The photoionization site serves as a seed electron for a new avalanche. As the avalanches coalesce, they form ionization wavefronts called streamers that propagate toward both the cathode (at maximum velocities of approximately 1×10^8 to 4×10^8 cm/s) and anode (at approximately 1.5 to 2 times the speed of the cathode-directed streamer) [14]. This process is shown in Figure 2.

The photons energetic enough to ionize a neutral (15.5 eV in N_2) need a high energy collision in order to form, and therefore, are not common. Second, high energy photons are readily absorbed in gas at atmospheric pressure [15]. Lower energy photons are more numerous and are not as readily absorbed, but do not have the energy to ionize a gas neutral directly. When a dielectric surface is introduced, however, an electron may be released from the surface when a lower energy photon strikes the surface. In polyethylene, for example, the band gap is 4 eV and work function is 4.5 eV [16], so an 8.5 eV photon should have the energy to release an electron from the dielectric surface, which will serve as a seed for a new avalanche. Because the lower energy photons associated with photoemission can travel farther before being absorbed than the higher energy photons associated with photoionization, it is postulated that an ionization wave next to a surface will travel with greater velocity than an ionization wave without a surface. Also, since the surface serves as a source of electrons, the number of electrons as a function of distance that the avalanche travels should also increase. This process is shown in Figure 3.

Of course the introduction of the surface introduces other processes that could serve as a sink or source of electrons. We will use the Monte Carlo code to evaluate the effect of each of several of these processes (such as secondary electron emission SEE). Another possible process, which is similar to photoemission, is photo-detachment of electrons from negative gaseous ions adhering to the dielectric surface. Such ions

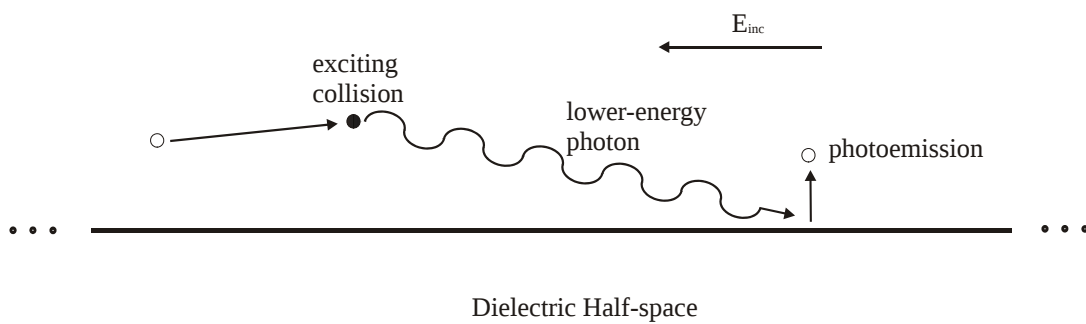


Figure 3. Dielectric photoemission process.

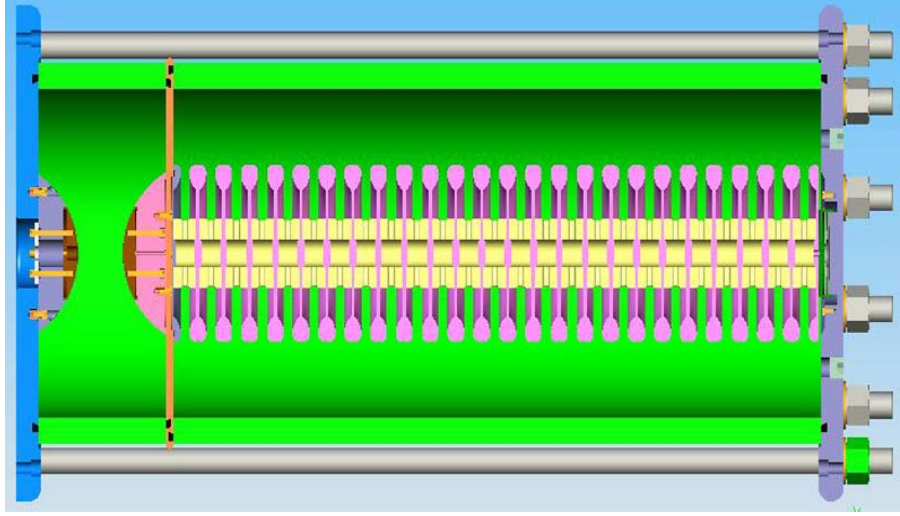


Figure 4. A laser-triggered high-voltage gas breakdown switch. The laser triggered gap is on the left end. The cascade gap region on the right is the intended breakdown path after the trigger gap is closed. The insulator housing is shown as the green colored region.

could be formed prior to the discharge from free electrons in the gap (from field emission or other ionizing collisions). It typically only requires a few eV to detach electrons from such negative ions. Such processes are present in the gas alone, but the presence of the surface could lead to a concentrated film of such ions. Nevertheless, because the number of electrons would be limited, it is not expected that this process would effect the primary avalanche growth, but would act a source of seed electrons and hence only influence secondary effects (velocity, discharge path). This final process was out of scope of the modeling in this report, but should be included if future work on secondary effects is conducted. Experiments with an additional normal DC bias field could be proposed in an attempt to prevent such an ionic film from forming.

1.2 Applications Involving Dielectric Surfaces

Three applications involving dielectric surfaces are now summarized.

1.2.1 Pulsed Power Switching

High voltage pulsed power gas switches are used in the pulse compression section of the Z accelerator. An example of such a switch is shown in Figure 4 (the length of such a switch is on the order of a meter). Switching is triggered by a laser which causes a closure of the left end gap, followed by a cascade breakdown of the rings on the right. Unfortunately failures in the form of insulator flashovers are sometimes observed [17] as shown in the open shutter image of Figure 5 and in the set of framing camera images of Figure 6 (the velocity involved is clearly quite high). This failure is surprising since the static field level on the housing is less than one half of that between the rings (there are very short time dynamic field enhancements along the insulator when the trigger gap closes). Such failures are of importance since they often lead to housing damage as shown in Figure 7.

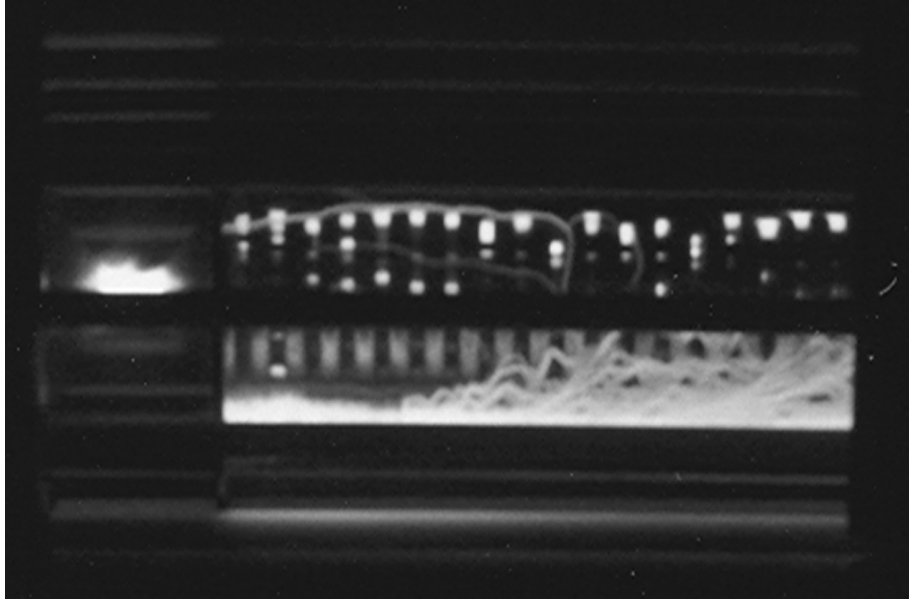


Figure 5. An open shutter photograph of an insulator housing flashover of the gas breakdown switch. The trigger gap on the left is clearly visible, along with the cascade gap discharges. However the tracks on the insulator clearly indicate abnormal behavior.

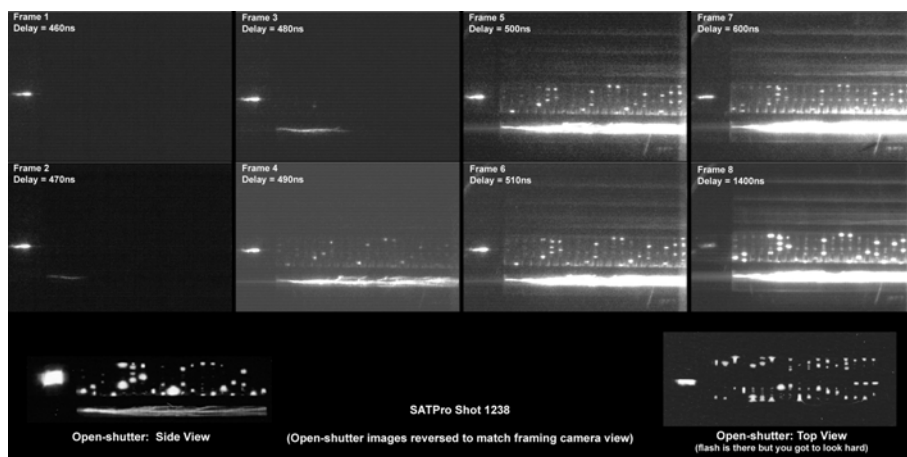


Figure 6. A set of framing camera images showing the rapid progression of the insulator housing discharge.



Figure 7. Damage to the dielectric housing caused by surface flashover.

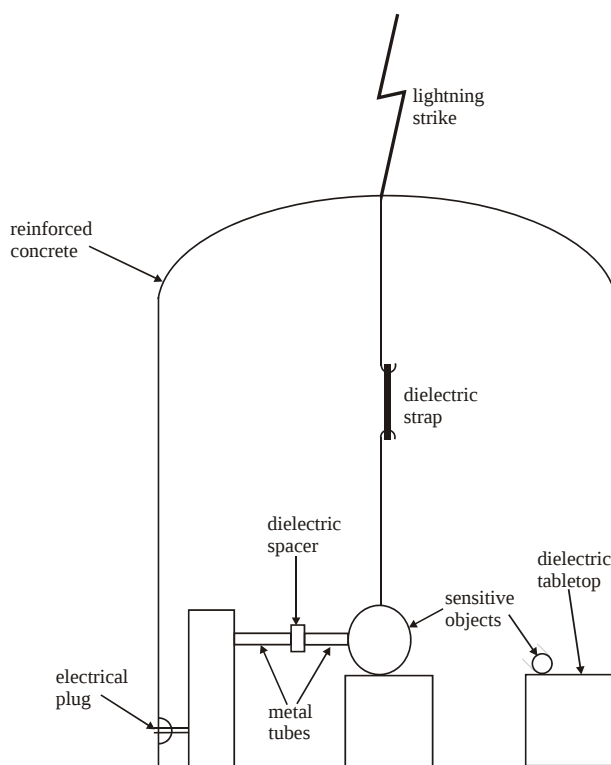


Figure 8. Breakdown across dielectrics in an explosive handling bay.

1.2.2 Explosive Safety

One of Sandia's responsibilities is to ensure safety while conducting certain operations involving high explosives even during extreme environments such as encountered during a lightning storm. The analysis procedure is to first determine the highest voltage present inside a reinforced, concrete structure that has been struck by lightning, as shown in Figure 8. In order to prevent breakdown to objects that are sensitive to current flowing through them a standoff distance is imposed between the sensitive objects and the place where the high voltage exists. The breakdown voltage as a function of electrode distance is well known for parallel-plate geometries and for certain non-uniform geometries (rod-plane, sphere-sphere, etc...) [18]. Many times, however, a dielectric surface is present in the vicinity of the electrodes, which modifies not only the fields between the electrodes, but serves as a possible source or sink of electrons during the breakdown process. In Figure 8, for example, we see that a dielectric strap isolates a crane hook, which is at a high voltage, from a sensitive object being lifted. We also see a dielectric spacer that isolates a piece of equipment from the sensitive object. The equipment is connected to the power grid and subject to high-voltage surges due to a lightning strike. Finally, we see a dielectric table surface, which may cause us to modify the imposed standoff distance from the high-voltage wall to the sensitive object.

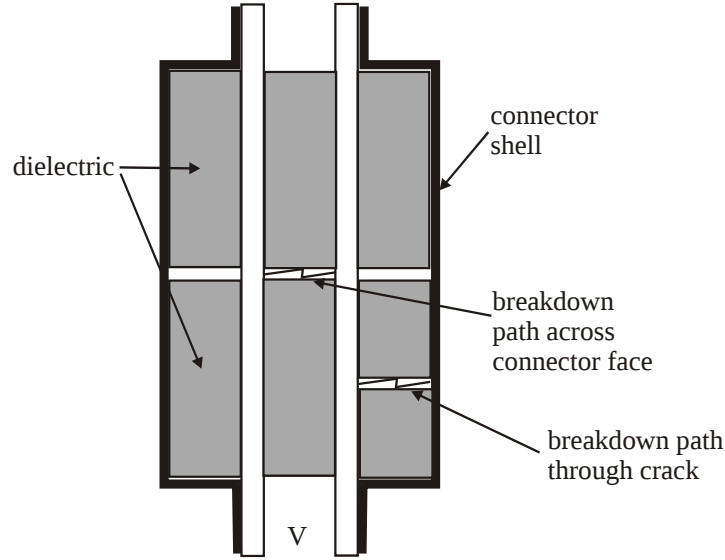


Figure 9. Possible connector breakdown paths.

Another problem is the prediction of the breakdown path in certain components that are exposed to high voltage due to a lightning strike or unintentionally applied AC power lines. Figure 9 shows a connector cross section where high voltage on one line breaks down to another line over the dielectric face of the connector. The wire-to-wire path may be of concern for safety. Another breakdown path shown in Figure 9 involves breakdown through wire insulation that has cracked due to aging. The wire-to-shell path may be of concern for reliability. In both of these cases, the dielectric surface is on both sides of the breakdown path.

1.2.3 TTU Experiments

Experiments were conducted at Texas Tech University on discharges across insulating surfaces [19], [20]. It was found that, even when the surface was grooved and the field bowed up away from the surface, discharges would track the surface for certain dielectric materials when the gas was electronegative. Figure 10 shows an example using SF₆, where the surface had a groove, and the discharge tracked the surface for Plexiglas but not for Teflon. More will be said about the threshold field levels along the paths in this experiment in the next major section. Suffice it to say that the discharge levels observed were not significantly different between the paths in this experiment.

1.3 Past Work In Literature

Evidence that photoemission affects breakdown across a dielectric surface can be found in the literature. Mahajan and Sudarshan [21] compared electron and ion avalanches in a parallel-plate gap bridged by different solid insulators to avalanches in an unbridged gap. The gap was 1 cm wide in nitrogen and the insulator materials included Plexiglas, polyethylene, Teflon, nylon and PVC. At the same time they measured the optical activity associated with the avalanche.

For most insulator samples tested, they found that the growth of the primary electron avalanche was slightly inhibited by the presence of the insulator. The growth of the primary ion avalanche was unaffected. Secondary avalanches in a Townsend type discharge were strongly suppressed by the presence of the insulator. Photon activity was also suppressed by the insulator. The suppression of the various

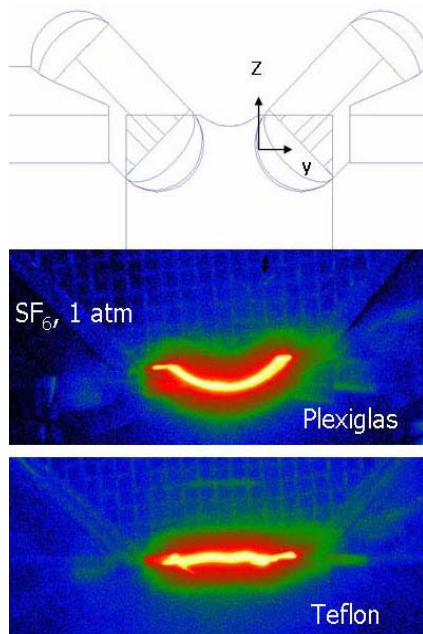


Figure 10. TTU grooved experimental fixture and discharge paths for two different dielectric materials.

avalanches was attributed to a buildup of surface charge on the insulator.

If the bridging insulator was made of nylon, the results were found to be quite different. The growth of the primary electron avalanche was much larger across the nylon insulator than in a plain gas gap. In the plain gas gap, the growth was exponential in nature, but this was not true for the nylon bridged gap. The presence of nylon substantially widened the pulsewidth of the avalanche. The ion avalanche remained unaffected. During the entire avalanche, there was photon activity. The nylon bridged gap had a breakdown voltage approximately half that of the unbridged gap.

Photoemission was proposed as the explanation for the strange behavior of nylon. The work function of a dielectric is low ($5 - 10$ eV) compared to the ionization potential of nitrogen (15 eV). The seeding of new avalanches due to photoemission causes the avalanche pulse to be spread out. Since there was no increase in number of ions, competing processes such as photoionization of the gas or additional electrons emitted from the cathode due to ion impact can be ruled out. If the avalanche was started a few millimeters away from the surface, the large electron avalanche was not observed indicating that the surface affects the breakdown. The authors did not explain why nylon was the only insulator to exhibit this behavior.

Tom et. al. [22] looked at flashover voltage of insulators bridging a nitrogen or argon gap between plane electrodes. The insulators were quartz filled epoxy resin, unfilled epoxy resin and Teflon. They found that if the insulator was pre-charged with positive or negative charge using a corona, the breakdown voltage was lower than that of an uncharged insulator. They attributed this to photoemission from the surface that provided an extra electron contribution to the primary avalanche.

Verhaart et. al. [23], [24] examined avalanche growth next to a Teflon insulator in N_2 , CO_2 , and SF_6 . They assumed that three processes were active in the avalanche: ionization, attachment and photoemission. They measured the total growth coefficient, obtained the ionization and attachment coefficient from literature and assumed the remaining component was due to photoemission from the insulator. They found no photoemission effect in CO_2 . They attributed this to the fact that the spectrum of CO_2 exhibits no emission in the energy range greater than 6 eV and Teflon has a small quantum yield when the photons have energies less than 6 eV. In N_2 photoemission was observed if the insulator was negatively charged prior to conducting the experiment. No photoemission was observed if the insulator was positively charged or was neutral. In SF_6 photoemission was observed no matter how the insulator was charged. Both N_2 and SF_6 have a spectrum with emission in the 10 eV range, where the quantum efficiency of Teflon is high enough to exhibit photoemission.

Jaksts and Cross [25] measured avalanches across polyethylene and Teflon insulators in N_2 . In an N_2 gap without an insulator they measured a primary avalanche and secondary avalanches that originated from the cathode due to photoemission. In the bridged gap, they measured additional avalanches that they attributed to photoemission from the insulator surface initiated by photons from the primary avalanche from an excited state having a lifetime of $20 \mu s$. Both spacers gave similar results, indicating that the exciting mechanism was the same in the gas. They also determined that the insulating surface did not affect the primary avalanche significantly.

Allen et. al. [26],[27],[28] studied the propagation of streamers across a parallel-plate air gap bridged by different solid insulators, 12 cm thick (insulator materials included Teflon, nylon and glazed ceramic). The streamer was formed by imposing a positive pulsed voltage on a point electrode located in a small aperture cut in the anode. They noted that the streamer propagated across the insulator with two components. One had the same speed as that of gas alone and the other was faster. The fast component was offered as evidence that photoemission was an important effect at the surface. They also measured the minimum field needed to propagate the streamer across the gap as being 400 kV/m in air. The bridged gaps needed a higher field to propagate the streamer.



Figure 11. View of electrodes buried in Lexan.

Past modeling efforts of streamers on surfaces include: Gallimberti [29], where the various interactions are discussed; Niemeyer [30], where the field along the streamer channel (sustaining) without and with a surface is cited; and a transmission-line model approach by Kanematsu [31].

2 STATIC BREAKDOWN CALCULATION ON TTU EXPERIMENT

When the gas switch – filled with electronegative SF₆ gas – experienced unexpected breakdown problems along its Lexan envelope we thought of an experiment that was conducted at Texas Tech University (TTU) in 2003, which seemed to indicate that a dielectric surface influenced the breakdown path if the gas was electronegative. The TTU experiment is shown in Figure 11. A pulsed voltage was applied across two electrodes that were buried in Lexan. The electrodes were configured so that the field lines tended to push the breakdown path away from the Lexan block as shown in Figure 12.

The actual breakdown path, however, seemed to depend on the gas surrounding the Lexan block. In nitrogen the breakdown path would follow the field lines and thus have a tendency to lift off the surface. Adding an electronegative gas (such as the oxygen in air) would cause breakdown to occur along the Lexan surface. This effect is shown in Figures 13 and 14. Even cutting a groove in the Lexan did not change this behavior. The breakdown path for a groove is shown in Figure 15 for nitrogen, which shows the path following the field lines. For air, the breakdown path follows the surface as shown in Figure 16.

One possible explanation for the tendency of the breakdown to hug the surface in air is that the fields are stronger near the surface and the different avalanche characteristics of nitrogen and air lead to different preferred breakdown paths because of it. We therefore used a combination of a method of moments code **Eiger_S** and integration of the ionization coefficient along directed paths in **Breakdown_Alpha** to predict the breakdown voltages along the paths as described in [32]. In Figure 17 the breakdown path is allowed to follow the field line. In air the breakdown voltage is predicted to be 26.6 kV. For nitrogen the breakdown voltage is predicted to be 28.8 kV. It is somewhat surprising that the predicted breakdown

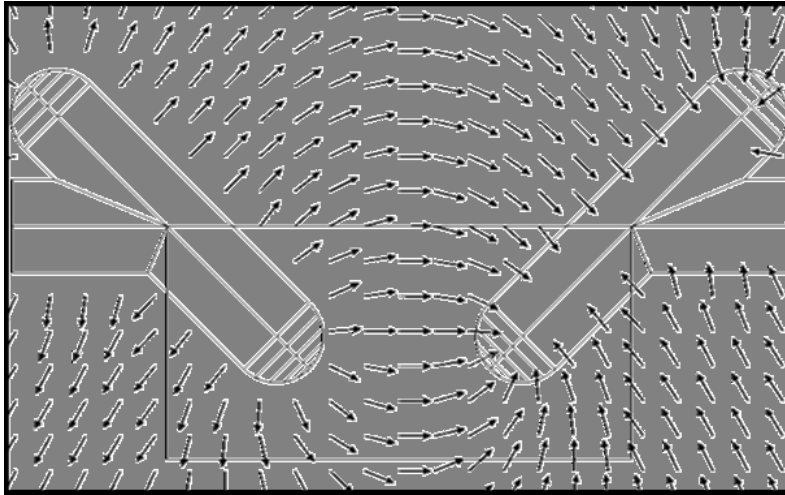


Figure 12. Field lines of TTU experiment.

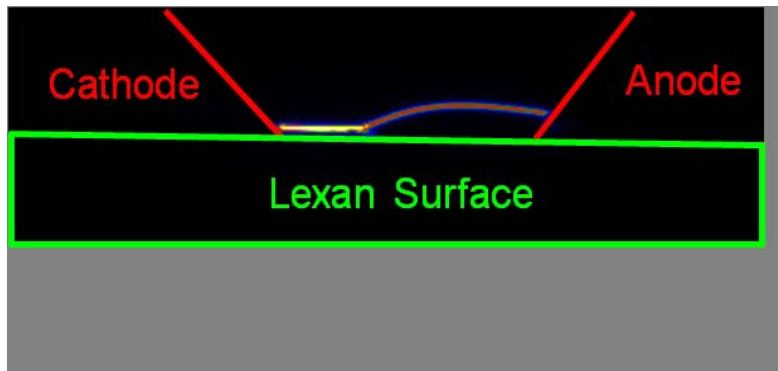


Figure 13. Breakdown in nitrogen lifts off the surface.

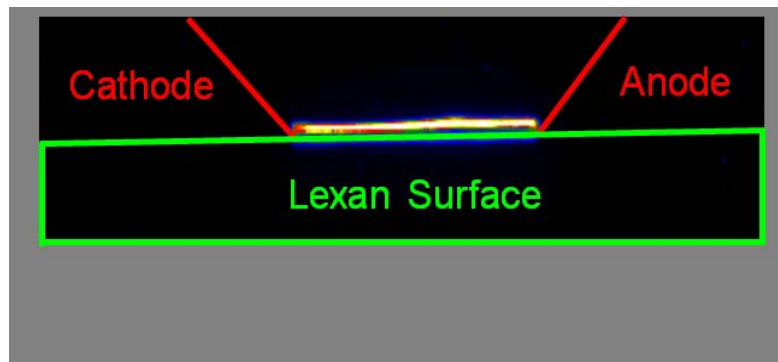


Figure 14. Breakdown in air hugs the surface.

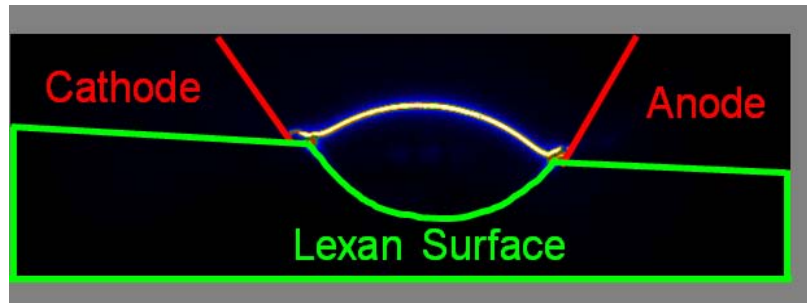


Figure 15. Breakdown path for nitrogen lifts off the grooved surface.

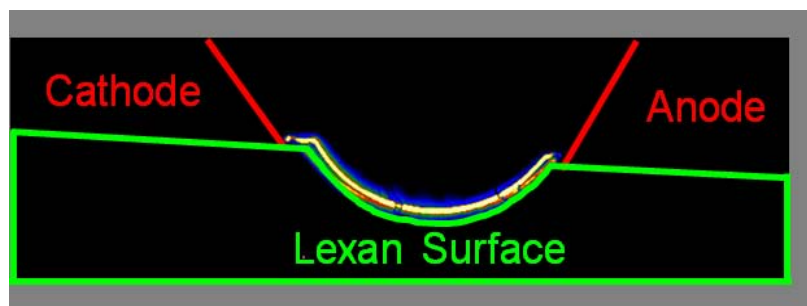


Figure 16. Breakdown path for air hugs the grooved surface.

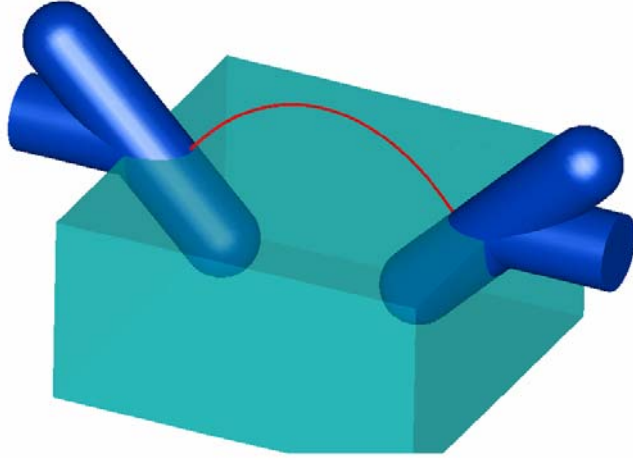


Figure 17. Breakdown path following the field line.

voltage in nitrogen is higher than in air, which is slightly electronegative. The answer lies in the fact that the primary ionization coefficient (α) is larger in air (at least the fit to the tabular data is) than for nitrogen in the larger E/p ranges. In Figure 18 we force the path to hug the surface. In the case of breakdown along the field line, the field vector is parallel to the breakdown path by definition. In the surface case, however, the breakdown path is not parallel to the field line. We first examined the case where the field that enters the α calculation is the component of the field parallel to the chosen breakdown path. In air the breakdown voltage is predicted to be 28.3 kV and in nitrogen it is predicted to be 30.9 kV. This suggests that from the perspective of looking at the fields alone, the preferred path for both gases is along the field line rather than hugging the surface. Next we examined the case where we take the field that enters the α calculation as the total field (parallel and perpendicular to the path). For this case in air the breakdown voltage is predicted to be 20.8 kV and for nitrogen it is predicted to be 22.6 kV. This indicates that the breakdown path along the surface is the preferred path, but both gases are behaving similarly. The larger total field next to the surface appears not to be the explanation for why an electronegative gas causes the breakdown path to hug the surface.

Figure 19 shows the calculated path following the field line for the grooved geometry and Figure 20 shows the path hugging the surface. Along the field line the predicted values are 26.6 kV for air and 28.8 kV for nitrogen. These are the same as for the non-grooved case, which indicates that the field distribution has not changed significantly. If we force the breakdown path to follow the surface and use the electric field parallel to the path to calculate α , we find that corona is predicted. The reason for this is that at two points along the path - symmetrical with respect to a point midway between the two electrodes - the field is completely perpendicular to the chosen breakdown path and drops below streamer sustaining field levels (4.7 kV/cm for air and 1.5 kV/cm for nitrogen). If we use the total field along the path to calculate α we find that the predicted breakdown values are 15.4 kV for air and 16.8 kV for nitrogen. These levels are smaller than the non-grooved case, which indicates that the fields are larger (closer to the electrodes); but again both gases show the same behavior, i.e., that breakdown in both gases should follow the surface if we believe that the total field drives the avalanche, or breakdown in both gases should follow the field line if we believe that the field parallel to the breakdown path should drive the avalanche.

We would actually expect that if seed electrons are present from photoionization (or from photoemission

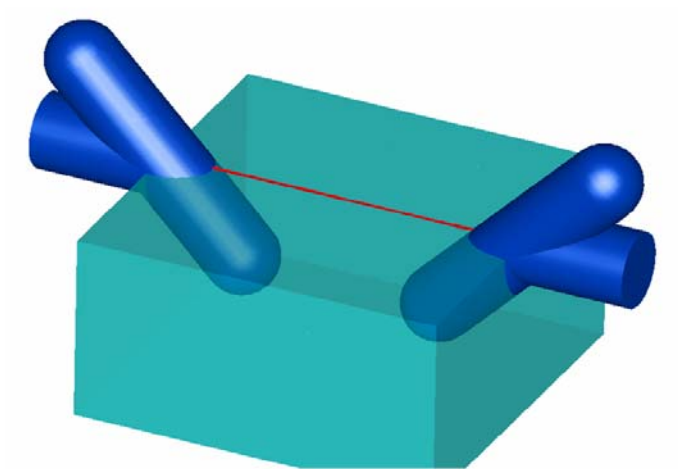


Figure 18. Breakdown path forced to hug surface.

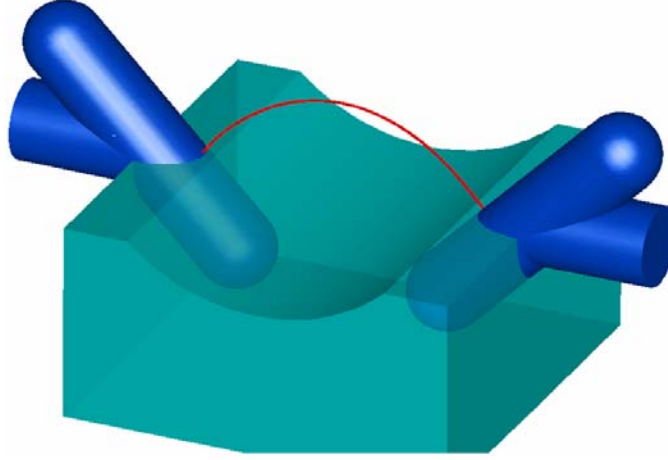


Figure 19. Breakdown path following the field line.

if the field points downward into the dielectric and is able to accelerate electrons off the surface into the gas), the total field would drive the ionization growth. In regions where the field vector points nearly normal to the surface, the avalanches are then in a direction nearly perpendicular to the path, with photoionization (or photoemission) creating new seeds in the direction along the path; in other words the ionization wave in these regions has avalanche activity in a direction perpendicular to the ionization wave direction. Unless an argument can be made that the standard discharge process (with avalanches in a direction along the path) are more favorable for nonattaching gases (perhaps due to increased ionization from collective electron drift along the path), whereas, the larger total fields available in certain regions along the surface path are more favorable for attaching gases (where either transverse or longitudinal drift is limited in range), or some other argument linked to the field can be found, we see from these simulations that the field level alone is not an explanation for the different paths taken in the two gases; we are then back to investigating the effects of surface photoemission (or possibly films of attached gas molecules on the surface) in our search for an explanation. On the other hand these simulations also indicate that the "average" field levels are not much different along the two paths, and hence we might term the discharge path in this experiment more of a secondary effect.

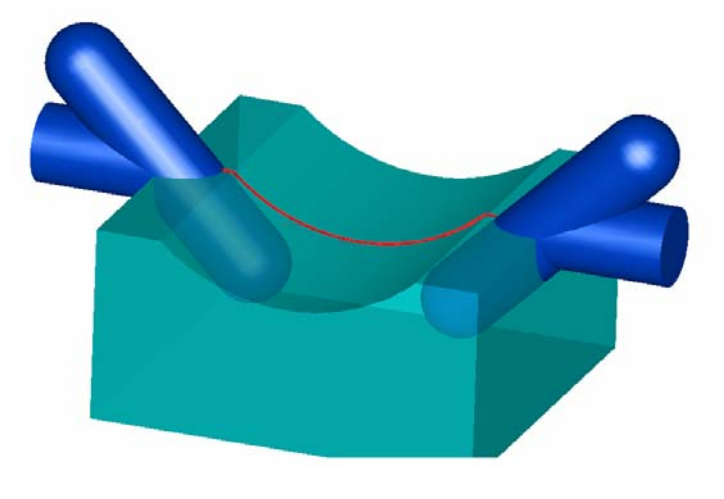


Figure 20. Breakdown path forced to hug surface.

3 ONE DIMENSIONAL FLUID MODEL

We begin the setup of the models in this report by giving the one dimensional fluid equations, which make clear the meaning of the rate coefficients and source terms. The one dimensional continuity equations are [33]

$$\frac{\partial N_e}{\partial t} = S + N_e \alpha |W_e| - N_e \eta |W_e| - N_e N_p \beta - \frac{\partial (N_e W_e)}{\partial x} + \frac{\partial}{\partial x} \left(D \frac{\partial N_e}{\partial x} \right) \quad (1)$$

$$\frac{\partial N_p}{\partial t} = S + N_e \alpha |W_e| - N_e N_p \beta - N_n N_p \beta - \frac{\partial (N_p W_p)}{\partial x} \quad (2)$$

$$\frac{\partial N_n}{\partial t} = N_e \eta |W_e| - N_p N_n \beta - \frac{\partial (N_n W_n)}{\partial x} \quad (3)$$

where N_e is the electron density, N_p is the positive ion density, N_n is the negative ion density, W_e is the electron velocity in the x direction, W_p is the positive ion velocity in the x direction, and W_n is the negative ion velocity in the x direction. The rate coefficients α and η are the primary ionization and attachment coefficients, β is the recombination coefficient and D is the diffusion coefficient. The source term S is the photoionization source term (on a surface there are source terms due to photoemission and secondary electron emission).

3.1 Photoionization Source Term

The source term S results from photoionization

$$S(x) = \sum_j \gamma_j \int_0^d \Omega(x-x') N_e(x') \alpha_j^*(x') |W_e(x')| e^{-\mu_j X} dx' \quad (4)$$

where γ_j is the secondary ionization coefficient for photoionization, α_j^* the excitation coefficient for ionizing radiation, μ_j is the photon absorption coefficient, $X = |x - x'|$, and Ω is the solid angle subtended at x' by the disc charge with channel radius r_c at x

$$\Omega(x-x') = \frac{1}{2} \left[1 - \frac{X}{\sqrt{r_c^2 + X^2}} \right] \quad (5)$$

This is actually the relative solid angle (normalized by the solid angle of a complete sphere 4π).

$$\Omega = \frac{1}{4\pi} \int_0^{\theta_0} \sin \theta d\theta \int_0^{2\pi} d\varphi = \frac{1}{2} (1 - \cos \theta_0) \quad (6)$$

$$\theta_0 = \arcsin \left(\frac{r_c}{\sqrt{r_c^2 + X^2}} \right) = \arccos \left(\frac{X}{\sqrt{r_c^2 + X^2}} \right) \quad (7)$$

3.1.1 Photoionization Secondary Excitation Coefficient

The excitation coefficient is taken to be proportional to the ionization coefficient [34], [35]

E/p	α'/p	η/p	$\alpha/p = (\alpha' - \eta)/p$
25.0	0.00120	0.00495	-0.00375
27.5	0.00205	0.00473	-0.00268
30.0	0.00340	0.00460	-0.00120
32.5	0.00560	0.00460	+0.00100
35.0	0.00880	0.00475	+0.00405
37.5	0.0130	0.00497	+0.0080
40.0	0.0190	0.00530	+0.0137
42.5	0.0260	0.00575	+0.0203
45.0	0.0340	0.00635	+0.0277
47.5	0.0460	0.00700	+0.0390
50.0	0.057	0.00780	+0.049
52.5	0.070	0.00870	+0.061
55.0	0.087	0.00967	+0.077
57.5	0.102	0.0108	+0.091
60.0	0.120	0.0119	+0.108
62.5	0.140		
65.0	0.170		

Table 1. Air ionization coefficients (from “High Voltage Engineering: Fundamentals,” E. Kuffel, et. al. [36]).

$$\alpha_j^* = c_j \alpha \quad (8)$$

where the constant of proportionality c_j is taken to be combined with the secondary ionization coefficient

$$\gamma'_j = \gamma_j c_j \quad (9)$$

3.2 Air Rates

The rate coefficients for air are given first.

3.2.1 Ionization And Attachment Coefficients

We take the fit to the effective ionization coefficient $(\alpha - \eta)$ to be [36], [37], [32]

$$(\alpha - \eta)/p \approx 0.005 \left\{ e^{-[p/E - 1/(31.25 \text{ V/(cm-T)})] 200 \text{ V/(cm-T)}} - 1 \right\} / (\text{cm-T}) , \quad E/p < 50 \text{ V/(cm-T)}$$

$$\approx 8.805 e^{-(p/E) 258.45 \text{ V/(cm-T)}} / (\text{cm-T}) , \quad 50 \text{ V/(cm-T)} < E/p < 200 \text{ V/(cm-T)}$$

$$\approx 15 e^{-(p/E) 365 \text{ V/(cm-T)}} / (\text{cm-T}) , \quad E/p > 200 \text{ V/(cm-T)} \quad (10)$$

The attachment coefficient is relatively insensitive to field for low fields

$$\eta/p \approx 0.005 / (\text{cm-T}) , \quad E/p < 40 \text{ V/(cm-T)} \quad (11)$$

Table 1 from [36] illustrates this effect. For higher field values the attachment coefficient grows slightly, as shown in the table, but the effective coefficient is overwhelmed by the ionization coefficient.

Notice that this formula and table give a critical field (where $\alpha - \eta = 0$) and reversal of sign (to positive values) when

$$E_c/p = 32.5 \text{ V}/(\text{cm} - \text{T}) = 25 \text{ kV}/(\text{cm} - \text{bar}) \quad (12)$$

3.2.2 Mobility

The mobility from Dutton [38] seems to be

$$\mu_e = 400 \text{ cm}^2/(\text{V} - \text{s}) = 0.04 \text{ m}^2/(\text{V} - \text{s}) \quad (13)$$

3.2.3 Diffusion Coefficient

In the large field near the streamer head we find the longitudinal diffusion coefficient [38]

$$D = 1800 \text{ cm}^2/\text{s} , 100 \text{ kV}/\text{cm} \quad (14)$$

This value is also used in 760 Torr nitrogen for streamer modeling [39], [40]

3.2.4 Recombination Coefficient

The recombination coefficient is taken as [41]

$$\beta = 2 \times 10^{-7} \text{ cm}^3/\text{s} \quad (15)$$

3.2.5 Photoionization Model

For air a more accurate photoionization model is available. The simple exponential decay is replaced by

$$\gamma'_j e^{-\mu_j X} \rightarrow \frac{p_q}{p + p_q} \xi \frac{\omega}{\alpha} f(X) \quad (16)$$

where the function f is given by [42]

$$f(r) = \frac{\exp(-k_1 p_{O_2} r) - \exp(-k_2 p_{O_2} r)}{r \ln(k_2/k_1)} \quad (17)$$

where

$$k_1 = 0.035 \text{ cm}^{-1} \text{ Torr}^{-1} \quad (18)$$

$$k_2 = 2 \text{ cm}^{-1} \text{ Torr}^{-1} \quad (19)$$

are the minimum and maximum absorption coefficients in oxygen for radiation in the range 98 – 102.5 nm, p_{O_2} is the partial pressure of oxygen (responsible for photon absorption in this range). Note that

$$\begin{aligned} \int_0^\infty f(r) dr &= \lim_{\varepsilon \rightarrow 0} \int_\varepsilon^\infty \frac{\exp(-k_1 p_{O_2} r) - \exp(-k_2 p_{O_2} r)}{r \ln(k_2/k_1)} dr \\ &= \frac{1}{\ln(k_2/k_1)} \lim_{\varepsilon \rightarrow 0} [E_1(k_1 p_{O_2} \varepsilon) - E_1(k_2 p_{O_2} \varepsilon)] = 1 \end{aligned} \quad (20)$$

where $E_1(x) = \int_x^\infty e^{-t} dt/t$ is the exponential integral function. The quenching pressure p_q has the form [42]

$$p_q = (n_0 \tau \langle \sigma v \rangle)^{-1} \quad (21)$$

where n_0 is the concentration of molecules at a pressure of 1 Torr, τ is the radiative lifetime, σ is the quenching cross section, and v is the relative velocity of the colliding particles. It appears that this relation arises because collisions between the excited atoms and other neutrals can transfer energy during the radiative lifetime and thereby quench the radiation process. The value is taken to be [42]

$$p_q = 30 \text{ T} \quad (22)$$

Values proposed for the factor $\xi\omega/\alpha$ are

$$\xi\omega/\alpha = 0.03, 0.1 \quad (23)$$

where the first value was proposed by [43] and the second value was suggested as an average by [44].

To connect this with another treatment of photoionization [45] (and to introduce the quantity used to model photoionization in kinetic simulations), we note that the number of photoions produced per ionizing collision per steradian per cm - Torr is [43]

$$\psi = \frac{p_q}{p + p_q} \left(\xi \frac{\omega}{\alpha} \right) \frac{f(r)}{4\pi p} \quad (24)$$

Hence we can write the total probability of producing a photoion for an ionizing collision as

$$P = 4\pi \int_0^\infty p\psi dr = \frac{p_q}{p + p_q} \left(\xi \frac{\omega}{\alpha} \right) \quad (25)$$

Penny [45] gave this probability for pressures p less than the quenching pressure (and ignoring the factor $p_q/(p + p_q)$) as $P = 0.02$, which is in the ballpark of the preceding lower limit. This function is shown in Figure 21.

For use in the calculations we write this in the form

$$4\pi p\psi = (\xi\omega/\alpha) \frac{p_q}{p + p_q} f(X) = \gamma_j^* \left[\frac{e^{-\mu_j^{(1)} X} - e^{-\mu_j^{(2)} X}}{X} \right] \quad (26)$$

where

$$S(x) = \int_0^d \Omega(x - x') N_e(x') \alpha(x') |W_e(x')| 4\pi p\psi dx'$$

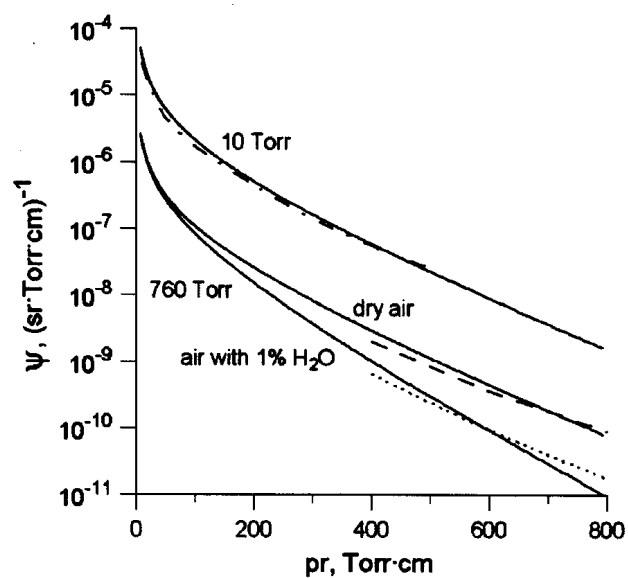


Figure 1. The function Ψ versus pressure times distance. Solid—calculation at pressures 10 and 760 Torr; dotted-dashed—[3]; dashed—[11], dry air; dotted—[11], air with 1% of H_2O .

Figure 21. Function ψ as a function of pr (from Naidis [43]).

$$= \int_0^d \Omega(x-x') N_e(x') \alpha(x') |W_e(x')| \frac{p_q}{p+p_q} (\xi\omega/\alpha) f(X) dx' \quad (27)$$

In the air simulations, below, we used the upper value $\xi\omega/\alpha = 0.1$.

3.2.6 Simplified Photoionization Model (Photoabsorption and Effective Secondary Coefficients)

Kulikovsky [44], [46] simplified the model for photoionization in air to a simple exponential at relatively small distances from the source of photons. He uses the photon absorption coefficient at one atmosphere, taken as [44]

$$\mu_j/p = 119.9/(\text{cm} - \text{bar}) \quad (28)$$

Thus in this case

$$4\pi p\psi = \gamma'_j e^{-\mu_j r} \quad (29)$$

and the value given is [44]

$$\gamma'_j/p = \frac{p_q}{p+p_q} 16.22/(\text{cm} - \text{bar}) \quad (30)$$

The photoionization source in this case is

$$S(x, E) = \gamma'_j \int_0^d \Omega(x-x') N_e(x') \alpha(E(x')) |W_e(E(x'))| e^{-\mu_j |x-x'|} dx' \quad (31)$$

We expect from this description that the probability of photoionization given an ionizing collision (removing the factor $N_e \alpha |W_e|$) is

$$P = 4\pi \int_0^\infty p\psi dr = \int_0^\infty \gamma'_j e^{-\mu_j r} dr = \gamma'_j/\mu_j \quad (32)$$

which has the value 0.1 at low pressures (10 T).

3.3 SF6 Rates

3.3.1 Ionization Coefficient

A fit to the ionization coefficient can be written as [47]

$$\alpha/N = 3.4473 \times 10^{34} (E/N)^{2.985} \text{ m}^2, \quad E/N < 4.6 \times 10^{-19} \text{ V} - \text{m}^2 = 460 \text{ Td}$$

$$= 11.269 (E/N)^{1.159} \text{ m}^2, \quad E/N > 4.6 \times 10^{-19} \text{ V} - \text{m}^2 = 460 \text{ Td} \quad (33)$$

where the units of E/N are MKS and suppressed in the arguments of the power laws. Note that the molecular density at $p = 1 \text{ bar} = 100 \text{ kPa}$ (one standard atmosphere is $P_0 = 101.325 \text{ kPa}$) and $T = 20^\circ \text{ C} = 293^\circ \text{ K}$, using $p = NkT$ is $N = 2.472 \times 10^{25} \text{ m}^{-3}$, where Boltzmann's constant is $k = 1.38066 \times 10^{-23} \text{ J/K}$.

3.3.2 Attachment Coefficient

The electron attachment coefficient can be written as [47]

$$\begin{aligned} \eta/N &= 2.0463 \times 10^{-20} - 0.25379 (E/N) + 1.4705 \times 10^{18} (E/N)^2 \\ &- 3.0078 \times 10^{36} (E/N)^3 \text{ m}^2, \quad 5.0 \times 10^{-20} \text{ V} - \text{m}^2 = 50 \text{ Td} < E/N < 2.0 \times 10^{-19} \text{ V} - \text{m}^2 = 200 \text{ Td} \\ &= 7.0 \times 10^{-21} \exp[-2.25 \times 10^{18} (E/N)] \text{ m}^2, \quad E/N > 2.0 \times 10^{-19} \text{ V} - \text{m}^2 = 200 \text{ Td} \end{aligned} \quad (34)$$

Notice that the critical field ($\alpha - \eta = 0$) from these formulas is at

$$E_c/p = 89 \text{ kV}/(\text{cm} - \text{bar}) \quad (35)$$

which corresponds to $E_c/N = 3.6 \times 10^{-19} \text{ V} - \text{m}^2 = 360 \text{ Td}$.

3.3.3 Electron Drift Velocity

The electron drift velocity can be written as [47]

$$w = 1.027 \times 10^{19} (E/N)^{0.7424} \text{ m/s}, \quad 10^{-20} \text{ V} - \text{m}^2 = 10 \text{ Td} < E/N < 2 \times 10^{-18} \text{ V} - \text{m}^2 = 2000 \text{ Td} \quad (36)$$

3.3.4 Electron Diffusion

The longitudinal electron diffusion coefficient can be written as [47]

$$D_L/\mu_e = 8.6488 \times 10^9 (E/N)^{1/2} \text{ V}, \quad E/N < 6.5 \times 10^{-19} \text{ V} - \text{m}^2 = 650 \text{ Td} \quad (37)$$

where the electron mobility can be found as

$$\mu_e = w/E \quad (38)$$

and thus

$$D_L N = 8.882 \times 10^{28} (E/N)^{0.2424} / (\text{m} - \text{s}), \quad 10 \text{ Td} < E/N < 650 \text{ Td} \quad (39)$$

3.3.5 Ionic Mobility

The ionic mobilities $\mu_{\pm}$ are determined from the normalized values $\mu_{\pm}^0$ from [47]

$$\mu = \mu^0 \frac{P_0}{P} \frac{T}{T_0} \quad (40)$$

where $P_0 = 101.325 \text{ kPa}$ is the standard atmospheric pressure and $T_0 = 273.16 \text{ K}$. Then for positive ions [47]

$$\begin{aligned}
\mu_+^0 &= 6.0 \times 10^{-5} \text{ m}^2\text{V}^{-1}/\text{s} , \ E/N < 1.2 \times 10^{-19} \text{ V} - \text{m}^2 = 120 \text{ Td} \\
&= [1.216 \times 10^{-5} \ln(E/N) + 5.89 \times 10^{-4}] \text{ m}^2\text{V}^{-1}/\text{s} , \\
1.2 \times 10^{-19} \text{ V} - \text{m}^2 = 120 \text{ Td} &< E/N < 3.5 \times 10^{-19} \text{ V} - \text{m}^2 = 350 \text{ Td} \\
&= -[1.897 \times 10^{-5} \ln(E/N) + 7.346 \times 10^{-4}] , \\
&E/N > 3.35 \times 10^{-19} \text{ V} - \text{m}^2 = 335 \text{ Td}
\end{aligned} \tag{41}$$

and for the negative ions [47]

$$\mu_-^0 = 1.69 \times 10^{32} (E/N)^2 + 5.3 \times 10^{-5} \text{ m}^2\text{V}^{-1}/\text{s} , \ E/N < 5.0 \times 10^{-19} \text{ V} - \text{m}^2 = 500 \text{ Td} \tag{42}$$

3.3.6 Recombination

The ion-volume recombination coefficient is [47]

$$\begin{aligned}
\beta &= K_r = 2.0 \times 10^{-13} (P)^{0.6336} \text{ m}^3/\text{s} , \ 1 \text{ kPa} \leq P \leq 39 \text{ kPa} \\
&= 2.28 \times 10^{-11} (P)^{-0.659} \text{ m}^3/\text{s} , \ 39 \text{ kPa} \leq P \leq 270 \text{ kPa} \\
&= 6.867 \times 10^{-10} (P)^{-1.279} \text{ m}^3/\text{s} , \ 270 \text{ kPa} \leq P \leq 2000 \text{ kPa}
\end{aligned} \tag{43}$$

where (P) is in kPa.

3.3.7 Photon Absorption Coefficient

The photon absorption coefficient is taken as [47]

$$\mu_j/N = (7.98 \pm 2.0) \times 10^{-23} \text{ m}^2 \tag{44}$$

or

$$\mu_j/p = (19.7 \pm 4.94) / (\text{cm} - \text{bar}) \tag{45}$$

Comparing this with the photoabsorption data in [48] for SF6 we see that this absorption distance corresponds to a wavelength of about 130 nm or 9.5 eV.

3.3.8 Photoionization Excitation Coefficient

According to Morrow [34], [35] we can take

$$\gamma'_j/p = 7.5 \times 10^{-4} / (\text{m} - \text{Pa}) = 0.75 / (\text{cm} - \text{bar}) \tag{46}$$

The photoionization source in this case is again

$$S(x, E) = \gamma'_j \int_0^d \Omega(x - x') N_e(x') \alpha(E(x')) |W_e(E(x'))| e^{-\mu_j |x - x'|} dx' \quad (47)$$

We expect from this description that the probability of photoionization given an ionizing collision (removing the factor $N_e \alpha |W_e|$) is

$$P = \int_0^\infty \gamma'_j e^{-\mu_j |x|} dx = \gamma'_j / \mu_j = 0.038 \quad (48)$$

similar to air.

4 KINETIC CALCULATIONS

In a volumetric breakdown calculation, the 1.5D code needs gas parameters that affect the growth of the streamer such as the Townsend first ionization coefficient (α) and, in an electronegative gas, the attachment coefficient (η). Expressions for these coefficients as a function of E/p exist in literature [32] and are summarized above.

4.1 Monte Carlo Calculation to Obtain Surface Parameters

When the streamer travels across a surface, processes come into play that modify the volumetric growth rates. Some of these processes are shown in Figure 22 for an anode directed streamer. The green block represents the dielectric surface. Inside the streamer the electrons, represented by circles with "e" inside, are clustered near the head, contributing to a large head field. The ions, both positive due to collision (α) and negative due to attachment (η) are formed in the head and migrate towards the streamer tail. The ions are represented in the figure by circles with "+" or "-" inside depending on the ion's charge. The fields of these charged particles cancel and lead to a smaller field in the tail. In the head the collisions release photons that are energetic enough to ionize a neutral. These photons, represented in the figure by a squiggly arrow, form seed electrons ahead of the streamer which then avalanche towards the head and contribute to the head electrons.

First and foremost, the surface will modify the field that the streamer experiences. In addition, we have accounted for two effects due to the surface: photoemission and secondary electron emission. Photoemission acts similarly to photoionization, but at a lower energy because the surface acts like a catalyst. In nitrogen, photons need to have an energy of at least 15 eV to have a chance to ionize a nitrogen neutral directly. If a surface is present, electrons can be emitted from the surface by 8 eV photons. Since there are many more collisions in the volume that emit 8 eV photons than there are collisions that emit 15 eV photons, the surface provides a source of electrons in front of the streamer head. This is shown in the figure by the rate α_s .

Secondary electron emission (SEE) is a process where an electron strikes the dielectric surface and depending on its energy either sticks to the surface or releases another electron. For the electron energies typical of a streamer, the electrons stick to the surface and the surface acts as an electron sink. This is shown in the figure by the rate η_s . Past studies indicate that for nitrogen the photoemission and SEE effects tend to cancel each other out and the rates near the surface are the same as the rates in the volume [11]. Since we don't know the effects of SEE and photoemission on the SF6 rates we ran the Monte Carlo code **breakdown** to determine them [10], [32]. We also obtained coefficients for nitrogen because we had most information about this gas. We used the nitrogen data as a substitute for air data in the 1.5D code.

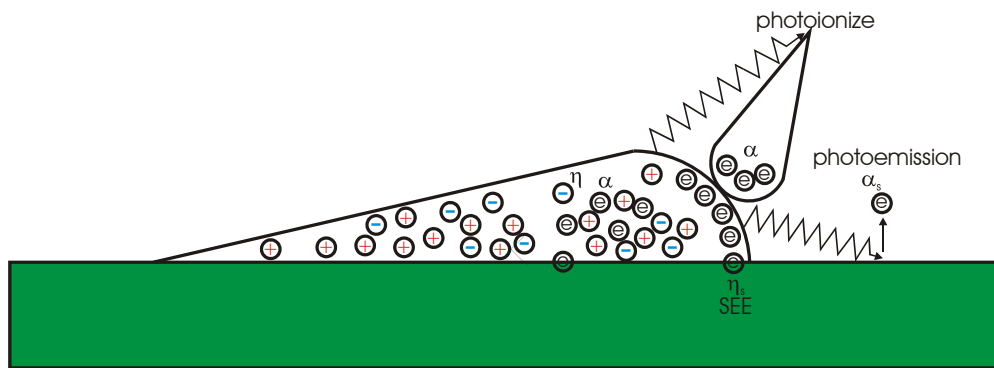


Figure 22. Processes for a streamer along a surface.

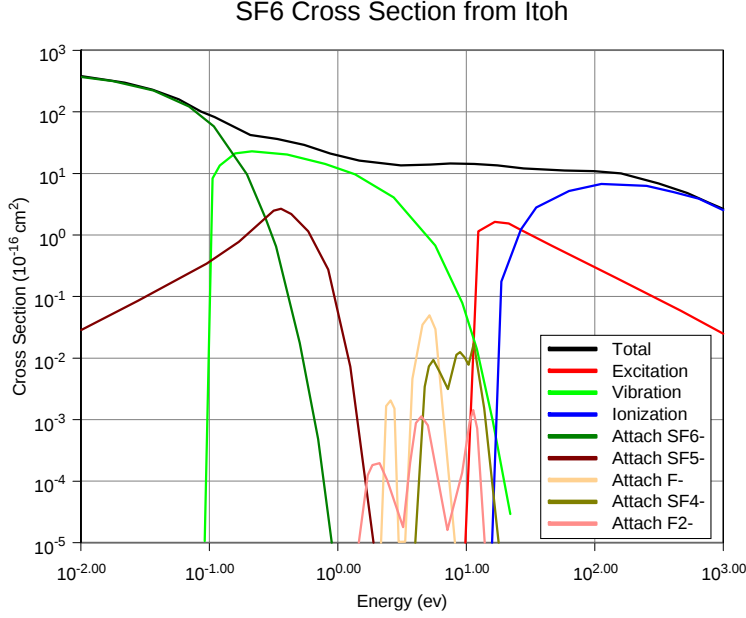


Figure 23. SF6 cross-sections used in kinetic simulations.

4.2 Monte Carlo Code Modifications for SF6

Since SF6 is an electronegative gas, we had to add the capability to handle attachment in **breakdown**. If an electron collides with the neutral background and an attaching collision occurs we create an ion with the negative charge of the electron. The ion's energy is the energy of the electron added to the thermal energy of the neutral. Finally, we delete the electron from the problem space.

The SF6 cross sections used in **breakdown** are from Itoh [49] and are shown in Figure 23. Included in the set are one each of cross sections for excitation, ionization and vibration. Also included are five attachment cross sections giving the cross-section for conversion to $\text{SF}_6^{(-)}$, $\text{SF}_5^{(-)}$, $\text{F}^{(-)}$, $\text{SF}_4^{(-)}$, and $\text{F}_2^{(-)}$. We do not differentiate between different types of attachment in the code. Christophorou has compiled data on cross-sections on SF6 from many sources and gives recommendations on which cross-section to use [50]. We discovered this data source too late to use, however.

Figures 24 and 25 show an example of a kinetic simulation in SF6.

The way we used **breakdown** to determine the effects of SEE and photoemission on electron growth rates is described in [11] for nitrogen. Here we will point out the differences that using SF6 as a gas entails. The type of gas does not enter the SEE calculation. It is only the energy of the electrons and material of the surface that are important.

The type of gas does enter the photoemission calculation and the problem with SF6 is that it hasn't been studied as extensively as nitrogen. Some of the data that we thought was sparse for nitrogen is non-existent for SF6 and we had to make many bounding assumptions. The first process important to photoemission is that an electron must collide with a neutral causing the neutral to be excited. In nitrogen, we had cross sections for eleven different excitations that would cause the neutral to emit in eleven different

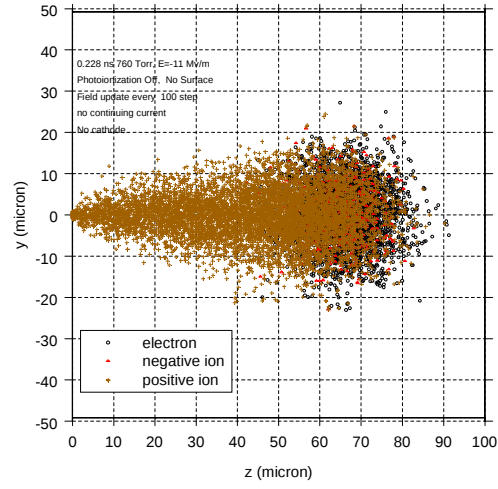


Figure 24. Avalanche in SF6.

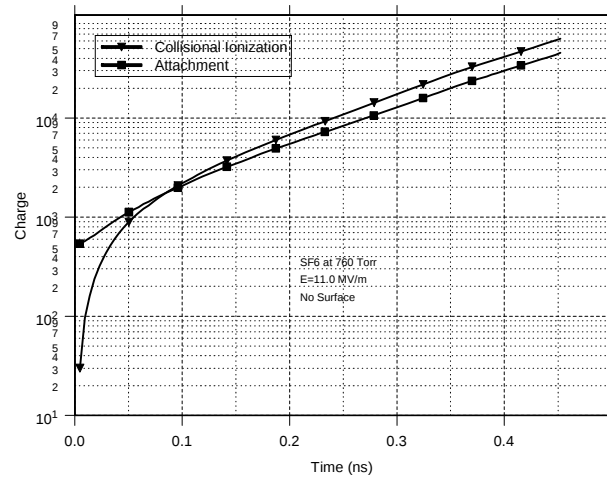


Figure 25. Example of SF6 ionization and attachment.

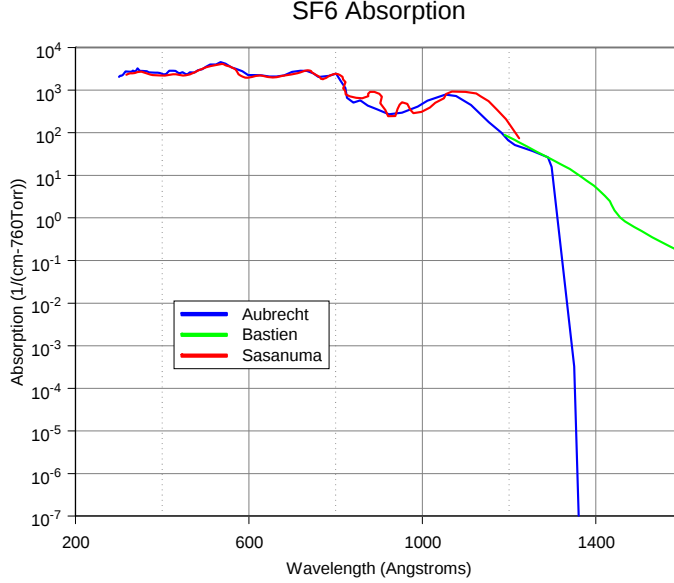


Figure 26. SF6 absorption.

spectral bands. In SF6, we have one excitation cross section. Once the neutral was excited in nitrogen we applied two bounding calculations to get the number of photons created and each of their energies: we created a single photon with the energy lost by the electron to the neutral in the collision (the threshold energy), or we used knowledge of the nitrogen spectrum to create a number of photons, each representing the neutral decaying to a lower energy state releasing a photon in a known band. In SF6, we don't know which energy level the neutral is excited to in the collision so we are forced to use the first method of creating a single photon with the threshold energy (9.8 eV). Once the photon is created, it must travel some distance through the gas. Along the way it is absorbed by the gas. Nitrogen absorption has been studied many times with special attention being paid to its band structure. We found three sources of absorption data for SF6 shown in Figure 26 [51], [52], [48]. Of these three, we chose the data by Aubrecht to put in the code because it was the most recent and agreed with the data from Sasanuma. Where it differs from Bastien is in the wavelength range greater than 1300 Å (9.5 eV). This is an important region because recall that we are generating a 9.8 eV excitation photon. Data for dielectric materials indicate that for photon energies between 7 and 10 eV the quantum yield – how many electrons are released from the surface per incident photon – increases several orders of magnitude. Transmission data in [19] indicates a curve that lies between Bastien and Aubrecht. By choosing the Aubrecht data we are emphasizing photoemission and increasing the ability of the surface to assist the breakdown process.

The last stage of the photoemission process consists of photons impacting a dielectric surface and releasing electrons. This depends on quantum yield of the surface and is independent of the type of gas. Details are discussed in [11].

4.3 Results

We wanted to get data that related the rate of electron generation from photoemission to the number of electrons being generated by collisional ionization similar to the photoionization data [45]. We therefore ran **breakdown** for various values of electric field over pressure (E/p) and kept track of the number of electrons generated due to photoemission dividing by the number of electrons generated by collisional

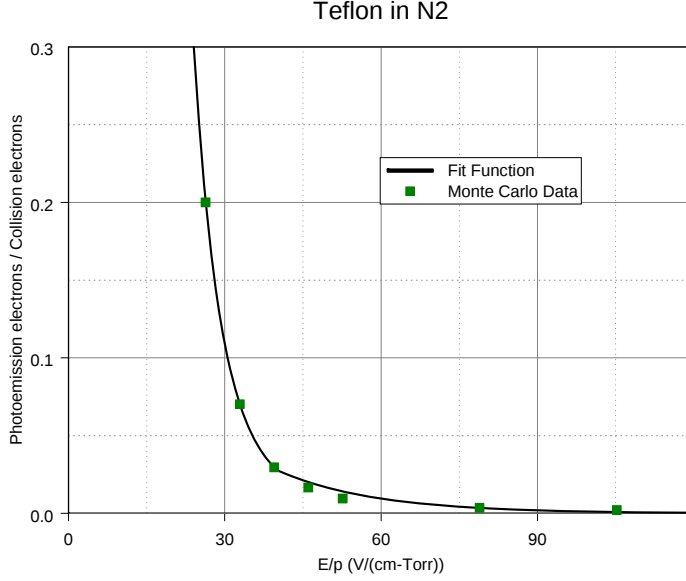


Figure 27. Photoemission for N2 and Teflon.

ionization. The results are shown green squares in Figure 27 for nitrogen and Figure 28 for SF6. The nitrogen data appears to vary exponentially with E/p and we used a feature of the graphing package **Axum** to fit an exponential curve to the data and extract a formula for that curve. The result was

$$\frac{\text{Photoemission electrons}}{\text{Collision electrons}} = \begin{cases} 81.59e^{(-0.2836E/p+0.0021E/p^2)} & \text{where } E/p < 40 \text{ V/cm-Torr} \\ 0.2372e^{(-0.0536E/p)} & \text{where } E/p > 40 \text{ V/cm-Torr} \end{cases} \quad (49)$$

The SF6 data does not show much variation with respect to E/p so we took the ratio as being constant:

$$\frac{\text{Photoemission electrons}}{\text{Collision electrons}} = 0.0006 \quad (50)$$

Note that by choosing to normalize the photoemitted by the collisional electrons (including attachment processes) we have brought the ionization coefficient α into the formulas for photoemission. If the ionization energies are significantly above the knee of the photoemission curve we would expect to see growth of this ratio for lower E/p levels. We observe this growth in air but not for SF6. This could be due to the inclusion of attachment in the kinetic calculations. We need to explore the presence of attachment in this calculation in the future. Connecting photoemission to the ionization coefficient was an expedient approach, since it then mimics photoionization in its implementation, however this could lead to a suppression of photoemission effects (particularly in SF6). In the future we might consider replacement of the collisional ionization coefficient by a collisional excitation coefficient (with energies near the knee of dielectric surface quantum yield curve 10 eV?) in the photoemission calculation.

For SEE we monitored the number of electrons captured by the dielectric as a function of time. This information along with the velocity of the avalanche allows us to extract an exponential parameter η_s for each E/p value. This quantity, normalized by pressure, is shown as green squares in Figure 29. The curve is a fit to the data similar in form to the exponential fits of the first ionization parameter α and is given for Teflon by the formula

$$\eta_s(E)/p = e^{-146 \text{ V/(cm-T)}/(E/p)} 0.0312 / (\text{cm} - \text{T}) \quad (51)$$

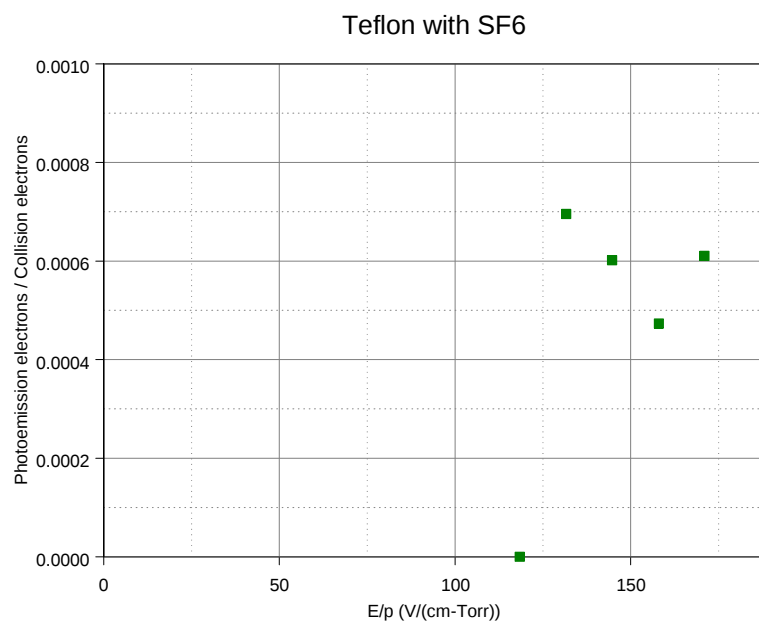


Figure 28. Photoemission for SF6 and Teflon.

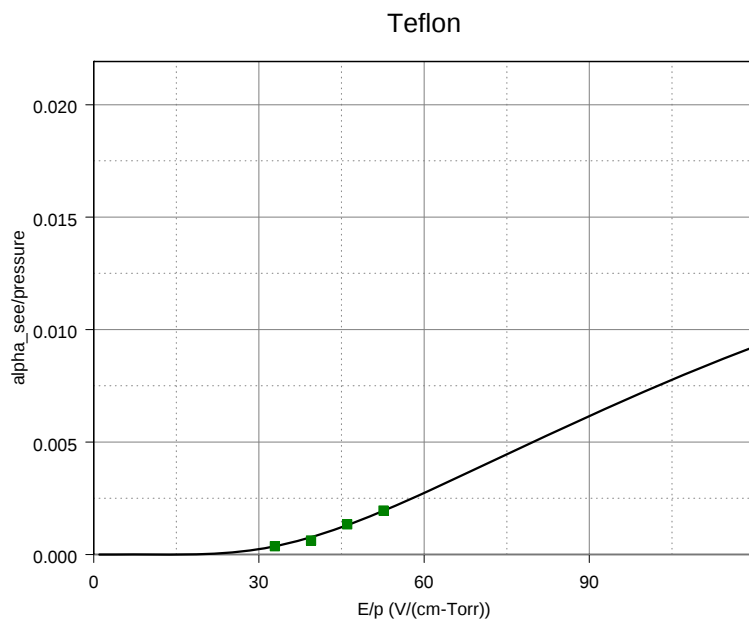


Figure 29. SEE for Teflon.

5 ONE-DIMENSIONAL FLUID AND 1.5D FIELD MODEL

This section summarizes relevant results from the papers by Gallimberti and Morrow [33], [34], [35], [53] on modeling of volumetric streamers and then treats the dielectric surface. The purpose of this model is to provide a means of examining ionization wave (streamer) sustaining field levels.

5.1 Axial Current And Charge Density

The source for the electric field is the total charge density

$$\rho = e(N_p - N_n - N_e) \quad (52)$$

where $e = 1.602189 \times 10^{-19}$ C is the magnitude of the electronic charge. The total axial current density is given by

$$J_x = e(N_p W_p - N_n W_n - N_e W_e) \quad (53)$$

5.2 Volume Electric Fields

The electric field is discussed for the volume case first and then for the surface. We consider the surface charge either uniformly distributed throughout a cylindrical disc (for each axial segment) or on the surface of a ring or tube (for each axial segment).

5.2.1 Disc Charges

For a cylinder with radius r_c of charge density $\rho(z')$, which is uniform in the cross section and extending from $z' = 0$ to $z' = d$, we can write the scalar potential as

$$\begin{aligned} \phi &= \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi}^{\pi} \frac{d\varphi'}{2\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \\ &= \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi+\varphi}^{\pi+\varphi} \frac{d\varphi'}{2\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\varphi - \varphi') + (z-z')^2}} \\ &= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr' \cos \varphi' + (z-z')^2}} \\ &= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{1}{\pi} \int_{-\pi}^0 \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr' \cos \varphi' + (z-z')^2}} \\ &= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{1}{\pi} \int_0^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr' \cos(\varphi' - \pi) + (z-z')^2}} \\ &= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{1}{\pi} \int_0^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r'^2 + 2rr' \cos \varphi' + (z-z')^2}} \\ &= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{r^2 + r'^2 + 2rr' \cos(2\varphi') + (z-z')^2}} \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{(r+r')^2 + (z-z')^2 - 4rr' \sin^2 \varphi'}} \\
&= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{1 - k^2 \sin^2 \varphi'}} \\
&= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} K(k)
\end{aligned} \tag{54}$$

where the complete elliptic integral of the first kind is

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \tag{55}$$

with argument

$$k = \frac{2\sqrt{rr'}}{\sqrt{(r+r')^2 + (z-z')^2}} \tag{56}$$

Now take the axial field

$$E_z = -\frac{\partial \phi}{\partial z} = -\frac{1}{2\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{\partial}{\partial z} \left[\frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} K(k) \right] \tag{57}$$

Using the identity

$$kK'(k) = \frac{1}{1-k^2} E(k) - K(k) \tag{58}$$

where the complete elliptic integral of the second kind is

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta \tag{59}$$

and

$$\frac{1}{k} \frac{\partial k}{\partial z} = -\frac{(z-z')}{(r+r')^2 + (z-z')^2} \tag{60}$$

$$1 - k^2 = \frac{(r-r')^2 + (z-z')^2}{(r+r')^2 + (z-z')^2} \tag{61}$$

$$\frac{\partial}{\partial z} \left[\frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} K(k) \right]$$

$$\begin{aligned}
&= \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} \left[\left\{ \frac{1}{1-k^2} E(k) - K(k) \right\} \frac{1}{k} \frac{\partial k}{\partial z} - \frac{(z-z')}{(r+r')^2 + (z-z')^2} K(k) \right] \\
&= - \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} \frac{(z-z')}{(r-r')^2 + (z-z')^2} E(k)
\end{aligned} \tag{62}$$

Now take the limit as the observation point moves to the center $r = 0$, $k = 0$, and use $E(0) = \pi/2$ to find

$$\begin{aligned}
E_z(0, z) &= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') (z-z') dz' \int_0^{r_c} \frac{r' dr'}{[r'^2 + (z-z')^2]^{3/2}} \\
&= \frac{1}{2\varepsilon_0} \int_0^d \rho(z') \left[\frac{z-z'}{|z-z'|} - \frac{z-z'}{\sqrt{r_c^2 + (z-z')^2}} \right] dz' \\
&= \frac{1}{2\varepsilon_0} \int_0^z \rho(z') \left[1 - \frac{z-z'}{\sqrt{r_c^2 + (z-z')^2}} \right] dz' + \frac{1}{2\varepsilon_0} \int_z^d \rho(z') \left[-1 - \frac{z-z'}{\sqrt{r_c^2 + (z-z')^2}} \right] dz' \\
&= \frac{1}{2\varepsilon_0} \int_{-z}^0 \rho(z+z') \left[1 + \frac{z'}{\sqrt{r_c^2 + z'^2}} \right] dz' + \frac{1}{2\varepsilon_0} \int_0^{d-z} \rho(z+z') \left[-1 + \frac{z'}{\sqrt{r_c^2 + z'^2}} \right] dz'
\end{aligned} \tag{63}$$

Note from charge balance that we expect

$$0 = \frac{1}{2\varepsilon_0} \int_{-z}^0 \rho(z+z') dz' + \frac{1}{2\varepsilon_0} \int_0^{d-z} \rho(z+z') dz' \tag{64}$$

The total field is the sum

$$E_z^{tot}(0, z) = E_{0z}(0, z) + E_z(0, z) \tag{65}$$

where $E_{0z}(0, z)$ is the external field created by the electrodes.

Constant Cylinder Charge

If we take the charge to be a pulse or constant in a cylinder over each

interval

$$z_{i'} - \frac{1}{2}\delta z_{i'-1/2} < z' < z_{i'} + \frac{1}{2}\delta z_{i'+1/2} \tag{66}$$

we have

$$E_z(0, z_i) = \frac{1}{2\varepsilon_0} \sum_{i'=1}^{i-1} \rho_{i'} \int_{z_{i'} - \delta z_{i'-1/2}/2}^{z_{i'} + \delta z_{i'+1/2}/2} \left[1 - \frac{z_i - z'}{\sqrt{r_c^2 + (z_i - z')^2}} \right] dz'$$

$$\begin{aligned}
& + \frac{1}{2\varepsilon_0} \rho_i \left\{ \int_{z_i - \delta z_{i-1/2}/2}^{z_i} \left[1 - \frac{z_i - z'}{\sqrt{r_c^2 + (z_i - z')^2}} \right] dz' + \int_{z_i}^{z_i + \delta z_{i+1/2}/2} \left[-1 - \frac{z_i - z'}{\sqrt{r_c^2 + (z_i - z')^2}} \right] dz' \right\} \\
& + \frac{1}{2\varepsilon_0} \sum_{i'=i+1}^I \rho_{i'} \int_{z_{i'} - \delta z_{i'-1/2}/2}^{z_{i'} + \delta z_{i'+1/2}/2} \left[-1 - \frac{z_i - z'}{\sqrt{r_c^2 + (z_i - z')^2}} \right] dz' \quad (67)
\end{aligned}$$

or

$$\begin{aligned}
E_z(0, z_i) &= \frac{1}{2\varepsilon_0} \sum_{i'=1}^{i-1} \rho_{i'} \int_{z_i - z_{i'} - \delta z_{i'+1/2}/2}^{z_i - z_{i'} + \delta z_{i'-1/2}/2} \left[1 - \frac{u}{\sqrt{r_c^2 + u^2}} \right] du \\
& + \frac{1}{2\varepsilon_0} \rho_i \left\{ \int_0^{\delta z_{i-1/2}/2} \left[1 - \frac{u}{\sqrt{r_c^2 + u^2}} \right] du + \int_{-\delta z_{i+1/2}/2}^0 \left[-1 - \frac{u}{\sqrt{r_c^2 + u^2}} \right] du \right\} \\
& + \frac{1}{2\varepsilon_0} \sum_{i'=i+1}^I \rho_{i'} \int_{z_i - z_{i'} - \delta z_{i'+1/2}/2}^{z_i - z_{i'} + \delta z_{i'-1/2}/2} \left[-1 - \frac{u}{\sqrt{r_c^2 + u^2}} \right] du \quad (68)
\end{aligned}$$

or

$$\begin{aligned}
& E_z(0, z_i) \\
&= \frac{1}{2\varepsilon_0} \sum_{i'=1}^{i-1} \rho_{i'} \left[\frac{1}{2} (\delta z_{i'+1/2} + \delta z_{i'-1/2}) - \sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} + \sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} \right] \\
& + \frac{1}{2\varepsilon_0} \rho_i \left\{ \delta z_{i-1/2}/2 - \sqrt{\delta z_{i-1/2}^2/4 + r_c^2} - \delta z_{i+1/2}/2 + \sqrt{r_c^2 + \delta z_{i+1/2}^2/4} \right\} \\
& + \frac{1}{2\varepsilon_0} \sum_{i'=i+1}^I \rho_{i'} \left[-\frac{1}{2} (\delta z_{i'+1/2} + \delta z_{i'-1/2}) - \sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} + \sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} \right] \quad (69)
\end{aligned}$$

or

$$\begin{aligned}
E_z(0, z_i) &= \frac{1}{2\varepsilon_0} \sum_{i'=1}^{i-1} \rho_{i'} \left[\delta z_{i'} - \sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} + \sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} \right] \\
& + \frac{1}{2\varepsilon_0} \rho_i \left\{ \delta z_{i-1/2}/2 - \sqrt{\delta z_{i-1/2}^2/4 + r_c^2} - \delta z_{i+1/2}/2 + \sqrt{r_c^2 + \delta z_{i+1/2}^2/4} \right\} \\
& + \frac{1}{2\varepsilon_0} \sum_{i'=i+1}^I \rho_{i'} \left[-\delta z_{i'} - \sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} + \sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} \right] \quad (70)
\end{aligned}$$

where

$$z_{i+1} - z_i = \delta z_{i+1/2} \quad (71)$$

$$\delta z_i = \frac{1}{2} (\delta z_{i+1/2} + \delta z_{i-1/2}) \quad (72)$$

and we note that the central $i = i'$ term is absent for uniform spacing. Note that if $\delta z_{i'} \ll |z_i - z_{i'}|$

$$E_z(0, z_i) \approx \frac{1}{2\varepsilon_0} \sum_{i'} \rho_{i'} \left[\operatorname{sgn}(z_i - z_{i'}) - \frac{z_i - z_{i'}}{\sqrt{r_c^2 + (z_i - z_{i'})^2}} \right] \delta z_{i'} \quad (73)$$

This is the same as an approximate evaluation of the integrals by the trapezoidal rule.

What if the channel radius is larger than the disc length $r \gg \delta z_{i'}$? In this case

$$\begin{aligned} \sqrt{r_c^2 + (z_i - z_{i'} \pm \delta z_{i' \mp 1/2}/2)^2} &\sim \sqrt{r_c^2 + (z_i - z_{i'})^2 \pm (z_i - z_{i'}) \delta z_{i' \mp 1/2}} \\ &\sim \sqrt{r_c^2 + (z_i - z_{i'})^2} \pm \frac{(z_i - z_{i'})}{\sqrt{r_c^2 + (z_i - z_{i'})^2}} \delta z_{i' \mp 1/2}/2 \end{aligned} \quad (74)$$

and

$$\begin{aligned} \sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} - \sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} &\sim -\frac{(z_i - z_{i'})}{\sqrt{r_c^2 + (z_i - z_{i'})^2}} (\delta z_{i'+1/2} + \delta z_{i'-1/2})/2 \\ &\sim -\frac{(z_i - z_{i'})}{\sqrt{r_c^2 + (z_i - z_{i'})^2}} \delta z_{i'} \end{aligned} \quad (75)$$

and we again obtain the asymptotic (or trapezoidal) form. Thus the criterion for switching to this form could more generally be written as

$$\sqrt{r_c^2 + (z_i - z_{i'})^2} \gg \delta z_{i'} \quad (76)$$

5.2.2 Tube Charges

We next consider the case where the charge is taken to be concentrated on the surface of a tube with surface charge density $\sigma(z')$ rather than throughout the volume. The potential is

$$\phi = \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi}^{\pi} r_c \frac{d\varphi'}{2\pi} \frac{\sigma(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$

$$\begin{aligned}
&= \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi+\varphi}^{\pi+\varphi} r_c \frac{d\varphi'}{2\pi} \frac{\sigma(z')}{\sqrt{r^2 + r_c^2 - 2rr_c \cos(\varphi - \varphi') + (z - z')^2}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r_c^2 - 2rr_c \cos \varphi' + (z - z')^2}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{\pi} \int_{-\pi}^0 \frac{d\varphi'}{\sqrt{r^2 + r_c^2 - 2rr_c \cos \varphi' + (z - z')^2}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{\pi} \int_0^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r_c^2 - 2rr_c \cos(\varphi' - \pi) + (z - z')^2}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{\pi} \int_0^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r_c^2 + 2rr_c \cos \varphi' + (z - z')^2}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{r^2 + r_c^2 + 2rr_c \cos(2\varphi') + (z - z')^2}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{(r + r_c)^2 + (z - z')^2 - 4rr_c \sin^2 \varphi'}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi \sqrt{(r + r_c)^2 + (z - z')^2}} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{1 - k^2 \sin^2 \varphi'}} \\
&= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi \sqrt{(r + r_c)^2 + (z - z')^2}} K(k)
\end{aligned} \tag{77}$$

$$k = \frac{2\sqrt{rr_c}}{\sqrt{(r + r_c)^2 + (z - z')^2}} \tag{78}$$

Now take the axial field

$$E_z = -\frac{\partial \phi}{\partial z} = -\frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{\partial}{\partial z} \left[\frac{2}{\pi \sqrt{(r + r_c)^2 + (z - z')^2}} K(k) \right] \tag{79}$$

Specializing to the axial electric field on axis we take $r = 0$, $k = 0$, and $E(0) = \pi/2$

$$E_z(0, z) = \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') \frac{(z - z')}{[r_c^2 + (z - z')^2]^{3/2}} dz' \tag{80}$$

Note from charge balance that we expect

$$0 = \frac{1}{2\varepsilon_0} \int_{-z}^0 \sigma(z+z') dz' + \frac{1}{2\varepsilon_0} \int_0^{d-z} \sigma(z+z') dz' \quad (81)$$

Constant Cylinder Charge
interval

If we take the charge to be a pulse or constant in a tube over each

$$z_{i'} - \frac{1}{2}\delta z_{i'-1/2} < z' < z_{i'} + \frac{1}{2}\delta z_{i'+1/2} \quad (82)$$

we have

$$\begin{aligned} E_z(0, z_i) &= \frac{r_c}{2\varepsilon_0} \sum_{i'=1}^I \sigma_{i'} \int_{z_{i'} - \delta z_{i'-1/2}/2}^{z_{i'} + \delta z_{i'+1/2}/2} \frac{(z_i - z')}{[r_c^2 + (z_i - z')^2]^{3/2}} dz' \\ &= \frac{r_c}{2\varepsilon_0} \sum_{i'=1}^I \sigma_{i'} \left[1/\sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} - 1/\sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} \right] \end{aligned} \quad (83)$$

where

$$z_{i+1} - z_i = \delta z_{i+1/2} \quad (84)$$

$$\delta z_i = \frac{1}{2} (\delta z_{i+1/2} + \delta z_{i-1/2}) \quad (85)$$

Note that if $\delta z_{i'} < |z_i - z_{i'}|$

$$E_z(0, z_i) \approx \frac{r_c}{2\varepsilon_0} \sum_{i'} \sigma_{i'} \frac{z_i - z_{i'}}{\{r_c^2 + (z_i - z_{i'})^2\}^{3/2}} \delta z_{i'} \quad (86)$$

At this point we ask how we relate the surface charge density $\sigma_{i'}$ to the volume charge density $\rho_{i'}$. Judging from Aleksandrov's paper [13] the connection must be made by equating these quantities to the charge per unit length $q_{i'}$ by means of

$$\pi r_c^2 \rho_{i'} \delta z_{i'} = 2\pi r_c \sigma_{i'} \delta z_{i'} = q_{i'} \delta z_{i'} \quad (87)$$

or

$$\pi r_c^2 \rho_{i'} = 2\pi r_c \sigma_{i'} = q_{i'} \quad (88)$$

Noting that

$$\rho = e(N_p - N_n - N_e) \quad (89)$$

we see that

$$\sigma = \left(\frac{1}{2}r_c\right) e (N_p - N_n - N_e) \quad (90)$$

5.3 Summary Of One Dimensional System

The equations for the volume case are $E = |E_x^{tot}|$, where it is often convention to change from z to x in the one-dimensional problem

$$\frac{\partial N_e}{\partial t} = S(x, E) + N_e [\alpha(E) - \eta(E)] |W_e(E)| - N_e N_p \beta - \frac{\partial [N_e W_e(E)]}{\partial x} + \frac{\partial}{\partial x} \left[D_L(E) \frac{\partial N_e}{\partial x} \right] \quad (91)$$

$$\frac{\partial N_p}{\partial t} = S + N_e \alpha(E) |W_e(E)| - N_e N_p \beta - N_n N_p \beta - \frac{\partial [N_p W_p(E)]}{\partial x} \quad (92)$$

$$\frac{\partial N_n}{\partial t} = N_e \eta(E) |W_e(E)| - N_p N_n \beta - \frac{\partial [N_n W_n(E)]}{\partial x} \quad (93)$$

5.3.1 Photoionization Computation

The computation of the photoionization is accomplished by means of

$$S(x, E) = \gamma'_j \int_0^d \frac{1}{2} \left[1 - \frac{|x - x'|}{\sqrt{r_c^2 + (x - x')^2}} \right] N_e(x') \alpha(E(x')) |W_e(E(x'))| e^{-\mu_j |x - x'|} dx' \quad (94)$$

or

$$S(x_i, E) = \gamma'_j \sum_{i'} \frac{1}{2} \left[1 - \frac{|x_i - x_{i'}|}{\sqrt{r_c^2 + (x_i - x_{i'})^2}} \right] N_e(x_{i'}) \alpha(E(x_{i'})) |W_e(E(x_{i'}))| e^{-\mu_j |x_i - x_{i'}|} \delta x_{i'} \quad (95)$$

5.3.2 Discretization

The problem is first discretized by selecting a grid x_i with $i = 1, \dots, I$ and

$$x_{i+1} - x_i = \delta x_{i+1/2} \quad (96)$$

$$\delta x_i = \frac{1}{2} (\delta x_{i+1/2} + \delta x_{i-1/2}) \quad (97)$$

5.3.3 Flux Corrected Algorithm

We use the flux corrected algorithm [54]. The canonical equation is taken as

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho w) + \frac{\partial}{\partial x}\left(D\frac{\partial \rho}{\partial x}\right) + A\rho w + S \quad (98)$$

where in this case ρ is the particle density, w is the particle velocity, D is the diffusion constant, A is a constant that can be field dependent, and S is a source term that may depend on ρw and the field.

The first step is to find an initial update to the density at time t_{n+1} from the value at t_n

$$\begin{aligned} \tilde{\rho}_i^{n+1} = & \rho_i^n - \frac{1}{2} \left[\frac{\delta x_{i+1/2}}{\delta x_i} \xi_{i+1/2} (\rho_{i+1}^n + \rho_i^n) - \frac{\delta x_{i-1/2}}{\delta x_i} \xi_{i-1/2} (\rho_i^n + \rho_{i-1}^n) \right] \\ & + \left[\frac{\delta x_{i+1/2}}{\delta x_i} v_{i+1/2} (\rho_{i+1}^n - \rho_i^n) - \frac{\delta x_{i-1/2}}{\delta x_i} v_{i-1/2} (\rho_i^n - \rho_{i-1}^n) \right] \end{aligned} \quad (99)$$

$$w_{i+1/2} = \frac{1}{2} (w_{i+1} + w_i) \quad (100)$$

$$\xi_{i+1/2} = w_{i+1/2} \delta t / \delta x_{i+1/2} \quad (101)$$

and the antidiffusive terms (we have replaced δx_i by δx_{i-1} in the denominator of the overall factor in the second equation)

$$\begin{aligned} \phi_{i+1/2} = & \mu_{i+1/2} \frac{\delta x_{i+1/2}}{\delta x_i} \left[\tilde{\rho}_{i+1}^{n+1} - \tilde{\rho}_i^{n+1} \right. \\ & \left. + \frac{1}{6} \left\{ -\frac{\delta x_{i+3/2}}{\delta x_{i+1}} (\rho_{i+2}^n - \rho_{i+1}^n) + \frac{\delta x_{i+1/2}}{\delta x_{i+1}} (\rho_{i+1}^n - \rho_i^n) + \frac{\delta x_{i+1/2}}{\delta x_i} (\rho_{i+1}^n - \rho_i^n) - \frac{\delta x_{i-1/2}}{\delta x_i} (\rho_i^n - \rho_{i-1}^n) \right\} \right] \end{aligned} \quad (102)$$

$$\begin{aligned} \phi_{i-1/2} = & \mu_{i-1/2} \frac{\delta x_{i-1/2}}{\delta x_i} \left[\tilde{\rho}_i^{n+1} - \tilde{\rho}_{i-1}^{n+1} \right. \\ & \left. + \frac{1}{6} \left\{ -\frac{\delta x_{i+1/2}}{\delta x_i} (\rho_{i+1}^n - \rho_i^n) + \frac{\delta x_{i-1/2}}{\delta x_i} (\rho_i^n - \rho_{i-1}^n) + \frac{\delta x_{i-1/2}}{\delta x_{i-1}} (\rho_i^n - \rho_{i-1}^n) - \frac{\delta x_{i-3/2}}{\delta x_{i-1}} (\rho_{i-1}^n - \rho_{i-2}^n) \right\} \right] \end{aligned} \quad (103)$$

$$v_{i+1/2} = \frac{1}{6} + \frac{1}{3} \varepsilon_{i+1/2}^2 \quad (104)$$

$$\mu_{i+1/2} = \frac{1}{6} \left(1 - \varepsilon_{i+1/2}^2 \right) \quad (105)$$

and thus (see below for limiting criterion to define $\tilde{\phi}_{i+1/2}$)

$$\rho_i^{n+1} = \tilde{\rho}_i^{n+1} - \tilde{\phi}_{i+1/2} + \tilde{\phi}_{i-1/2} \quad (106)$$

To include diffusive term replace v by v'

$$v'_{i+1/2} = v_{i+1/2} + \eta_{i+1/2} \quad (107)$$

$$\eta_{i+1/2} = \delta t D_{i+1/2} / \delta x_{i+1/2}^2 \quad (108)$$

$$D_{i+1/2} = \frac{1}{2} (D_i + D_{i+1}) \quad (109)$$

The limited antidiffusive fluxes are found from the algorithm [55]

$$\tilde{\phi}_{i+1/2} = \phi_{i+1/2} C_{i+1/2}, 0 \leq C_{i+1/2} \leq 1 \quad (110)$$

$$C_{i+1/2} = \left\{ \begin{array}{l} \min(R_{i+1}^+, R_i^-) , \phi_{i+1/2} \geq 0 \\ \min(R_i^+, R_{i+1}^-) , \phi_{i+1/2} < 0 \end{array} \right\} \quad (111)$$

$$R_i^\pm = \left\{ \begin{array}{l} \min(1, Q_i^\pm / P_i^\pm) , P_i^\pm > 0 \\ 0 , P_i^\pm = 0 \end{array} \right\} \quad (112)$$

$$Q_i^+ = (\rho_i^{\max} - \tilde{\rho}_i) \Delta x_i \quad (113)$$

$$P_i^+ = \max(0, \phi_{i-1/2}) - \min(0, \phi_{i+1/2}) \text{ the sum of all antidiffusive fluxes into grid point } i \quad (114)$$

$$Q_i^- = (\tilde{\rho}_i - \rho_i^{\min}) \Delta x_i \quad (115)$$

$$P_i^- = \max(0, \phi_{i+1/2}) - \min(0, \phi_{i-1/2}) \text{ the sum of all antidiffusive fluxes from grid point } i \quad (116)$$

$$\text{set } \phi_{i+1/2} = 0 \text{ if } \phi_{i+1/2} (\tilde{\rho}_{i+1}^{n+1} - \tilde{\rho}_i^{n+1}) < 0 \quad (117)$$

$$\text{and either } \phi_{i+1/2} (\tilde{\rho}_{i+2}^{n+1} - \tilde{\rho}_{i+1}^{n+1}) < 0 \quad (118)$$

$$\text{or } \phi_{i+1/2} (\tilde{\rho}_i^{n+1} - \tilde{\rho}_{i-1}^{n+1}) < 0 \quad (119)$$

$$\tilde{\rho}_i^a = \max(\rho_i^n, \tilde{\rho}_i^{n+1}) \quad (120)$$

$$\tilde{\rho}_i^{\max} = \max(\tilde{\rho}_{i-1}^a, \rho_i^n, \tilde{\rho}_{i+1}^a) \quad (121)$$

$$\tilde{\rho}_i^b = \min(\rho_i^n, \tilde{\rho}_i^{n+1}) \quad (122)$$

$$\tilde{\rho}_i^{\min} = \min \left(\tilde{\rho}_{i-1}^b, \tilde{\rho}_i^b, \tilde{\rho}_{i+1}^b \right) \quad (123)$$

To include the remaining terms we take

$$A\rho w + S = A(E_i^n) \rho_i^n w_i^n + S(x_i, E_i^n) \quad (124)$$

$$E_i^n = |E_x(x_i, t_n)| \quad (125)$$

5.3.4 Zalesak Modification

The modification of the basic Boris and Book algorithm by Zalesak is now given. The problem we wish to solve is

$$\rho_t + f_x = 0 \quad (126)$$

$$f = \rho w \quad (127)$$

A finite difference approximation to this equation is in conservation (or flux) form when it can be written as

$$\rho_i^{n+1} = \rho_i^n - \Delta x_i^{-1} [F_{i+1/2} - F_{i-1/2}] \quad (128)$$

where w and f are defined at the spatial grid points x_i and temporal grid points t^n and $\Delta x_i = (x_{i+1} - x_{i-1})/2$. The $F_{i+1/2}$ are called transportive fluxes, and are functions of f at one or more of the time levels t^n . The functional dependence of F on f defines the integration scheme (leapfrog, Lax-Wendroff, Crank Nicholson, donor cell, etc.).

It is well known that higher order (order 2 and above) schemes for numerically integrating (126) suffer from dispersive ripples in ρ , particularly near steep gradients in ρ . Lower order schemes, such as donor cell, Lax-Friedrichs, or high order with a zeroth order diffusion added, produce no ripples but suffer from excessive numerical diffusion. Flux-corrected transport (FCT) is a technique developed by Boris and Book which embodies the best of both of the above worlds. FCT constructs the net transportive flux point by point (non linearly) as a weighted average of a flux computed by a low order scheme and a flux computed by a high order scheme. The weighting is done in a manner which insures that the high order flux is used to the greatest extent possible without introducing ripples. This weighting procedure is referred to as flux-correction of flux-limiting. The result is a family of transport algorithms capable of resolving moving contact discontinuities and shock fronts without undershoot and overshoot. The procedure is as follows:

1. Compute $F_{i+1/2}^L$, the transportive flux given by some low order scheme guaranteed to give monotonic (ripple free) results for the problem at hand. The low order leapfrog-trapezoidal algorithm is a donor cell plus a zeroth order diffusive flux with coefficient 1/8

$$F_{i+1/2}^L = w_{i+1/2} \rho_{i+1/2}^{DC} - \frac{1}{8} (x_{i+1} - x_i) (\rho_{i+1}^0 - \rho_i^0) \Delta t^{-1} \quad (129)$$

where

$$w_{i+1/2} = \frac{1}{2} (w_i + w_{i+1}) \quad (130)$$

$$\rho_{i+1/2}^{DC} = \begin{cases} \rho_i^0, & \text{if } w_{i+1/2} \geq 0 \\ \rho_{i+1}^0, & \text{if } w_{i+1/2} < 0 \end{cases} \quad (131)$$

$$\rho_i^0 = \begin{cases} \rho_i^{n-1}, & \text{for leapfrog step} \\ \rho_i^n, & \text{for trapezoidal step} \end{cases} \quad (132)$$

2. Compute $F_{i+1/2}^H$, the transportive flux given by some high order scheme;

3. Define the antidiffusive flux

$$A_{i+1/2} = F_{i+1/2}^H - F_{i+1/2}^L \quad (133)$$

4. Compute the updated low order (transported and diffused) solution:

$$\rho_i^{td} = \rho_i^n - \Delta x_i^{-1} \left[F_{i+1/2}^L - F_{i-1/2}^L \right] \quad (134)$$

5. Limit the $A_{i+1/2}$ in a manner such that ρ^{n+1} as computed in step 6 below is free of extreme not found in ρ^{td} or ρ^n :

$$A_{i+1/2}^C = C_{i+1/2} A_{i+1/2}, \quad 0 \leq C_{i+1/2} \leq 1 \quad (135)$$

6. Apply the limited antidiffusive fluxes:

$$\rho_i^{n+1} = \rho_i^{td} - \Delta x_i^{-1} \left[A_{i+1/2}^C - A_{i-1/2}^C \right] \quad (136)$$

The problem is discretized by selecting a grid x_i with $i = 1, \dots, I$ and

$$x_{i+1} - x_i = \delta x_{i+1/2} \quad (137)$$

$$\delta x_i = \frac{1}{2} (\delta x_{i+1/2} + \delta x_{i-1/2}) \quad (138)$$

We want to solve

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} (\rho w) = 0 \quad (139)$$

$$\rho(x, 0) = g(x) \quad (140)$$

Let the x domain be divided into cells of width Δx where ρ_i and w_i are the density and the velocity at the center of each cell. Note that for uniform spacing

$$x_{i+1} - x_i = \Delta x = \delta x_{i+1/2} \quad (141)$$

$$\delta x_i = \frac{1}{2} (\delta x_{i+1/2} + \delta x_{i-1/2}) = \Delta x \quad (142)$$

Now at time step m and time $t = m\Delta t$ we assume that these variables are known on the grid. The objective is to compute the density one time step later ρ_i^{m+1} . First a diffused density is calculated using a low order scheme for computing the spatial derivative. An example is the donor cell method

$$\rho_i^D = \rho_i^m - \frac{\Delta t}{\Delta x} (f_{i+1/2} - f_{i-1/2}) \quad (143)$$

where

$$f_{i+1/2} = \begin{cases} w_{i+1/2}^m \rho_i^m, & w_{i+1/2}^m \geq 0 \\ w_{i+1/2}^m \rho_{i+1}^m, & w_{i+1/2}^m < 0 \end{cases} \quad (144)$$

and

$$w_{i+1/2}^m = \frac{w_i^m + w_{i+1}^m}{2} \quad (145)$$

$f_{i1/2}$ is the low order flux at the boundary between the i and $i + 1$ cells.

Second, a higher order scheme is used to compute the spatial derivative in the transport equation, using higher order fluxes. Using the Leapfrog-Trapezoidal scheme to obtain the higher order fluxes, and intermediate density is calculated with the expression

$$\rho_i^{int} = \rho_i^{m=1} - \frac{2\Delta t}{\Delta x} (F_{i+1/2} - F_{i-1/2}) \quad (146)$$

where for example in the fourth order

$$F_{i+1/2}(f) = \frac{7}{12} (f_{i+1} + f_i) - \frac{1}{12} (f_{i+2} + f_{i-1}) \quad (147)$$

The high order flux $F_{i+1/2}^H$ is then obtained as follows

$$F_{i+1/2}^H = F_{i+1/2}(f^S) \quad (148)$$

where

$$f_i^S = \rho_i^S w_i^m \quad (149)$$

$$\rho_i^S = \frac{\rho_i^m + \rho_i^{int}}{2} \quad (150)$$

this density profile contains ripples.

In the final step of the FCTZ, the high order fluxes and the diffused solution, ρ_i^D , are used in conjunction with a nonlinear filter to obtain the density profile at time $(m + 1)\Delta t$. The objective is to adjust the fluxes in and out of a cell so as to decrease the diffusion introduced by the low order scheme without introducing ripples into the density profile. The density at time $t^{m+1} = (m + 1)\Delta t$ is obtained from the equation

$$\rho_i^{m+1} = \rho_i^D - \frac{\Delta t}{\Delta x} [A_{i+1/2}^C - A_{i-1/2}^C] \quad (151)$$

where

$$A_{i+1/2}^C = C_{i+1/2} A_{i+1/2} \quad (152)$$

with

$$0 \leq C_{i+1/2} \leq 1 \text{ and } A_{i+1/2} = F_{i+1/2}^H - f_{i+1/2} \quad (153)$$

where $A_{i+1/2}$ is the difference between high and low order fluxes. It corresponds to the maximum amount of anti-diffusion flux available for reducing the diffusion introduced by the low order scheme. The coefficients $C_{i+1/2}$ determine the amount of anti-diffusion fluxes to be used. The fluxes $A_{i+1/2}^C$ are determined as follows.

If $A_{i+1/2} (\rho_{i+1}^D - \rho_i^D) < 0$ and either $A_{i+1/2} (\rho_{i+2}^D - \rho_{i+1}^D) < 0$, or $A_{i+1/2} (\rho_i^D - \rho_{i-1}^D) < 0$ then

$$A_{i+1/2} = 0 \quad (154)$$

The purpose of this criterion is to ignore any antidiffusive flux that tends to decrease the slope of continuously increasing or decreasing portions of the density profile. That is, the high order fluxes can only steepen a profile. The coefficients are obtained from the recipe

$$C_{i+1/2} = \begin{cases} \min(R_{i+1}^+, R_i^-) & \text{if } A_{i+1/2} \geq 0 \\ \min(R_i^+, R_{i+1}^-) & \text{if } A_{i+1/2} < 0 \end{cases} \quad (155)$$

where

$$R_i^+ = \begin{cases} \min(1, Q_i^+ / P_i^+) & \text{if } P_i^+ > 0 \\ 0 & \text{if } P_i^+ = 0 \end{cases} \quad (156)$$

$$R_i^- = \begin{cases} \min(1, Q_i^- / P_i^-) & \text{if } P_i^- > 0 \\ 0 & \text{if } P_i^- = 0 \end{cases} \quad (157)$$

and

$$P_i^+ = \max(0, A_{i-1/2}) - \min(0, A_{i+1/2}) \quad (158)$$

$$Q_i^+ = (\rho_i^{\max} - \rho_i^D) \frac{\Delta x}{\Delta t} \quad (159)$$

$$P_i^- = \max(0, A_{i+1/2}) - \min(0, A_{i-1/2}) \quad (160)$$

$$Q_i^- = (\rho_i^D - \rho_i^{\min}) \frac{\Delta x}{\Delta t} \quad (161)$$

with

$$\rho_i^{\max} = \max(\rho_{i-1}^a, \rho_i^a, \rho_{i+1}^a) \quad (162)$$

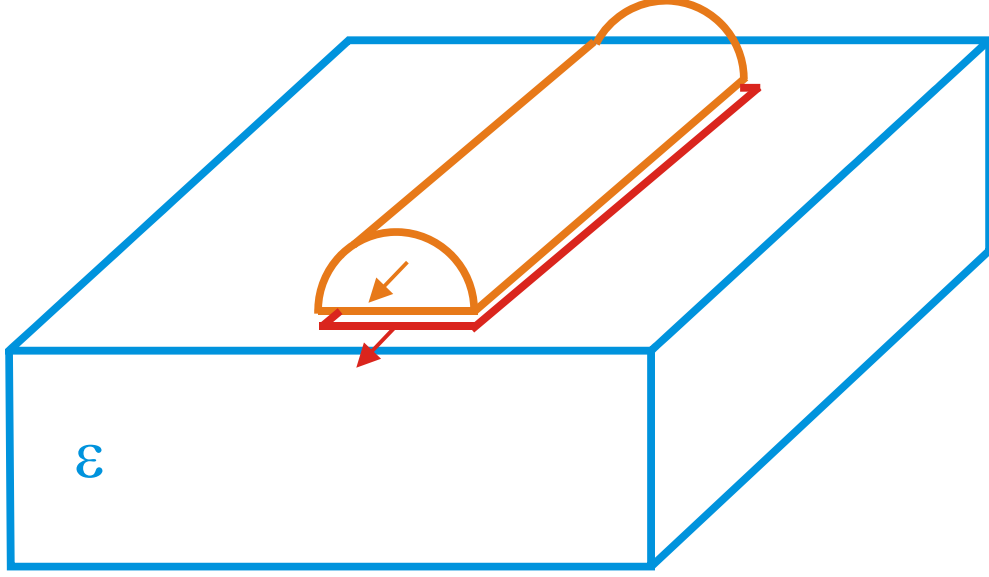


Figure 30. Surface charge model consisting of semicylindrical charge in gas and surface strip charge.

$$\rho_i^{\min} = \min(\rho_{i-1}^b, \rho_i^b, \rho_{i+1}^b) \quad (163)$$

$$\rho_i^a = \max(\rho_i^m, \rho_i^D) \quad (164)$$

$$\rho_i^b = \min(\rho_i^m, \rho_i^D) \quad (165)$$

The + and - superscripts refer to fluxes into and out of a cell, respectively. The last set define upper $\rho_i^{\max}$ and lower $\rho_i^{\min}$ bounds for the density in the i th cell at time $(m+1)\Delta t$. These bounds are determined from the densities at time $m\Delta t$ and the diffused densities in the $i-1$, i , and $i+1$ cells. This criterion has been selected to prevent the generation of ripples by requiring that local maxima and minima not be created. Thus, the maximum anti-diffusion fluxes that can be used Q_i^+ and Q_i^- are then obtained from the difference between the bounds and the diffused density at time $(m+1)\Delta t$. Note that these fluxes cannot be greater than the maximum anti-diffusion fluxes available, namely, the A_i 's. The R_i 's as defined above insure this condition. They represent the fraction of the total antidiffusion flux available in the i th cell that is to be used.

5.4 Surface Electric Fields

When the surface is added the preceding charge distributions become a half disc and a half ring or tube. With the surface present there is also charge on the insulator below the streamer. For the disc model we take this to be an extending strip of charge with the same width as the disc diameter. For the ring model the surface charge is taken as two extending line charges (at the edges of the half disc and the surface).

5.4.1 Half Cylinder Charge

Suppose we consider the half cylinder charge

$$\begin{aligned}
\phi &= \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi}^0 \frac{d\varphi'}{2\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \\
&= \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi}^0 \frac{d\varphi'}{2\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\varphi - \varphi') + (z-z')^2}}
\end{aligned} \tag{166}$$

Now arbitrarily setting the observation point to $\varphi = 0$ (since we intend to take $r = 0$ in the end anyway)

$$\begin{aligned}
\phi(r, 0, z) &= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{1}{\pi} \int_{-\pi}^0 \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr' \cos \varphi' + (z-z')^2}} \\
&= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{1}{\pi} \int_0^\pi \frac{d\varphi'}{\sqrt{r^2 + r'^2 - 2rr' \cos(\varphi' - \pi) + (z-z')^2}} \\
&= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{1}{\pi} \int_0^\pi \frac{d\varphi'}{\sqrt{r^2 + r'^2 + 2rr' \cos \varphi' + (z-z')^2}} \\
&= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{r^2 + r'^2 + 2rr' \cos(2\varphi') + (z-z')^2}} \\
&= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{(r+r')^2 + (z-z')^2 - 4rr' \sin^2 \varphi'}} \\
&= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{1 - k^2 \sin^2 \varphi'}} \\
&= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} K(k)
\end{aligned} \tag{167}$$

$$k = \frac{2\sqrt{rr'}}{\sqrt{(r+r')^2 + (z-z')^2}} \tag{168}$$

Now take the axial field

$$E_z(r, 0, z) = -\frac{\partial \phi}{\partial z}(r, 0, z) = -\frac{1}{4\varepsilon_0} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{\partial}{\partial z} \left[\frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} K(k) \right]$$

Now take $r = 0$, $k = 0$, and $E(0) = \pi/2$

$$\begin{aligned}
E_z(0, 0, z) &= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') (z - z') dz' \int_0^{r_c} \frac{r' dr'}{\left[r'^2 + (z - z')^2\right]^{3/2}} \\
&= \frac{1}{4\varepsilon_0} \int_0^d \rho(z') \left[\frac{z - z'}{|z - z'|} - \frac{z - z'}{\sqrt{r_c^2 + (z - z')^2}} \right] dz' \\
&= \frac{1}{4\varepsilon_0} \int_0^z \rho(z') \left[1 - \frac{z - z'}{\sqrt{r_c^2 + (z - z')^2}} \right] dz' + \frac{1}{4\varepsilon_0} \int_z^d \rho(z') \left[-1 - \frac{z - z'}{\sqrt{r_c^2 + (z - z')^2}} \right] dz' \\
&= \frac{1}{4\varepsilon_0} \int_{-z}^0 \rho(z + z') \left[1 + \frac{z'}{\sqrt{r_c^2 + z'^2}} \right] dz' + \frac{1}{4\varepsilon_0} \int_0^{d-z} \rho(z + z') \left[-1 + \frac{z'}{\sqrt{r_c^2 + z'^2}} \right] dz' \quad (169)
\end{aligned}$$

which is one half the full cylinder result, which could have been anticipated directly from symmetry (the preceding derivation may be useful if other field components are desired in the half space case).

5.4.2 Dielectric Images

The dielectric images

$$\begin{aligned}
\phi_i &= \frac{1}{2\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d dz' \int_0^\pi \frac{d\varphi'}{2\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\varphi - \varphi') + (z - z')^2}} \\
&= \frac{1}{2\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d dz' \int_0^\pi \frac{d\varphi'}{2\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\varphi - \varphi') + (z - z')^2}} \quad (170)
\end{aligned}$$

Setting $\varphi = -\pi$ (since we are eventually letting $r = 0$)

$$\begin{aligned}
\phi(r, -\pi, z) &= \frac{1}{2\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d dz' \int_0^\pi \frac{d\varphi'}{2\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\pi + \varphi') + (z - z')^2}} \\
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d dz' \int_0^\pi \frac{d\varphi'}{\pi} \int_0^{r_c} r' dr' \frac{\rho(z')}{\sqrt{r^2 + r'^2 + 2rr' \cos \varphi' + (z - z')^2}} \\
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{r^2 + r'^2 + 2rr' \cos(2\varphi') + (z - z')^2}} \\
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{(r + r')^2 + (z - z')^2 - 4rr' \sin^2 \varphi'}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{1 - k^2 \sin^2 \varphi'}} \\
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d \rho(z') dz' \int_0^{r_c} r' dr' \frac{2}{\pi \sqrt{(r+r')^2 + (z-z')^2}} K(k)
\end{aligned} \tag{171}$$

Now take $r = 0$, $k = 0$, and $E(0) = \pi/2$

$$\begin{aligned}
E_z(0, 0, z) &= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d \rho(z') (z - z') dz' \int_0^{r_c} \frac{r' dr'}{[r'^2 + (z - z')^2]^{3/2}} \\
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^d \rho(z') \left[\frac{z - z'}{|z - z'|} - \frac{z - z'}{\sqrt{r_c^2 + (z - z')^2}} \right] dz' \\
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^z \rho(z') \left[1 - \frac{z - z'}{\sqrt{r_c^2 + (z - z')^2}} \right] dz' \\
&\quad + \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_z^d \rho(z') \left[-1 - \frac{z - z'}{\sqrt{r_c^2 + (z - z')^2}} \right] dz' \\
&= \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_{-z}^0 \rho(z + z') \left[1 + \frac{z'}{\sqrt{r_c^2 + z'^2}} \right] dz' \\
&\quad + \frac{1}{4\varepsilon_0} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \int_0^{d-z} \rho(z + z') \left[-1 + \frac{z'}{\sqrt{r_c^2 + z'^2}} \right] dz'
\end{aligned} \tag{172}$$

5.4.3 Total Disc Charge and Field

The problem with the half disc and image then has a field which is

$$\frac{1}{2} \left[1 + \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \right] = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon} \tag{173}$$

times the full space result.

5.4.4 Surface Strip Charge

The potential from the strip is

$$\phi = \frac{1}{4\pi\epsilon_0} \int_0^d dz' \int_{-r_c}^{r_c} dx' \int_0^\Delta dy' \frac{\rho_s(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \quad (174)$$

Setting $y = 0$ on the surface of the dielectric

$$\begin{aligned} \phi(x, 0, z) &\approx \frac{\Delta}{4\pi\epsilon_0} \int_0^d \rho_s(z') dz' \int_{-r_c}^{r_c} \frac{dx'}{\sqrt{(x-x')^2 + (z-z')^2}} \\ &\approx \frac{\Delta}{4\pi\epsilon_0} \int_0^d \rho_s(z') dz' \int_{(x-r_c)/|z-z'|}^{(x+r_c)/|z-z'|} \frac{du}{\sqrt{u^2 + 1}} \\ &\approx \frac{\Delta}{4\pi\epsilon_0} \int_0^d \rho_s(z') dz' [\text{Arcsinh}((x+r_c)/|z-z'|) - \text{Arcsinh}((x-r_c)/|z-z'|)] \\ &\approx \frac{\Delta}{4\pi\epsilon_0} \int_0^d \rho_s(z') dz' \ln \left[\frac{(x+r_c) + \sqrt{(x+r_c)^2 + (z-z')^2}}{(x-r_c) + \sqrt{(x-r_c)^2 + (z-z')^2}} \right] \end{aligned} \quad (175)$$

and thus at the strip center

$$\phi(0, 0, z) \approx \frac{\Delta}{4\pi\epsilon_0} \int_0^d \rho_s(z') \ln \left[\frac{\sqrt{r_c^2 + (z-z')^2} + r_c}{\sqrt{r_c^2 + (z-z')^2} - r_c} \right] dz' \quad (176)$$

The corresponding field contribution from the strip charge is

$$E_z(0, 0, z) = -\frac{\partial\phi}{\partial z}(0, 0, z) \approx \frac{\Delta r_c}{2\pi\epsilon_0} \int_0^d \rho_s(z') \frac{dz'/(z-z')}{\sqrt{r_c^2 + (z-z')^2}} \quad (177)$$

This integral appears to converge only in a principal value sense as $z' \rightarrow z$. If we replace $\Delta\rho_s$ by σ_s the surface charge density

$$E_z(0, 0, z) = -\frac{\partial\phi}{\partial z}(0, 0, z) \approx \frac{r_c}{2\pi\epsilon_0} \int_0^d \sigma_s(z') \frac{dz'/(z-z')}{\sqrt{r_c^2 + (z-z')^2}} \quad (178)$$

Including a reflection in the dielectric surface gives

$$E_z(0, 0, z) \approx \frac{2\epsilon_0}{\epsilon + \epsilon_0} \frac{r_c}{2\pi\epsilon_0} \int_0^d \sigma_s(z') \frac{dz'/(z-z')}{\sqrt{r_c^2 + (z-z')^2}} \quad (179)$$

If we take the surface charge density to be constant on each of the finite length strips we can write this as

$$E_z(0, 0, z_i) \approx \frac{2\epsilon_0}{\epsilon + \epsilon_0} \frac{r_c}{2\pi\epsilon_0} \sum_{i'=1}^I \sigma_{si'} \int_{z_{i'} - \delta z_{i'-1/2}/2}^{z_{i'} + \delta z_{i'+1/2}/2} \frac{dz'/(z_i - z')}{\sqrt{r_c^2 + (z_i - z')^2}}$$

$$\begin{aligned}
&\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{r_c}{2\pi\varepsilon_0} \sum_{i'=1}^I \sigma_{si'} \int_{z_i - z_{i'} - \delta z_{i'+1/2}/2}^{z_i - z_{i'} + \delta z_{i'-1/2}/2} \frac{du/u}{\sqrt{r_c^2 + u^2}} \\
&\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{1}{2\pi\varepsilon_0} \sum_{i'=2}^{I-1} \sigma_{si'} \ln \left| \frac{z_i - z_{i'} + \delta z_{i'-1/2}/2}{z_i - z_{i'} - \delta z_{i'+1/2}/2} \frac{r_c + \sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2}}{r_c + \sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2}} \right|
\end{aligned} \tag{180}$$

where we used the identity

$$\int \frac{dx}{x\sqrt{a^2 + x^2}} = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{a^2 + x^2}} \right| \tag{181}$$

Note that if $i = i'$

$$\ln \left| \frac{\delta z_{i'-1/2}/2}{\delta z_{i'+1/2}/2} \frac{r_c + \sqrt{r_c^2 + (\delta z_{i'+1/2}/2)^2}}{r_c + \sqrt{r_c^2 + (\delta z_{i'-1/2}/2)^2}} \right| \tag{182}$$

and this vanishes for a uniform grid with $\delta z_{i'-1/2} = \delta z_{i'+1/2}$. It also becomes small when $r_c \gg \delta z_{i'-1/2}, \delta z_{i'+1/2}$ as well as when $r_c \rightarrow 0$.

Note that if $(\delta_{i'+1/2} + \delta_{i'-1/2})/2 = \delta z_{i'} \ll |z_i - z_{i'}|$

$$\begin{aligned}
E_z(0, 0, z_i) &\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{1}{2\pi\varepsilon_0} \sum_{i'=2}^{I-1} \sigma_{si'} \ln \left| \frac{1 + \frac{\delta z_{i'-1/2}/2}{(z_i - z_{i'})} r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2} \sqrt{1 - 2 \frac{(z_i - z_{i'})\delta z_{i'+1/2}/2}{\sqrt{r_c^2 + (z_i - z_{i'})^2}^2}}}{1 - \frac{\delta z_{i'+1/2}/2}{(z_i - z_{i'})} r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2} \sqrt{1 + 2 \frac{(z_i - z_{i'})\delta z_{i'-1/2}/2}{\sqrt{r_c^2 + (z_i - z_{i'})^2}^2}}} \right| \\
&\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{1}{2\pi\varepsilon_0} \sum_{i'=2}^{I-1} \sigma_{si'} \ln \left| \frac{1 + \frac{\delta z_{i'-1/2}/2}{(z_i - z_{i'})} r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2} - \frac{(z_i - z_{i'})\delta z_{i'+1/2}/2}{\sqrt{r_c^2 + (z_i - z_{i'})^2}}}{1 - \frac{\delta z_{i'+1/2}/2}{(z_i - z_{i'})} r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2} + \frac{(z_i - z_{i'})\delta z_{i'-1/2}/2}{\sqrt{r_c^2 + (z_i - z_{i'})^2}}} \right| \\
&\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{1}{2\pi\varepsilon_0} \sum_{i'=2}^{I-1} \sigma_{si'} \ln \left| \frac{1 + \frac{\delta z_{i'-1/2}/2}{(z_i - z_{i'})} \frac{1 - \frac{(z_i - z_{i'})\delta z_{i'+1/2}/2}{[r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2}] \sqrt{r_c^2 + (z_i - z_{i'})^2}}}{1 - \frac{\delta z_{i'+1/2}/2}{(z_i - z_{i'})} \frac{1 + \frac{(z_i - z_{i'})\delta z_{i'-1/2}/2}{[r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2}] \sqrt{r_c^2 + (z_i - z_{i'})^2}}}{1 + \frac{(z_i - z_{i'})\delta z_{i'+1/2}/2}{[r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2}] \sqrt{r_c^2 + (z_i - z_{i'})^2}}} \right| \\
&\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{1}{2\pi\varepsilon_0} \sum_{i'=2}^{I-1} \sigma_{si'} \ln \left| 1 + \frac{\delta z_{i'}}{(z_i - z_{i'})} - \frac{(z_i - z_{i'})\delta z_{i'}}{[r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2}] \sqrt{r_c^2 + (z_i - z_{i'})^2}} \right| \\
&\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{1}{2\pi\varepsilon_0} \sum_{i'=2}^{I-1} \sigma_{si'} \left[\frac{1}{(z_i - z_{i'})} - \frac{(z_i - z_{i'})}{\left\{ r_c + \sqrt{r_c^2 + (z_i - z_{i'})^2} \right\} \sqrt{r_c^2 + (z_i - z_{i'})^2}} \right] \delta z_{i'}
\end{aligned}$$

$$\approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{r_c}{2\pi\varepsilon_0} \sum_{i'=2}^{I-1} \sigma_{si'} \frac{\delta z_{i'}/(z_i - z_{i'})}{\sqrt{r_c^2 + (z_i - z_{i'})^2}} \quad (183)$$

Thus the approximate version is

$$E_z(0, 0, z_i) \approx \frac{2\varepsilon_0}{\varepsilon + \varepsilon_0} \frac{r_c}{2\pi\varepsilon_0} \sum_{i'} \sigma_{si'} \frac{\delta z_{i'}/(z_i - z_{i'})}{\sqrt{r_c^2 + (z_i - z_{i'})^2}} \quad (184)$$

5.5 Tube Charges

We next consider the case where the charge is taken to be concentrated on the surface of a tube rather than throughout the volume.

5.5.1 Semi-Circular Tube

In the case of a semicircular tube we have

$$\begin{aligned} \phi &= \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi}^0 r_c \frac{d\varphi'}{2\pi} \frac{\sigma(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \\ &= \frac{1}{2\varepsilon_0} \int_0^d dz' \int_{-\pi+\varphi}^{\varphi} r_c \frac{d\varphi'}{2\pi} \frac{\sigma(z')}{\sqrt{r^2 + r_c^2 - 2rr_c \cos(\varphi - \varphi') + (z-z')^2}} \\ &= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{2\pi} \int_{-\pi}^0 \frac{d\varphi'}{\sqrt{r^2 + r_c^2 - 2rr_c \cos \varphi' + (z-z')^2}} \\ &= \frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{\pi} \int_{-\pi}^0 \frac{d\varphi'}{\sqrt{r^2 + r_c^2 - 2rr_c \cos \varphi' + (z-z')^2}} \\ &= \frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{\pi} \int_0^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r_c^2 - 2rr_c \cos(\varphi' - \pi) + (z-z')^2}} \\ &= \frac{r_c}{2\varepsilon_0} \int_0^d \sigma(z') dz' \frac{1}{\pi} \int_0^{\pi} \frac{d\varphi'}{\sqrt{r^2 + r_c^2 + 2rr_c \cos \varphi' + (z-z')^2}} \\ &= \frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{r^2 + r_c^2 + 2rr_c \cos(2\varphi') + (z-z')^2}} \\ &= \frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{(r+r_c)^2 + (z-z')^2 - 4rr_c \sin^2 \varphi'}} \end{aligned}$$

$$\begin{aligned}
&= \frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi \sqrt{(r+r_c)^2 + (z-z')^2}} \int_0^{\pi/2} \frac{d\varphi'}{\sqrt{1-k^2 \sin^2 \varphi'}} \\
&= \frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') dz' \frac{2}{\pi \sqrt{(r+r_c)^2 + (z-z')^2}} K(k)
\end{aligned} \tag{185}$$

$$k = \frac{2\sqrt{rr_c}}{\sqrt{(r+r_c)^2 + (z-z')^2}} \tag{186}$$

Now take the axial field

$$E_z = -\frac{\partial \phi}{\partial z} = -\frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') dz' \frac{\partial}{\partial z} \left[\frac{2}{\pi \sqrt{(r+r_c)^2 + (z-z')^2}} K(k) \right] \tag{187}$$

Specializing to the axial electric field on axis we take $r = 0$, $k = 0$, and $E(0) = \pi/2$

$$E_z(0, z) = \frac{r_c}{4\varepsilon_0} \int_0^d \sigma(z') \frac{(z-z')}{[r_c^2 + (z-z')^2]^{3/2}} dz' \tag{188}$$

Note from charge balance that we expect

$$0 = \frac{1}{4\varepsilon_0} \int_{-z}^0 \sigma(z+z') dz' + \frac{1}{4\varepsilon_0} \int_0^{d-z} \sigma(z+z') dz' \tag{189}$$

Constant Half Cylinder Charge
each interval

If we take the charge to be a pulse or constant in a tube over

$$z_{i'} - \frac{1}{2}\delta z_{i'-1/2} < z' < z_{i'} + \frac{1}{2}\delta z_{i'+1/2} \tag{190}$$

we have

$$\begin{aligned}
E_z(0, z_i) &= \frac{r_c}{4\varepsilon_0} \sum_{i'=1}^I \sigma_{i'} \int_{z_{i'} - \delta z_{i'-1/2}/2}^{z_{i'} + \delta z_{i'+1/2}/2} \frac{(z_i - z')}{[r_c^2 + (z_i - z')^2]^{3/2}} dz' \\
&= \frac{r_c}{4\varepsilon_0} \sum_{i'=1}^I \sigma_{i'} \left[1/\sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} - 1/\sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} \right]
\end{aligned} \tag{191}$$

where

$$z_{i+1} - z_i = \delta z_{i+1/2} \tag{192}$$

$$\delta z_i = \frac{1}{2} (\delta z_{i+1/2} + \delta z_{i-1/2}) \quad (193)$$

Note that if $\delta z_{i'} \ll |z_i - z_{i'}|$

$$E_z(0, z_i) \approx \frac{r_c}{4\epsilon_0} \sum_{i'} \sigma_{i'} \frac{z_i - z_{i'}}{\{r_c^2 + (z_i - z_{i'})^2\}^{3/2}} \delta z_{i'} \quad (194)$$

5.5.2 Total Tube Charge and Field

The problem with the half tube and image then has a field which is

$$\frac{1}{2} \left[1 + \frac{\epsilon_0 - \epsilon}{\epsilon_0 + \epsilon} \right] = \frac{\epsilon_0}{\epsilon_0 + \epsilon} \quad (195)$$

times the full space result.

5.5.3 Surface Line Charges

Following the tube charge for the volume part of the streamer, the surface charge is now simplified to twin line charges. The potential in this case is thus

$$\phi = \frac{1}{4\pi\epsilon_0} \int_0^d q_s(z') \left[\frac{1}{\sqrt{(x - r_c)^2 + y^2 + (z - z')^2}} + \frac{1}{\sqrt{(x + r_c)^2 + y^2 + (z - z')^2}} \right] dz' \quad (196)$$

where the filament charge is connected to the strip surface charge density by means of

$$q_s = r_c \sigma_s \quad (197)$$

The axial field is

$$E_z(x, y, z) = -\frac{\partial \phi}{\partial z} \\ = \frac{1}{4\pi\epsilon_0} \int_0^d q_s(z') \frac{\partial}{\partial z'} \left[\frac{1}{\sqrt{(x - r_c)^2 + y^2 + (z - z')^2}} + \frac{1}{\sqrt{(x + r_c)^2 + y^2 + (z - z')^2}} \right] dz' \quad (198)$$

If we take the charge to be a pulse or constant in a line over the interval

$$z_{i'} - \frac{1}{2} \delta z_{i'-1/2} < z' < z_{i'} + \frac{1}{2} \delta z_{i'+1/2} \quad (199)$$

we have on axis

$$E_z(0, 0, z_i) = \frac{1}{2\pi\epsilon_0} \sum_{i'=1}^I q_{si'} \int_0^d \frac{\partial}{\partial z'} \left[\frac{1}{\sqrt{r_c^2 + (z_i - z')^2}} \right] dz'$$

$$\begin{aligned}
&= \frac{1}{2\pi\epsilon_0} \sum_{i'=1}^I q_{si'} \int_0^d \frac{(z_i - z')}{\left\{r_c^2 + (z_i - z')^2\right\}^{3/2}} dz' \\
&= -\frac{1}{2\pi\epsilon_0} \sum_{i'=1}^I q_{si'} \int_{z_i - z_{i'} - \delta_{i'+1/2}/2}^{z_i - z_{i'} + \delta_{i'-1/2}/2} \frac{\partial}{\partial u} \left(\frac{1}{\sqrt{r_c^2 + u^2}} \right) du \\
&= \frac{1}{2\pi\epsilon_0} \sum_{i'=1}^I q_{si'} \left[\frac{1}{\sqrt{r_c^2 + (z_i - z_{i'} - \delta_{i'+1/2}/2)^2}} - \frac{1}{\sqrt{r_c^2 + (z_i - z_{i'} + \delta_{i'-1/2}/2)^2}} \right] \quad (200)
\end{aligned}$$

Note that if $(\delta_{i'+1/2} + \delta_{i'-1/2})/2 = \delta z_{i'} \ll |z_i - z_{i'}|$ there is cancellation. It also becomes small when $r_c \gg \delta z_{i'-1/2}, \delta z_{i'+1/2}$ as well as when $r_c \rightarrow 0$. In these cases we write

$$E_z(0, 0, z_i) \approx \frac{1}{2\pi\epsilon_0} \sum_{i'=1}^I q_{si'} \frac{(z_i - z_{i'})}{\left\{r_c^2 + (z_i - z_{i'})^2\right\}^{3/2}} \delta z_{i'} \quad (201)$$

5.6 Summary Of Surface Equations

We take the fluid equations with surface charge rates due to SEE attachment or emission as (note with the photoemission included it is possible for N_s to be negative!)

$$\frac{\partial N_s}{\partial t} = \eta_s(E) N_e \ell_n |W_e(E)| - S_{pe}(x, E) \ell_n \quad (202)$$

$$\frac{\partial N_p}{\partial t} = S + N_e \alpha(E) |W_e(E)| - N_e N_p \beta - N_n N_p \beta - \frac{\partial [N_p W_p(E)]}{\partial x} \quad (203)$$

$$\frac{\partial N_n}{\partial t} = N_e \eta(E) |W_e(E)| - N_p N_n \beta - \frac{\partial [N_n W_n(E)]}{\partial x} \quad (204)$$

$$\frac{\partial N_e}{\partial t} = S(x, E) + S_{pe}(x, E) + N_e [\alpha(E) - \eta(E) - \eta_s(E)] |W_e(E)|$$

$$-N_e N_p \beta - \frac{\partial [N_e W_e(E)]}{\partial x} + \frac{\partial}{\partial x} \left[D_L(E) \frac{\partial N_e}{\partial x} \right] \quad (205)$$

where N_e , N_p are volume number densities and N_s is a surface number density. The effective normal dimension ℓ_n is taken as

$$\ell_n = A/P \quad (206)$$

where A is the effective streamer cross sectional area and P is the effective streamer surface cross length. In the case of a half disc of radius r_c we have $A = \pi r_c^2/2$ and $P = 2r_c$ so that $\ell_n = \pi r_c/4$.

Note that the volume and surface charge densities are

$$\rho = e(N_p - N_n - N_e) \quad (207)$$

$$\sigma_s = -eN_s \quad (208)$$

and the surface secondary electron emission coefficient from the preceding section on the kinetic simulation of Teflon is

$$\eta_s(E)/p = e^{-146 \text{ V}/(\text{cm-T})/(E/p)} 0.0312/(\text{cm-T}) \quad (209)$$

5.6.1 Air Photoemission

The photoemission process is fit by the function introduced in air photoionization [42]

$$f_e(r) = \frac{\exp(-k_{e1}p_{O_2}r) - \exp(-k_{e2}p_{O_2}r)}{r \ln(k_{e2}/k_{e1})} \quad (210)$$

where here we take the absorption coefficients to be those of the partial pressure of oxygen over the band from the knee of the quantum yield curve of the surface (Teflon 10 eV) (we actually use minima of the 1100 - 1225 Å band [56] to enhance the photon propagation distance)

$$k_{e1} = 0.0033 \text{ cm}^{-1}\text{Torr}^{-1} \quad (211)$$

to the preceding value (energy range to 12.65 eV or 98 nm)

$$k_{e2} = 2 \text{ cm}^{-1}\text{Torr}^{-1} \quad (212)$$

For air we also use the model ($p_q = 30 \text{ Torr}$ [42])

$$4\pi p\psi = (\xi\omega/\alpha)_e \frac{p_q}{p + p_q} f_e(X) = \gamma_{ej}^* \left[\frac{e^{-\mu_{ej}^{(1)}X} - e^{-\mu_{ej}^{(2)}X}}{X} \right] \quad (213)$$

where for photoemission we take from the kinetic simulation (note that the kinetic simulation does not explicitly account for quenching of the radiation states so the preceding formula includes the appropriate ratio, and in addition, the kinetic simulation operates at relatively early times where the particle avalanche swarm is relatively short)

$$\begin{aligned} (\xi\omega/\alpha)_e &= 81.59e^{-(E/p)0.2836 \text{ (cm-T)}/V + (E/p)^2 0.002099 \text{ (cm-T)}/V}, \quad E/p < 40 \text{ V}/(\text{cm-T}) \\ &= 0.2372e^{-(E/p)0.05363 \text{ V}/(\text{cm-T})}, \quad E/p > 40 \text{ V}/(\text{cm-T}) \end{aligned} \quad (214)$$

5.6.2 SF6 Photoemission

For SF6 we use the photoionization function

$$4\pi p\psi = \gamma'_{ej} e^{-\mu_{ej}X} \quad (215)$$

The emission coefficient for SF6 and Teflon is estimated to be (this is the fraction of collisional ionization

leading to photoemission)

$$\gamma'_{ej}/\mu_{ej} = 6 \times 10^{-4} \quad (216)$$

and using the value of μ_j for photoionization (this was done because the μ_j coefficient above corresponded to the value in SF6 for 9.5 eV, which is a reasonable value to use considering the location of the knee of the Teflon quantum yield curve)

$$\mu_{ej}/p = (19.7 \pm 4.94) / (\text{cm} - \text{bar}) \quad (217)$$

$$\gamma_{ej}/p = 6 \times 10^{-4} \times 1970 \text{ m}^{-1}\text{bar}^{-1} = 1.182 \text{ m}^{-1}\text{bar}^{-1} \quad (218)$$

6 MODEL RESULTS

Here we examine the results from the models. Although both types of charge models were examined (ring-line and disc-strip models), the disc-strip model was used in the following results. We include the ring-line model since it forms a better link with the radial expansion section (although we did not observe the stability advantage for the ring-line model noted in the literature for the axial propagation).

6.1 Critical Fields

The critical field levels without a surface present, where $\alpha - \eta = 0$ have already been mentioned above. These are $E_c/p = 32.5 \text{ V}/(\text{cm} - \text{T}) = 25 \text{ kV}/(\text{cm} - \text{bar})$ in air and $E_c/p = 89 \text{ kV}/(\text{cm} - \text{bar})$ in SF6. For the surface emission to significantly change these levels we would need to see the results in Figures 27 or 28 be significant fractions of unity in the vicinity of these critical field levels. Because they are much smaller than unity we conclude that the surface has only a small effect; the secondary electron attachment (which in this energy range results in a loss of electrons) could actually result in a slight increase in critical field level as noted previously in nitrogen [11].

6.2 Calculation of Sustaining Fields

One quantity of interest for streamers is its sustaining field, which is defined as the level of applied field required to maintain the streamer once it is formed [14], [57]. In this section we report on the results of using the 1.5D streamer code (discussed previously) to determine the sustaining field for streamers in both SF6 and air and both in a volume and in the presence of a surface. In all cases we started the solution with $10^9 \text{ electrons/cm}^3$ and 10^9 ions/cm^3 in a single cell located at $x = 5 \text{ mm}$, the pressure was set at 1 bar, the temperature was set to 20° C , the time step was set to 1 picosecond and each cell was 10 microns long. Since this is a 1.5D solution, we had to assume a value for the streamer radius. We therefore assumed various values in the range from $50 \text{ }\mu\text{m}$ to $600 \text{ }\mu\text{m}$ to assess the effect that the radius has on the calculated value of the sustaining field. The section on radial expansion, below, will comment further on this radius.

To calculate the sustaining field we use the following procedure. Initially, we set the applied field to a high value for a few nanoseconds. During this time, the electrons will avalanche and form two streamer heads that propagate in opposite directions toward the anode and cathode. The heads rapidly grow to a level where the fields in their vicinity are a factor of three to four times larger than the applied field. Both cathode- and anode-directed streamers at this point are well-formed and propagating strongly. We then quickly decrease the applied field to a much smaller value and observe the behavior of the streamer head.

The charge in the streamer heads will immediately decrease. If the charge in the streamer head continues to steadily decrease, the lower field that we picked is too small to sustain the streamer. On the other hand, if the charge in the head begins to increase again, the lower field that we picked is too large. We vary the lower field value until we find the field where the head charge remains constant and we declare this to be the sustaining field. Our values of sustaining fields were found to within 1 kV/cm.

Finding the sustaining field is a tedious procedure and it is necessary to reduce the data being generated as much as possible. With this in mind, let us look at an example calculation. In this example, the streamers are formed in air by applying a 75 kV/cm field for 3.6 ns, reducing the field linearly until it reaches 15 kV/cm at 3.8 ns, and keeping it at 15 kV/cm from 3.8 ns to the end of the simulation at 20 ns. The streamer radius was set to 50 μm . A plot of the quantities of interest as a function of distance is shown for various times (4, 10, 15 and 20 ns) in Figures 31 through 34. The electron density (N_e), positive ion density (N_p), negative ion density (N_n) and the total charge density ($N_{tot} = N_p - N_e - N_n$) are plotted with respect to the left ordinate axis. The electric field in the x direction (E_x) is plotted with respect to the right ordinate axis. If we focus on the electric field, we note that it is a good indicator of the streamer head position, namely, the cathode- and anode-directed streamers are located at the two places where the field value goes through a sharp minimum (the x field here is negative, the magnitudes of the electric field are really experiencing maximum values). Therefore, instead of looking at individual plots of the charge density at different times, we can plot the position of the these two field minima as a function of time as shown in Figure 35.

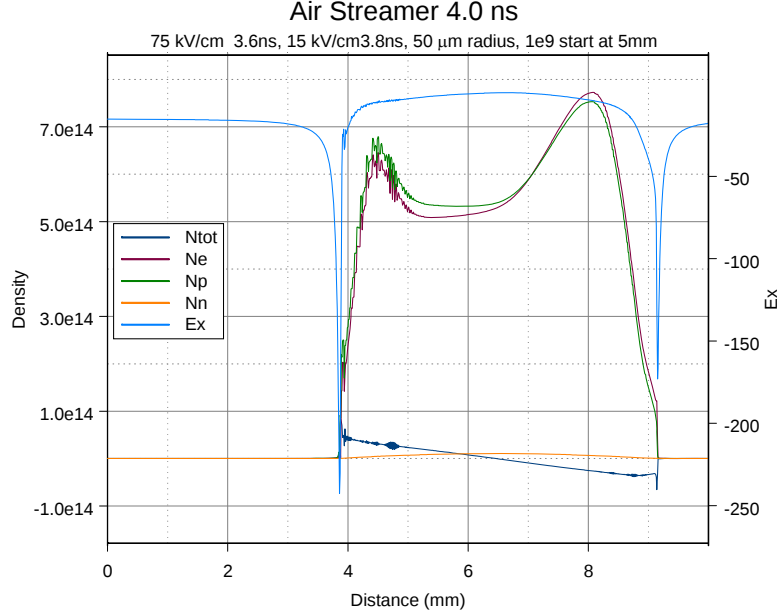


Figure 31. Air streamer at 4 ns.

In Figure 35 the anode is located to the right and the cathode is located to the left. The plot shows the position of the two field minima (starting at 5 mm) as they evolve in time over 20 ns. The anode-directed streamer is in blue and the cathode-directed streamer is in red. The sizes of the symbols along each of the plots indicate the magnitude of the electric field minimum. Unfortunately, differences in symbol size are hard to distinguish, particularly when the field peak is relatively constant as it is near the sustaining field. Therefore, the same information is shown Figure 36, which plots the peak magnitude as

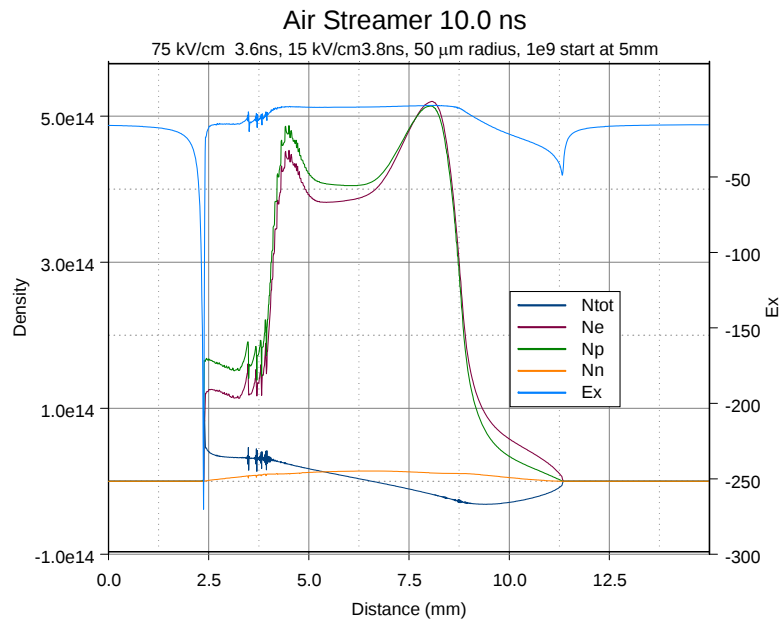


Figure 32. Air streamer at 10 ns.

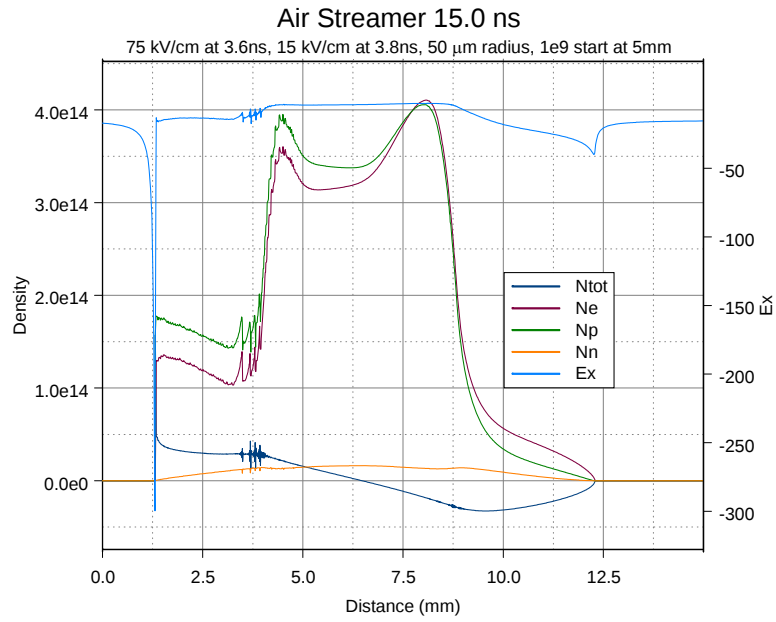


Figure 33. Air streamer at 15 ns.

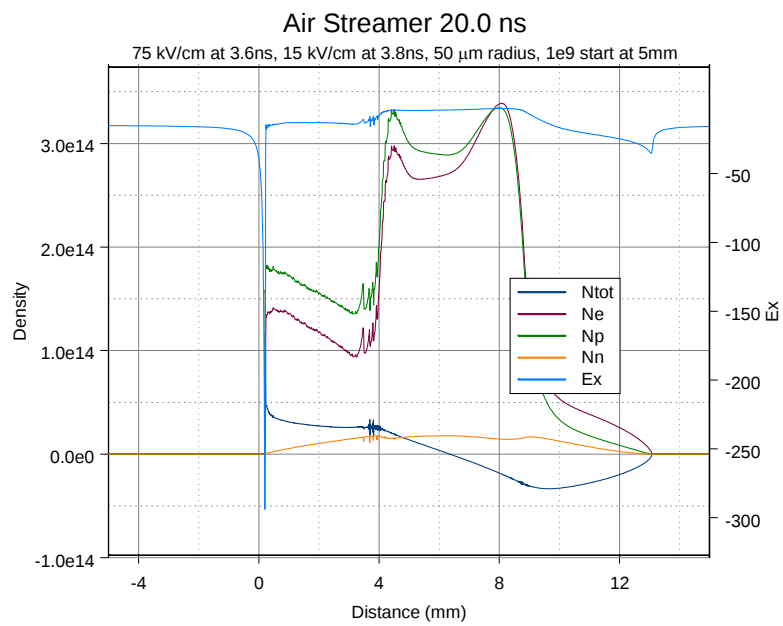


Figure 34. Air streamer at 20 ns.

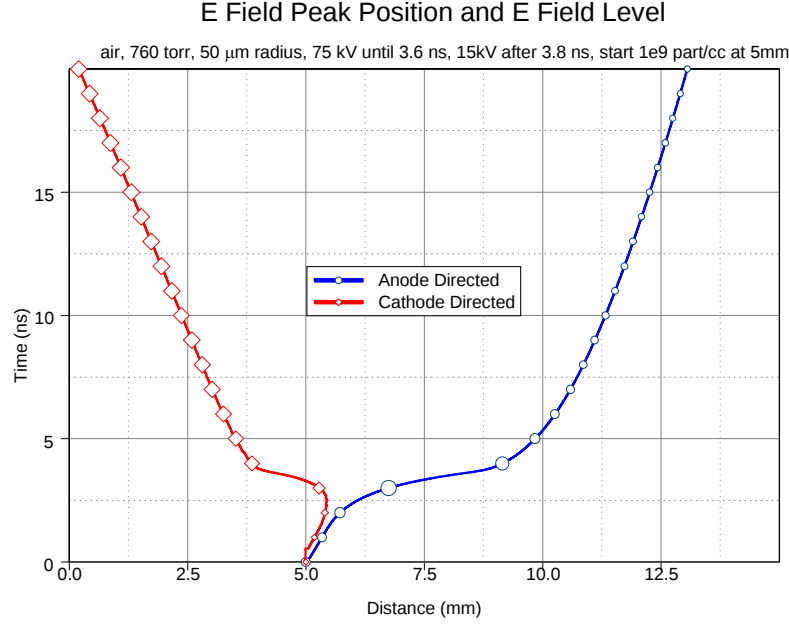


Figure 35. E field peak position versus time 50 μm , 1e9.

a function of time. The inverse slopes of the plots in Figure 35 give the velocities of the streamer heads, which, again for convenience, are plotted as a function of time in Figure 37. Positive velocity is in the direction of propagation so the anode-directed streamer has a positive velocity in the $+x$ direction and the cathode-directed streamer has a positive velocity in the $-x$ direction.

We can interpret Figure 35 by noting that initially both streamers propagate toward the anode with drift velocity. At around 3 ns the peak field increases in magnitude for both streamers and photoionization causes the cathode-directed streamer to change directions and propagate toward the cathode. The velocities of both streamers increase as indicated by the change of slope. At around 4 ns the applied field has been reduced to 15 kV/cm and the streamers slow down, which is again indicated by a change in slope. Along the anode-directed streamer at times greater than 4 ns, the reduction in symbol size shows that the peak field magnitude is decreasing. This is confirmed by the blue plot in Figure 36 and can be seen in the behavior of the anode-directed minimum in Figures 31 through 34. The slope continues to steepen, which indicates that the anode-directed streamer is slowing. This is confirmed by the blue plot in Figure 37. We conclude from this example that 15 kV/cm is too small to sustain an anode-directed streamer. Along the cathode-directed streamer the symbol size is difficult to distinguish so we must look at the red plot in Figure 36. This indicates that the field peak magnitude is increasing in general but oscillating from step to step. This oscillation is a characteristic of the grid size (10 μm) versus the width of the streamer head and is mitigated by gridding on a finer scale. The larger streamer radii do not have this oscillation because the width of the streamer head is much larger than the chosen grid size. We will say more on this later in Subsection 6.8. The cathode-directed streamer after 7 ns continues at a constant velocity as seen in Figure 37. We conclude from this example that the sustaining field for a cathode-directed streamer is smaller than 15 kV/cm.

We found that for most cases, the most accurate way to determine the sustaining voltage was to look

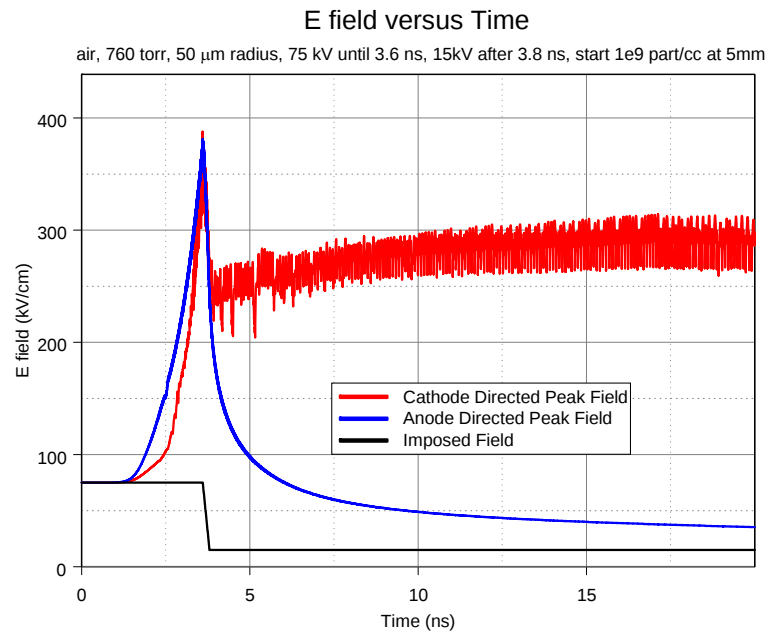


Figure 36. E field peak magnitude vs. time 50 μm .

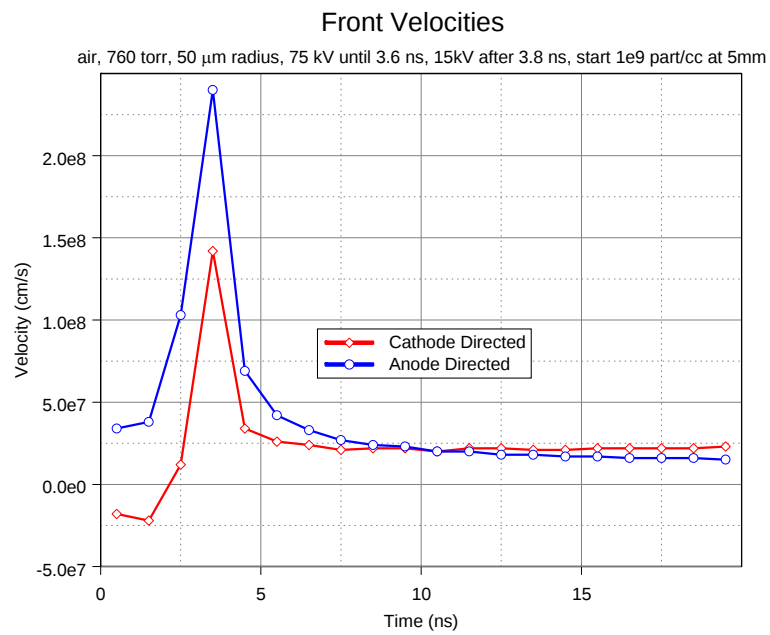


Figure 37. Velocity of E field peak vs. time 50 μm .

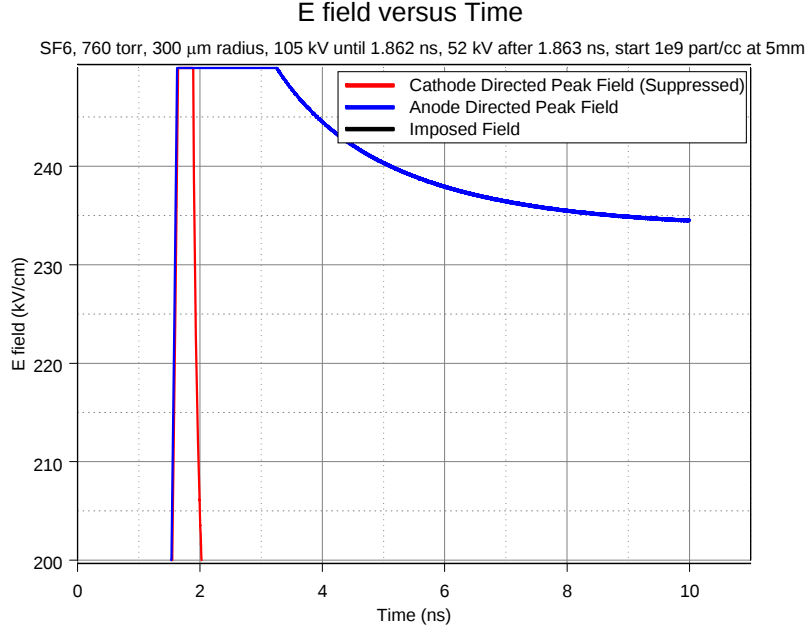


Figure 38. Anode-directed streamer peak magnitude vs. time for 52 kV sustaining field.

at the magnitude of peak field versus time. Figures 38 through 40 show a series of simulations to find the anode-directed sustaining field in SF6. We initiate the streamer by setting the field to 105 kV/cm for 1.862 ns. In Figure 38 the applied field is decreased to 52 kV/cm. Note that the anode-directed peak field continues to decrease until the simulation ends at 10 ns. In Figure 39 the applied field is decreased to 53 kV/cm. Here the anode-directed peak field initially decreases but then levels out until the simulation ends at 12 ns. Finally, in Figure 40 the applied field is decreased to 54 kV/cm. For this field the anode-directed peak field initially decreases, but then begins to increase as the simulation ends at 12 ns. This series of simulations indicates that the anode-directed sustaining field is at 53 kV/cm.

For the sake of efficiency, one is tempted to make a decision at the earliest time possible as to whether the peak is decreasing or increasing. We found, however, that if the simulation is halted too early, the peak doesn't have sufficient time to increase. In the previous series, for example, at 6 ns all three peak values are decreasing. This could have caused the modeler to miss the transition and spend an inordinate amount of time searching for it in the wrong range of fields.

6.3 Air

Using the technique discussed in the previous subsection we determined the sustaining voltage for anode- and cathode directed streamers in air. We initiated the simulation with 10^9 electrons/cm³ and 10^9 ions/cm³ in a single cell at located at $x = 5$ mm, pressure was set to 1 bar, temperature was set to 20⁰ C, the time step was set to 1 picosecond and each cell was 10 μm long. We varied the streamer radius from 50 μm to 600 μm to assess the effect that the radius has on the sustaining field calculated.

The critical field (the field level required for an avalanche to grow) for air at 760 Torr is 25 kV/cm [36]. Therefore, we set the applied field higher than the critical field at 70 kV/cm for a time sufficient to allow the anode-directed streamer peak field to grow to 300 kV/cm (about four times the level of the applied field). Our initial plan was to use the same applied field for all sustaining field calculations. Conceptually,

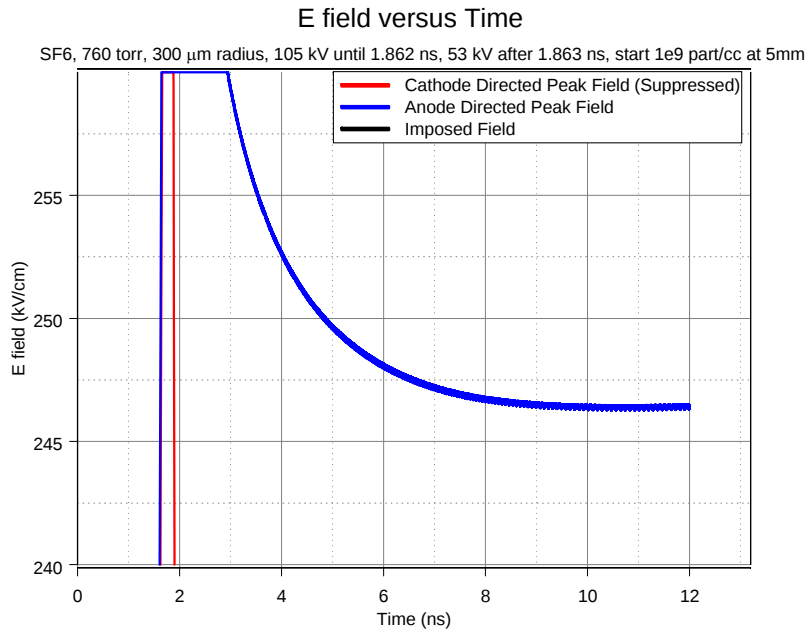


Figure 39. Anode-directed streamer peak magnitude vs. time for 53 kV sustaining field.

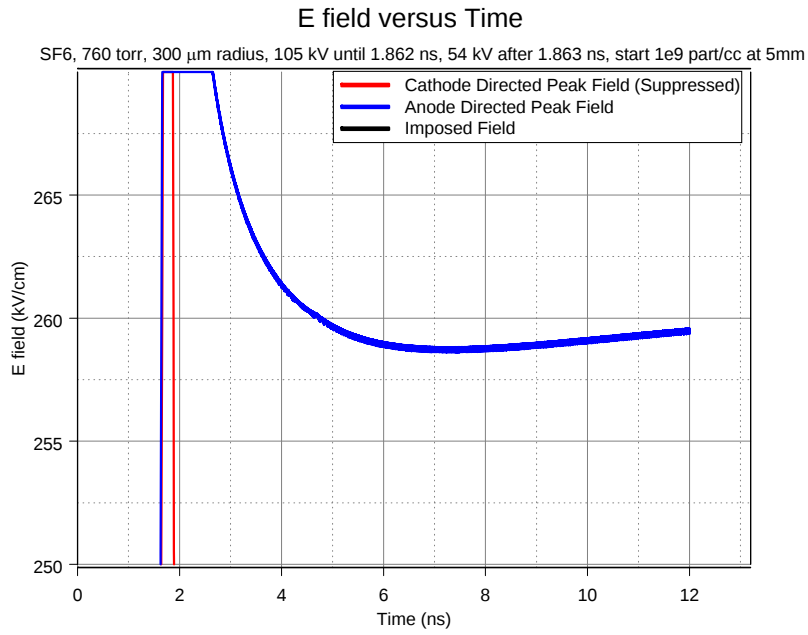


Figure 40. Anode-directed streamer peak magnitude vs. time for 54 kV sustaining field.

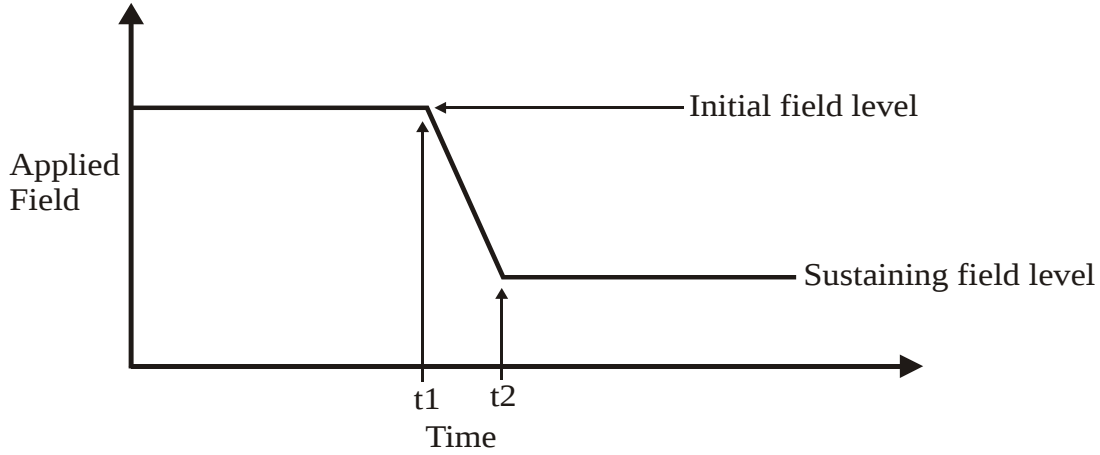


Figure 41. Applied field waveform.

the applied field is shown in Figure 41. We set the initial field to 70 kV/cm until the first breakpoint ($t_1 = 2.35$ ns), then decreased linearly over the next 0.2 ns to the postulated sustained field level and kept it at that level from $t_2 = 2.55$ ns until the end of the simulation. The problem, we discovered, was that as we changed the assumed streamer radius, the peak field level attained at 2.35 ns would change. We decided that it was more important to keep the starting peak field the same between runs so we changed the timing of the applied field to make this occur. The timing we finally chose is shown in Table 2. The first column is the assumed streamer radius along with if the streamer is anode-directed or cathode-directed. The second column is the time t_1 where the 70 kV/cm ends and the linear decrease begins. The time t_1 is when the cathode- or anode-directed streamer field (depending on which sustaining field we are searching for) is greater than 300 kV/cm for the first time. We had the applied field decrease to sustaining field levels in one time step so t_2 is 1 picosecond later than t_1 . This caused the total field to drop immediately. Originally, when we had the applied field drop occur over 0.2 ns, the head field would continue to increase for some number of timesteps and it was difficult to control the field level finally attained.

In air, we know experimentally that the sustaining field for the anode-directed streamer (10–20 kV/cm) is much larger than that of the cathode-directed streamer (5 kV/cm). Therefore, when searching for the anode-directed sustaining level computationally we had to suppress the evolution of the cathode-directed streamer once it formed. Otherwise, the cathode-directed streamer would grow exponentially and dominate the problem until the simulation became unstable. To suppress the cathode-directed streamer, we manipulated the applied field spatially by setting it to zero for $x < 0$ mm. This allowed the cathode-directed streamer to form and influence the anode-directed streamer in the beginning when they were close to one another, but after it crossed $x = 0$ mm it started to extinguish.

A natural question to ask is how far away should the cathode-directed streamer be before we suppress it? We did some studies where we set the applied field to zero at $x < 0$ mm and compared the results to those where the applied field was set to zero at $x < -10$ mm. The differences in how the heads propagated were negligible for the small radius cases (50 and 100 μm). For the larger radii, the head propagation is affected. Figure 42 shows the path that the anode-directed streamer takes when the cathode-directed streamer is suppressed at $x < 0$ mm, while Figure 43 shows the same information when the cathode-directed streamer is suppressed at $x < -10$ mm. It is clear that the suppressed, cathode-directed streamer will be different in the two figures. But note that at 15 ns, the anode-directed streamer is at 54 mm in Figure 42

Radius (μm)	t1 (ns)
50 (cathode)	2.284
50 (anode)	2.277
100 (cathode)	2.361
100 (anode)	2.341
300 (cathode)	2.482
300 (anode)	2.479
600 (cathode)	2.547
600 (anode)	2.552

Table 2. Time for 300 kV/cm field.

and at 62 mm in Figure 43. At 15 ns, the anode-directed streamer head is at 120 kV/cm in Figure 44 and at 130 kV/cm in Figure 45. The sustaining field levels, however, are insensitive enough that only for the 600 μm radius case did the sustaining levels change: from 15 kV/cm for suppression at $x < 0$ mm to 16 kV/cm for suppression at $x < -10$ mm. Based on this study, we decided to suppress the cathode-directed streamer starting at $x = 0$ mm.

While attempting to find the sustaining levels, we noticed that for all gases, the anode-directed streamer often had a very sharp transition where the sustaining field could be found easily to within 1 kV/cm as shown previously in Figures 38 - 40. The cathode-directed streamer does not have as sharp a transition, especially for the 50 and 100 μm radii. One characteristic of the cathode-directed streamer in air, which helps in determining the sustaining level, is that it stops propagating at and below the sustaining field level. This is shown for the 300 μm radius streamer in Figures 46 - 49. Figure 46 shows the cathode-directed peak field when the guess at the sustaining field is too small. We see the expected decrease in head field as time increases. Figure 47 shows the cathode-directed streamer propagating to -2.5 mm in the first 3 ns and then stopping. This causes the red line that represents the cathode-directed streamer to be vertical. The anode-directed streamer continues to propagate, as seen in the non-vertical nature of the blue line. Figure 48 shows the cathode-directed peak field when the guess at the sustaining field is too big. We see the expected increase in head field as time increases. From the non-vertical nature of the red line in Figure 49, we see that the cathode-directed streamer no longer stops after the first 3 ns. In this 300 μm example, both head field level and speed of the cathode-directed streamer showed a sharp transition between 3 and 4 kV/cm. For smaller streamer radii, 100 μm for example, the head field level is falling at 1 kV/cm and rising at 4 kV/cm but in between is rather constant. The peak doesn't propagate, though, until the field level reaches 4 kV/cm.

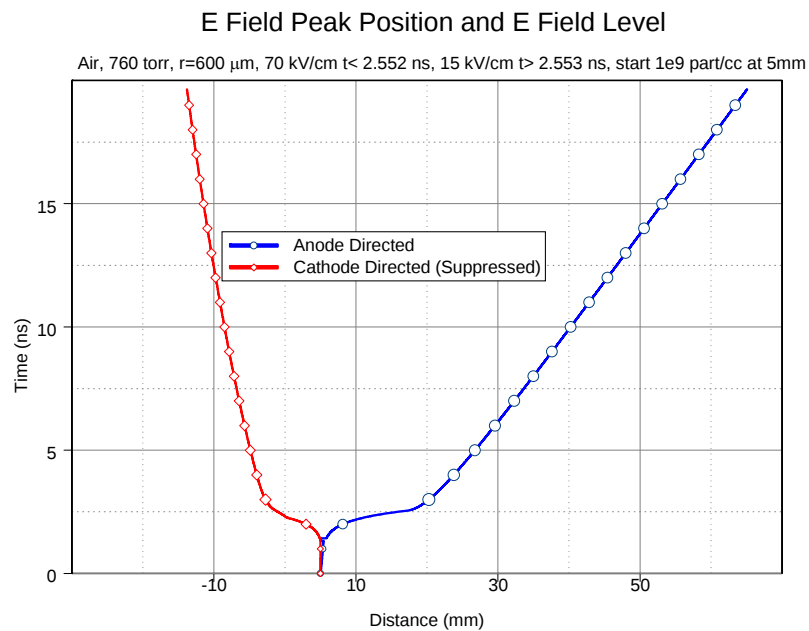


Figure 42. $600\ \mu\text{m}$ radius streamer with cathode-directed streamer suppressed at $x < 0\ \text{mm}$.

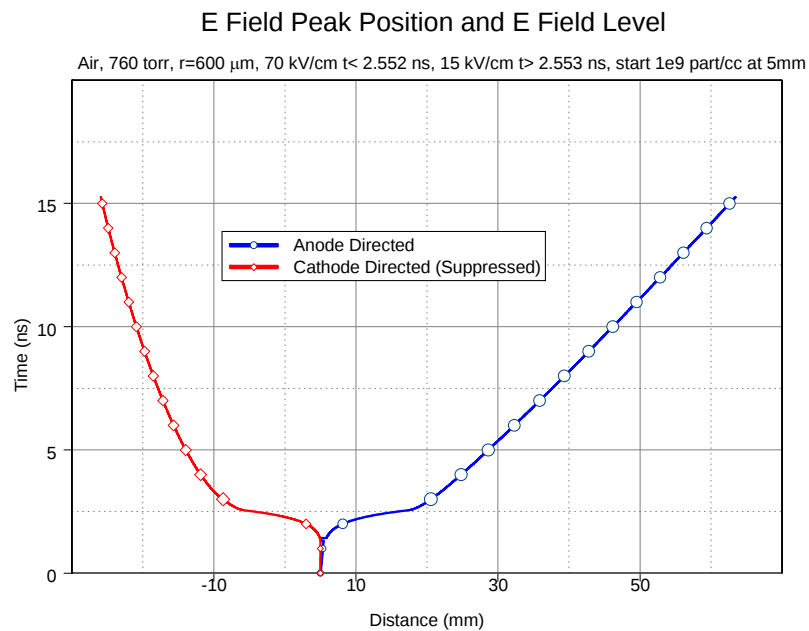


Figure 43. $600\ \mu\text{m}$ radius streamer with cathode-directed streamer suppressed at $x < -10\ \text{mm}$.

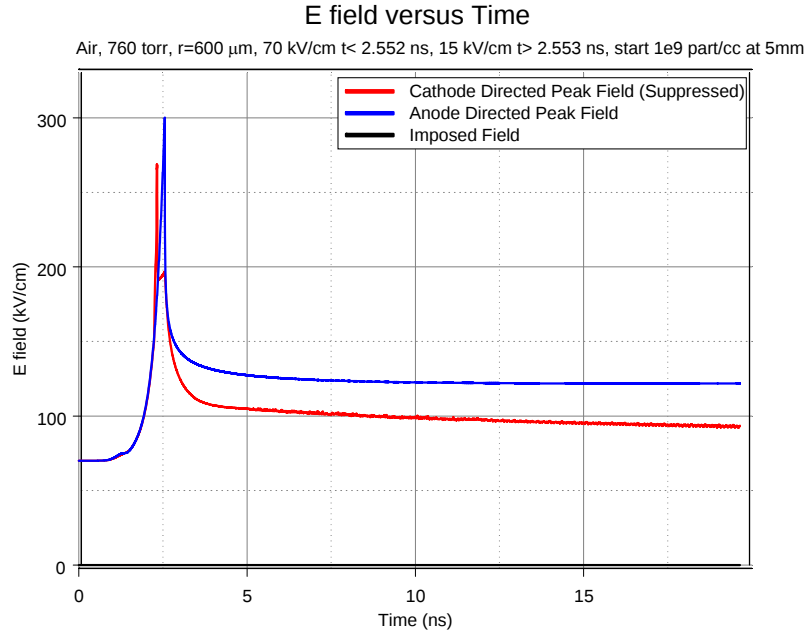


Figure 44. $600\ \mu\text{m}$ radius streamer with cathode-directed streamer suppressed at $x < 0\ \text{mm}$.

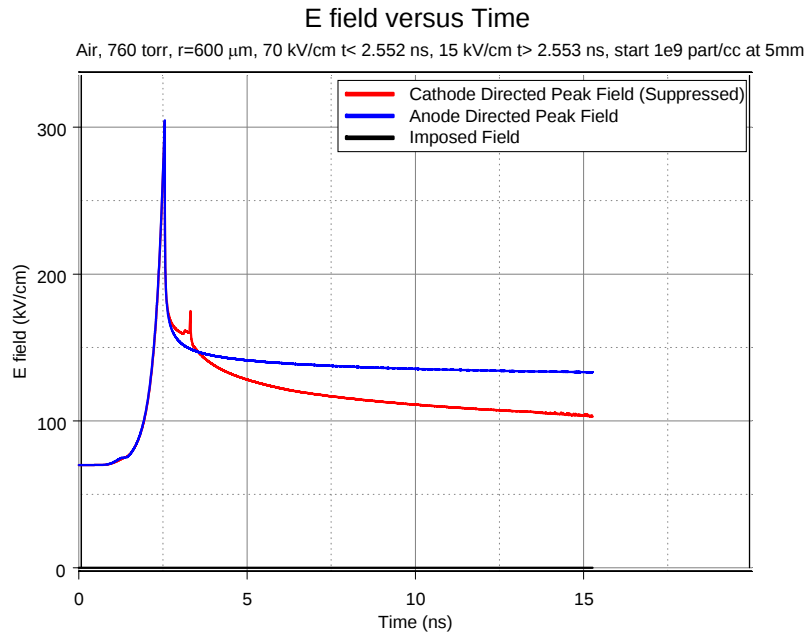


Figure 45. $600\ \mu\text{m}$ radius streamer with cathode-directed streamer suppressed at $x < -10\ \text{mm}$.

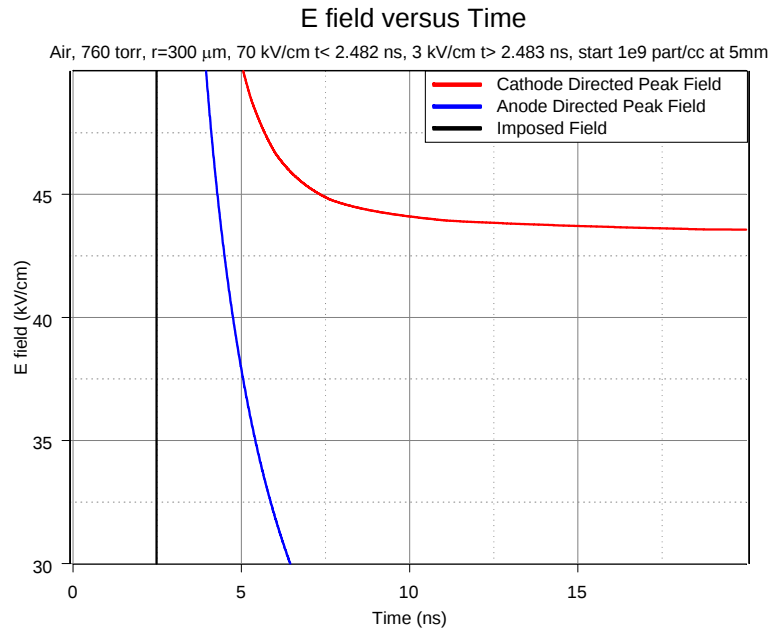


Figure 46. Cathode-directed streamer below sustaining field level.

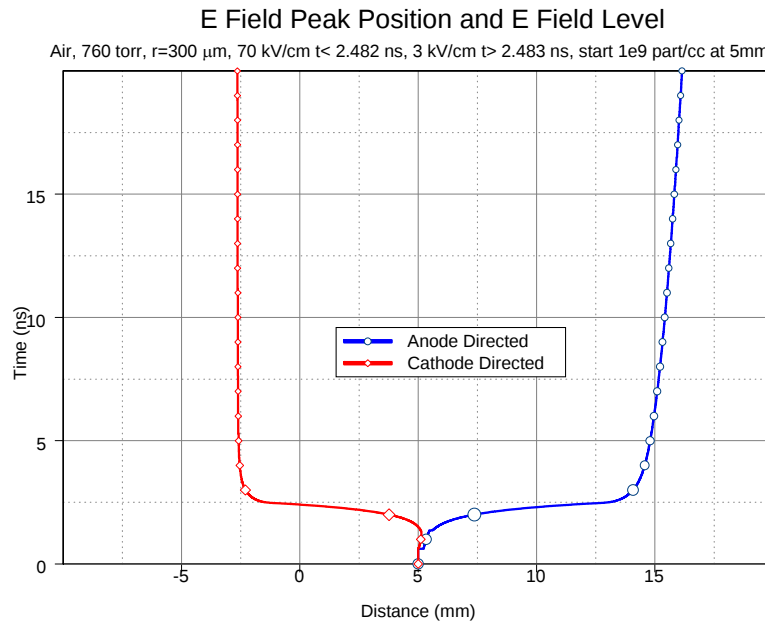


Figure 47. Cathode-directed streamer stops below sustaining field level.

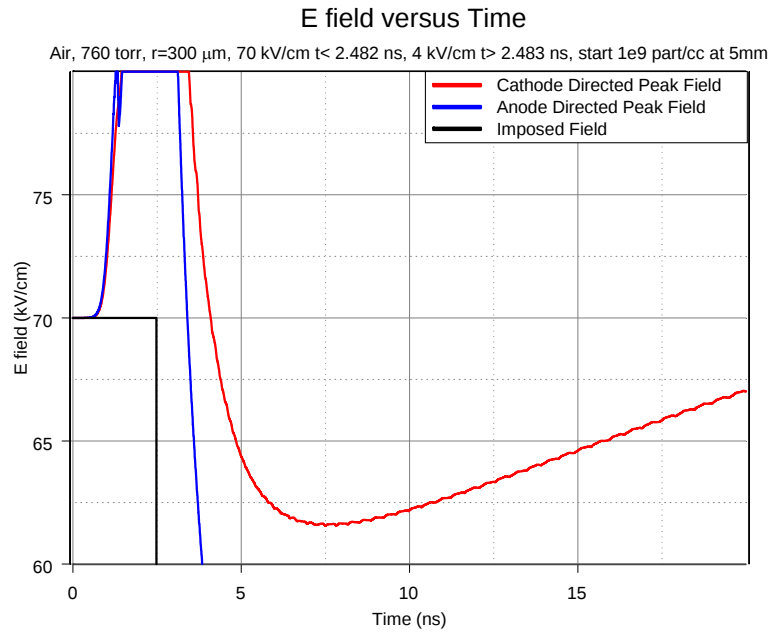


Figure 48. Cathode-directed streamer above sustaining field.

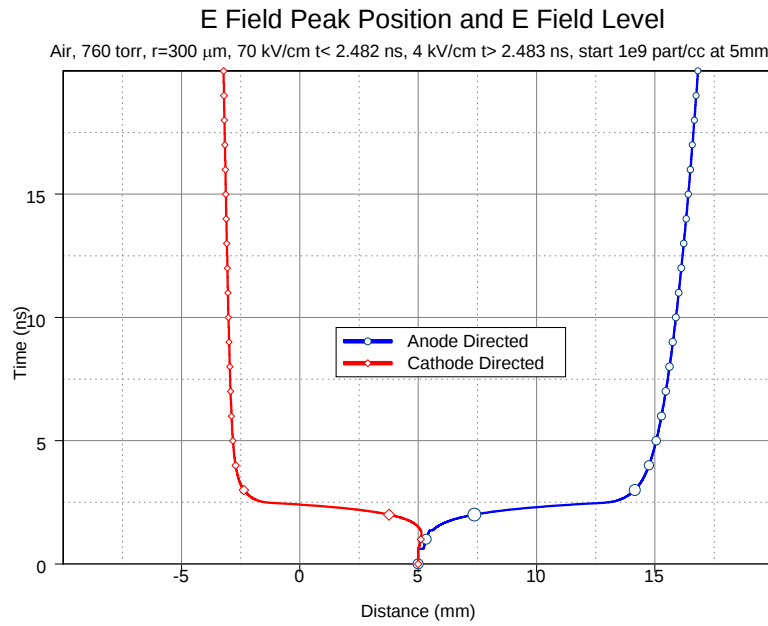


Figure 49. Cathode-directed streamer propagates above sustaining field level.

The sustaining fields for the cathode- and anode-directed streamers are shown in Table 3 for various assumed streamer radii. As discussed above, for the cathode-directed streamer we define the transition where the streamer starts to propagate as well as increases in head field level. If the head field decreases at one sustaining field and increases at a field 1 kV/cm higher we give the sustaining field as a range spanning 1 kV/cm. If the head field decreases for one sustaining field level, is constant at a sustaining field 1 kV/cm higher and increases at a sustaining field increased a further 1 kV/cm, we give the sustaining field as the single value where the head field is constant. We note that the calculated cathode-directed sustaining field is remarkably consistent across a wide range of assumed streamer radii. Literature gives this value as between 4.5 and 5 kV/cm [14]. Our level is about 1 kV/cm less at between 3 and 4 kV/cm. Literature also says that the anode-directed streamer requires higher fields than cathode-directed ones and we see this in our results comparing columns 2 and 3 of Table 3.

Our results show that the predicted anode-directed streamer sustaining field decreases as the assumed streamer radius increases. There is more ambiguity in measuring the anode-directed sustaining field than there is for the cathode-directed streamer because a field high enough to study an anode-directed streamer sustaining level (20 kV/cm) will also excite and be far above the cathode-directed streamer sustaining level (5 kV/cm). Literature gives experimental results that range between 10 and 16 kV/cm [14]. Unfortunately, except for the 600 μm result, our calculated anode-directed sustaining field are above this range. Note that both anode- and cathode-directed streamer sustaining fields are below critical field (25 kV/cm). The anode-directed sustaining field is more than 50% of critical field, while the cathode-directed sustaining field is 10% – 15% of critical field.

Radius (μm)	Cathode-directed Field (kV/cm)	Anode-directed Field (kV/cm)
50	4-5	27-28
100	3-4	26-27
300	3-4	18-19
600	3-4	15

Table 3. Air sustaining field.

6.4 Air and Surface

We next use the techniques discussed in the previous section to calculate the sustaining field when the streamer is next to a Teflon surface. As in the previous section we allow the head field in the cathode- or anode-directed streamer to increase until it reaches 300 kV/cm and then drop the applied field to sustaining level in 1 picosecond. Table 4 shows the time t_1 for each radius and streamer direction.

One item of interest when dealing with surfaces is that as a rule, when a surface is not present, the cathode- and anode-directed streamer head fields grow at nearly the same rate under the influence of the initial 70 kV/cm applied field. This can be seen in Figure 44 where the red and blue lines lie on top of one another prior to 2 ns. Figure 50 shows that when a surface is added, the field level in the anode-directed streamer peak grows faster than the cathode-directed streamer peak. This led to a problem in the 600 μm radius cathode-directed streamer case. The anode-directed streamer grew large enough in early time to suppress the cathode-directed streamer, so the cathode-directed streamer never attained the required level of 300 kV/cm. We tried various tricks to circumvent this problem, like suppressing the anode-directed streamer spatially at 7 mm rather than at 10 mm, but this was not sufficient and given the fact that the large-radius streamers affect each other's propagation characteristics at distances larger than 10 mm, it didn't seem prudent to move the suppression to within 1 mm of the streamer starting point. We decided, therefore, not to run the 600 μm simulation for the cathode-directed sustaining field.

Results of the simulation are shown in Table 5. We note that the presence of the surface does not appear to affect the sustaining field level significantly (by more than 2 kV/cm).

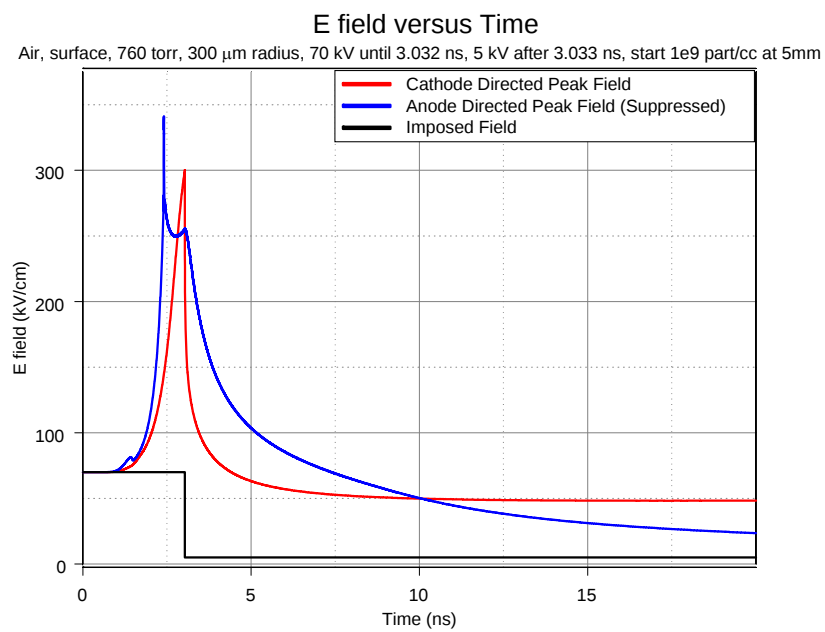


Figure 50. Anode-directed streamer grows faster than cathode-directed streamer.

Radius (μm)	t_1 (ns)
50 (cathode)	2.487
50 (anode)	2.417
100 (cathode)	2.616
100 (anode)	2.426
300 (cathode)	3.032
300 (anode)	2.429
600 (cathode)	NA
600 (anode)	2.340

Table 4. Time for 300 kV/cm field.

6.5 SF6

We next change the gas of interest to SF6, which is highly electronegative. The critical field (the level that the field needs to be for an avalanche to start) for SF6 at 760 Torr is 89.7 kV/cm [36]. Therefore, we set the applied field higher than the critical field at 105 kV/cm, let the streamer head field level build to 450 kV/cm (this is the same ratio to the applied field as the air case $-450/105 = 300/70$), and drop the field level to a guess of the sustaining field level in 1 picosecond. The time needed for the streamer head to build to 450 kV/cm is given in Table 6 for the various assumed streamer radii and streamer direction.

Unlike streamers in air, for small radii the SF6 results do not show any semblance of a sharp transition for the cathode-directed streamer. This is shown in Figures 51 - 54 for the 50 μm radius case. For this radius, when the sustaining field is set at any value between 54 and 65 kV/cm, the peak field at the head of the cathode-directed streamer remained relatively constant. When the sustaining field was dropped to 52 kV/cm, the head field would begin to decrease near the end of the run between 18 and 20 ns as seen in Figure 51. At 65 kV/cm the peak field of the streamer increases initially but levels out to a constant value as seen in Figure 53. When the sustaining field is increased to 66 kV/cm, as seen in Figure 54, the head field steadily increases. Further, as seen in Figure 55, the cathode-directed streamer at or below threshold continues to propagate so we can't use this as a factor to help us determine the threshold, like we did in air. The result is that for SF6 we cannot find the sustaining level for 50 and 100 μm radii cathode-directed streamers in SF6 to an accuracy better than 10 kV/cm. For larger radii streamers and for anode-directed streamers, we again can find the sustaining level to within 1 kV/cm. This is reflected in Table 7.

Table 7 shows the sustaining fields calculated for SF6. Note that unlike air SF6 does not have a disparity in sustaining field levels between the anode- and cathode-directed streamers. The sustaining fields are much higher than in air (about 60% of the critical field of SF6). As the assumed streamer radius increase, both the cathode- and anode-directed sustaining fields decrease, but the anode-directed sustaining field decreases more than the cathode-directed field.

6.6 SF6 and Surface

In this section we study the effect that a Teflon surface has on the calculated sustaining fields in SF6. Unfortunately, we were unable to obtain results using our previous streamer head field (450 kV/cm) and dropping to sustaining level in 1 picosecond. This was due in part to the fact that like in air, when a surface is added to the SF6, the anode-directed streamer grows more rapidly than the cathode-directed streamer and begins to affect the streamer propagation. In general, for the cathode-directed cases we dropped to sustaining field in 200 picoseconds and for the anode-directed cases we dropped to sustaining field in 1 picosecond. Table 8 shows the applied field characteristics (t_1 and t_2) for a given radius and streamer direction and the resulting electric field attained in the head (column 4).

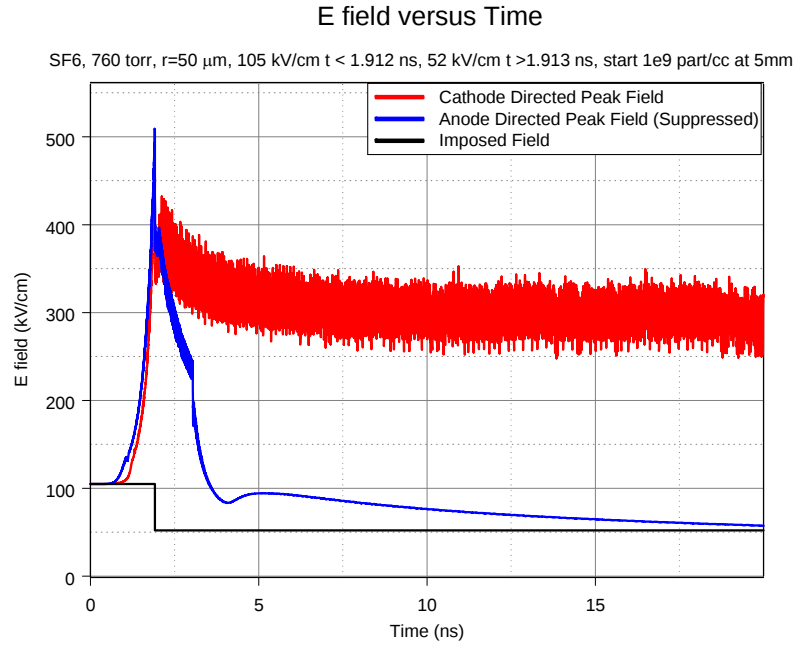


Figure 51. Cathode-directed streamer decays in a 52 kV/cm sustaining field.

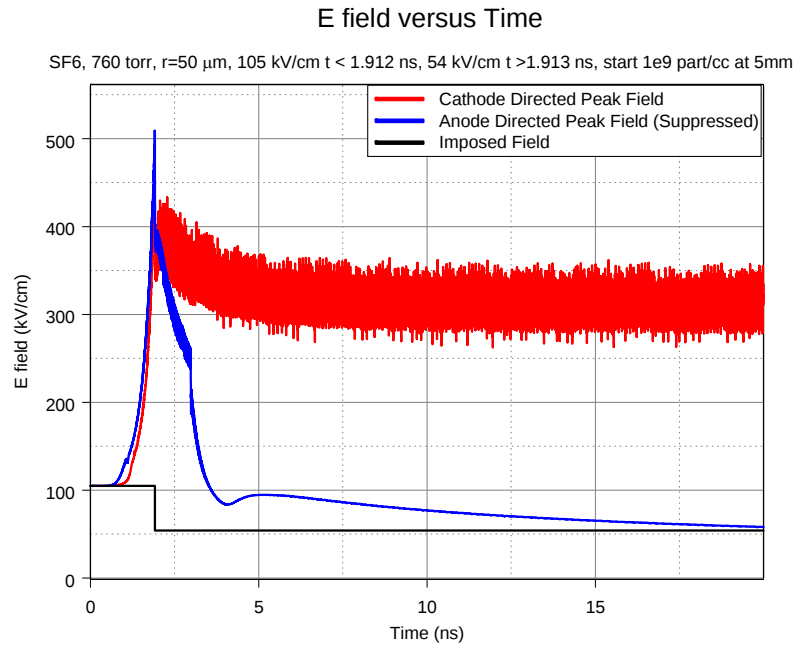


Figure 52. Cathode-directed streamer is constant in a 54 kV/cm sustaining field.

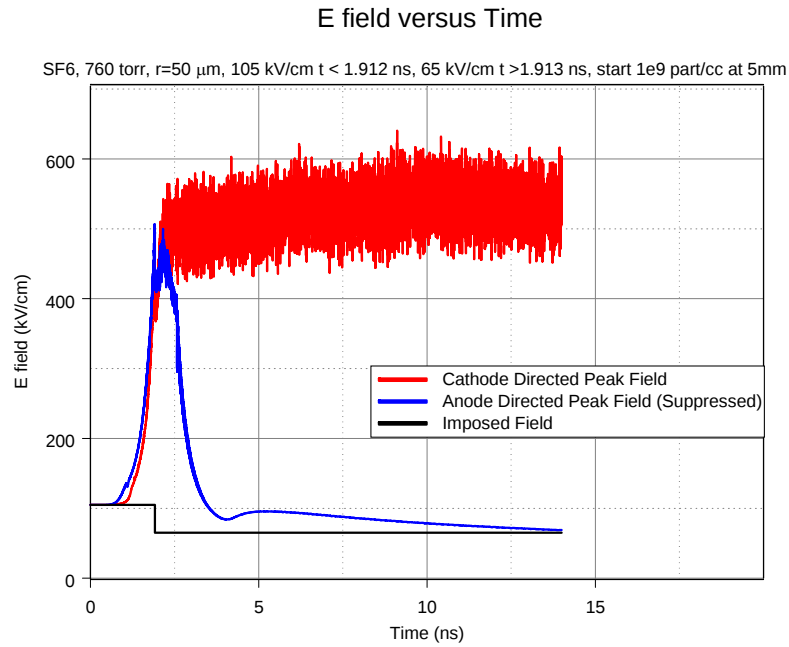


Figure 53. Cathode-directed streamer continues to be constant in a 65 kV/cm sustaining field.

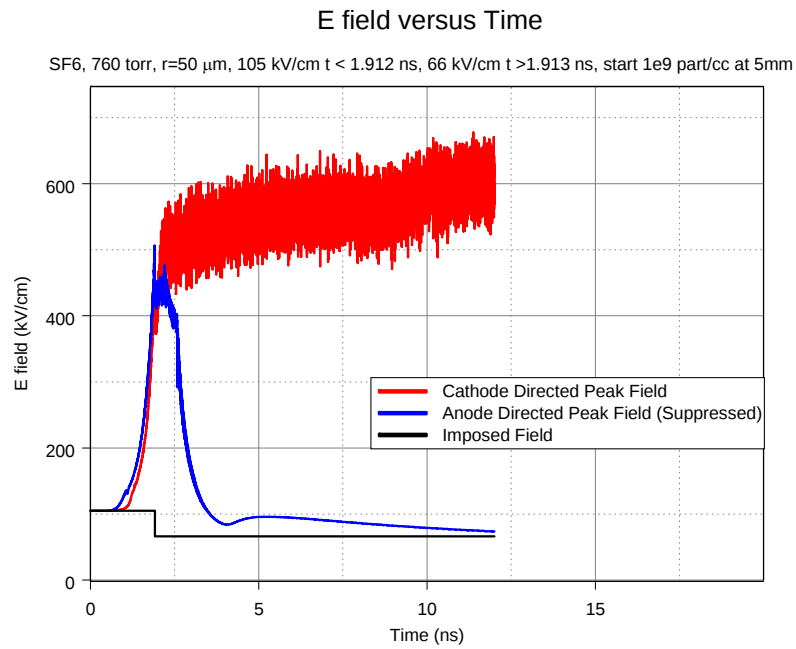


Figure 54. Cathode-directed streamer grows in a 66 kV/cm sustaining field.

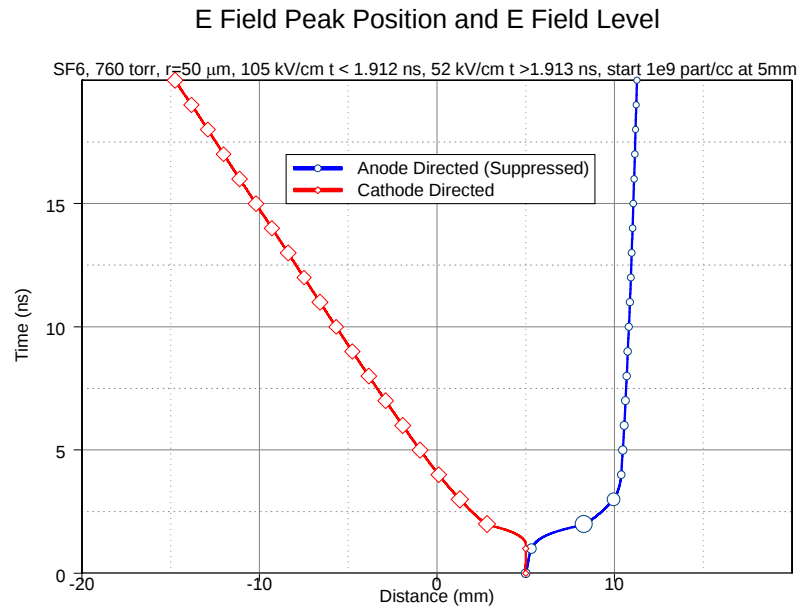


Figure 55. Cathode-directed streamer propagates below sustaining field.

Radius (μm)	Cathode-directed Field (kV/cm)	Anode-directed Field (kV/cm)
50	4-5	29-30
100	4-5	28
300	5	15-16
600	not run	12-13

Table 5. Air and surface sustaining field.

Table 9 shows the resulting streamer sustaining fields for SF6 and a Teflon surface. As with SF6 alone, we have trouble in determining the sustaining field for both cathode- and anode-directed streamer when the assumed streamer radius is 50 μm . The 100 μm radius and larger streamers all have a transition that happens within 1 kV/cm. The behavior with radius for SF6 and a surface is similar to the behavior of SF6 alone: an increase of radius causes the sustaining field to decrease. The cathode-directed sustaining field for the SF6 and surface is higher than that for the SF6 alone. The anode-directed sustaining field for SF6 and surface is slightly lower than the SF6 alone for the larger radii. This results, especially for the larger radii, in an anode-directed sustaining field that is approximately half that of the cathode-directed sustaining field. This is different from the behavior of a surface on air in that the surface seems to influence the sustaining levels and set the levels opposite that of air (anode-directed is less than cathode-directed).

6.7 High Pressure SF6

In this section we briefly study the effect that high pressure has on the calculated sustaining fields in SF6. In this case we chose a single radius (300 μm) and a pressure of 2 bar at 20⁰ C. We hypothesized that the field quantities such as critical field and sustaining field should scale as E/p . Since in Subsection 6.5 we used 105 kV/cm as an initial field, in this case we nearly doubled that value to 200 kV/cm. We allowed the head field to increase to a value of 800 kV/cm before dropping the applied field to the postulated sustaining field value in 1 ps. For the cathode-directed sustaining field we applied the initial field for 1.368 ns and for the anode-directed sustaining field we applied the initial field for 1.371 ns. The cathode-directed sustaining field calculated was between 70 and 88 kV/cm, which is 1.3 to 1.7 times the sustaining field of 52 kV/cm found in subsection 6.5 for SF6 at a pressure of 1 bar. The anode-directed sustaining field calculated was 98 kV/cm, which is 1.8 times the sustaining field of 54 kV/cm for SF6 at 1 bar. These results indicate that doubling the pressure does not quite double the sustaining field.

6.8 Run Times

All the sustaining field results were obtained by running the 1.5D streamer code on Sandia's Thunderbird computer. Finding the sustaining field meant repeatedly running the code with different guesses for the sustaining field. We ran 960 jobs over a period of seven months requiring a total of 14,660 hours of processor time. The majority of the time was spent in finding the sustaining fields for the 300 μm and 600 μm radii SF6 cases. Figure 56 shows the run time on a single processor of the Thunderbird in minutes per nanosecond of simulation time as a function of number of grid points required. Thus since we were using a grid size of 10 μm , 8000 grid points would cover a distance of 80 mm. If we were running a simulation of streamers without a surface, each nanosecond of simulation required 80 minutes of processor time. If a surface was included, each nanosecond of simulation time requires 250 minutes of processor time. We would usually simulate 20 ns so a volume simulation would take 27 hours and a simulation involving a surface would take 83 hours. The large radii streamers travel faster than the small radii streamers, so the initial applied field for a 50 μm streamer radius would push the anode- and cathode-directed streamer heads 5 mm apart while for a 600 μm radius the initial field would push the streamer heads 40 mm apart. In air, after the applied field dropped to sustaining level, the cathode-directed streamer slowed to a halt or propagated very slowly so the size of the simulation space did not need to be increased significantly. In air,

Radius (μm)	t1 (ns)
50 (cathode)	1.912
50 (anode)	1.876
100 (cathode)	1.873
100 (anode)	1.843
300 (cathode)	1.925
300 (anode)	1.910
600 (cathode)	1.999
600 (anode)	1.983

Table 6. Time for 450 kV/cm field.

for the cathode-directed streamer, we were forced to suppress the anode-directed streamer spatially. This kept the anode-directed streamer from propagating with any speed and limited the size of the simulation space. In SF6 when the applied field drops to sustaining level, both streamer heads continue to propagate at a high rate of speed ($10^8 - 10^9$ cm/s) which causes the simulation space to become large (300 mm at 10 ns).

To limit the grid size and speed the simulation we would suppress the cathode-directed streamer spatially when searching for the anode-directed sustaining field and visa versa. Our check on this approximation was described previously in sub-section 6.3.

Another technique to limit the grid size is to do the early-time part of the simulation (the first 10 ns, for example) using a smaller grid, write a restart file, then increase the extent of the grid for the next section of time. We would always keep the end of the grid ahead of the approaching streamer head. This technique gives about a factor of two to three speed-up, but there is some approximation involved here as well. If the entire simulation space is gridded – say -50 mm to 50 mm, for example – the charges in the regions near the ends of the grid will evolve over the entire time that the simulation runs. If we use the restart technique we would, for example, start by gridding the space from -25 mm to 25 mm. When the streamer head passed 20 mm we would stop the simulation, write a restart file and use the restart file as the initial conditions for the -25 mm to 25 mm section of a larger (-50 mm to 50 mm) grid. Unfortunately, we don't know the initial conditions of the 25 to 50 mm and -50 to -25 mm sections of the new grid so we set the charge density of the unknown sections of the new larger grid to be constant and equal to the charge density of the end cell of the original grid. This technique worked most of the time, but a few times the plots of the peak head field as a function of time would change in slope after a restart, possibly because we let the head approach too close to the end before restarting with a new grid. When this would happen, because we were looking for small changes of this nature and we didn't want to get confused by use of a time-saving technique, we would start the simulations over from the beginning using a larger grid. The ultimate expression of this technique would be to track the heads and continually grow the grid to keep just ahead of the streamer heads.

Another way of limiting the grid size is to increase the $10\ \mu\text{m}$ element size to 20 or $40\ \mu\text{m}$. With this in mind we performed a study to see what was required to resolve the head of the streamer with different radii. We discovered that for a $50\ \mu\text{m}$ radius a $20\ \mu\text{m}$ cell size put a single point to resolve the streamer head electric field peak as seen in Figure 57. This is probably a minimum resolution level. The $10\ \mu\text{m}$ and $5\ \mu\text{m}$ cell size resolves the peak better as seen in Figures 58 and 59. Note that although the peak is resolved better with a smaller cell size and the electric field being calculated is stable at around -500 kV/cm, the position of the peak continues to change. It is as if finer discretization causes the peak to slow down. Here the position at 2.5 ns into the simulation changes from 2.77 mm for $20\ \mu\text{m}$ cell size to 3.02 mm for $10\ \mu\text{m}$ cell size to 3.14 mm for $5\ \mu\text{m}$ cell size, to 3.46 mm for $2.5\ \mu\text{m}$ cell size. There is something in this algorithm that causes the head position to be sensitive to the grid even when it is chosen relatively

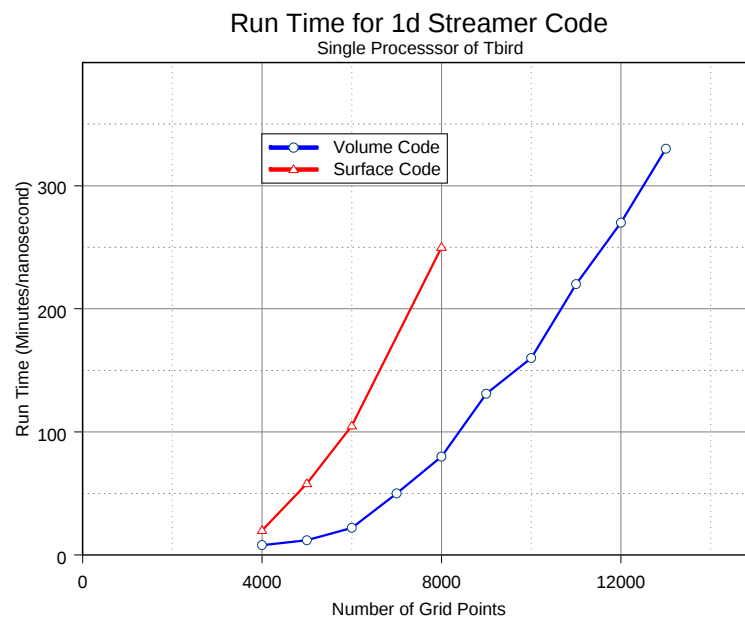


Figure 56. Tbird run times for surface and volume streamer codes.

Radius (μm)	Cathode-directed Field (kV/cm)	Anode-directed Field (kV/cm)
50	54-66	71
100	50-61	63
300	52	54
600	51-52	52

Table 7. SF6 sustaining field.

fine (does not appear to converge). This trend becomes worse when the radius becomes larger as seen in Figures 60 and 61 for 300 μm radius. Both 20 and 10 μm cell size resolves the head but the position of the head and to a lesser extent the magnitude of the head field change with discretization. We are looking into why this occurs.

7 RADIAL EXPANSION OF HEAD

The treatment in Aleksandrov and Bazelyan of radial head expansion is now discussed [58], [59], [60], [13], [61], [62]. The method is to carry out an additional drift and avalanche step, also accounting for ionization wave expansion with new seed electrons arising from photoionization, at the end of the axial advance in which the radius of the channel is expanded.

7.1 Axial Tube Field

The treatment here uses the field representation of a ring or tube charge

$$\begin{aligned}
E_z(0, z_i) &= \frac{r_c}{2\epsilon_0} \sum_{i'=1}^I \sigma_{i'} \left[1/\sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} - 1/\sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} \right] \\
&= \frac{1}{4\pi\epsilon_0} \sum_{i'=1}^I q_{ci'} \left[1/\sqrt{r_c^2 + (z_i - z_{i'} - \delta z_{i'+1/2}/2)^2} - 1/\sqrt{r_c^2 + (z_i - z_{i'} + \delta z_{i'-1/2}/2)^2} \right] \quad (219)
\end{aligned}$$

where $q_{ci'} = 2\pi r \sigma_{i'}$ is the charge per unit length along the channel with radius r_c .

7.2 Radial Cylindrical Shell Field

We study the radial expansion of the channel with a simplification of the field, assuming the charge $q_{ci'}$ is slowly varying along the streamer length. The problem is then approximated as having a field distribution of a infinite line charge with charge per unit length q_c . The radial field is then

$$E_r(r) \sim \frac{q_c}{2\pi\epsilon_0 r} \quad (220)$$

We begin the problem with a small radius [13] $r_0 = 0.015$ cm and examine the radial expansion of the streamer channel. The essential characteristic of the radial electric field is the inverse strength with channel radius. The ionization coefficient decreases rapidly with field strength and thus the radial avalanche will rapidly halt. We grid out to a large radius $R = 5 - 10$ mm.

Note that if we use this cylindrical field to advance the radial avalanche, the cylindrical field at large

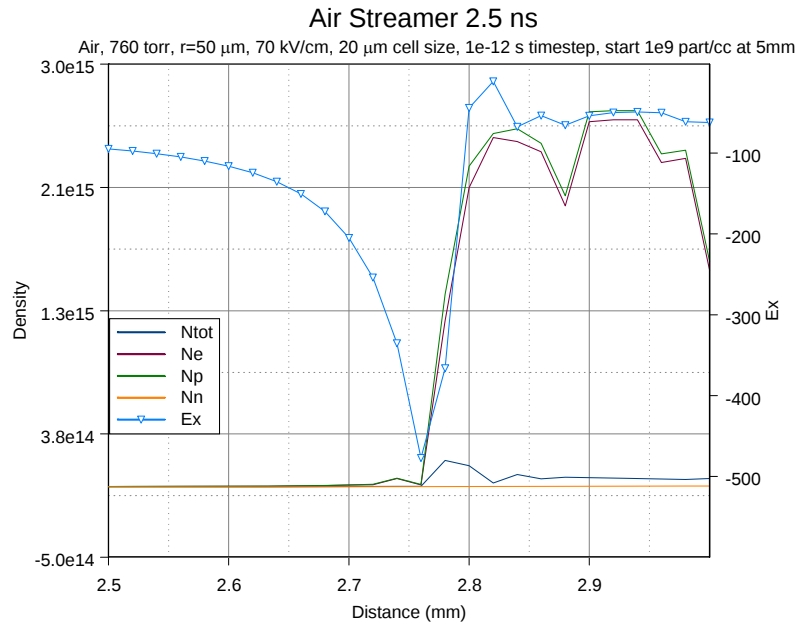


Figure 57. Cathode-directed streamer with 20 μm cell size, 50 μm radius at 2.5 ns.

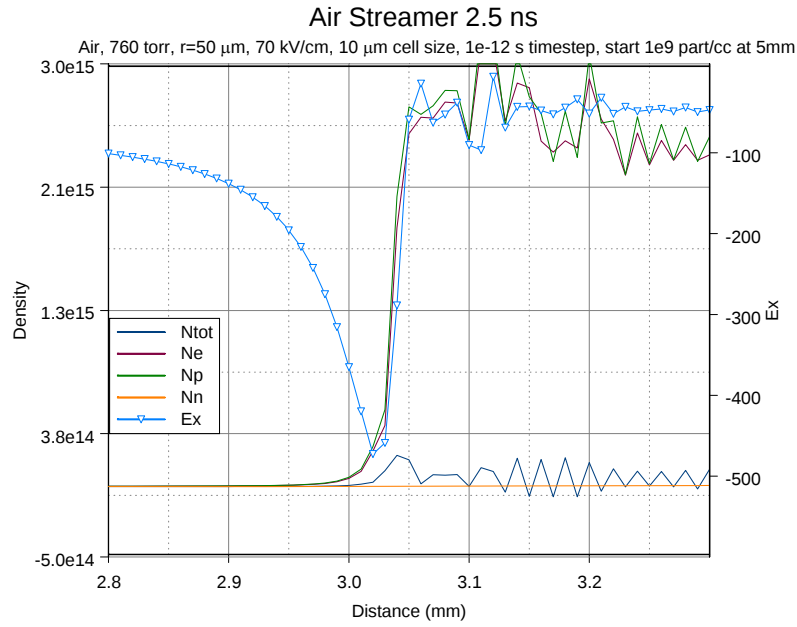


Figure 58. Cathode-directed streamer with 10 μm cell size, 50 μm radius at 2.5 ns.

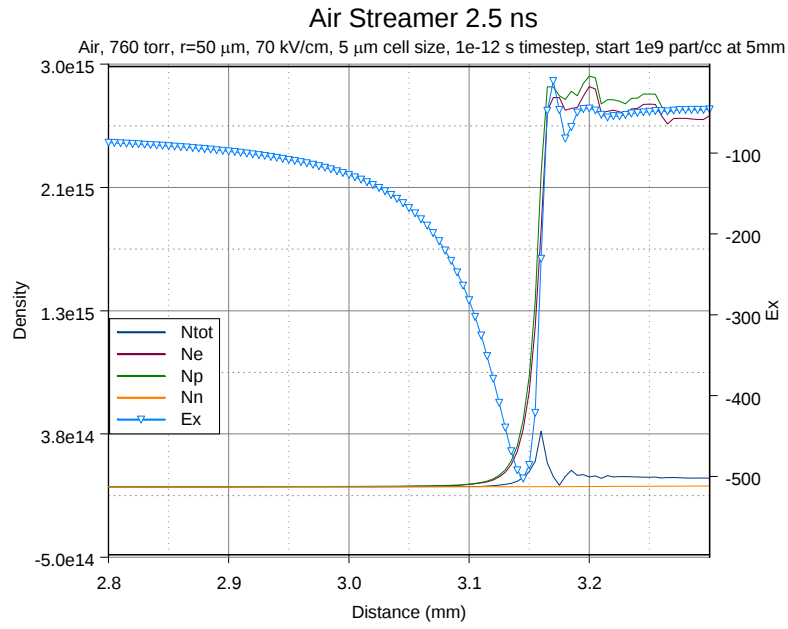


Figure 59. Cathode-directed streamer with 5 μm cell size, 50 μm radius at 2.5 ns.

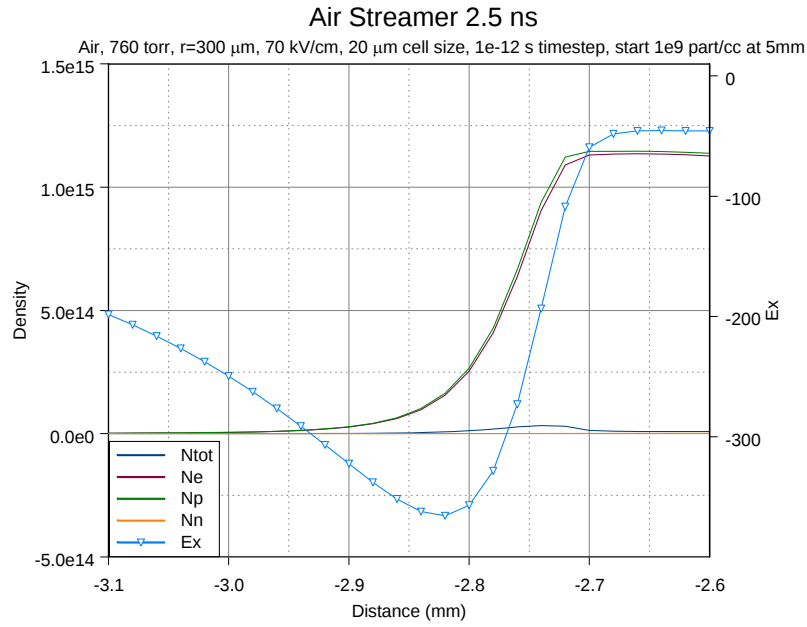


Figure 60. Cathode-directed streamer with 20 μm cell size, 300 μm radius at 2.5 ns.

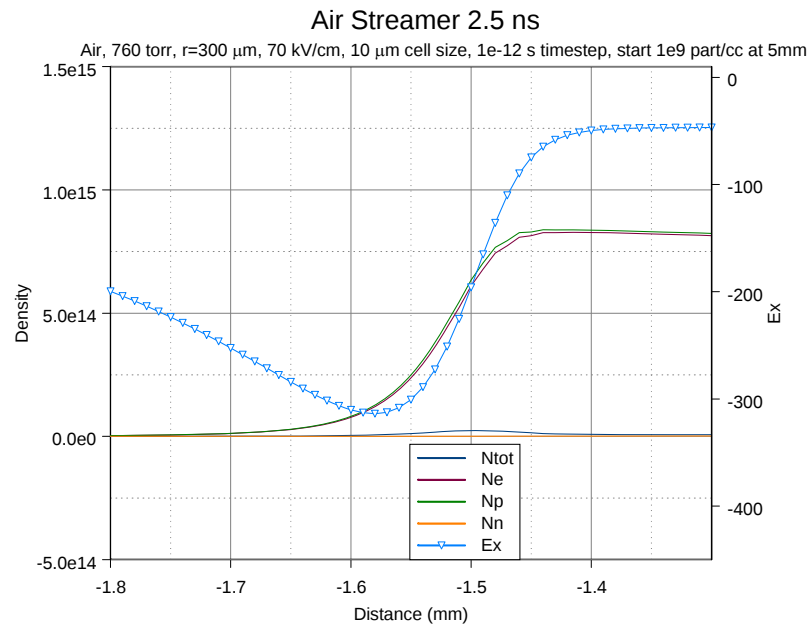


Figure 61. Cathode-directed streamer with 10 μm cell size, 300 μm radius at 2.5 ns.

Radius (μm)	t1 (ns)	t2 (ns)	Head E field (kV/cm)
50 (cathode)	2.101	2.301	400
50 (anode)	2.101	2.102	550
100 (cathode)	2.046	2.246	400
100 (anode)	2.046	2.047	600
300 (cathode)	1.976	2.176	300
300 (anode)	1.700	1.701	300
600 (cathode)	2.300	2.500	300
600 (anode)	1.639	1.640	300

Table 8. Times and initial field for SF6 and surface.

radii does not need to be re-evaluated during the radial step because the ionization is charge neutral. However because of radial charge screening the field within the avalanche will be modified. To account for this field we use

$$\nabla^2 \phi = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) = -\frac{1}{r} \frac{\partial}{\partial r} (r E_r) = -\rho / \varepsilon_0 \quad (221)$$

and thus

$$r E_r = \frac{1}{\varepsilon_0} \int_0^r \rho(r') r' dr' \quad (222)$$

$$\rho(r) r = e (r N_p - r N_n - r N_e) \quad (223)$$

Now we can discretize the unknown, taken as the product $\rho(r_i) r_i$ or $r_i N_j(r_i)$, in the radial direction. The problem is discretized by selecting a grid r_i with $i = 1, \dots, I$ and

$$r_{i+1} - r_i = \delta r_{i+1/2} \quad (224)$$

$$\delta r_i = \frac{1}{2} (\delta r_{i+1/2} + \delta r_{i-1/2}) \quad (225)$$

and for convenience we set

$$\delta r_1 = \frac{1}{2} \delta r_{3/2} \text{ and } \delta r_I = \frac{1}{2} \delta r_{I-1/2} \quad (226)$$

Thus the trapezoidal rule gives

$$\int_a^b f(x) dx = \sum_{i=2}^I \frac{1}{2} [f(x_i) + f(x_{i-1})] \delta x_{i-1/2} = \sum_{i=2}^{I-1} f(x_i) \delta x_i \quad (227)$$

or

$$r \rho = \sum_{i'=1}^I r_{i'} \rho(r_{i'}) \quad (228)$$

or

Radius (μm)	Cathode-directed Field (kV/cm)	Anode-directed Field (kV/cm)
50	62-82	74-82
100	79	73
300	67	47
600	71	37-38

Table 9. SF6 and surface sustaining field.

$$\varepsilon_0 r_i E_r(r_i) = \sum_{i'=2}^i \frac{1}{2} [r_{i'} \rho(r_{i'}) + r_{i'-1} \rho(r_{i'-1})] \delta r_{i-1/2} = \sum_{i'=2}^{i-1} r_{i'} \rho(r_{i'}) \delta r_{i'} + \frac{1}{2} r_i \rho(r_i) \delta r_{i-1/2} \quad (229)$$

7.3 Selection Of New Radius

After running this calculation we need to select a new radius and charge (this would enable us to go back to the 1.5D calculation). To do this it seems reasonable to take

$$q_c = 2\pi \int_{r_0}^R r \rho dr = 2\pi \sum_{i'=2}^{I-1} r_i \rho(r_i) \delta r_i \quad (230)$$

For the new radius $r_0^{(n)}$ we could look for the peak of $r_i \rho(r_i)$ or perform the average

$$r_0^{(n)} q_c = 2\pi \sum_{i'=2}^{I-1} r_i^2 \rho(r_i) \delta r_i \quad (231)$$

We will use the average operation because for the anode directed streamer head we expect electron drift to take place and possibly spread out the radial charge density profile. In fact it could be desirable to use a disc model on the anode head and a tube model only on the cathode head. However the attachment process should make the shell model more valid since electron drift is eventually eliminated in favor of the slower ionic drift. Perhaps, a more accurate calculation is the integration over each radial shell (using the midpoint value for the charge density)

$$\begin{aligned} r_0^{(n)} q_c &= 2\pi \sum_{i'=2}^{I-1} [r_i \rho(r_i)] \frac{1}{2} \left[(r_i + \delta r_{i+1/2}/2)^2 - (r_i - \delta r_{i-1/2}/2)^2 \right] \\ &= 2\pi \sum_{i'=2}^{I-1} [r_i \rho(r_i)] \left[r_i \frac{1}{2} (\delta r_{i+1/2} + \delta r_{i-1/2}) + \frac{1}{8} (\delta r_{i+1/2}^2 - \delta r_{i-1/2}^2) \right] \\ &= 2\pi \sum_{i'=2}^{I-1} [r_i \rho(r_i)] \left[r_i \delta r_i + \frac{1}{8} (\delta r_{i+1/2}^2 - \delta r_{i-1/2}^2) \right] \end{aligned} \quad (232)$$

For uniform spacing this is identical to the preceding value.

7.4 Radial Expansion With Photoionization

The source term S results from photoionization

$$S(x) = \sum_j \gamma_j \int_0^R S_r(r-r') N_e(r') \alpha_j^*(r') |W_e(r')| e^{-\mu_j|r-r'|} dr' \quad (233)$$

where γ_j is the secondary ionization coefficient for photoionization, α_j^* the excitation coefficient for ionizing radiation, μ_j is the photon absorption coefficient and S_r is the radial spreading factor for a source at r' by the charge at r

$$S_r = \frac{1}{2} \frac{r'}{r} \quad (234)$$

(Note that $rS_r \rightarrow 1/2$ when the densities are scaled by the radial coordinate to form the unknowns.) We rewrite this as

$$\begin{aligned} S(x) &= \sum_j \gamma'_j \int_0^R S_r(r-r') N_e(r') \alpha(r') |W_e(r')| e^{-\mu_j|r-r'|} dr' \\ &= \int_0^R S_r(r-r') N_e(r') \alpha(r') |W_e(r')| 4\pi r' \psi dr' \\ &= \int_0^R S_r(r-r') N_e(r') \alpha(r') |W_e(r')| \frac{p_q}{p+p_q} (\xi\omega/\alpha) f(r) dr' \end{aligned} \quad (235)$$

where the final for is used for air.

7.5 Continuity Equations In Radial Coordinates

The vector form of the continuity equations are

$$\frac{\partial N_e}{\partial t} = S + N_e \alpha |W_e| - N_e \eta |W_e| - N_e N_p \beta - \nabla \cdot (N_e \underline{W}_e - D \nabla N_e) \quad (236)$$

$$\frac{\partial N_p}{\partial t} = S + N_e \alpha |W_e| - N_e N_p \beta - N_n N_p \beta - \nabla \cdot (N_p \underline{W}_p) \quad (237)$$

$$\frac{\partial N_n}{\partial t} = N_e \eta |W_e| - N_p N_n \beta - \nabla \cdot (N_n \underline{W}_n) \quad (238)$$

or in the radial coordinate

$$\frac{\partial N_e}{\partial t} = S + N_e \alpha |W_e| - N_e \eta |W_e| - N_e N_p \beta - \frac{\partial (r N_e W_e)}{r \partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left(r D \frac{\partial N_e}{\partial r} \right) \quad (239)$$

$$\frac{\partial N_p}{\partial t} = S + N_e \alpha |W_e| - N_e N_p \beta - N_n N_p \beta - \frac{\partial (r N_p W_p)}{r \partial r} \quad (240)$$

$$\frac{\partial N_n}{\partial t} = N_e \eta |W_e| - N_p N_n \beta - \frac{\partial (r N_n W_n)}{r \partial r} \quad (241)$$

or if we regard rN as the unknown in this radial problem

$$\frac{\partial(rN_e)}{\partial t} = rS + (rN_e)\alpha|W_e| - (rN_e)\eta|W_e| - \frac{1}{r}(rN_e)(rN_p)\beta - \frac{\partial(rN_eW_e)}{\partial r} + \frac{\partial}{\partial r} \left[D \frac{\partial(rN_e)}{\partial r} - \frac{1}{r} D(rN_e) \right] \quad (242)$$

$$\frac{\partial(rN_p)}{\partial t} = rS + (rN_e)\alpha|W_e| - \frac{1}{r}(rN_e)(rN_p)\beta - \frac{1}{r}(rN_n)(rN_p)\beta - \frac{\partial(rN_pW_p)}{\partial r} \quad (243)$$

$$\frac{\partial(rN_n)}{\partial t} = (rN_e)\eta|W_e| - \frac{1}{r}(rN_p)(rN_n)\beta - \frac{\partial(rN_nW_n)}{\partial r} \quad (244)$$

The presence of the β/r and D/r terms appear to require a non zero starting radius (in our case r_0). The photoionization source is also scaled with radius. The extra electron diffusion term in the first of the three equations can be lumped in with the velocity term.

In this report we will examine the radial expansion to get a feel for the differences between air and SF6 but will not combine it with the axial ionization wave propagation. If we were to combine this radial expansion with the axial propagation we might collect radial runs to form a lookup table with different field strengths, densities, radii, time histories, etc. We may also be able to limit the axial region where the radial expansion calculation is necessary (Aleksandrov [13] stated that the radial expansion was terminated when the charge density behind the front dropped below $5 \times 10^{11} \text{ cm}^{-3}$).

We have compared radial expansion using electron seeding, with the level suggested by Aleksandrov [13] $N_e = 10^8 \text{ cm}^{-3}$, but have found that levels approaching 10^{11} cm^{-3} seemed to be necessary to obtain agreement with photoionization in this radial expansion problem. Although the photoionization calculation slows the calculation down (compared to seeding) we report these levels here (it is possible to run them in a reasonable amount of time in the radial case only).

We actually grid down to a radius of $100 \mu\text{m}$ but apply the seed electrons (or positive ions) in a single unknown shell ($\delta r = 2.5 \mu\text{m}$ thick) at $r_0 = 150 \mu\text{m}$. We use uniform gridding with $1000 - 4000$ unknowns over the radial region from r_0 to R , maintaining the radial unknown thicknesses at $\delta r = 2.5 \mu\text{m}$. The simulation curves are labeled with an initial charge density, which is the equivalent uniform density in a cylinder of charge with radius r_c , having the same total charge per unit length as was concentrated in the thin shell of the simulation (thus $2\pi r \delta r N_{shell} = N_j \pi r_c^2$). Anode directed means that $N_j = N_e$ and cathode directed means that $N_j = N_p$. We apply the flux corrected algorithm to integrate the radial equations.

Figures 62 and 63 show radial expansion examples in air for anode and cathode polarities. In the cathode case (with a shell of positive ionic charge) the solution region was seeded with free electrons with density $N_e = 10^8 \text{ cm}^{-3}$ to get the streamer going.

Figures 64, 65, 66, 67, 68, 69, 70, and 71 show examples of radial expansion in SF6. In this case the radius, electron density, negative ion, and positive ion density are shown for anode and cathode polarities.

Notice that the radial expansion is similar in SF6 but slightly behind the air expansion. In air it was noted that a fixed radius of $300 \mu\text{m}$ had similar axial propagation behavior to the expanded radius streamer model [13]. Thus from these radial comparisons we see that SF6 has similar radial channel extent to that of air.

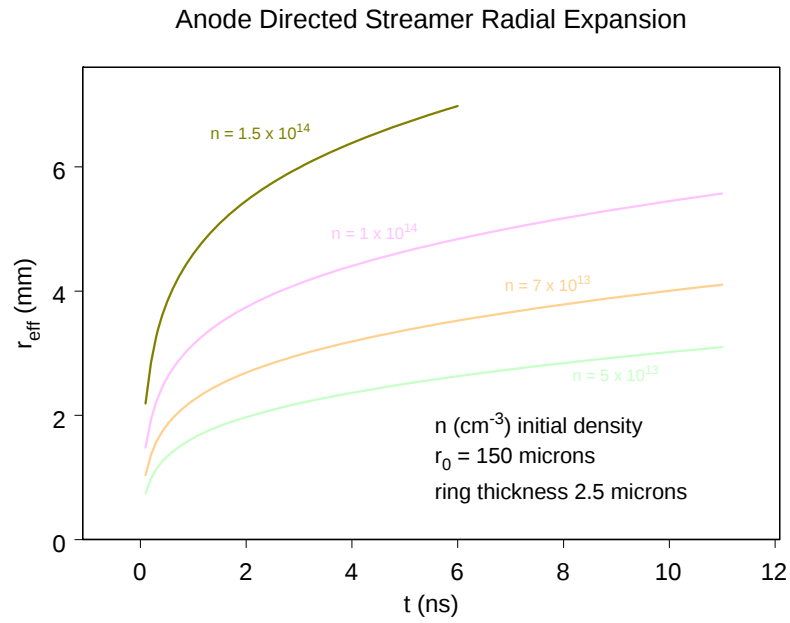


Figure 62. Radial expansion in air driven by various effective disc electron densities associated with the anode end of the streamer channel. Photoionization is present and the actual electron density is collapsed to a $2.5 \mu\text{m}$ shell at $r_0 = 150 \mu\text{m}$.

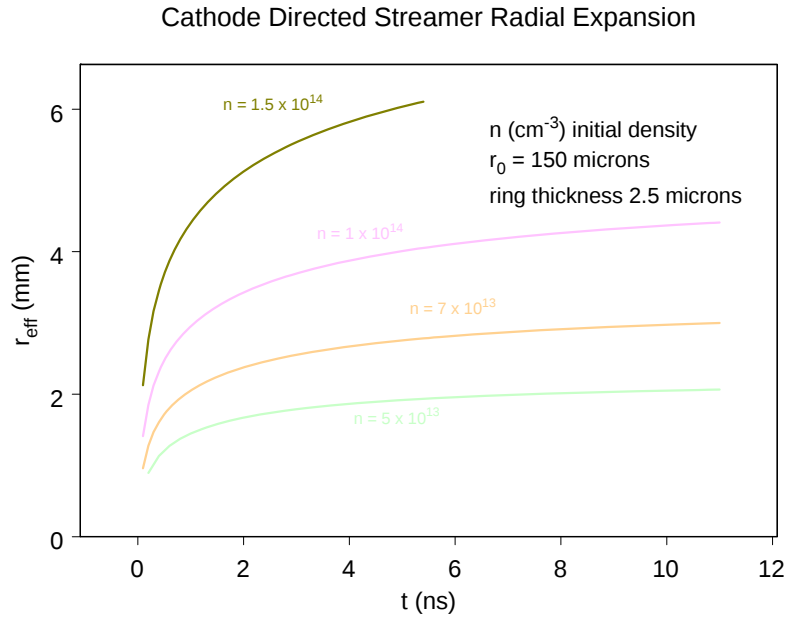


Figure 63. Radial expansion in air driven by various effective disc positive ion densities associated with the cathode end of the streamer channel. Photoionization is present and the actual positive ion density is collapsed to a $2.5 \mu\text{m}$ shell at $r_0 = 150 \mu\text{m}$. Seed electrons with density 10^8 cm^{-3} are present in the space to initiate the avalanche.

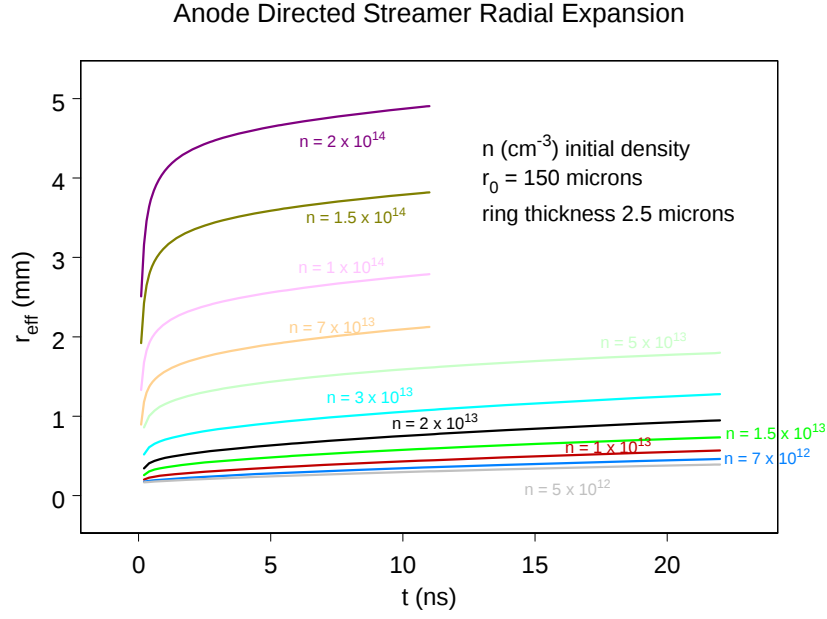


Figure 64. Radial expansion in SF6 driven by various effective disc electron densities associated with the anode end of the streamer channel. Photoionization is present and the actual electron density is collapsed to a $2.5 \mu\text{m}$ shell at $r_0 = 150 \mu\text{m}$.

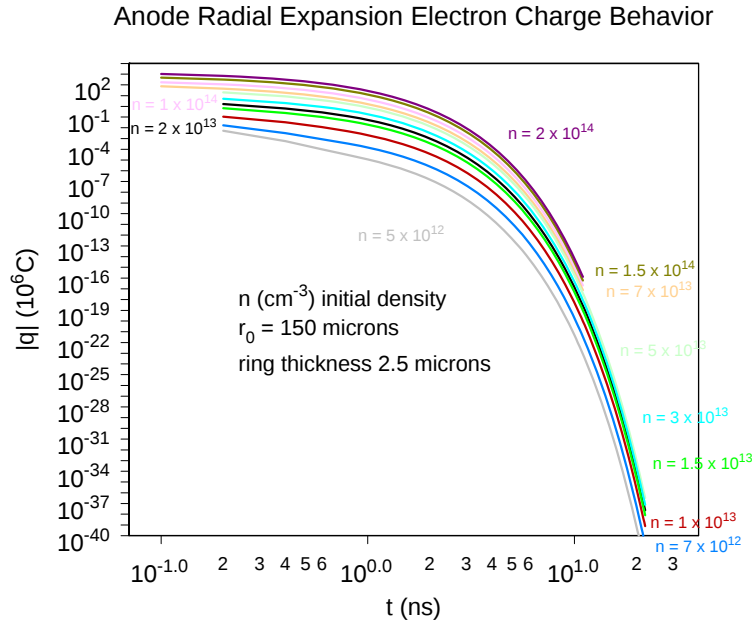


Figure 65. Electron charge magnitude behavior in SF6 during anode radial expansion.

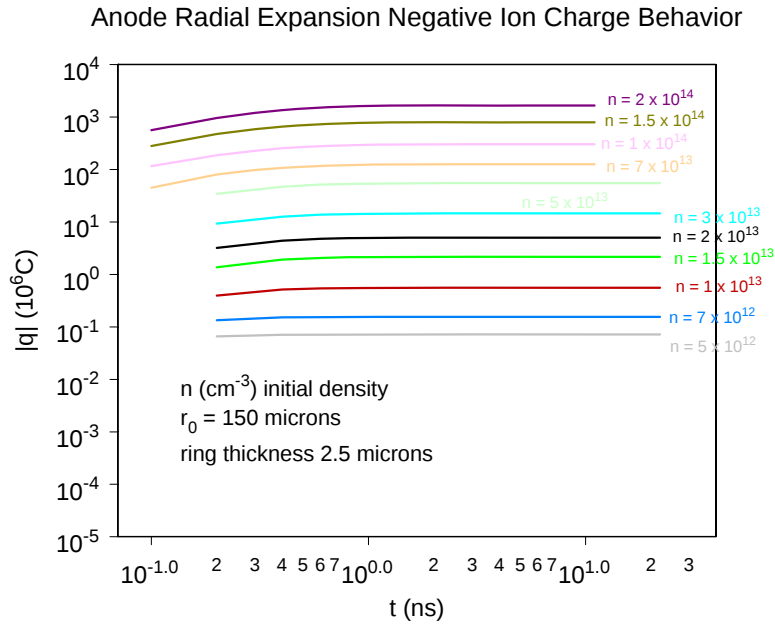


Figure 66. Negative ion charge magnitude behavior in SF6 during anode radial expansion.

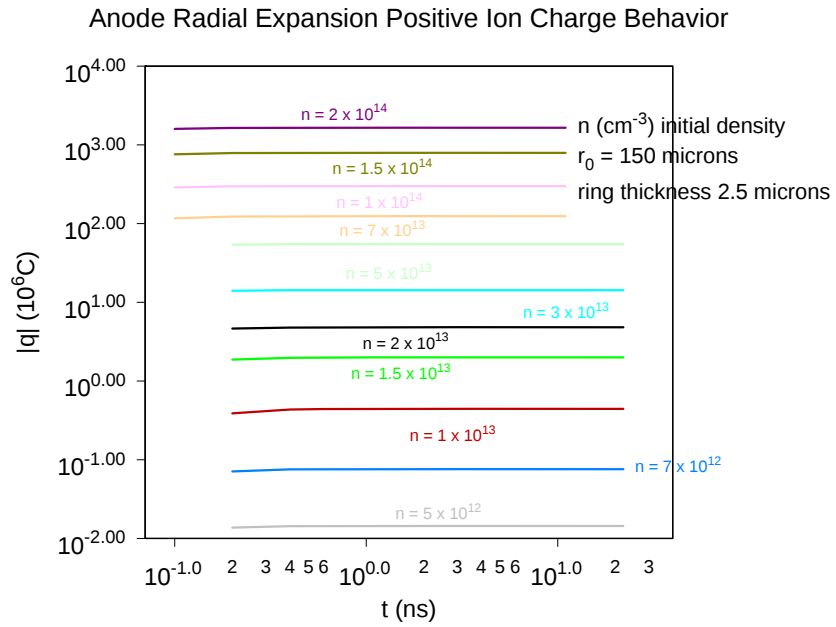


Figure 67. Positive ion charge behavior in SF6 during anode radial expansion.

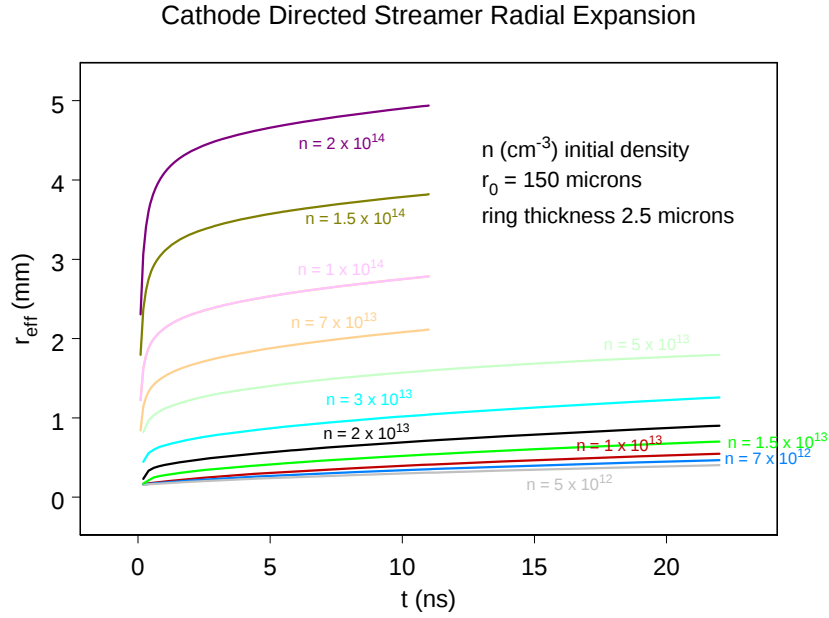


Figure 68. Radial expansion in SF6 driven by various effective disc positive ion densities associated with the cathode end of the streamer channel. Photoionization is present and the actual positive ion density is collapsed to a $2.5 \mu\text{m}$ shell at $r_0 = 150 \mu\text{m}$. Seed electrons with density 10^8 cm^{-3} are present in the space to initiate the avalanche.

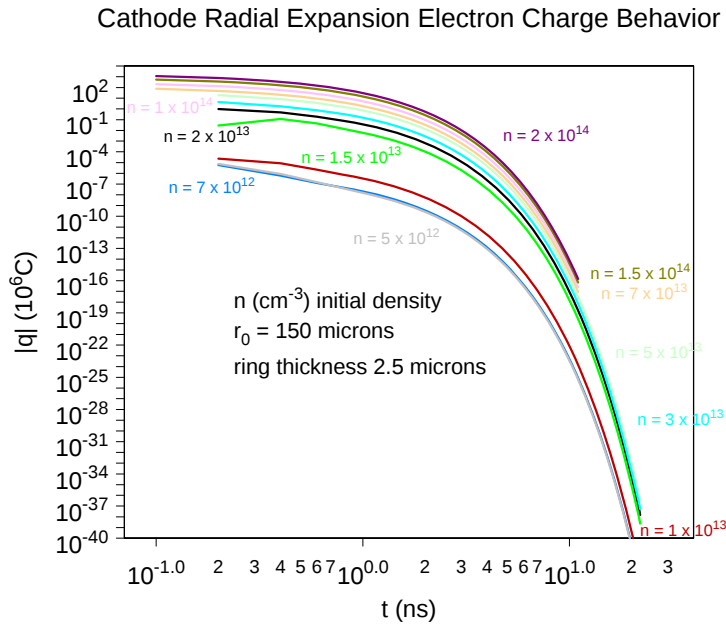


Figure 69. Electron charge magnitude behavior in SF6 during cathode radial expansion.

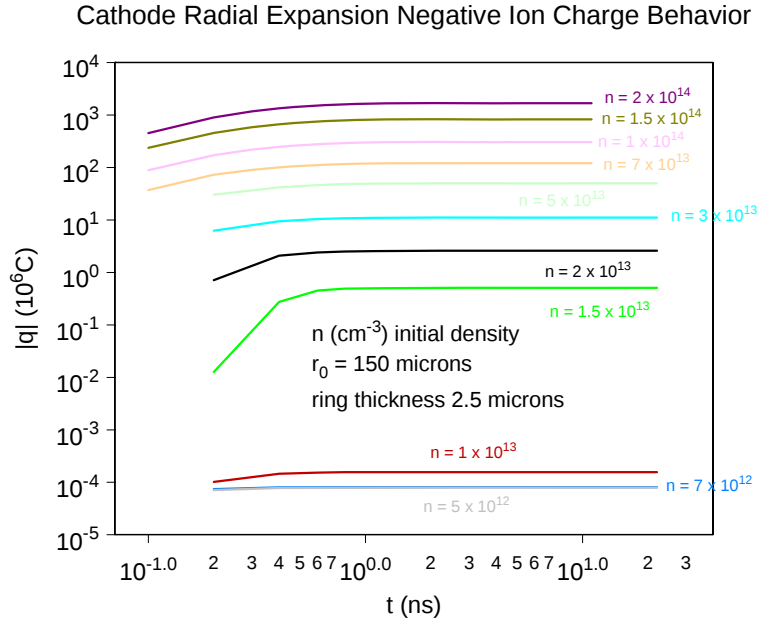


Figure 70. Negative ion charge magnitude behavior in SF6 during cathode radial expansion.

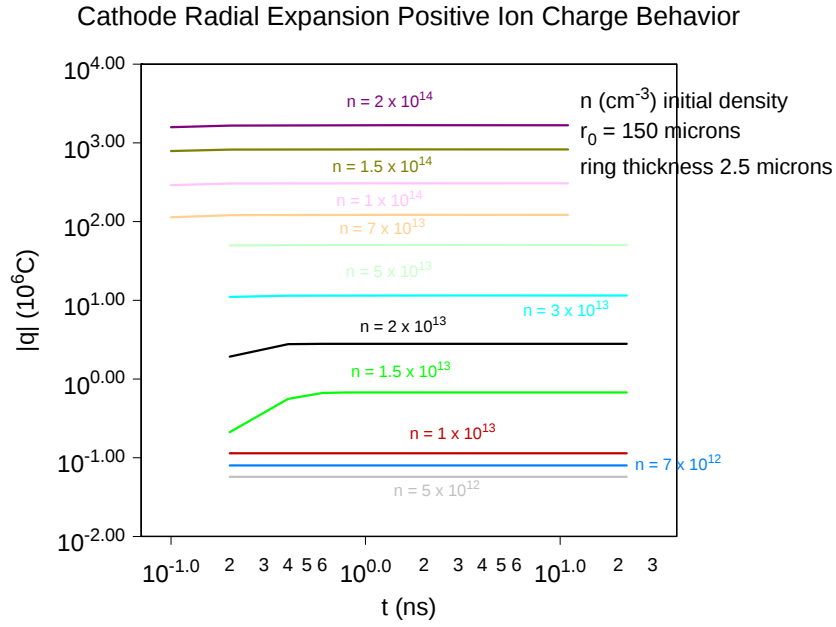


Figure 71. Positive ion charge behavior in SF6 during cathode radial expansion.

Monte Carlo Simulation of Streamer on Surface

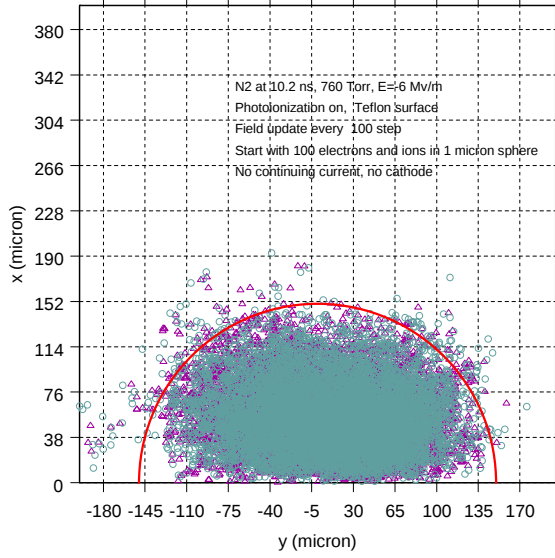


Figure 72. Monte Carlo simulation of an avalanche near a surface.

8 TWO DIMENSIONAL RADIAL EXPANSION

The 1.5D fluid code assumes, in a volumetric breakdown, that the cross-section of the streamer is a circular disk or circular ring. From symmetry considerations, not considering branching, this assumption is reasonable. In breakdown across a surface, the 1.5D code assumes the cross-section of the streamer is a half-circle. When considering the implementation of Aleksandrov's radial expansion algorithm for surface breakdown, a question arose if the surface influences the cross-section of the streamer enough to cause it to deviate from the half-circle assumption.

Monte Carlo simulations of the avalanche in early time show a cross-section that is close to that of a half circle. Figure 72 shows the distribution of electrons (cyan circles) and positive ions (magenta triangles) at 10.2 ns. The view is along the axis of the avalanche. The red line in the figure is a circle of radius 150 μm for comparison. The distribution deviates slightly from the half-circle in the vicinity of the surface near the outer boundary of the avalanche.

8.1 The Two Dimensional Fluid Code

In order to understand how the cross-section of the streamer evolves, we constructed a code named **fluid** based on the two-dimensional, Flux Corrected Transport (FCT) algorithm in a Cartesian coordinate system. The code generally follows the work by Dhali, but in a Cartesian rather than a cylindrical coordinate system [63]. We applied Kunhardt's improvements to the FCT algorithm [64] in each of the two dimensions ($\hat{x}, \hat{y}$) and the parts of the FCT algorithm specific to two dimensions follow Zalesak [55].

We solve the continuity equations from the preceding one-dimensional section, generalized to 2 dimensions

$$\frac{\partial N_e}{\partial t} = S + N_e \alpha |W_e| - N_e \eta |W_e| - N_e N_p \beta_{ep} - \nabla \cdot (\underline{W}_e N_e) + D_e \nabla^2 N_e \quad (245)$$

$$\frac{\partial N_p}{\partial t} = S + N_e \alpha |W_e| - N_e N_p \beta_{ep} - N_n N_p \beta_{np} - \nabla \cdot (\underline{W}_p N_p) \quad (246)$$

$$\frac{\partial N_n}{\partial t} = N_e \eta |W_e| - N_n N_p \beta_{np} - \nabla \cdot (\underline{W}_n N_n) \quad (247)$$

where N_e , N_p and N_n are the number densities of the electrons, positive ions and negative ions respectively, $\underline{W}_e = \hat{x}W_{ex} + \hat{y}W_{ey}$ is the velocity of the electrons, and $\underline{W}_p$ and $\underline{W}_n$ are the velocities of the positive and negative ions respectively, $|W_e|$ is the magnitude of the electron velocity, α is the Townsend first ionization coefficient, η is the attachment coefficient, β_{ep} is the recombination coefficient of electrons and positive ions, β_{pn} is the recombination coefficient between positive and negative ions, and D_e is the diffusion coefficient. Coefficients such as α and the velocity are dependent on the electric field $\underline{E}$, which is obtained from the electric potential Φ using the equation

$$\underline{E} = -\nabla \Phi \quad (248)$$

The electric potential is obtained by solving the Poisson equation

$$\nabla^2 \Phi = -\frac{q_e}{\varepsilon_0} (N_p - N_e - N_n) \quad (249)$$

where q_e is the magnitude of the electric charge and ε_0 is the permittivity of free-space.

8.1.1 Convection algorithm

Note that for each time step, we first move all three types of particles across the cell boundaries by convection; apply diffusion only to the electrons; use the rates to change the particle density in each cell and finally update the field in each cell. In this sub-section we will describe the convection step, which is based on the FCT algorithm as improved by Kunhardt [64]. To explain the convection step we will apply it to a general charge density N , which represents all three of the particle types,

$$\frac{\partial N}{\partial t} = -\nabla \cdot (\underline{W} N) \quad (250)$$

This algorithm is implemented in the subroutine **update_density_convection**.

Low-order step The first step of an FCT algorithm is to calculate an updated density based on a low-order scheme for computing the spatial derivative. Finite-differencing Equation 250 in time we obtain the density at the next time step ($t + \Delta t$) based on quantities known at the present time step. For convenience we will index the time step with the letter m

$$N^{m+1} = N^m - \Delta t \frac{\partial}{\partial y} (W_y^m N^m) - \Delta t \frac{\partial}{\partial x} (W_x^m N^m) \quad (251)$$

Referring to Figure 73, we next finite-difference the spatial derivatives using a low-order method and obtain for density in cell (i,j)

$$N_L^{m+1}(i,j) = N^m(i,j) - \frac{1}{V_{i,j}} \left[f_{i+1/2,j}^m - f_{i-1/2,j}^m + g_{i,j+1/2}^m - g_{i,j-1/2}^m \right] \quad (252)$$

where the subscript L on the left-hand side denotes that the density update was obtained by a low-order method. $V_{i,j} = \Delta y_i \Delta x_j$ is the volume of cell (i,j) and g and f are the flux of particles crossing the boundaries of cell (i,j) and are given by

$$f_{i+1/2,j}^m = \begin{cases} W_y^m(i+1/2,j) N^m(i,j) \Delta x_j \Delta t & \text{if } W_y^m(i+1/2,j) \geq 0 \\ W_y^m(i+1/2,j) N^m(i+1,j) \Delta x_j \Delta t & \text{if } W_y^m(i+1/2,j) < 0 \end{cases} \quad (253)$$

$$g_{i,j+1/2}^m = \begin{cases} W_x^m(i, j+1/2)N^m(i, j)\Delta y_i\Delta t & \text{if } W_x^m(i, j+1/2) \geq 0 \\ W_x^m(i, j+1/2)N^m(i, j+1)\Delta y_i\Delta t & \text{if } W_x^m(i, j+1/2) < 0 \end{cases} \quad (254)$$

The velocity, which is known in the center of each cell, is interpolated to the border between two cells. In the original FCT algorithm this was done using a simple average, but we used a linear interpolation of the natural logarithm of the velocity to obtain a more accurate representation of the flux [64]. At this point we will specialize the discussion to interpolating W_x . The W_y interpolation is done the same way using the same subroutines. With this in mind, let us designate W_j to be the velocity at the center of cell (i, j) and Δx to be the width of cell (i, j) . We will not write the x subscript and since the i index does not change we will not write the i index either. We scale by the constant w_0 so that the logarithmic interpolation scheme works for any velocity range:

$$w_j = \frac{W_j}{w_0} \quad (255)$$

If $w_j \geq 0$ and $w_{j+1} \geq 0$ then let

$$U(x) = \ln[1 + w(x)] \quad (256)$$

Expand $U(x)$ around x_j in two terms of a Taylor series

$$U(x) = U(x_j) + (x - x_j) \frac{dU(x_j)}{dx} \quad (257)$$

let $x' = (x - x_j)$ then

$$U(x' + x_j) = U(x_j) + x' \frac{dU(x_j)}{dx} \quad (258)$$

substituting for $U(x)$ and finite-differencing the derivative in x we obtain

$$\ln[1 + w(x' + x_j)] = \ln[1 + w_j] + x' \left\{ \frac{\ln[1 + w_{j+1}] - \ln[1 + w_j]}{\Delta x} \right\} \quad (259)$$

Therefore,

$$\begin{aligned} \ln[1 + w(x' + x_j)] &= \ln[1 + w_j] + \ln[e^{b_j x'}] \\ &= \ln\{[1 + w_j] e^{b_j x'}\} \end{aligned} \quad (260)$$

where

$$b_j = \frac{\ln[1 + w_{j+1}] - \ln[1 + w_j]}{\Delta x} \quad (261)$$

Re-arranging

$$w(x' + x_j) = [1 + w_j] e^{b_j x'} - 1 \quad (262)$$

If $w_j < 0$ and $w_{j+1} \geq 0$ then we add a constant $2|w_j|$ to the argument of the natural logarithm to make it always greater than 0, i.e.,

$$U(x) = \ln[1 + 2|w_j| + w(x)] \quad (263)$$

then

$$w(x' + x_j) = [1 + |w_j|] e^{b_j x'} - 2|w_j| - 1 \quad (264)$$

where

$$b_j = \frac{\ln[1 + 2|w_j| + w_{j+1}] - \ln[1 + |w_j|]}{\Delta x} \quad (265)$$

If $w_j \geq 0$ and $w_{j+1} < 0$ then we add a constant $2|w_{j+1}|$ to the argument of the natural logarithm to

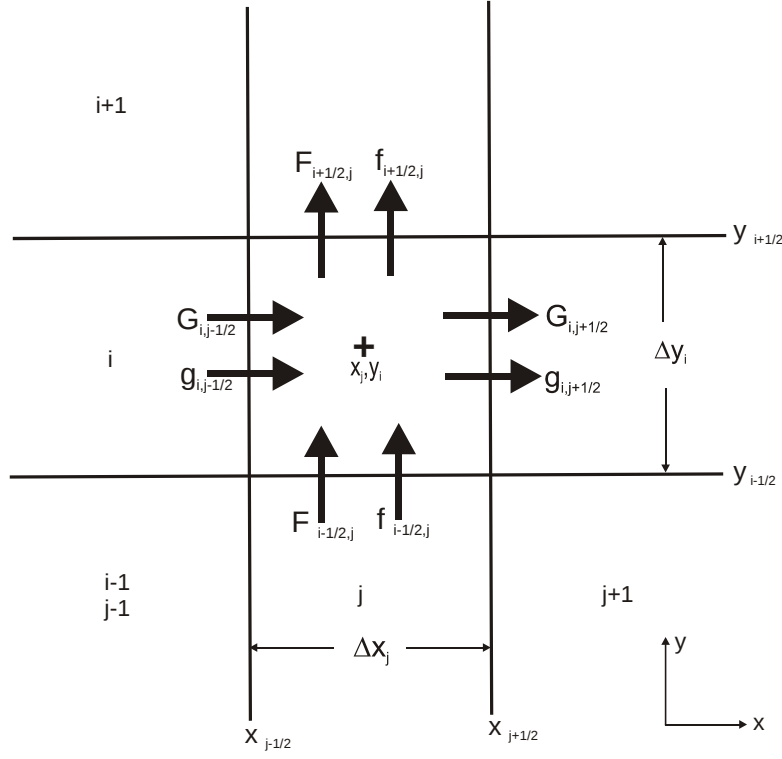


Figure 73. Grid nomenclature.

make it greater than 0, i.e.,

$$U(x) = \ln[1 + 2|w_{j+1}| + w(x)] \quad (266)$$

then

$$w(x' + x_j) = [1 + 2|w_{j+1}| + w_j] e^{b_j x'} - 2|w_{j+1}| - 1 \quad (267)$$

where

$$b_j = \frac{\ln[1 + |w_{j+1}|] - \ln[1 + 2|w_{j+1}| + w_j]}{\Delta x} \quad (268)$$

If $w_j < 0$ and $w_{j+1} < 0$ then

$$U(x) = \ln[1 - w(x)] \quad (269)$$

then

$$w(x' + x_j) = -\left\{[1 + |w_j|] e^{b_j x'} - 1\right\} \quad (270)$$

where

$$b_j = \frac{\ln[1 + |w_{j+1}|] - \ln[1 + |w_j|]}{\Delta x} \quad (271)$$

The above calculations are found in the subroutine **interpolate_velocity**.

High-order step The next step of an FCT algorithm is to update the density in cell (i, j) using a high-order method. This is done in two steps. The first task of step one is to calculate an initial intermediate density $N_I^{m+1}(i, j)$:

$$N_I^{m+1}(i, j) = N^m(i, j) - \frac{1}{2V_{i,j}} \left[F_{i+1/2,j}^m - F_{i-1/2,j}^m + G_{i,j+1/2}^m - G_{i,j-1/2}^m \right] \quad (272)$$

where F and G are a high-order calculation of the particles crossing the boundary of cell (i, j) .

As with the low-order calculation we will focus on the G calculation in x , knowing that F is calculated identically. Instead of using more of the surrounding cells to get a high-order flux calculation [63], Kunhardt uses an accurate calculation of the flux crossing the boundary using just the cells adjacent to the boundary. In order to obtain $G_{i,j+1/2}$ we fit the density to an exponentially varying function in order to account for the rapid spatial variation of the streamer front. We scale N by a constant n_0 so that the logarithmic interpolation scheme works for any density range:

$$n(i, j) = \frac{N(i, j)}{n_0} \quad (273)$$

Like we did above with the velocity we expand the function

$$V(x) = \ln[1 + n(x)] \quad (274)$$

Note that $n(x)$ is always greater than zero so we do not have to consider the different cases like we had to for velocity. We obtain

$$n(x' + x_j) = [1 + n_j^c] e^{a_j x'} - 1 \quad (275)$$

where n_j^c is the density at the center of the cell and

$$a_j = \frac{\ln[1 + n_{j+1}] - \ln[1 + n_j]}{\Delta x} \quad (276)$$

where n_j is the average cell density calculated at time step. This calculation is performed in the subroutine **interpolate_density**.

For the high-order calculation (but not for the low-order) Kunhardt makes a distinction between the average density (n_j) in a cell and the density in the center of the cell (n_j^c). If the density does not vary significantly from cell to cell, these two values are the approximately same, but if a_j is large, we have to find the center density from the average density. It is the average density that is calculated in the low-order step. We can find the center density by

$$\begin{aligned} \Delta x [1 + n_j] &= \int_{-\Delta x/2}^{\Delta x/2} (1 + n_j^c) e^{a_j x'} dx' \\ &= (1 + n_j^c) \frac{2}{a_j} \sinh\left(\frac{a_j \Delta x}{2}\right) \end{aligned} \quad (277)$$

therefore

$$n_j^c = \begin{cases} \frac{a_j \Delta x [1 + n_j]}{2 \sinh\left(\frac{a_j \Delta x}{2}\right)} - 1 & \text{if } a_j \neq 0 \\ n_j & \text{if } a_j = 0 \end{cases} \quad (278)$$

Likewise, we assume that the velocity from the low-order calculation is the average velocity across the cell and using the same procedure calculate the velocity at the center of the cell for the high-order calculation

$$w_j^c = \begin{cases} \frac{b_j \Delta x [1 + w_j]}{2 \sinh\left(\frac{b_j \Delta x}{2}\right)} - 1 & \text{if } b_j \neq 0 \\ w_j & \text{if } b_j = 0 \end{cases} \quad (279)$$

For each Δt time step, mass will cross the boundary between the j and $j + 1$ cell. Since we know the distribution of velocity over the cell we can compute what portion of cell j crosses the boundary. Referring to Figure 74 this is the portion of the cells between x_t and $\Delta x/2$. Our next step is to calculate x_t by solving the following equation

$$w(x_t + x_j) \Delta t = \frac{\Delta x}{2} - x_t \quad (280)$$

Note that x_t is dependent on the sign of w_j^c and w_{j+1}^c . Assuming that $w_j \geq 0$ and $w_{j+1} \geq 0$ and substituting

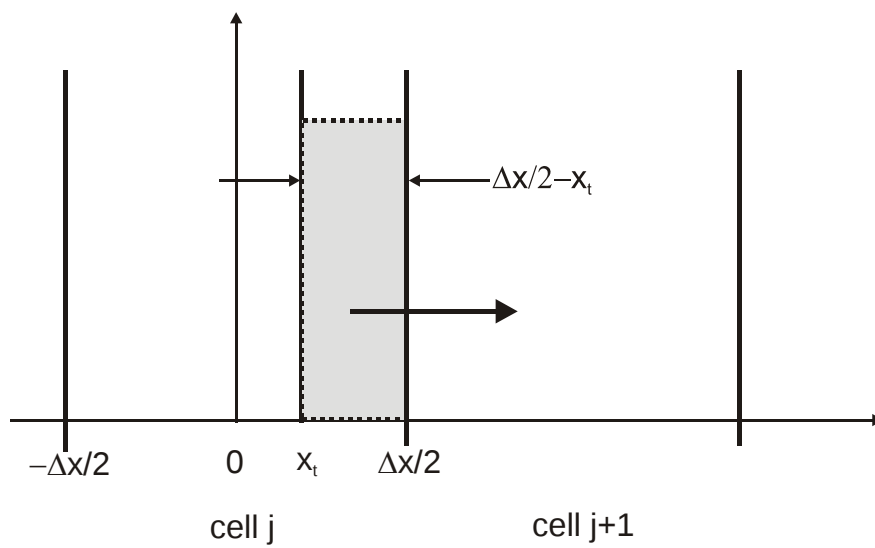


Figure 74. Mass transfer across cell boundary.

for $w(x_t + x_j)$ we obtain

$$\{[1 + w_j^c] e^{b_j x_t} - 1\} \Delta t = \frac{\Delta x}{2} - x_t \quad (281)$$

re-arranging we obtain the non-linear equation

$$[1 + w_j^c] \Delta t e^{b_j x_t} + x_t - \Delta t - \frac{\Delta x}{2} = 0 \quad (282)$$

Substitute a small argument approximation for the exponential

$$e^{b_j x_t} \sim 1 + b_j x_t \quad (283)$$

to obtain

$$[1 + w_j^c] \Delta t [1 + b_j x_t] + x_t = \Delta t + \frac{\Delta x}{2} \quad (284)$$

or

$$x_t = \frac{\frac{\Delta x}{2} - w_j^c \Delta t}{1 + [1 + w_j^c] b_j \Delta t} \quad (285)$$

The other cases are derived similarly using expression for the velocity from Section 1 so we will just give the results. If $w_j < 0$ and $w_{j+1} \geq 0$ then

$$x_t = \frac{\frac{\Delta x}{2} + |w_j^c| \Delta t}{1 + [1 + |w_j^c|] b_j \Delta t} \quad (286)$$

If $w_j^c \geq 0$ and $w_{j+1}^c < 0$ then

$$x_t = \frac{\frac{\Delta x}{2} - w_j^c \Delta t}{1 + [1 + 2|w_{j+1}^c| + w_j^c] b_j \Delta t} \quad (287)$$

If $w_j^c < 0$ and $w_{j+1}^c < 0$ then

$$x_t = \frac{\frac{\Delta x}{2} + |w_j^c| \Delta t}{1 - [1 + |w_j^c|] b_j \Delta t} \quad (288)$$

Once we have the value of x_t we integrate the expression for mass from x_t to $\Delta x/2$ to obtain an expression for the mass crossing $j + 1/2$ boundary

$$G_{i,j+1/2} = \Delta y \int_{x_t}^{\Delta x/2} [(1 + n_j^c) e^{a_j x'} - 1] dx' \quad (289)$$

or

$$G_{i,j+1/2} = \begin{cases} \left\{ \frac{(1+n_j^c)}{a_j} (e^{a_j \Delta x/2} - e^{a_j x_t}) - \frac{\Delta x}{2} + x_t \right\} \Delta y & \text{if } a_j \neq 0 \\ n_j^c \left(\frac{\Delta x}{2} - x_t \right) \Delta y & \text{if } a_j = 0 \end{cases} \quad (290)$$

The high-order flux is calculated in the subroutine **get_high_order_flux**.

After obtaining the initial intermediate density using Equation 272 and normalizing by n_0 to obtain $n_I^{m+1}(i, j)$ in each cell, we average with the density at the center of each cell. In obtaining the center density we have to take into account both dimensions. Since this is different from Kunhardt's one-dimensional analysis, details are given below

$$n_x^c(i, j) = \begin{cases} \frac{a_j \Delta x [1 + n(i, j)]}{2 \sinh\left(\frac{a_j \Delta x}{2}\right)} - 1 & \text{if } a_j \neq 0 \\ n(i, j) & \text{if } a_j = 0 \end{cases} \quad (291)$$

where

$$a_j = \frac{\ln[1 + n(i, j + 1)] - \ln[1 + n(i, j)]}{\Delta x} \quad (292)$$

and

$$n_y^c(i, j) = \begin{cases} \frac{a_i \Delta y [1 + n(i, j)]}{2 \sinh(\frac{a_i \Delta y}{2})} - 1 & \text{if } a_i \neq 0 \\ n(i, j) & \text{if } a_i = 0 \end{cases} \quad (293)$$

where

$$a_i = \frac{\ln[1 + n(i + 1, j)] - \ln[1 + n(i, j)]}{\Delta y} \quad (294)$$

then we average

$$n^c(i, j) = \frac{n_x^c(i, j) + n_y^c(i, j)}{2} \quad (295)$$

the final intermediate density is given by

$$n_{int}(i, j) = \max \left[\frac{n_I(i, j) + n^c(i, j)}{2}, 1 \right] \quad (296)$$

We used the value of 1 rather than the standard 0 for the floor value of n_{int} for convenience. We plot the results on a log scale and zeros would cause the graphing software to fail. The second step in the high-order scheme is to re-calculate F and G , but use the final intermediate density $n_{int}(i, j)$ and $w^c(i, j)$ in Equation 290. We designate the high-order flux as $F_H^m(i + 1/2, j)$ and $G_H^m(i, j + 1/2)$.

Flux limiting step At this point the important quantities that we have calculated are the low-order flux (f, g) , the high-order flux (F_H, G_H) , the density at the present time step $N^m(i, j)$ and the updated low-order density $N_L^{m+1}(i, j)$. From now on we will follow the standard 2D, flux-correction algorithm as given by Zalesak [55]. We calculate the anti-flux, given as

$$\begin{aligned} A_{i+1/2, j} &= F_H(i + 1/2, j) - f_{i+1/2, j} \\ A_{i, j+1/2} &= G_H(i, j + 1/2) - g_{i, j+1/2} \end{aligned} \quad (297)$$

We then modify the anti-flux in the following manner

$$\begin{aligned} A_{i+1/2, j} &= 0 \\ \text{if } A_{i+1/2, j} (N_L^{m+1}(i + 1, j) - N_L^{m+1}(i, j)) &< 0 \text{ and} \\ \text{either } A_{i+1/2, j} (N_L^{m+1}(i + 2, j) - N_L^{m+1}(i + 1, j)) &< 0 \text{ or } A_{i+1/2, j} (N_L^{m+1}(i, j) - N_L^{m+1}(i - 1, j)) < 0 \end{aligned} \quad (298)$$

$$\begin{aligned} A_{i, j+1/2} &= 0 \\ \text{if } A_{i, j+1/2} (N_L^{m+1}(i, j + 1) - N_L^{m+1}(i, j)) &< 0 \text{ and} \\ \text{either } A_{i, j+1/2} (N_L^{m+1}(i, j + 2) - N_L^{m+1}(i, j + 1)) &< 0 \text{ or } A_{i, j+1/2} (N_L^{m+1}(i, j) - N_L^{m+1}(i, j - 1)) < 0 \end{aligned} \quad (299)$$

We next limit the anti-flux. We first have to compute some intermediate quantities:

$$N_{i, j}^a = \max [N^m(i, j), N_L^{m+1}(i, j)] \quad (300)$$

$$N_{i, j}^{\max} = \max [N_{i-1, j}^a, N_{i, j}^a, N_{i+, j}^a, N_{i, j-1}^a, N_{i, j+1}^a] \quad (301)$$

$$N_{i, j}^b = \min [N^m(i, j), N_L^{m+1}(i, j)] \quad (302)$$

$$N_{i, j}^{\min} = \min [N_{i-1, j}^b, N_{i, j}^b, N_{i+, j}^b, N_{i, j-1}^b, N_{i, j+1}^b] \quad (303)$$

$$P_{i,j}^+ = \max [0, A_{i-1/2,j}] - \min [0, A_{i+1/2,j}] + \max [0, A_{i,j-1/2}] - \min [0, A_{i,j+1/2}] \quad (304)$$

$$Q_{i,j}^+ = [N_{i,j}^{\max} - N_L^{m+1}(i,j)] \Delta V_{i,j} \quad (305)$$

$$R_{i,j}^+ = \begin{cases} \min \left[1, \frac{Q_{i,j}^+}{P_{i,j}^+} \right] & \text{if } P_{i,j}^+ > 0 \\ 0 & \text{if } P_{i,j}^+ = 0 \end{cases} \quad (306)$$

$$P_{i,j}^- = \max [0, A_{i+1/2,j}] - \min [0, A_{i-1/2,j}] + \max [0, A_{i,j+1/2}] - \min [0, A_{i,j-1/2}] \quad (307)$$

$$Q_{i,j}^- = [N_L^{m+1}(i,j) - N_{i,j}^{\min}] \Delta V_{i,j} \quad (308)$$

$$R_{i,j}^- = \begin{cases} \min \left[1, \frac{Q_{i,j}^-}{P_{i,j}^-} \right] & \text{if } P_{i,j}^- > 0 \\ 0 & \text{if } P_{i,j}^- = 0 \end{cases} \quad (309)$$

Then

$$C_{i+1/2,j} = \begin{cases} \min [R_{i+1,j}^+, R_{i,j}^-] & \text{if } A_{i+1/2,j} \geq 0 \\ \min [R_{i,j}^+, R_{i+1,j}^-] & \text{if } A_{i+1/2,j} < 0 \end{cases} \quad (310)$$

$$C_{i,j+1/2} = \begin{cases} \min [R_{i,j+1}^+, R_{i,j}^-] & \text{if } A_{i,j+1/2} \geq 0 \\ \min [R_{i,j}^+, R_{i,j+1}^-] & \text{if } A_{i,j+1/2} < 0 \end{cases} \quad (311)$$

and we can find the limited anti-flux.

$$A_{i+1/2,j}^C = C_{i+1/2,j} A_{i+1/2,j} \quad \text{where } 0 \leq C_{i+1/2,j} \leq 1 \quad (312)$$

$$A_{i,j+1/2}^C = C_{i,j+1/2} A_{i,j+1/2} \quad \text{where } 0 \leq C_{i,j+1/2} \leq 1$$

Finally, we update the low-order density with the limited anti-flux to obtain the final density.

$$N^{m+1}(i,j) = N_L^{m+1}(i,j) - \frac{1}{V_{i,j}} [A_{i+1/2,j}^C - A_{i-1/2,j}^C + A_{i,j+1/2}^C - A_{i,j-1/2}^C] \quad (313)$$

8.1.2 Diffusion algorithm

The diffusion portion of Equation 245 we calculate as a separate update of N_e after we apply the convection algorithm following the method of Dhali [63].

$$\frac{\partial N_e}{\partial t} = D_e \nabla^2 N_e \quad (314)$$

or finite-differencing in time and space:

$$\begin{aligned} N_e^{m+1}(i,j) &= N_e^m(i,j) + \Delta t D_e \frac{[N_e^m(i,j+1) - 2N_e^m(i,j) + N_e^m(i,j-1)]}{\Delta x^2} \\ &\quad + \Delta t D_e \frac{[N_e^m(i+1,j) - 2N_e^m(i,j) + N_e^m(i-1,j)]}{\Delta y^2} \end{aligned} \quad (315)$$

To be specific, the $N_e^m(i,j)$ on the right hand side of Equation 315 is the final density from the convection section ($N_e^{m+1}(i,j)$).

8.1.3 Source algorithm

After the diffusion algorithm is applied to the electrons we update all three types of density due to ionization. We used temporary arrays to hold the present density of electrons, positive and negative ions

so that this update maintains charge neutrality.

$$\frac{\partial N_e}{\partial t} = [\alpha |W_e| - \eta |W_e| - N_p \beta_{ep}] N_e + S \quad (316)$$

Dhali used a second-order scheme, but we will use a simpler first-order scheme. Finite-differencing in time we obtain

$$N_e^{m+1}(i, j) = N_e^m(i, j) + \Delta t \{K_e(i, j) N_e^m(i, j) + S_e(i, j)\} \quad (317)$$

where K_e is the coefficient

$$K_e(i, j) = (\alpha(i, j) - \eta(i, j)) |W_e(i, j)| - N_p^m(i, j) \beta_{ep}(i, j) \quad (318)$$

The quantity S_e is a place-holder for a source of electrons such as photoionization. At the present time we handle photoionization by means of background seeding with electrons so $S(i, j) = 0$.

Similarly, for positive ions we obtain

$$N_p^{m+1}(i, j) = N_p^m(i, j) + \Delta t \{K_p(i, j) N_p^m(i, j) + S_p(i, j)\} \quad (319)$$

where

$$K_p(i, j) = -\beta_{ep}(i, j) N_e^m(i, j) - \beta_{pn}(i, j) N_n^m(i, j) \quad (320)$$

$$S_p(i, j) = \alpha(i, j) |W_e(i, j)| N_e^m(i, j) \quad (321)$$

and for negative ions we obtain

$$N_n^{m+1}(i, j) = N_n^m(i, j) + \Delta t \{K_n(i, j) N_n^m(i, j) + S_n(i, j)\} \quad (322)$$

$$K_n(i, j) = -\beta_{pn}(i, j) N_p^m(i, j) \quad (323)$$

$$S_n(i, j) = \eta(i, j) |W_e(i, j)| N_e^m(i, j) \quad (324)$$

8.1.4 Electric Field

The electric field was solved using the SLATEC [65] routine SEPX4, which solves the following equation

$$A \frac{\partial^2 U(x, y)}{\partial x^2} + B \frac{\partial U(x, y)}{\partial x} + C U(x, y) + \frac{\partial^2 U(x, y)}{\partial y^2} = G(x, y) \quad (325)$$

where A, B , and C are constants and G is a known function of x and y . If $A = 1$, $B = C = 0$ and

$$G(x, y) = \frac{q_e}{\epsilon_0} (N_e(x, y) + N_n(x, y) - N_p(x, y)) \quad (326)$$

we can use SEPX4 to solve Equation 249. Figure 75 shows the results of applying SEPX4 (labeled Poisson) to a circular cylinder of charge, which can be solved analytically. A 20 mm by 20 mm space is gridded to 0.1 mm resolution. The cylinder has a radius of 0.55 mm. Within the cylinder the charge density is $N_e = N_n = 0.0$ and $N_p = 1.0 \times 10^{14}$ positive ions/cm³.

Once we have the electric potential throughout the grid, we can finite-difference Equation 248 to obtain the electric field.

$$E_x = \frac{\Phi(i, j+1) - \Phi(i, j-1)}{x(i, j+1) - x(i, j-1)} \quad (327)$$

$$E_y = \frac{\Phi(i+1, j) - \Phi(i-1, j)}{x(i+1, j) - x(i-1, j)} \quad (328)$$

Figure 76 shows a comparison between the E_x field obtained by finite-differencing the potential shown in Figure 75 and an analytical expression for E_x due to a circular cylinder of charge.

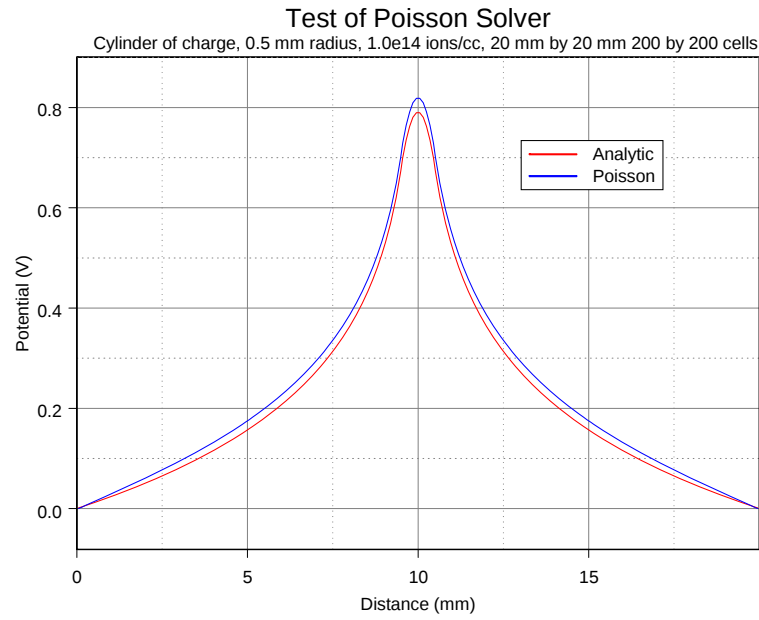


Figure 75. Test case using SEPX4 to calculate potential.

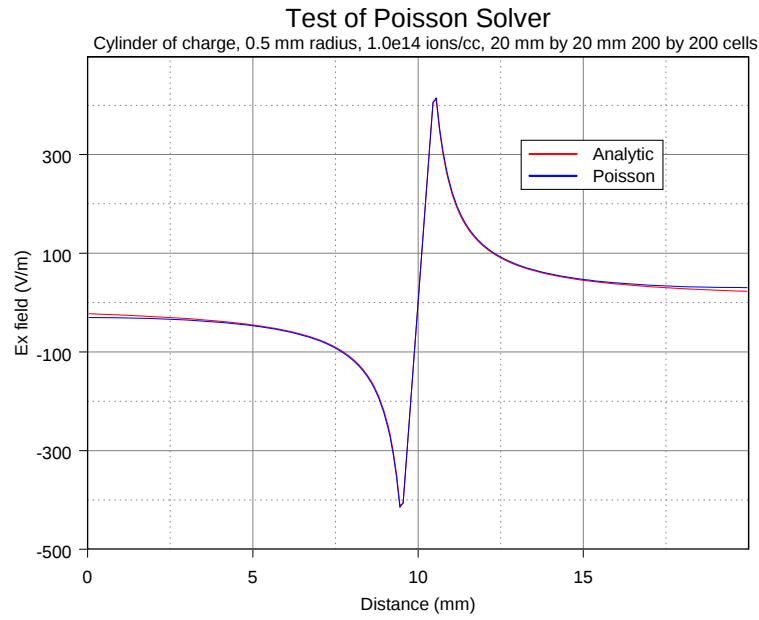


Figure 76. E_x as a function of x obtained by finite-difference.

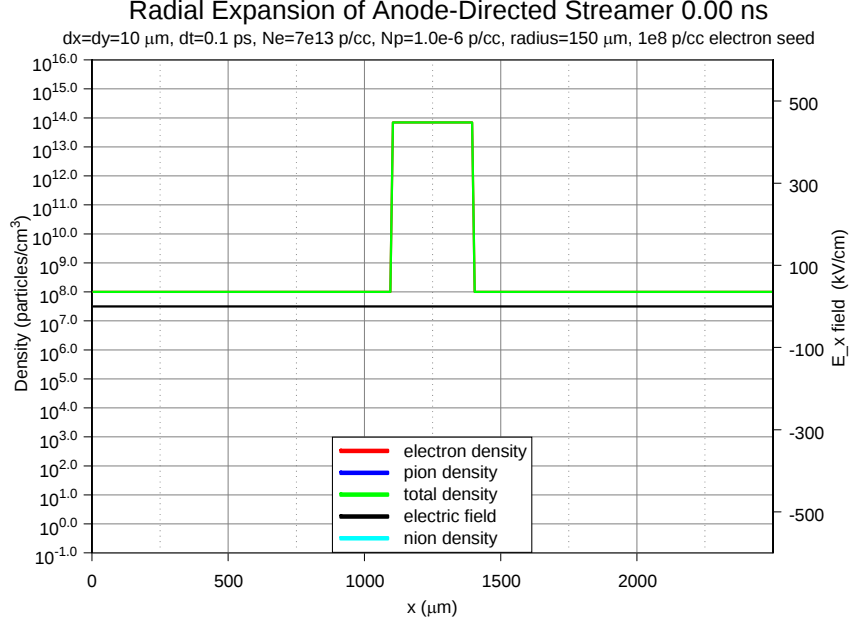


Figure 77. Initial conditions of the anode-directed streamer.

If the streamer propagates across a dielectric surface we calculate the field by combining SEPX4 and image theory. We double the size of the computation space in the y direction. In the top half of the computation space, we place the net charge

$$N_{net}(x, y) = \frac{q_e}{\varepsilon_0} (N_e(x, y) + N_n(x, y) - N_p(x, y)) \quad (329)$$

In the bottom half of the space we place an image charge that accounts for the dielectric

$$N_{image}(x, -y) = \frac{1 - \varepsilon_r}{1 + \varepsilon_r} \frac{q_e}{\varepsilon_0} [N_e(x, y) + N_n(x, y) - N_p(x, y)] \quad (330)$$

where ε_r is the relative permittivity of the dielectric. For example, Teflon has $\varepsilon_r = 2.1$.

8.1.5 Results

In this subsection are the results for some runs of the 2D code. The plan was to run the axial 1.5D code until a streamer was formed, then take the electron and positive ion densities to drive the 2D code. Eventually, we settled for just capturing the difference between the values of N_e and N_p that were calculated by the 1.5D code. For the anode-directed streamer the electrons have a higher density than the positive ions so we set $N_e = 7 \times 10^{13}$ electrons/cm³ and $N_p = 1.0 \times 10^{-6}$ ions/cm³. The time step was 0.1 ps. We chose a 2500 μm by 2500 μm computation space discretized into 10 μm by 10 μm cells. We did not include photoionization in the calculation, but instead accounted for it by seeding 1.0×10^8 electrons/cm³ throughout the entire computation space. Inside a circular cylinder of radius 150 μm , centered at the point $x = 1250 \mu\text{m}$, $y = 1250 \mu\text{m}$ we place an electron density of 7.0×10^{13} electrons/cm³ (higher than the background). The initial conditions are shown in Figure 77 for a cut across the center of the computation space at $y = 1250 \mu\text{m}$.

Figure 78 shows the density (left axis) and E_x as a function of x at the cut $y = 1250 \mu\text{m}$ at 0.01 ns. Figure 79 is a contour plot of the $\log_{10}$ of the net density ($N_p - N_e - N_n$) across the entire computation

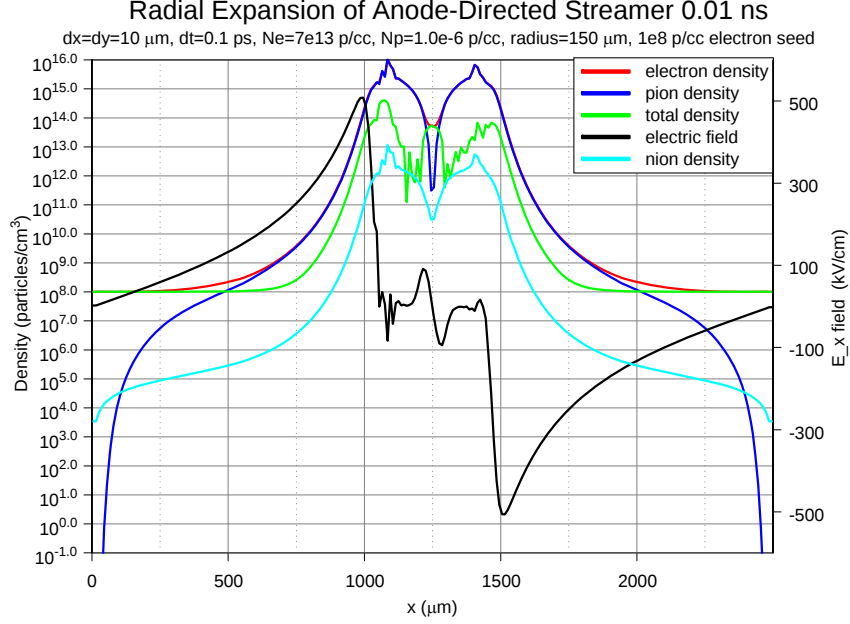


Figure 78. Anode-directed streamer at 0.01 ns.

space. The random contours in the center of the plot are noise as seen in the center portion of the green line of Figure 78. Note that the streamer is expanding symmetrically as it should, there is no bias being manifested due to the grid. Figures 80 and 81 show the same information but for a time of 0.05 ns. Most of the expansion happens in the first 0.05 ns, then the expansion slows considerably.

Plots of the total density and electric field of Figure 80 are repeated in Figure 82 to compare with results of the 1D radial code discussed in the preceding section. The 1D radial code results are given in Figure 83. Both codes calculate the electric field peak to be slightly less than 200 kV/cm. The E field peak of the 2D code is at 1850 μm , but remember that the center of the simulation is at 1250 μm so the peak has moved a distance of 600 μm from the center. The 1D code has its E field peak at 700 μm . The total density is somewhat different between the two. The 2D code calculates approximately 1.8×10^{13} particles/cm³ as the peak density, the 1D code calculates 1.2×10^{13} particles/cm³ as the peak density.

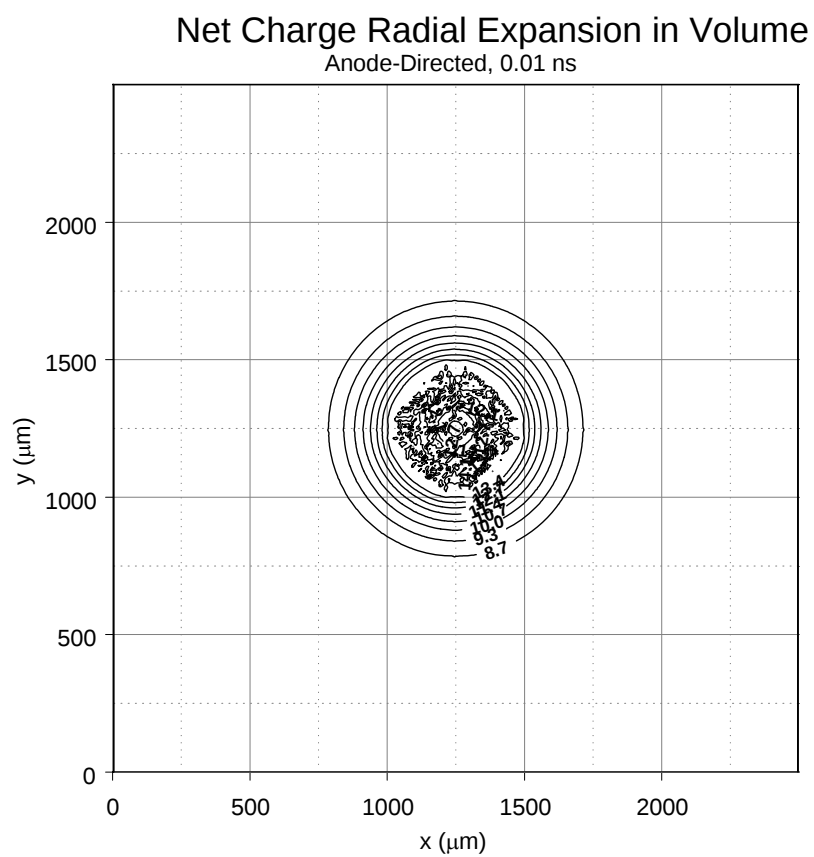


Figure 79. Contour plot of anode-directed streamer at 0.01 ns.

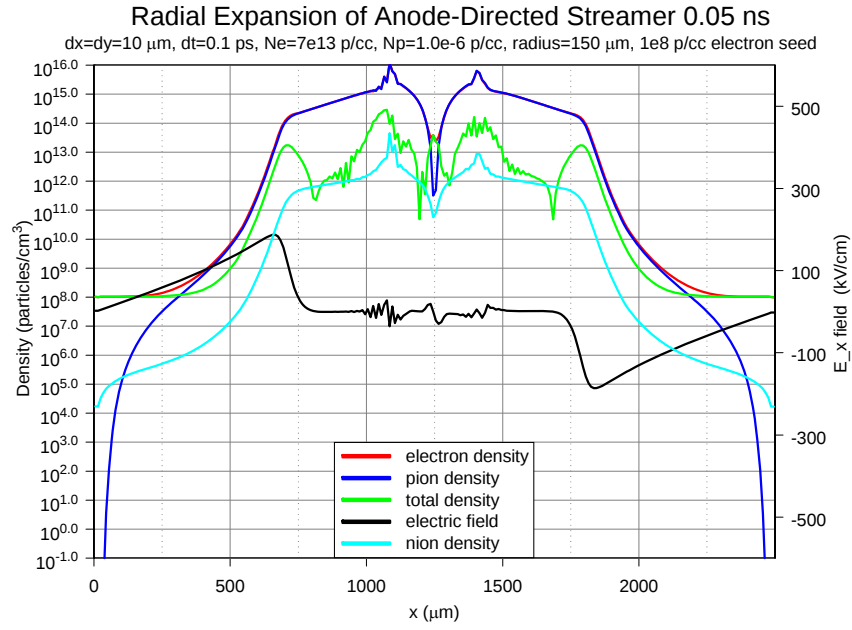


Figure 80. Anode-directed streamer at 0.05 ns.

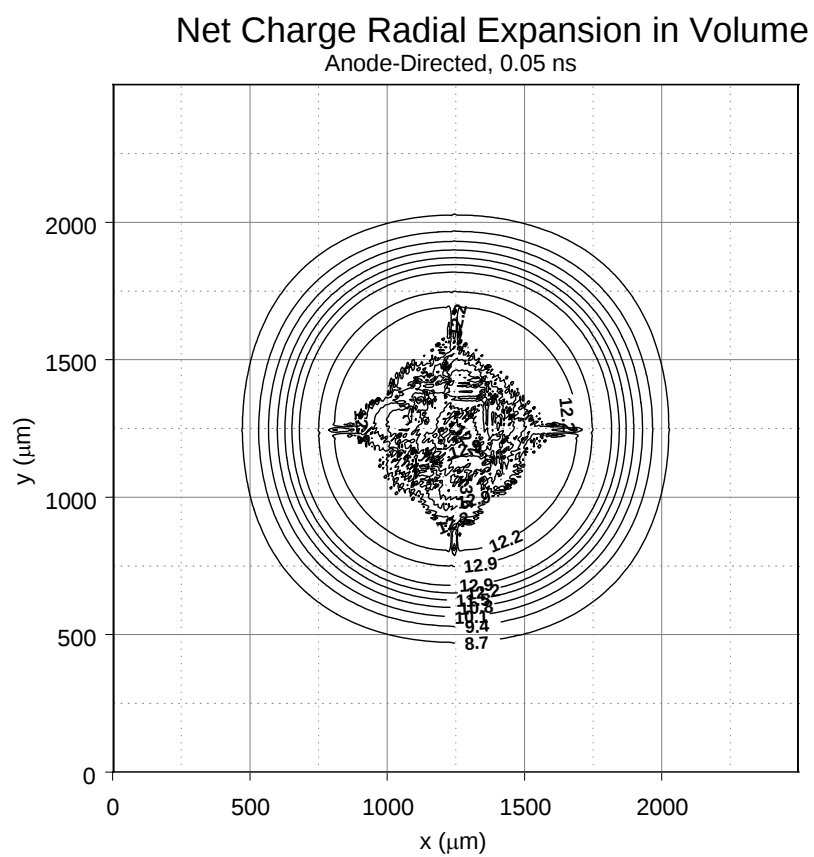


Figure 81. Contour plot of anode-directed streamer at 0.05 ns.

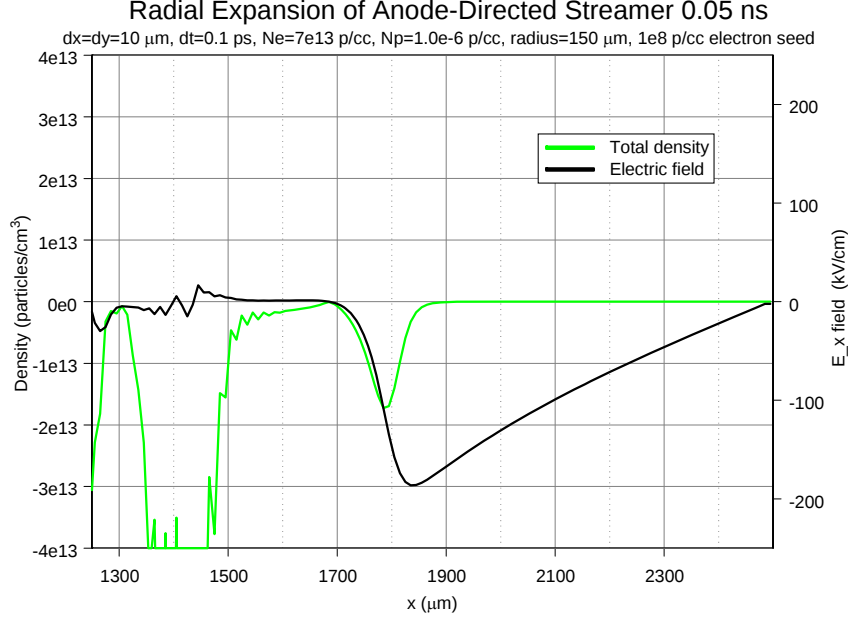


Figure 82. Anode-directed streamer at 0.05 ns calculated with 2D code.

Next we examine results for the cathode-directed streamer. Again we discretize a $2500 \mu\text{m}$ by $2500 \mu\text{m}$ space with 250 by 250 cells giving a cell size of $10 \mu\text{m}$ by $10 \mu\text{m}$. The time step is 0.1 ps. We again seed the entire space with a background of $N_e = 1 \times 10^8$ electrons/ cm^3 . Within a circular cylinder of radius $150 \mu\text{m}$ centered at $x = 1250 \mu\text{m}$, $y = 1250 \mu\text{m}$, we insert a positive ion density $N_p = 7 \times 10^{13}$ positive ions/ cm^3 . These initial conditions are shown in Figure 84 for a cut across the center of the cylinder along $y = 1250 \mu\text{m}$. Figure 85 shows the density (left axis) and E_x field (right axis) as a function of x at 0.01 ns, and Figure 86 shows the same information at 0.1 ns.

Figure 87 is a repeat of Figure 86, but plots just the total charge density (shown in green) and the electric field (in black). We compare these quantities to a calculation of radial expansion using the 1D code from the preceding section, which is shown in Figure 88. The E_x field peak is slightly below 150 kV/cm in both figures. Note that the center of the simulation in Figure 86 is at $1250 \mu\text{m}$, so at 0.1 ns, the peak of E_x has moved $1900 \mu\text{m} - 1250 \mu\text{m} = 650 \mu\text{m}$ from center. In Figure 88 the center is at 0.0 so the peak of E_x is $900 \mu\text{m}$ from the center. The peak of the total charge density is twice as high (1.4×10^{13} particles/ cm^3) in Figure 86 as in Figure 88 (0.7×10^{13} particles/ cm^3). It should be recalled that the 1D code begins with a shell of charge whereas the 2D code begins with a cylinder of charge (both having the same total charge per unit length). Electron seeding was used in the comparisons of this section for both codes.

8.2 Radial Expansion On Surface

Finally, just to show our progress thus far, Figure 89 shows a contour plot of how the net charge of a streamer expands next to a surface. Again the problem space is $2500 \mu\text{m}$ by $2500 \mu\text{m}$ with $10 \mu\text{m}$ by $10 \mu\text{m}$ cells. The time step is 0.1 ps. We seed the entire space with a background of $N_e = 1 \times 10^8$ electrons/ cm^3 . Within a half-circular cylinder of radius $150 \mu\text{m}$ centered at $x = 1250 \mu\text{m}$, $y = 0 \mu\text{m}$, we insert an electron density of $N_e = 1 \times 10^{13}$ electrons/ cm^3 and $N_p = 1 \times 10^{-6}$ positive ions/ cm^3 . The surface is assumed to be Teflon. The distribution gives an indication that a half circle remains a good approximation of the streamer cross-section even after the avalanche phase of breakdown. These results must be used with

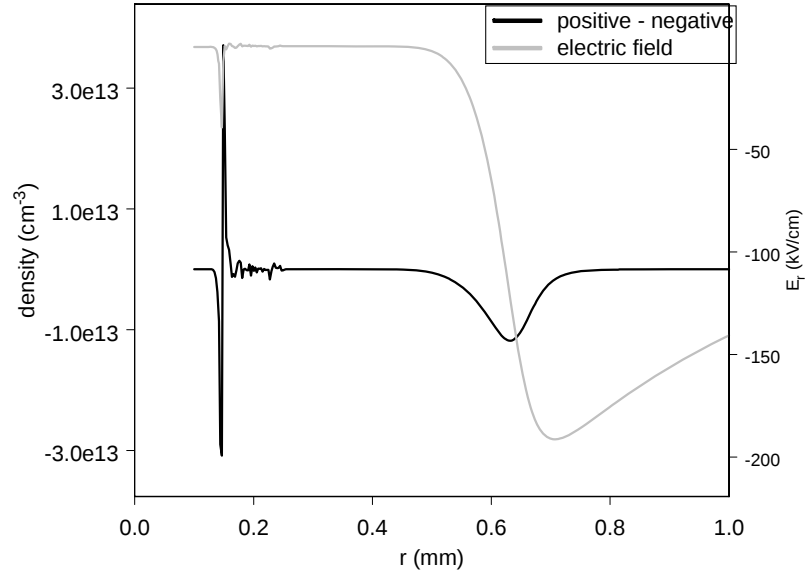


Figure 83. Radial expansion of anode-directed streamer at 0.05 ns calculated by 1D code.

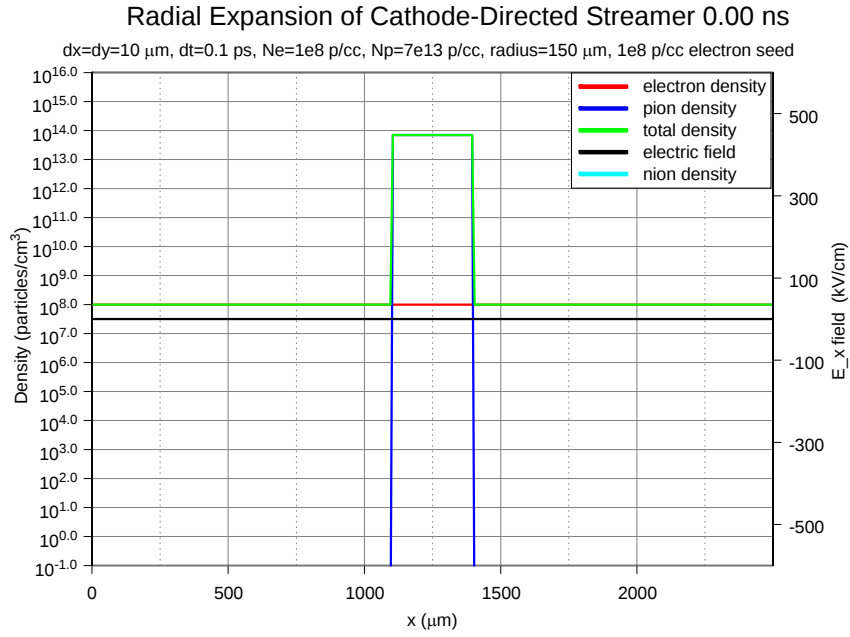


Figure 84. Initial condition of cathode-directed streamer.

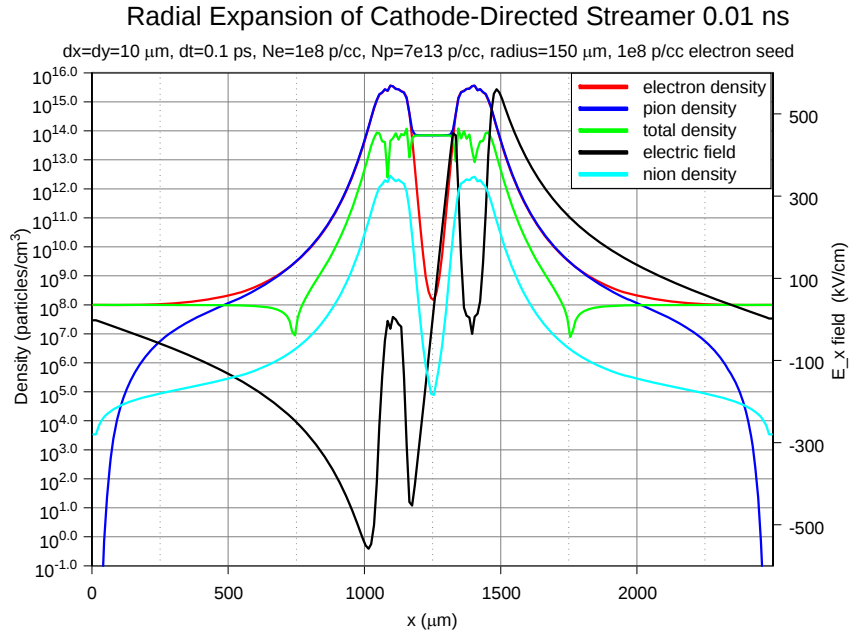


Figure 85. Cathode-directed streamer at 0.01 ns.

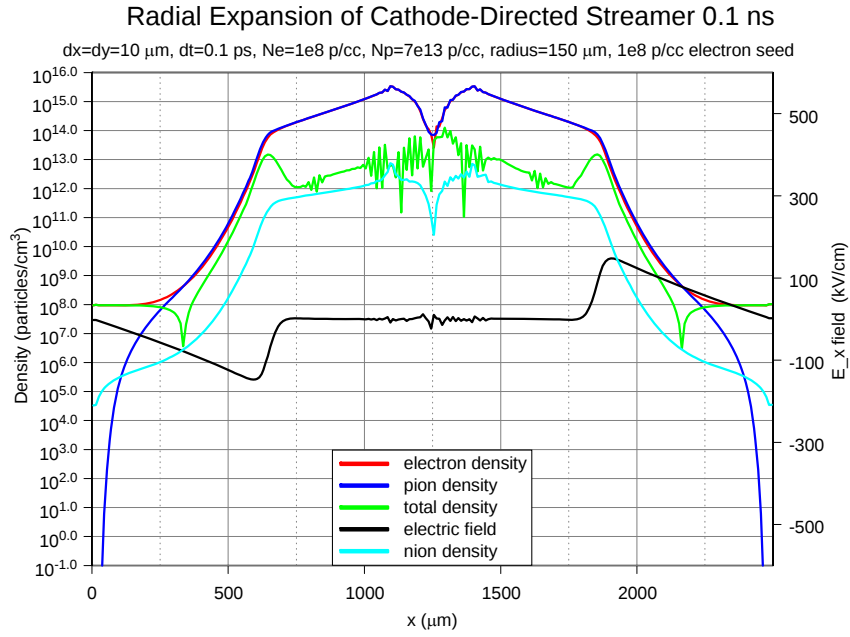


Figure 86. Cathode-directed streamer at 0.1 ns.

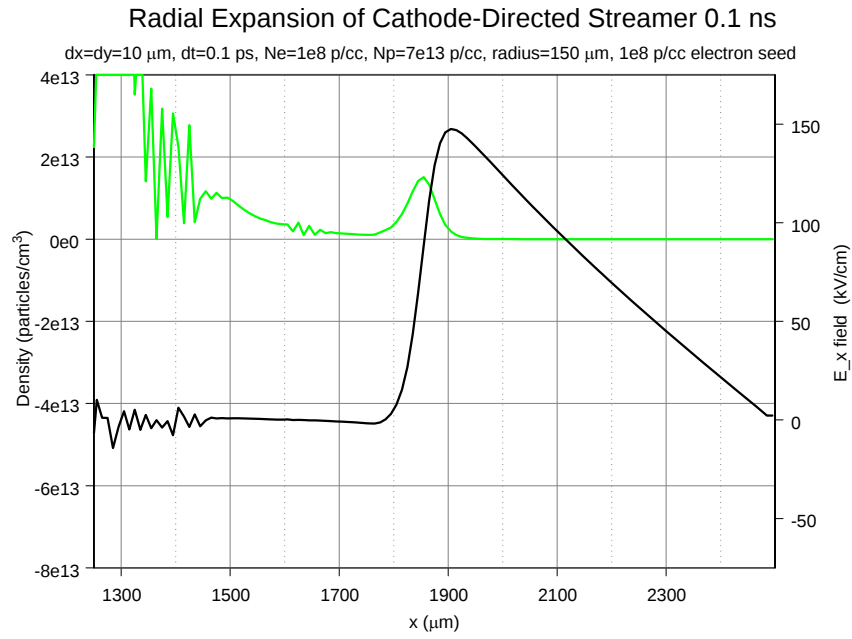


Figure 87. Cathode-directed streamer at 0.1 ns calculated with 2D code.

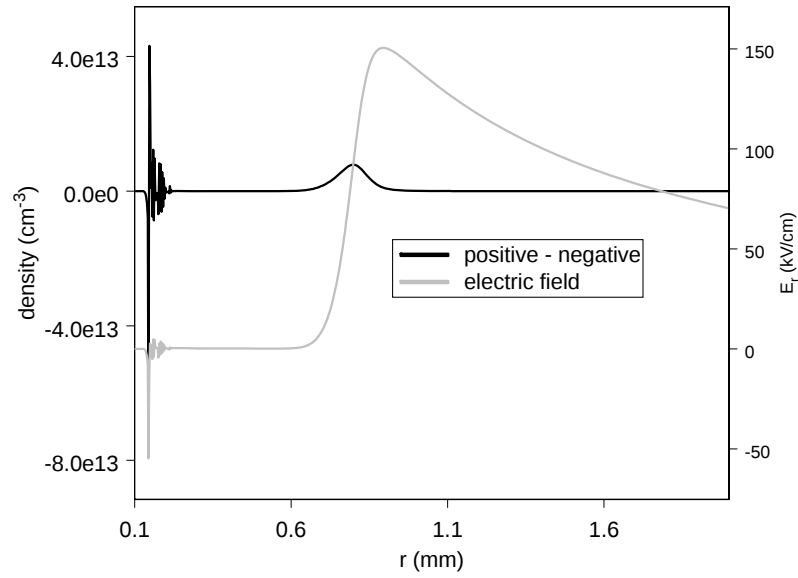


Figure 88. Radial expansion of cathode-directed streamer at 0.1 ns calculated by 1D code.

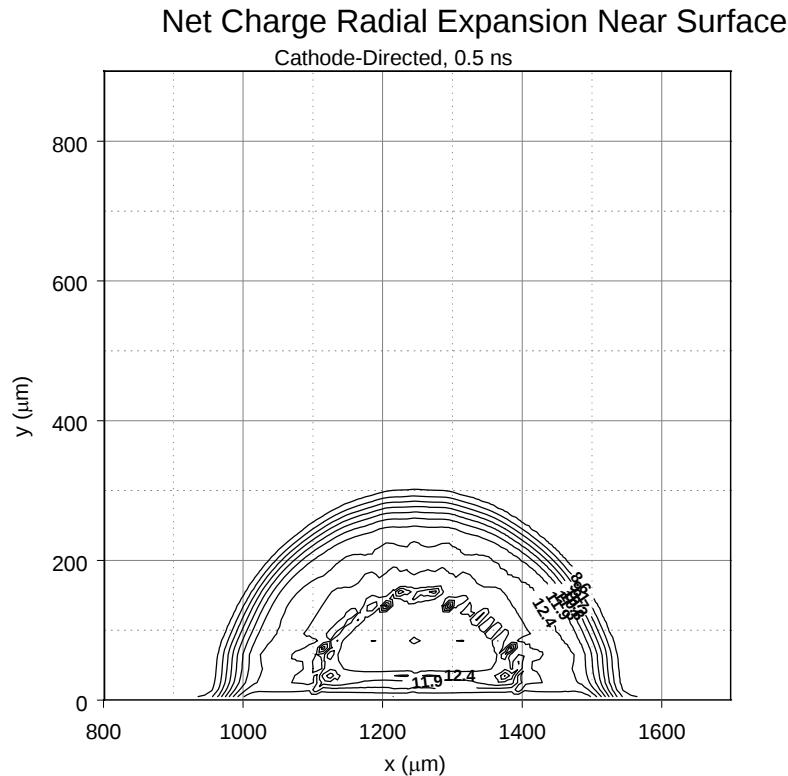


Figure 89. Contour plot of total charge density next to Teflon surface.

caution, however, because all of the surface physics has not yet been included. We have not accounted for photo-emission from the surface (acting like a source of electrons) and secondary electron emission from the surface (acting like a sink of electrons). We hope to account for these effects in the near future.

9 EXPERIMENTS WITH UNIFORM FIELD

The purpose of the experiments was to validate the theory as it evolved. Two separate experimental setups were used. The first was specifically designed to study the electrical breakdown across surfaces immersed in a high pressure gas and mandated a custom setup. It was used to study volume breakdown and flashovers across surfaces in the same chamber with the same drivers. It is referred to as the “Surface Flashover Experiment”. A major effort was directed at measuring the velocity of propagation along a surface immersed in an electronegative gas at pressures between one and three atmospheres. During these tests, it was recognized that we might use the concept of a “sustaining field” in our efforts to discriminate between dominant mechanisms which will be discussed in the next section.

Testing with pulses of nanosecond duration required precise impedance matching throughout the system. The high voltages we are interested in demand dimensions on the order of centimeters. Since nanosecond pulses have frequency content in the gigahertz range, experiments in the nanosecond regime must use microwave techniques to obtain useful data. The introduction of a gap into a uniform impedance line results in a time dependent mismatch, as the gap transitions from an open circuit to a short circuit and back again. In reality, the gap transitions from a large impedance to a very small impedance exponentially

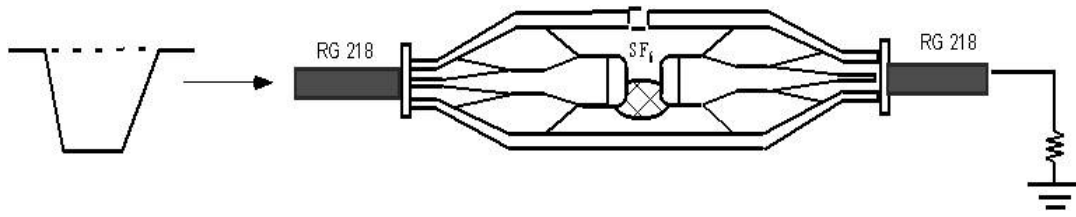


Figure 90. A representation of the flashover experiment showing the surface between the electrodes.

with time. The introduction of a surface into the half space of the gap makes the resistive fall even more complicated.

The experimental arrangement consists of a nanosecond pulse generator applied to a planar two-electrode self-breaking gap. Typical experiments have a surface inserted halfway between the two electrodes and immersed in a gas at a pressure between 1 and 3 atmospheres as shown in Fig 90. Three different gases were tested. Sulfur hexafluoride (SF_6), a strongly electronegative gas, dry air, a weakly electronegative gas and nitrogen, which is not electronegative. Control experiments were recorded without the surface so only the volumetric breakdown in the gas was operative and comparisons with and without the surface could be made.

The details of the experimental arrangement are shown in Fig. 91. The data acquisition system is configured where the master delay generator sets a timing datum from which all measurements are referenced. This allows relative times to be calculated directly between any two signals.

The high voltage pulse generator is a custom unit originally designed to simultaneously trigger multiple spark gaps through long lengths of high voltage cables by Maxwell Laboratories, now part of L-3 Communications Pulse Sciences Division. A fast $0.1\mu\text{F}$ capacitor is charged by a resistor mounted in an axial feedthrough and connected to the high voltage electrode. When the trigger pulse is applied to the floating electrode, the capacitor is switched to ground by a single, low inductance pressurized gas switch. A schematic of the pulse generator is shown in Fig. 92. For these experiments, a single cable was used at the output, producing a risetime of 4 ns and an adjustable peak voltage. The first pulse from the generator is shown in Fig. 93.

During testing with the TG 125, testing was confined to the first pulse only. The TG 125 was used because it provided a low jitter, high voltage pulse for testing. However, it is not a pulse generator, it is a trigger generator, and when the gap does not fire, multiple pulses are produced. The voltage and current monitors are co-located and allow a clear picture of the time to breakdown. An overlay in time of the voltage and current measured just prior to the gap is shown in Fig. 94. The capacitance of the spark gap is on the order of a few pF, and the capacitive displacement current can be seen on the current pulse, signifying the arrival of the charging pulse. The voltage rings, and eventually the gap fails, and current flows. The gap distance is decreased until the gap fails during the pulse duration.

The TG-125 is connected to the chamber by a length of RG 218 cable and triggered by a L-3 40230 trigger unit. The 40230 has its own jitter specification of less than 3 ns from the signal it gets from the master trigger generator. We find the jitter to be even better than specified and have measured it to be 1.5 ns, resulting in very low jitter pulsed power. The TG-125 is charged through an internal 20 kohm charge resistor to a maximum voltage of $\pm 125\text{ kV}$ by a polarity-reversible Glassman power supply. The polarity of the 40230 trigger unit is set to produce a trigger pulse in the opposite polarity to the TG-125 charge. The TG-125 charge voltage is monitored with a Spellman high impedance resistive divider. The charge rate of the Glassman power supply is controlled with a ramp waveform produced with an Agilent arbitrary waveform generator (ARB). The charge path also includes a power supply isolation relay and a

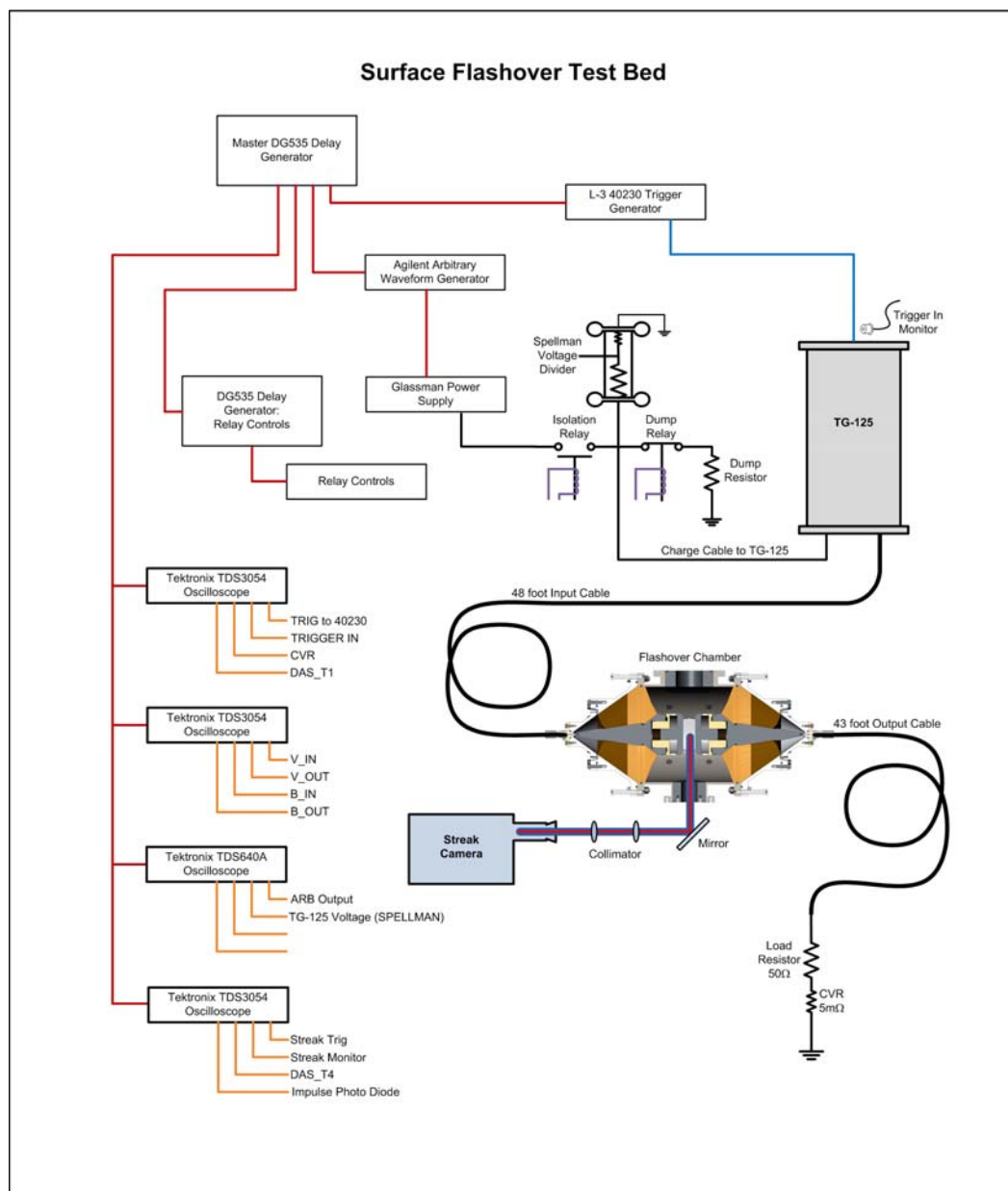


Figure 91. The physical layout of the surface flashover experiment.

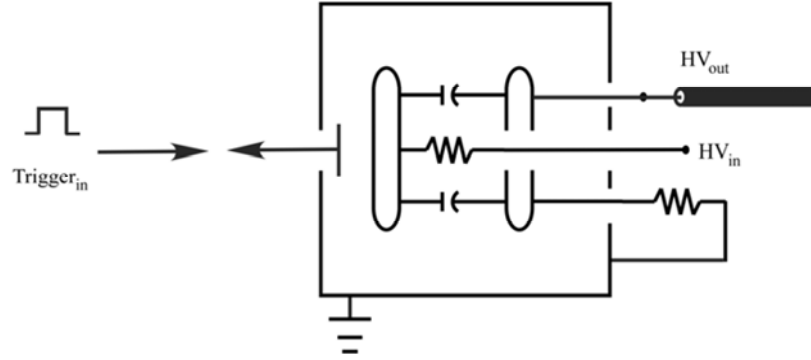


Figure 92. The pulsed power is provided by a custom TG 125 trigger generator initiated by a L-3 40230 trigger. The resultant low jitter was necessary to capture the fast discharge with the streak camera.

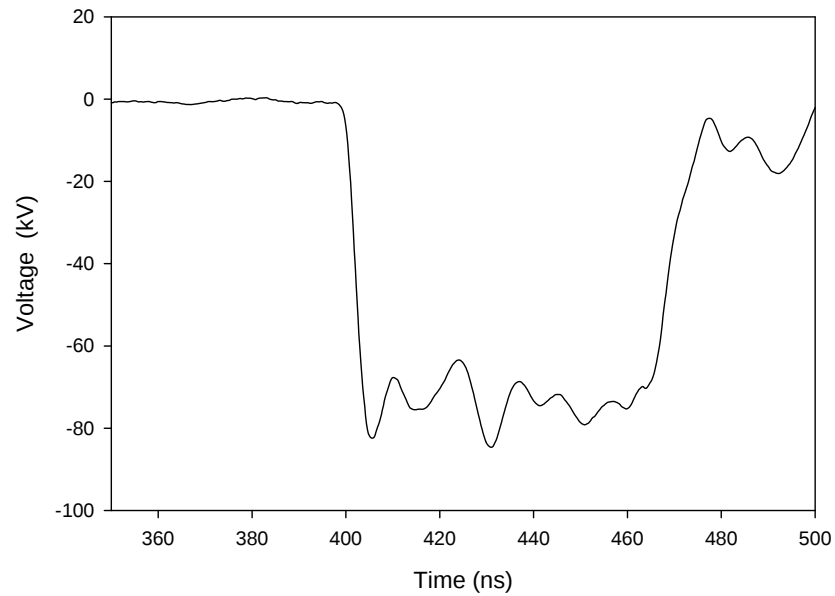


Figure 93. The first pulse from the TG-125 trigger generator.

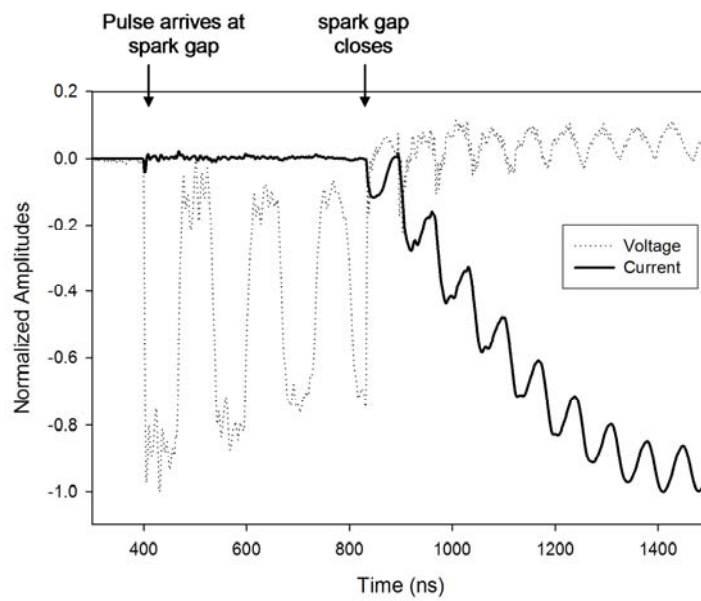


Figure 94. The voltage and current of the TG125 when the gap does not breakdown. The multiple pulses are because the TG 125 is a trigger generator and charges the transmission line that connects it to the chamber.



Figure 95. The load tank contains the resistive termination for the discharge chamber. The load is a solid resistor of the type which has been shown to maintain a pure resistance to a fraction of a nanosecond.

circuit dump relay. The trigger generator is insulated with sulfur hexafluoride at a pressure which varies with the charge voltage.

The chamber is also terminated in a cable and matched solid resistor which provides transit time isolation from the test gap with no reflections. The load is a 50 ohm oil-saturated HVR resistor disk assembly situated in a small oil reservoir, shown in Fig. 95. The load is attached directly to the CVR mounted in the base plate of that reservoir.

A discharge chamber, shown in Fig. 96, has been designed to study surface flashover of surfaces in electronegative gases at pressures up to 10 atm and accommodate any number of surface materials. The outer conductor is made of stainless steel to accommodate the large volume of high pressure gas. The chamber is impedance matched throughout to allow for very fast pulse propagation and the impedance is chosen to be 50 ohms to mate to commercial cable for moderate voltages. Strategic cuts are made throughout the fixture to accommodate connecting to different generators; the fixture is capable of holding off voltages up to 1.4 MV. There are four large ports located radially around the center of the chamber for optical and physical access and six smaller ports for diagnostic monitors and gas connections. The incarnation that is tested here hooks to the RG-214 cable to accommodate the pulser.

The design of the surface flashover fixture has a number of requirements. It must (1) hold the maximum working pressure with a suitable factor of safety to allow for operation near personnel, (2) it must propagate the maximum desired voltage without failing in the radial direction, and (3) it must produce a uniform electric field in the gap while accommodating an insulating sample with good electrical contacts. The uniform field in the gap is achieved by making the electrodes large compared to the gap which also serves to wedge the inner conductor into the insulating plastic. A fourth constraint is imposed by testing with high voltage pulses of nanosecond duration. The chamber must be impedance matched throughout.

Impedance matching may be maintained by decreasing the outer conductor in proportion to the inner conductor, but this would introduce additional difficulties. Insulator faces which are parallel to the electric field have an increased likelihood of breaking down along that surface. Moreover, the fields necessary

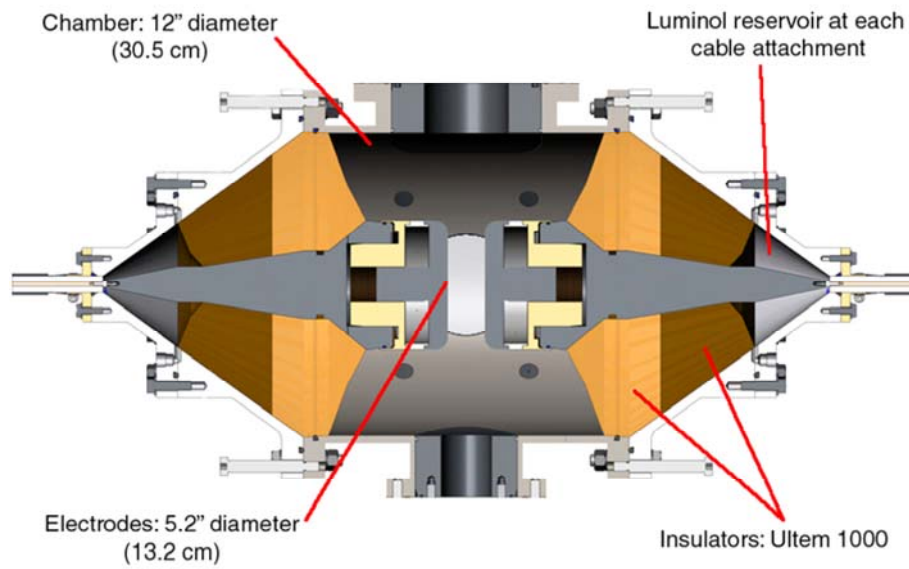


Figure 96. A crosssectional view of the discharge chamber for high pressure surface flashover experiments.

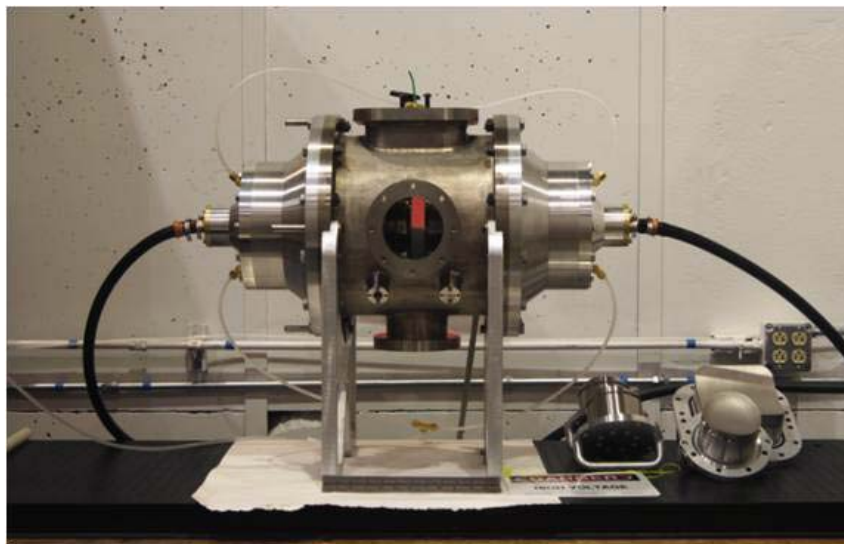


Figure 97. The discharge chamber is capable of withstanding gas pressures up to 10 atmospheres.

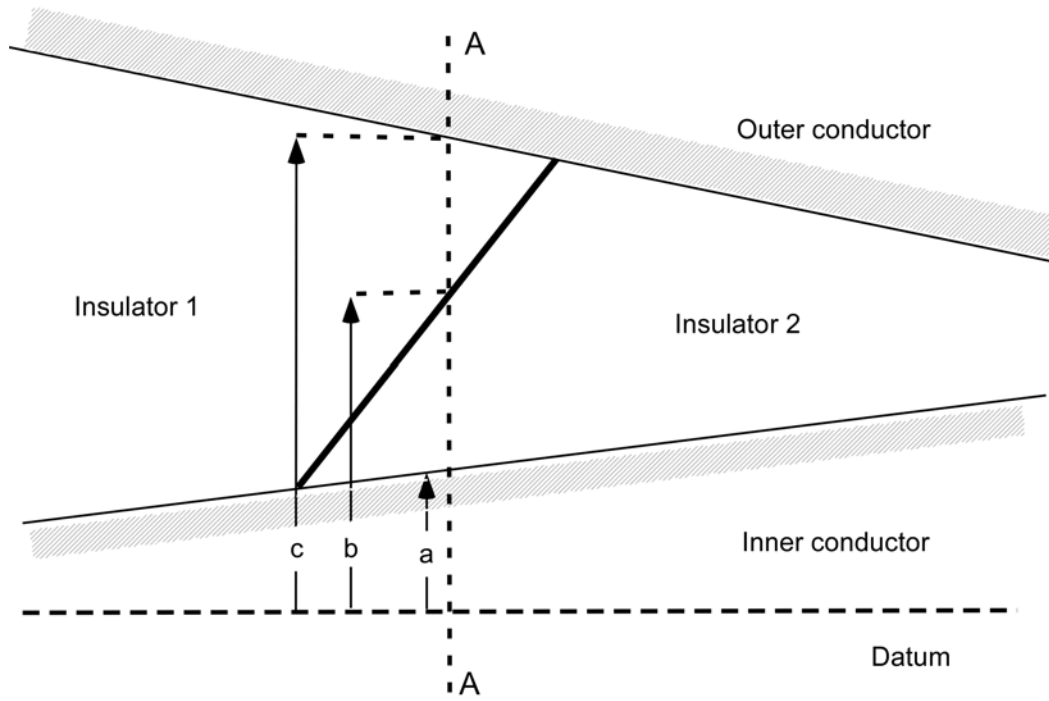


Figure 98. Along with the dielectric constant, the shape of the interface is used to maintain the electrical impedance throughout the chamber.

to produce the surface flashover are poorly understood and no rule of thumb exists. The impedance at any point along a transmission line depends only on the dielectric properties of the insulators and the dimensions. To go from the section containing gas, with a relative dielectric constant of 1, to a plastic, the impedance may be maintained by changing the dimensions of the interface as well as the inner and outer conductors, as illustrated in Fig. 98

The impedance of the transmission line with a dielectric interface and a varying ratio of inner to outer conductor radii can be calculated along any cut A-A perpendicular to the direction of propagation by

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\ln\left(\frac{c}{a}\right) \left[\frac{\ln\left(\frac{b}{a}\right)}{\epsilon_2} + \frac{\ln\left(\frac{c}{b}\right)}{\epsilon_1} \right]} \quad (331)$$

where a is the radial dimension of the inner conductor, c is the radial dimension of the outer conductor, b is the radial dimension of the interface, and ϵ_1 and ϵ_2 are the relative dielectric constants on either side of the interface.

This relation is true for any cut along the interface. It can be seen from the above relation that using straight line variations for the dimensions a , b and c cannot possibly yield a perfect impedance match because of the logarithmic dependence on the dimensions. The variation in impedance occurs midway along the interface, as seen in Fig. 99 and may be minimized by keeping the length of the interface along the propagation direction short. However, because the interface is also a pressure support and must hold off the voltage, this is not a free design parameter. The insulator used is called ULTEM, an amorphous thermoplastic polyetherimide material which combines exceptional mechanical strength and electrical properties. The relative dielectric constant is 3.15. The impedance along the ULTEM/ gas interface was

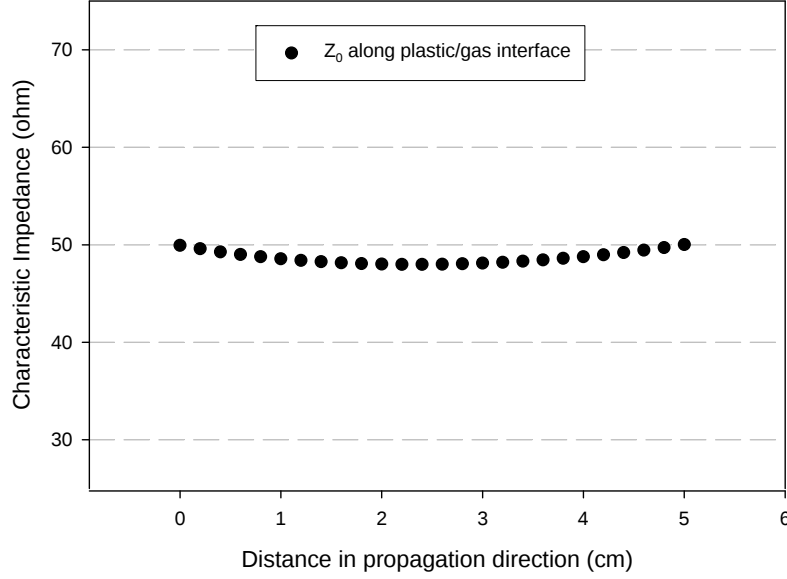


Figure 99. The impedance across the torlon/gas interface inside the discharge chamber.

calculated to range from 48.75 to 50.0.

The characteristic impedance match was verified using a cable pulse with a fast rise time and the gap of the chamber shorted. An overlay of the input waveform and the output into a matched load is shown in Fig. 100. We estimate that the chamber is matched to within 1%.

From Equation 332, the breakdown criteria is met provided

$$E_{BR} \geq \frac{V}{a \ln\left(\frac{c}{a}\right)} \quad (332)$$

within a section with one insulator.

The cable from the pulsed power attaches to the discharge chamber with an oil filled mating section. This allows a void free region with small dimensions and high electrical breakdown strength. The oil is a product called Luminol which is almost clear, has good electrical breakdown strength and a relative dielectric constant of 2.2. The interface between the Luminol and the ULTEM was matched to 50 ohms .

The gap is formed by an interruption in the center conductor of the test chamber. The condition for uniform electric field in the gap region is met by making the electrodes large relative to the gap length. The detail for electrode geometry is shown in Fig. 101. The inner conductor is made from stainless steel to accommodate the large force which results when the maximum working pressure acts on the large electrode area. A custom “Bal Seal” is used to maintain good current contact under high pressures. A wave disk spring washer is used to insure there is a positive contact pressure when the surface is installed to minimize the possibility of an air gap between the electrode and the surface. The gap may be adjusted by using appropriate spacers, designed to withstand the pressure and has a range of 0.5 to 3 cm.

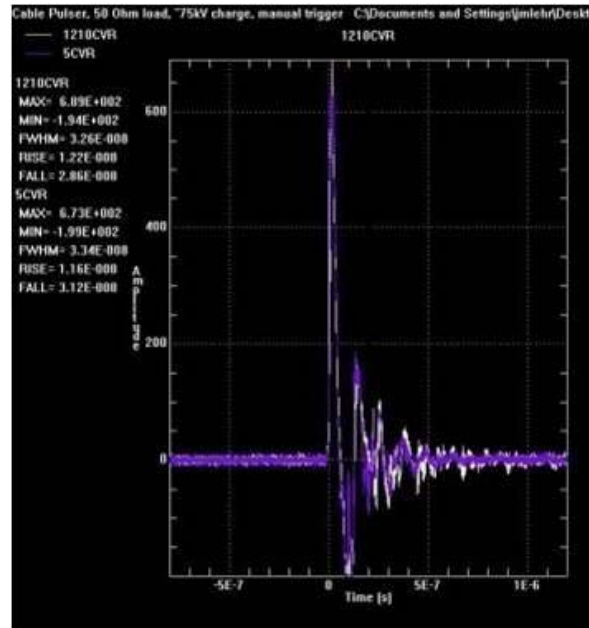


Figure 100. An overlay of a pulse with a fast risetime measured at the input and at the load show the excellent impedance match when the test gap is shorted.

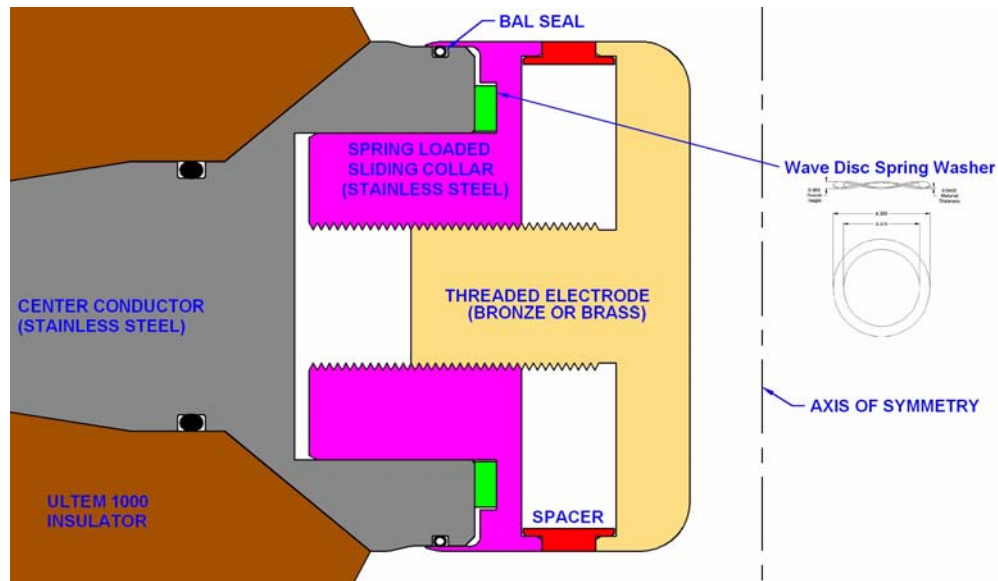


Figure 101. Details of the electrode design to insure good electrical contact between the inner conductor and the insulating test sample, which is not shown.

Before breakdown occurs the gap acts as a capacitance. During the design phase, this was a significant concern because the electrodes are large and the gap may be small. However, capacitance of the gap was calculated to be sufficiently small to permit the full voltage to develop across the gap in less than a nanosecond in all testing scenarios.

The experiment was designed to give correlated electrical and optical measurements. The electrical diagnostics are interspersed throughout the system. A Spellman HVD-100 voltage divider monitors the charge voltage to the TG-125. The 40230 trigger unit is monitored by an un-calibrated capacitive voltage divider (V-dot) at its connection to the TG 125. This monitor gives timing information and has been used to verify pulsed power operation. There are a number of access ports on the discharge chamber. Four of these small chamber ports hold voltage and current probes situated on either side of the flashover gap to monitor input and output voltage and current.

The optical diagnostics consist of still (open shutter) photography, a gated CCD camera and streak photography. The open shutter cameras, both film and digital may be located at the top or the side port windows. A streak camera is used to measure the time resolved light emission of the discharge and is located in a screen box adjacent to the experiment to shield it from electromagnetic interference from the pulsed power. The optical path to the chamber includes a turning mirror and a two-lens collimator.

9.1 Threshold Electric Field Levels

Threshold field levels are of extreme interest because they are a fundamental parameter of the breakdown process and, thus, are useful for comparing theory with experiment. Critical breakdown levels are known to be very sensitive to experimental conditions such as electrode material (even away from the Townsend regime, when streamers are operative), gas purity, voltage pulsewidth, humidity, etc. Within these bounds, however, the prediction of the critical value for the onset of electrical breakdown is important because it is fundamental to the chain of events which must occur for a conducting channel to fully form to carry the circuit current through an insulator. In other words, if the critical fields are predicted, the proposed mechanisms are operative along with their relative importance and the theory is mostly right.

The critical levels are also of supreme engineering importance because it is the starting value with which designers use to size equipment, ultimately determining the inductance of the system. For bulk insulators, the threshold values are well understood. For pulses longer than a few microseconds, the DC values are generally used. For gases, the critical electrical field levels are known to be dependent on both the gas pressure and the pulse width of the applied voltage pulse, with shorter pulses producing higher values of the critical fields [66].

It is widely thought that the introduction of surface into an insulating media at high pressure (atmospheric and above) greatly reduces the critical level for breakdown. This belief stems from an extrapolation from the vacuum insulator experience (where it is certainly true) and from operational observation. Although high voltage machines and devices are typically insulated with liquids or gases, it is observed that when they fail unexpectedly, it is along an insulating surface. The prevailing rule of thumb among high voltage equipment designers is to design for the electric field strength of the insulator that the surface is immersed in with a hefty factor of safety.

A case in point (in this case in liquid instead of gas) is the output transmission lines in the system assessment module for the Z Refurbishment project [67]. The water insulated triplate transmission lines are supported with high density polyethylene rods and designed to the field strength of purified water. The triplate was operated over 800 shots before failure, but when it failed, it was along the insulating support. Figure 102 shows open shutter photographs of the machine operation. The upper photo shows a routine shot. Note the many partial discharges occurring under the considerable applied voltage of 4.6 MV on the

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Avg. Breakdown Field (kV/cm)
1 atm	77.5	10	91
2 atm	87	12	102
3 atm	95.5	12	112

Table 10. Commercial Dry Air.

output transmission line. No activity is shown in the vicinity of the support rod until it failed, shown in the bottom photograph. Figure 103 shows the damage pattern along the surface of the rod. Inspection of the damage pattern indicated the failure may have initiated in the midpoint along the structure and not at either electrode.

Threshold values for the onset of electrical breakdown across an insulator have been measured for Teflon and high density polyethylene (HDPE) surfaces immersed in a high pressure gaseous environment. The gases used were commercial dry air and nitrogen at pressures of 1, 2 and 3 absolute atmospheres and SF6 at 1 and 2 atmospheres. The threshold values for the gas alone (volumetric breakdown) have also been measured for comparison with a negative polarity voltage.

A small initiation pin is added to the electrode on the ground side. The input pulsed voltage is measured at the derivative voltage monitor located near the high voltage electrode. When the voltage pulse arrives at the open circuited gap, the pulse is reflected back onto itself and may produce twice the voltage amplitude of the initial pulse. The peak voltage is the breakdown voltage. A typical input voltage waveform is shown in Figure 104. The time to breakdown is measured by normalizing the input voltage waveform and the output current waveform, as shown in Figure 105. Since all the waveforms are referenced to a single reference, the process is straightforward.

9.1.1 Volume Results

The threshold breakdown fields of the gaseous insulators were measured in the surface flashover discharge chamber under nearly identical conditions as would be used when samples were included in order to separate the effects of the gas and the surface. The gap length for the volume experiments is $g = 8.5$ mm with a $h = 1$ mm field enhancement pin. It should be noted that there are insufficient shots to provide delay times with a high degree of confidence. These values are given in Tables 10, 11, 12. Note that the “Average Breakdown Field” in the tables of this threshold section is actually the external plane field (without the pin) V/g rather the average field $V/(g - h)$ (see nonuniform field section below). In this threshold subsection these two values only differ by the factor $1 - h/g = 0.88$.

9.1.2 Teflon Surfaces

Teflon is a preferred material for applications using SF6 where surface flashover is likely and where mechanical strength is not needed. A report [23] on surface flashover in an SF6 atmosphere performed at the Dutch company KEMA recorded evidence of anomalous behavior of the discharge along a Teflon surface. It was found that in a significant fraction of the flashover events, the surface was not damaged. For some applications, this result may have important engineering implications. For instance, in switches when a flashover occurs, the surface is damaged so that it no longer holds off the full voltage. In a pulsed power machine, this manifests as a prefire- the switching of the current before command trigger which can cause a lost shot at minimum, and may result in substantial damage. In these investigations, threshold levels, delay times and streak photography have been measured in air and SF6 at several atmospheres.

The threshold measurements were taken using virgin Teflon surfaces with a 64 finish. They are nominally 1 cm in length. The threshold voltage levels and breakdown delay times are summarized in Table



Figure 102. Open shutter pictures showing normal operation (upper) for multimegavolt voltages and a failure along the support rod in the output transmission line section.

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Avg. Breakdown Field (kV/cm)
1 atm	88.5	15	104
2 atm	130	40	153

Table 11. Sulfur Hexafluoride (SF₆).

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Average Breakdown Field (kV/cm)
1 atm	78.5	30	92.3
2 atm	96	12	113
3 atm	106	16	125

Table 12. Nitrogen.



Figure 103. The damage along the rod after a surface failure.

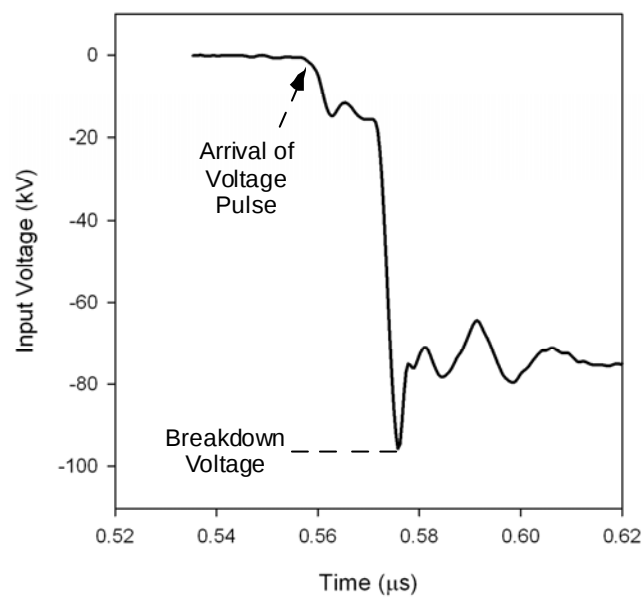


Figure 104. The voltage signal measured at the input of the gap when the gap breaks.

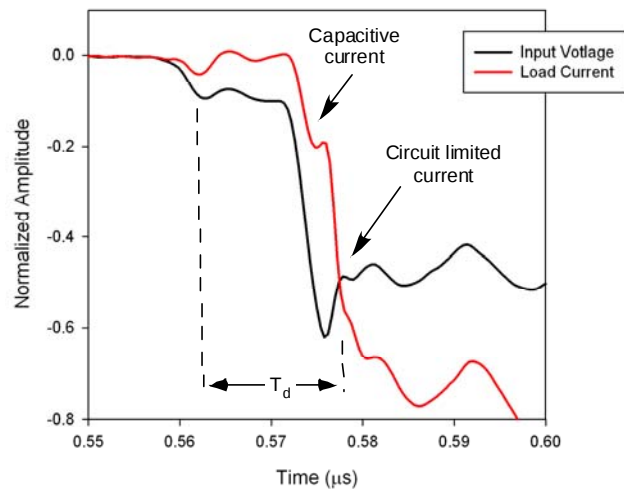


Figure 105. The input voltage and load current are normalized and overlaid to find the time to breakdown of the gap.

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Avg. Breakdown Field (kV/cm)
1 atm	88	33	88
2 atm	100	15	100
3 atm	104	17.5	104

Table 13. Teflon Surface immersed in Commercial Dry Air.

13 for breakdown in air at pressures of 1, 2 and 3 atmospheres.

The average electric field is computed for the surface flashover along the Teflon surface and the air volume at each of the three pressures, and summarized in Table 14.

In comparing this data with that for the volume, we find the threshold values for surface flashover across a Teflon insulator are only slightly lower than those obtained in volumetric air and the delay times are somewhat higher as shown in Tables 15 and 16.

Once again, we see that the threshold values for surface flashover across a Teflon surface in SF6 is the approximate threshold for breakdown in a volume of SF6 at the same pressure.

9.1.3 High Density Polyethylene

High Density Polyethylene (HDPE) is a widely used material in high voltage applications. It combines good electrical properties with good mechanical strength. It is low cost and easily machined.

The threshold voltage levels and breakdown delay times are summarized in the following Table 17 for breakdown in air at pressures of 1, 2 and 3 atmospheres.

The average electric field, V/d , is computed for the surface flashover along the Teflon surface and the air volume at each of the three pressures, and summarized in Table 18.

In comparing this data with that for the volume, we find the threshold values for surface flashover across a HDPE insulator are only slightly lower than those obtained in volumetric air similar to what was observed with Teflon. The delay times on the other hand are significantly higher.

9.1.4 Summary Of Threshold Fields

The threshold values are summarized in Table 23 for air, Table 24 for SF6 and Table 25 for nitrogen. The percent change is the change from the electrical breakdown value in the volume. In each case, the introduction of the surface lowers the threshold value by a small amount, lending credence to the practice of using the electrical strength of the insulating fluid, with a factor of safety, even in the vicinity of a surface. It can be noted that the small field enhancement pin may play a role and future experiments will also compare threshold values in a truly uniform field. It is known that SF6 has a high breakdown strength relative to air, but these measurements show high values for air as well. This will be examined further in the future.

9.1.5 Effect of Flashover History

In high voltage engineering, the conventional wisdom states that once a surface has failed via surface flashover, it loses its insulating ability and can no longer hold off voltage. This has been attributed to an increased conductivity along the path the surface discharge followed, but the chemistry has not been proven. In practice, a surface flashover is usually only discovered when it stops holding off voltage. For

P(atm)	Avg. Breakdown Field (kV/cm) Teflon/Air	Avg. Breakdown Field (kV/cm) Volumetric Air
1	88	91
2	100	102
3	104	112

Table 14. Threshold Electrical Breakdown Field for air.

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Avg. Breakdown Field (kV/cm)
1 atm	95.5	34	95.5
2 atm	147	34	147

Table 15. Teflon Surface immersed in SF₆.

P(atm)	Avg. Breakdown Field (kV/cm) Teflon/SF ₆	Avg. Breakdown Field (kV/cm) Volumetric SF ₆
1	95.5	104
2	147	153

Table 16. Threshold Electrical Breakdown Field for SF₆.

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Avg. Breakdown Field (kV/cm)
1 atm	86	40	86
2 atm	96	50	96
3 atm	99	38	99

Table 17. HDPE Surfaces immersed in Commercial Dry Air.

P(atm)	Avg. Breakdown Field (kV/cm) HDPE/Air	Avg. Breakdown Field (kV/cm) Volumetric Air
1	86	91
2	96	102
3	99	112

Table 18. Threshold Electrical Breakdown Field for air.

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Avg. Breakdown Field (kV/cm)
1 atm	94	30	94
2 atm	146	35.5	146

Table 19. HDPE surface immersed in SF₆.

P(atm)	Avg. Breakdown Field (kV/cm) HDPE/SF ₆	Avg. Breakdown Field (kV/cm) Volumetric SF ₆
1	94	104
2	146	153

Table 20. Threshold Electrical Breakdown Field for HDPE/SF₆.

Gas Pressure	Breakdown Voltage (kV)	Delay Time (ns)	Avg. Breakdown Field (kV/cm)
1 atm	81	36	81
2 atm	98	17	98
3 atm	109	17	109

Table 21. HDPE Surfaces immersed in Commercial Dry Nitrogen.

P(atm)	Avg. Breakdown Field (kV/cm) HDPE/N ₂	Avg. Breakdown Field (kV/cm) Volumetric N ₂
1	90	105
2	109	128
3	121	141

Table 22. Threshold Electrical Breakdown Field for nitrogen.

Pressure	Volume E_{BR} (kV/cm)	Teflon E_{BR} (kV/cm)	Teflon % change	HDPE E_{BR} (kV/cm)	% change
1 atm	91	88	3	86	5
2 atm	102	100	2	96	6
3 atm	112	104	7	99	12

Table 23. Summary of threshold values in Air

Pressure	Volume E_{BR} (kV/cm)	Teflon E_{BR} (kV/cm)	Teflon % change	HDPE E_{BR} (kV/cm)	% change
1 atm	104	95.5	8	94	10
2 atm	153	142	8	146	5

Table 24. Summary of threshold values for SF₆.

Pressure	Volume E_{BR} (kV/cm)	HDPE E_{BR} (kV/cm)	HDPE % change
1 atm	92.3	81	12
2 atm	113	98	13
3 atm	125	109	13

Table 25. Summary of threshold values in N₂.

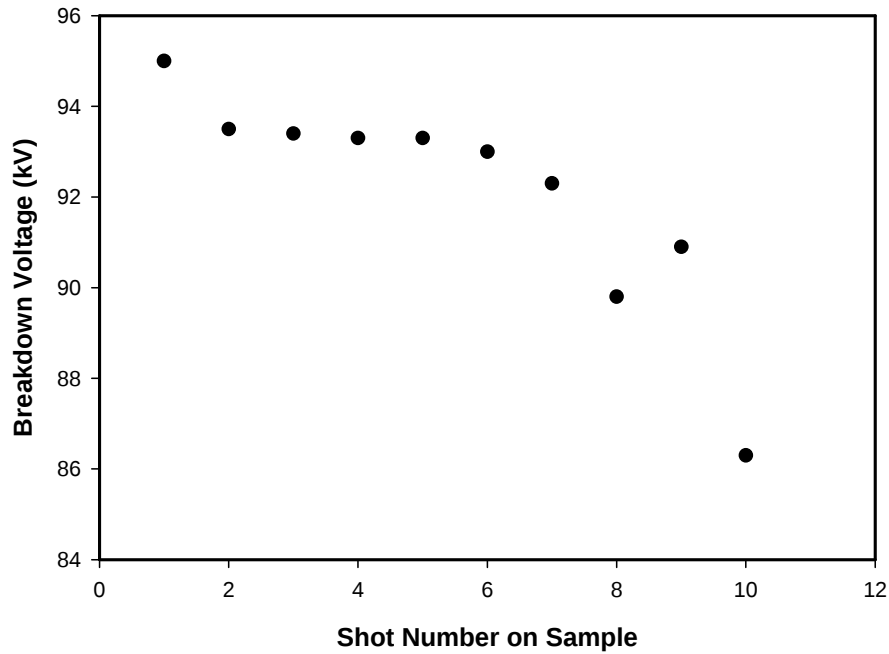


Figure 106. The electrical breakdown voltage of a HDPE surface in atmospheric air degrades with successive flashovers.

instance, in the Z machine operation, it was long thought that when the Intermediate Store output voltage decreased, the LTGS must have failed and, upon inspection, it was always the case. During the ZR upgrade, serendipitously, spare open shutter cameras were installed to view the LTGS during the system assessment test program (SATPro), which showed the LTGS was flashing over many times before the failure could be detected in the electrical signals. This has prompted a test to quantify the degradation in performance of a surface with sequential flashover events.

The previous voltage threshold tests were performed on virgin (unflashed) surfaces for each test. For this series, a virgin high density polyethylene (HDPE) surface with a very smooth (32) finish, is installed and made to flashover under the influence of a positive polarity voltage pulse. Ten sequential shots are taken on a single surface and the breakdown voltage is recorded. The test is done in air and in SF₆ at a pressure of 10 psig.

The results for air are shown in Figure 106 . The decrease in hold off voltage is consistent and shows indications of continuing to go down, which is consistent with other experimental data taken during the course of this effort. The decrease in voltage is smooth and decreases the hold off voltage by approximately 10% in 10 shots.

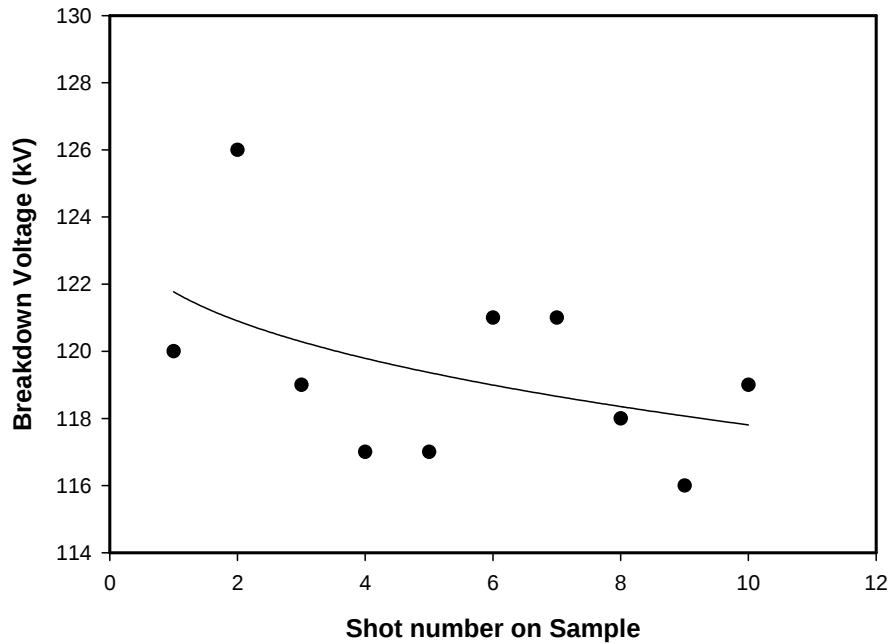


Figure 107. The threshold for electrical breakdown across a HDPE surface in atmospheric SF₆ decreases for successive flashovers with significant scatter.

The results for SF₆ are shown in Figure 107. The decrease in hold off voltage is clear, and the decrease within those 10 shots is, like air, about 10%, but the decrease is much more erratic than the smooth decrease shown for air.

Measurements of the threshold for breakdown voltage across a surface in a high pressure gas, and their comparison to thresholds for breakdown where only the gas is present, might lead to the conclusion that the surface does not play any role. These results, showing the lowering of the threshold voltage in subsequent voltage applications, indicate that not only does the surface play some role, but that the type of gas is also important.

This is bolstered by a shot series taken in the course of our camera being setup. After numerous shots on a surface, it was failing to hold off the voltage. Damage on the insulating surface is evidenced by discoloration located at the enhancement pin. Since these shots were done only to work on the camera, rather than take out the surface sample, the surface was simply pushed to one side by about 2 cm, leaving a “fresh” surface at the enhancement pin. We initially found the discharge in 10 psig of SF₆ followed the old discharge path as shown in Figure 108. This is very curious since SF₆ is known to be susceptible to electrical field enhancements and this open shutter photograph shows the discharge preferring the damage path on the surface to the field enhancement.

Additional shots show that this does not happen in air. Figure 109 shows sequential shots with both air and SF₆ at 10 psig. The shot numbers are written by hand in the lower left hand corner of each Polaroid snapshot. The first, Shot SF286 is a discharge in 10 psig (7 psia) of SF₆ which follows the damage path from previous shots instead of the field enhancement pin. The next shot is in air at 10 psig and shows the discharge is located at the field enhancement again. SF288 verifies that the discharge in air stays located at the enhancement pin. SF289 is in SF₆ and the discharge again moves to the damage on the surface

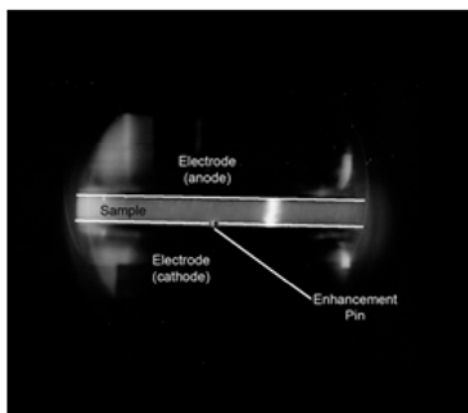


Figure 108. A open shutter photograph of a surface discharge in SF6 along a high density polyethylene surface where the discharge follows the damage path instead of the field enhancement.

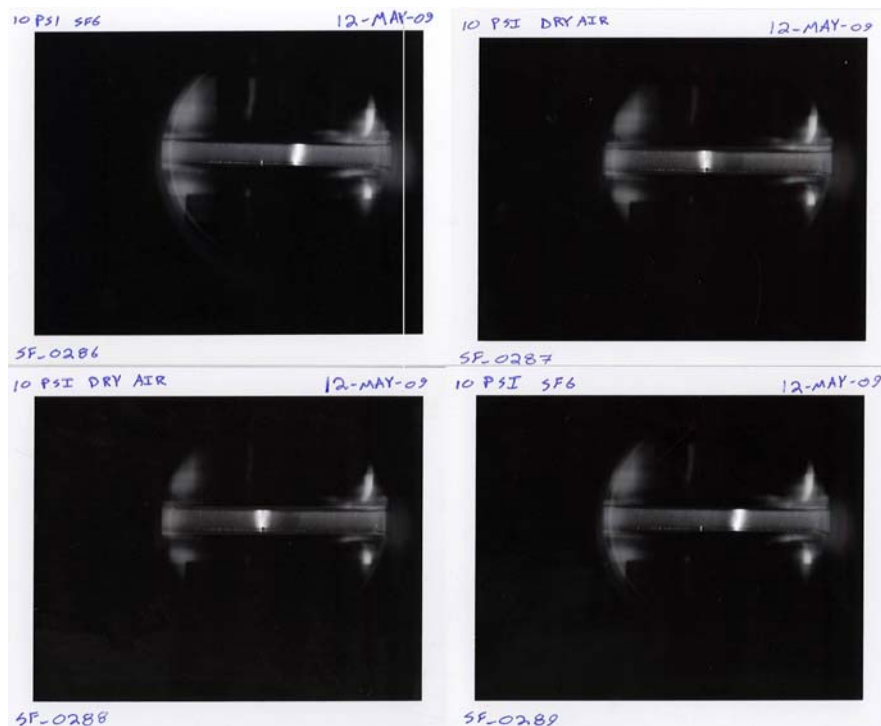


Figure 109. Open shutter photography showing the discharge in SF6 prefers to follow the old discharge path rather than the field enhancement while air prefers the pin.

from previous flashovers. It can be noted that the SF6 at 10 psig data followed the damage path 100% of occurrences and discharges in air occurred at the enhancement pin 100% of the time. It should also be noted that the damage was severe enough to cause a notable loss of voltage holdoff capability before this series was performed. The mechanism is undetermined but indicated the interaction of the gas and the surface materials are intricately linked. Moreover, the damage may be specific to a gas type.

It can also be noted that these results are consistent with the LTGS failures observed on SATPro. In SF6, the decrease in threshold voltage values for surface flashover is erratic, explaining why the LTGS can have a surface flashover then normal operation on subsequent shots. Eventually, however, the surface becomes sufficiently damaged so that it will no longer hold off the full applied voltage, causing the Intermediate Store to be switched early which can be seen in the voltage waveforms.

These open shutter pictures also relate to another observation: there is little additional inductance introduced by the flashover. The LTGS was developed to provide precise, low jitter switching for low inductance, very high voltage pulsed power drivers. The inductance of the switch is added directly to the overall inductance of the system and affects the risetime. The “backbone” section of the LTGS lowers the inductance by promoting multi-channeling which cuts the inductance of a single spark channel considerably. In the output waveforms of the cited LTGS failures, the output waveforms remain unchanged, indicating the failed LTGSs have the same approximate inductance as in normal operation. The open shutter photographs show that the discharge in these experiments spreads over the surface somewhat, resulting in a lower inductance configuration than one would see in a spark channel in a volume discharge.

9.2 Streak Photography

Streak cameras convert incident light into electrons and send them through a high speed sweep. The electrons are swept from top to bottom in order to determine the intensity distribution of the incident light in relation to the time. The light passes through a slit and forms an image on the photocathode of the streak tube. The photocathode converts the light to electrons and these electrons are directed towards the phosphor screen by accelerating electrodes. A high speed, high voltage pulse is applied to the sweep electrodes so that the electrons are swept in the direction from top to bottom. The swept electrons are directed towards the phosphor screen where they are converted back into light. The optical image produced on the phosphor screen is the “streak image”, and this serves the intensity distribution image on the phosphor screen in the vertical axis direction as time domain. In this way the temporal intensity distribution is converted to spatial intensity distribution on the phosphor screen. An image intensifier utilizes a multichannel plate intensifier to amplify the streak image.

Two different streak cameras were used on this experiment: an EG&G unit and a Hammamatsu. The EG&G camera is one that was originally developed in the 1960s and was very high grade, which we used extensively on volume discharges in atmospheric air. The Streak tube is an EG&G model L-CA-24 (S/N 208) with a 50 mm input surface and amplified with a 50 mm Micro channel plate intensifier (MCP) and a 300 μm slit. The image is recorded on a CCD camera. The optical setup gives a spatial resolution of 26.9 μm per pixel and a temporal resolution range of 20.7 – 285.7 ps per pixel. All streak data is taken with a temporal reference provided by an EG&G calibration source model CAL4IS which fires an 820 nm, 20 mW laser diode at a rate of once per 4 ns. In addition, absolute timing is obtained from an externally triggered laser diode. The timing between the applied voltage and the ramp of the streak camera is shown in Figure 110. The relation of the streak image to the test gap is illustrated in Figure 111. The streak image gives the relation between distance and time, from which velocity may be extracted.

In the recent past, the streamer velocity was used as a parameter to compare theory and experiments for volumetric discharges. The experimental voltage waveform was used to drive the simulation, allowing a direct comparison between theory and experiment. The velocity of propagation of light emitted from a discharge in atmospheric laboratory air with a gap of 1.27 cm, was measured with a streak camera. The streak image is shown in Figure 112, where the horizontal axis corresponds to gap distance and the vertical axis to time. The scale is shown in the upper quadrant of the streak image. The discharge initiates at the cathode (at the bottom of the streak data) and travels towards the anode with a constant velocity. Before the anode directed streamer crosses the gap, a cathode directed streamer starts and results in acceleration. The velocities were obtained from the streak image by digitizing the coordinates by hand and computing the velocities at various times after initiation. The values of instantaneous velocities derived from the computations and the digitized streak data were compared and agreed to within 20%.

It was assumed that the streamer velocity would be a good parameter to compare for breakdown via surface flashover as well, even though such measurements are sparse in the literature, and those were mainly in the microsecond time regime. The EG&G streak camera was configured for use with the surface discharge chamber looking perpendicular to both the surface and the discharge path, as shown in Figure 113. The resulting streak image with the camera is shown in Figures 114 and 115.

We conclude that the discharge across the surface is much faster than the discharge in a volumetric gas and this streak camera is too slow to capture the flashover events. The image is radically different than that obtained in volumetric air. The image was digitized by hand but the large change in shape of the discharge is suspect. We concluded that the EG&G camera was not sufficiently precise for this measurement.

We were able to borrow another camera that not only has a significantly better resolution but also

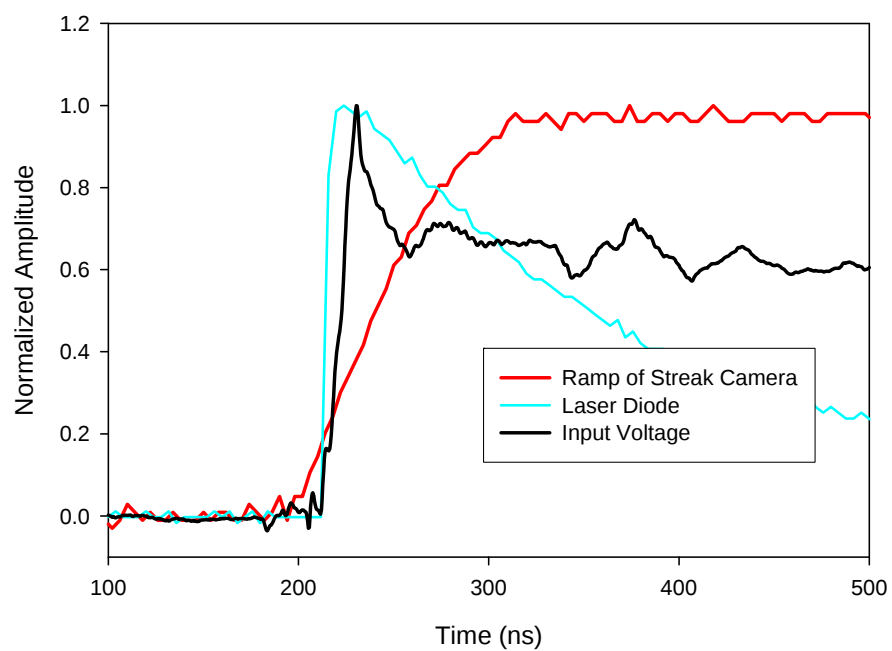


Figure 110. An overlay of the normalized electrical signals showing the relative timing of the applied voltage, the laser diode and the ramp of the streak camera. the ramp of the streak camera shows it is capturing the region of interest.

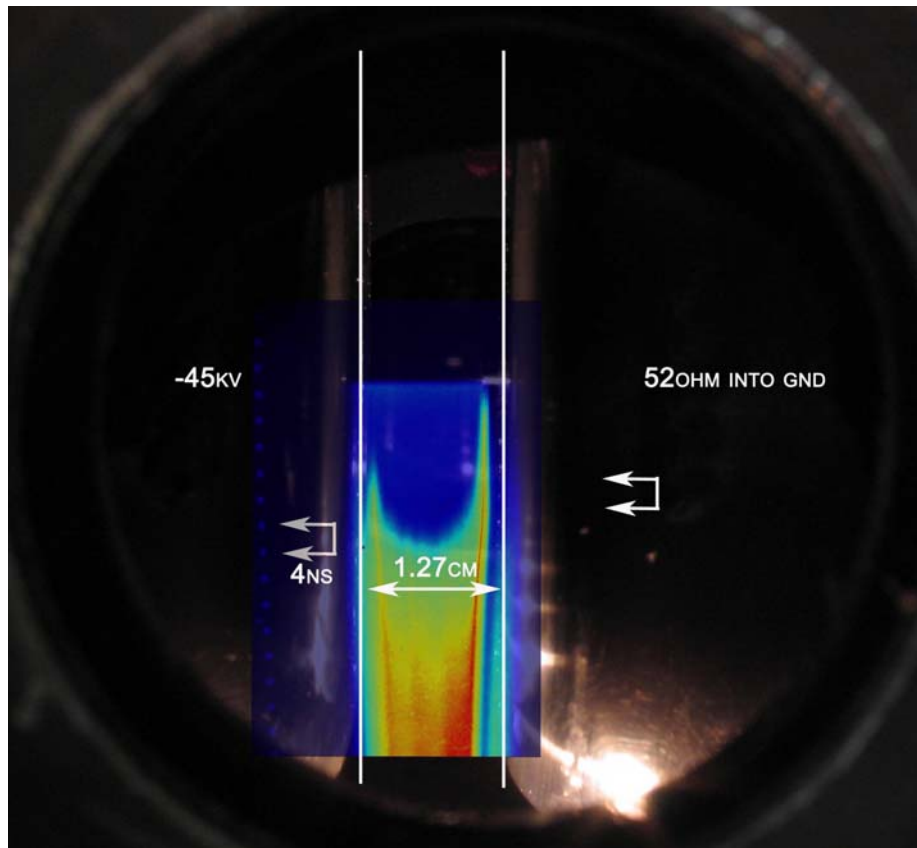


Figure 111. An overlay of a streak image with the test gap shows the relation between the time evolution of light emission within the gap for a volume discharge. The field enhancement pin which initiates the discharge is located on the electrode on the right hand side.

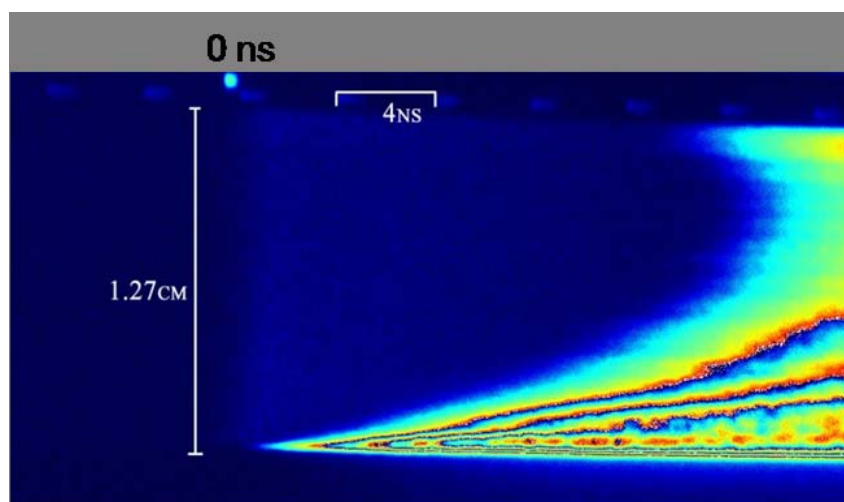


Figure 112. A streak image of a volumetric discharge in laboratory air.

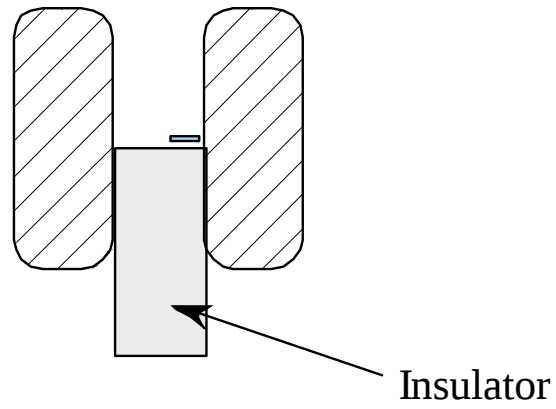


Figure 113. The view of the surface discharge gap as as seen from the streak camera.

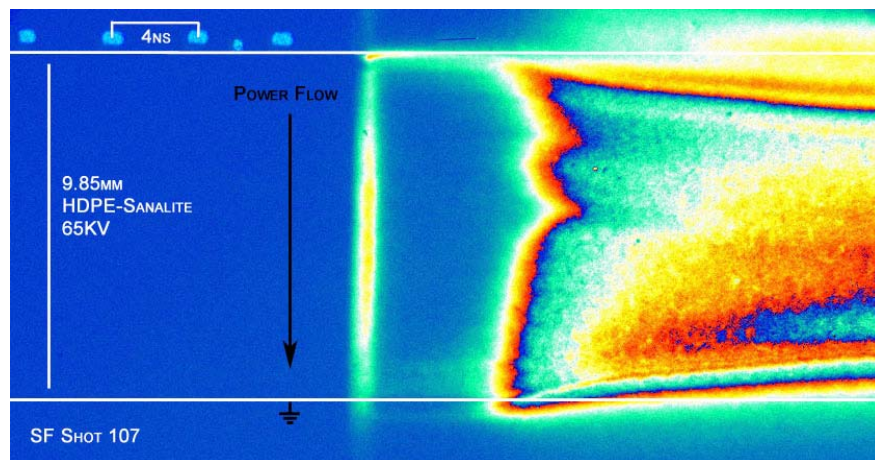


Figure 114. The streak image obtained with the EG&G camera of a discharge across a HDPE surface in air.

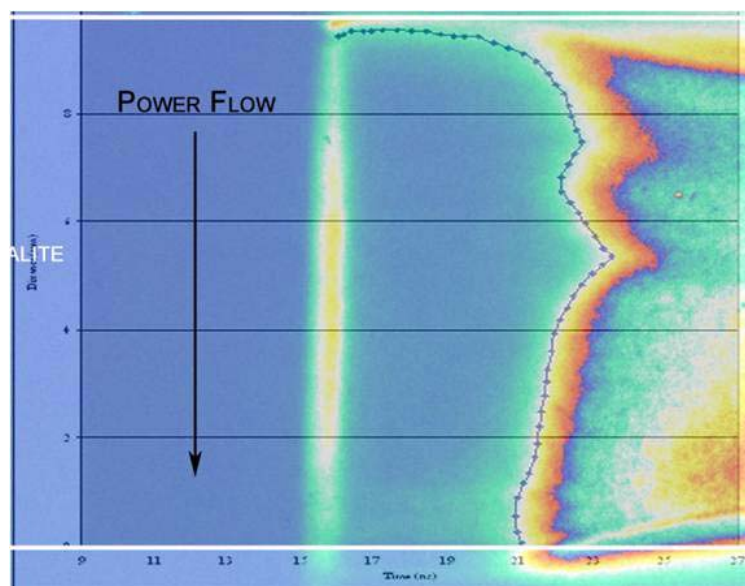


Figure 115. The streak image of an atmospheric air discharge across a HDPE surface was hand digitized to get distance versus time data. The shape of the curve is radially different than that measured in volumetric air and much faster.

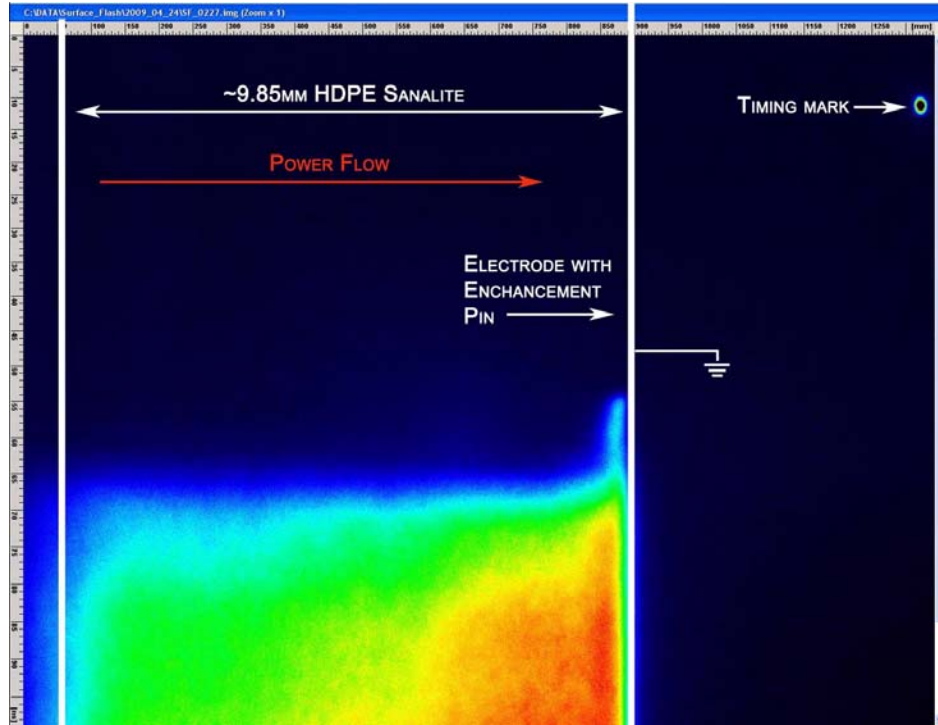


Figure 116. A screen shot from the Hammamatsu fast streak camera showing the timing mark as well as its relation to the gap geometry.

had an integrated data analysis package. The camera was set up to have an external timing mark so the streak images could be correlated to the electrical signals. The Hammamatsu C770 Streak camera is a state of the art streak camera with the capability to handle a large number of photoelectrons, allowing precise measurement with high dynamic range. It comes with a large photocathode (17 mm) which provides excellent spatial resolution. The time window on the streak image can be set from 0.5 ns to 1 ms of sweep ranges. The timing resolution is better than 5 ps and has an optical gain which varies from 10 to 1000. The Hammamatsu is used in conjunction with the HPD Temporal Analyzer, a high performance digital image processing system designed to read out images from the phosphor screen of Hammamatsu streak cameras. It provides remote control of all streak camera functions via GPIB and USB interfaces and enables precise acquisition and quantitative analysis of two dimensional streak data including a full range of data correction and calibration functions. A screen image of the streak camera is shown in Figure 116 illustrating the internal scales and external timing mark as well as the relation to the gap geometry.

The data processing associated with this fast streak camera was an evolving process throughout the project. Initially, we sought a fast comb, like the one we used with the EGG streak camera, to provide absolute timing for the Hammamatsu. We found that our 4 ns comb was a specialty item from the 1960s that could not be purchased, and the capability to make them has disappeared. Initially we used the comb from the EG&G camera on the Hammamatsu, but initial data showed the discharge across a surface is so fast that the 4 ns between timing markers was typically the time record of the streak image. Instead, a photodiode was used to correlate the streak data with the voltage signals. Initial streak data was similar to that found in the literature, where the information is mostly qualitative due to the lack of an absolute reference. Also, reflections from the inner metallic surfaces of the chamber were a concern. We were interested in the affect of pressure and surface material on the breakdown process and, thus, were consequently interested in making qualitative comparisons between datasets. We found that this could be done by extracting the light intensity levels recorded by the streak camera. Various levels were extracted

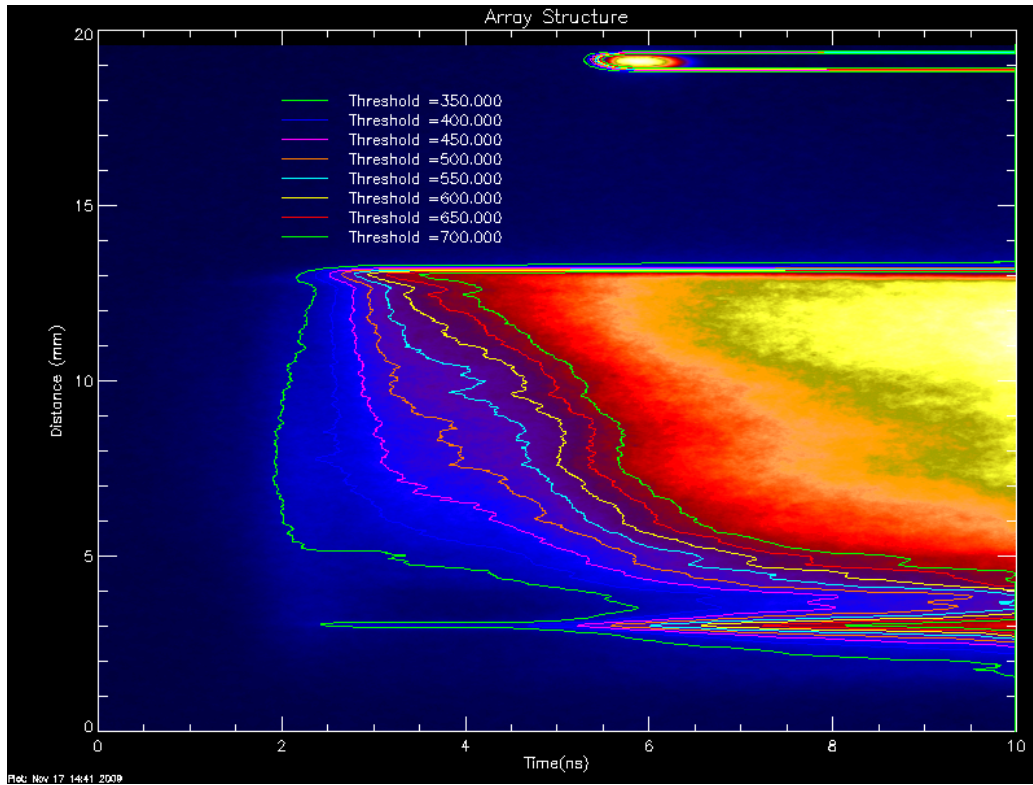


Figure 117. In processing the data of a streak image, the light intensity levels are extracted and then overlaid with the image.

using an IDL routine and re-plotted on the image as shown in Fig. 117. The streak image data can be presented as a function of spatial position and time for a given intensity level.

Late in the testing, we considered whether we could be measuring light from the far end of the chamber – in the same line of sight as the camera. We installed a light absorbing material from Thorlabs to address this reflection. While no strong correlation was detected, it is better to remove the possibility of a reflection. The possibility of low light levels and the effect of a light leak or other extraneous light was considered and resulted in the removal of the background radiation from the image. This was done by recording an image prior to the flashover event. This “background” can then be digitally subtracted from the streak image. Figure 118 is the same image as Figure 117 with the background radiation removed.

The small effect this has is shown in Figure 119. A comparison of intensity levels with and without the background removed makes little difference. Removing the background, however, remains a good practice which will continue to be incorporated into our data reduction technique.

It can be seen in the streak images that the time window is relatively large compared to the discharge velocity. The window must be chosen wide enough in time to account for the total time of the discharge and the jitter associated with the system, including the pulsed power. This is illustrated in Figure 120. Often, several shots were required to capture a streak image of a surface flashover.

9.2.1 Velocity Results

We have found that the introduction of a surface into a volume of high pressure gas greatly reduces the

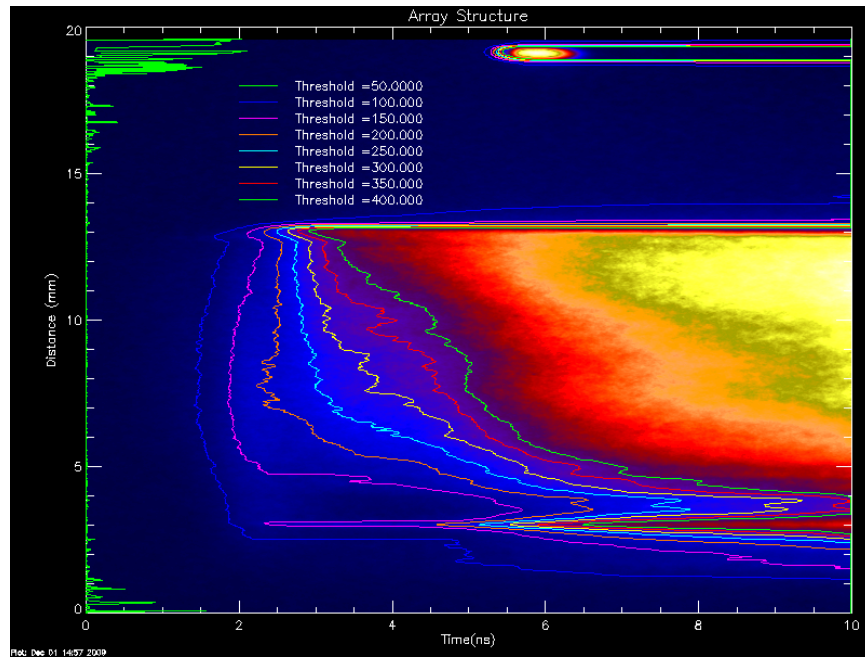


Figure 118. The streak image of the previous figure with the background radiation removed.

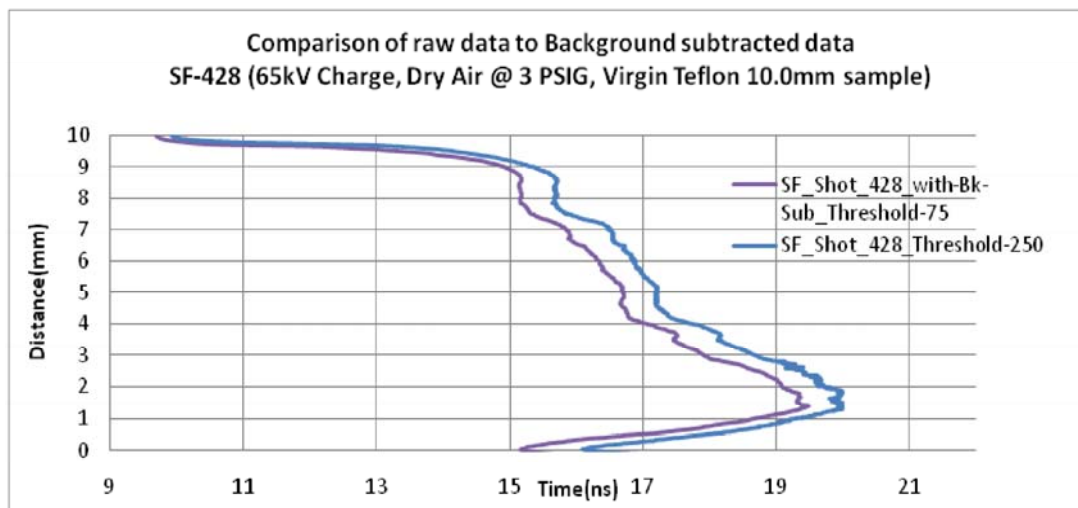


Figure 119. The extracted light intensity levels for a surface flashover shot with and without the background removed.

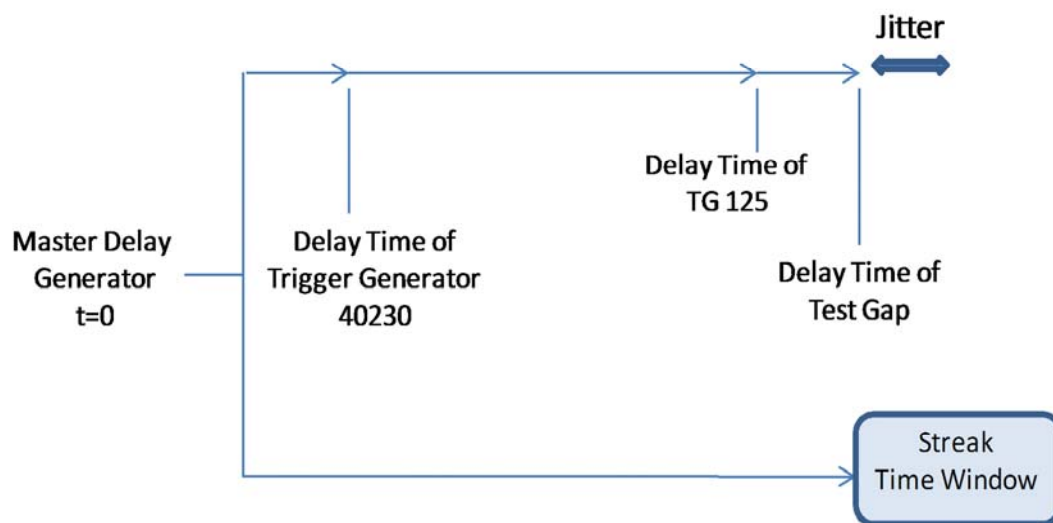


Figure 120. A schematic describing the effect of timing jitter in the system on the time resolution of the streak camera.

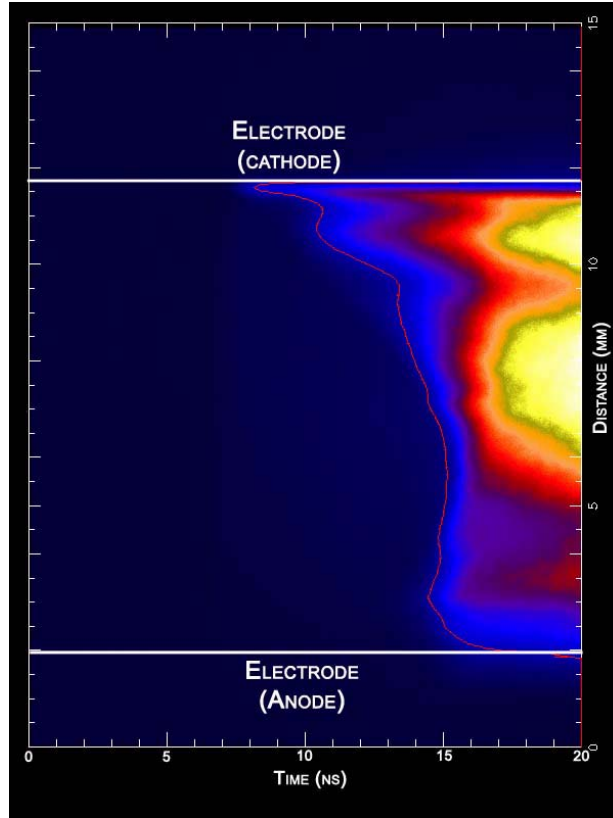


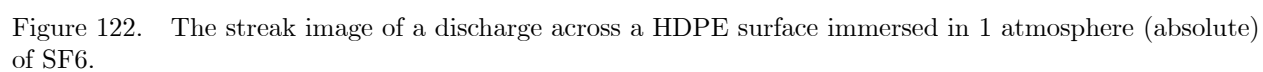
Figure 121. The streak image of a volume breakdown in SF6 at a pressure of 1 atmosphere (absolute).

time it takes for light to transverse the gap. Our images show this is true in air, SF6 and nitrogen, in both polarities, occurs in both Teflon and HDPE insulating surfaces, and at each of the pressures tested.

Representative data is shown in Figure 121 which is a volume breakdown in atmospheric SF6. The discharge initiates at the cathode and transverses the gap in about 5 ns. The streak image of this breakdown may be compared to the discharge when a HDPE surface is introduced, as shown in Figure 122. Using the data reduction techniques described here, a direct comparison can be made, as shown in Figure 123. The time to transverse the gap reduces from 5 ns to just a fraction of a ns. The overall jitter of the system must be reduced to resolve the velocity of the surface discharge further.

10 EXPERIMENTS WITH NONUNIFORM FIELD

In a uniform electrode geometry, the breakdown may be characterized by the critical electric field strength, above which breakdown occurs. In a non-uniform field, the discharge initiates in the vicinity of a high electric field point above the critical field, but the streamer, once it is formed, must pass through a low field region caused by the non-uniformity of the gap before bridging the gap. Here we intend to use the voltage collapse across the gap as an indicator of streamer propagation across the gap, in an attempt to determine sustaining field. A streamer's sustaining field, sometimes called a stability field, is the background electric field in which a streamer propagates without gaining or losing charge. Measurements of the streamer sustaining field are sparse and controversial [5], [68], [69], [57].



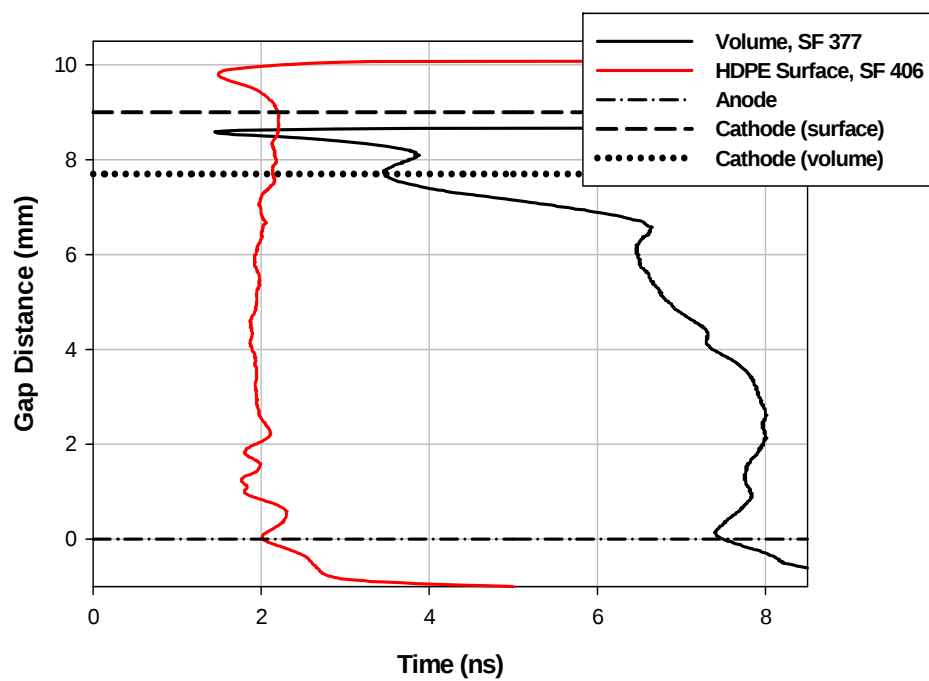


Figure 123. A comparison between intensity levels for the streak photographs of the two previous figures. The introduction of the surface speeds up the discharge by an order of magnitude.

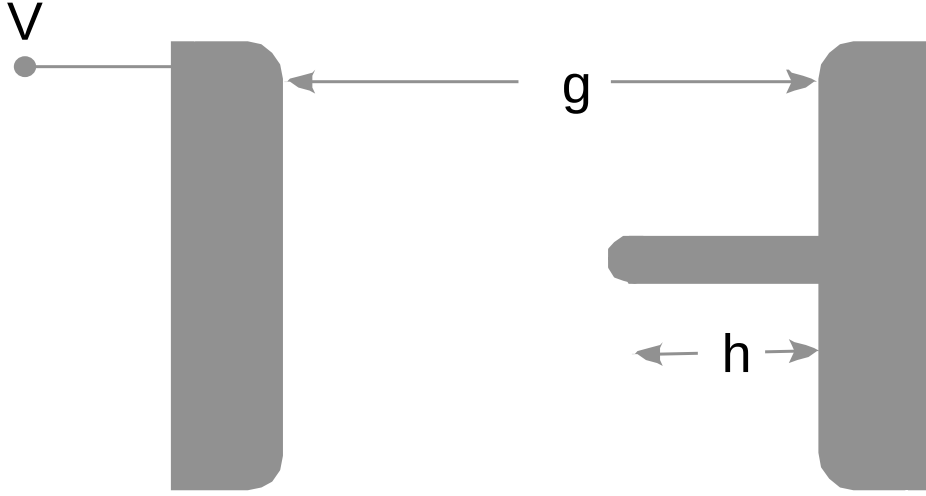


Figure 124. Illustration of the symbols used in calculating the minimum field. The diameter of the pin, $2a$, is 1 mm and the height is 10 mm in this experiment.

10.1 Description of Experiment

We measured the sustaining field in air and sulfur hexafluoride (SF6) by altering the electrode geometry of an experimental fixture we had used to investigate electrical breakdown in gases in the nanosecond regime. In the modeling effort, the sustaining field is obtained by allowing the streamer to form under the influence of an electric field, then dropping the driving electric field over a very short time to a much lower field. The field where the streamer neither grows nor dies is the minimum sustaining field. Experimentally, this is replicated by initiating a streamer with a small rod between parallel plate electrodes and applying a rectangular voltage pulse with a fast risetime and flat top. This electrode arrangement spatially distributes the electric field so that it first initiates the discharge then guides it through the sustaining field level. The electric field decreases with distance away from the field enhancement point. For the voltage across the gap to collapse, the discharge must transverse the low field region. The minimum electric field which will continue to advance the streamer is its sustaining field.

The electric field for the electrode arrangement used in the experiments has been derived analytically. The parameters used are shown in Figure 124 , where the pin has a diameter $2a$ and height h , and the electrodes are separated by a distance, g .

The minimum field in the gap occurs at the planar electrode, where $z = g$ and is given for our experiment and electrode geometry, $h/a = 20$ and the quantities $q_1 = 0.0822204$ and $Q_1 = 0.04580947$ by the formula

$$E_z^{tot}(0, g) \approx E_0 \left[1 - 2q_1 \ln \left(\frac{g+h}{g-h} \right) + 2q_1 \left(\frac{2gh}{g^2 - h^2} \right) + 2Q_1 \frac{4gh^3}{(g^2 - h^2)^2} \right] \quad (333)$$

where the external (without the pin) plane is

$$E_0 = V/g \quad (334)$$

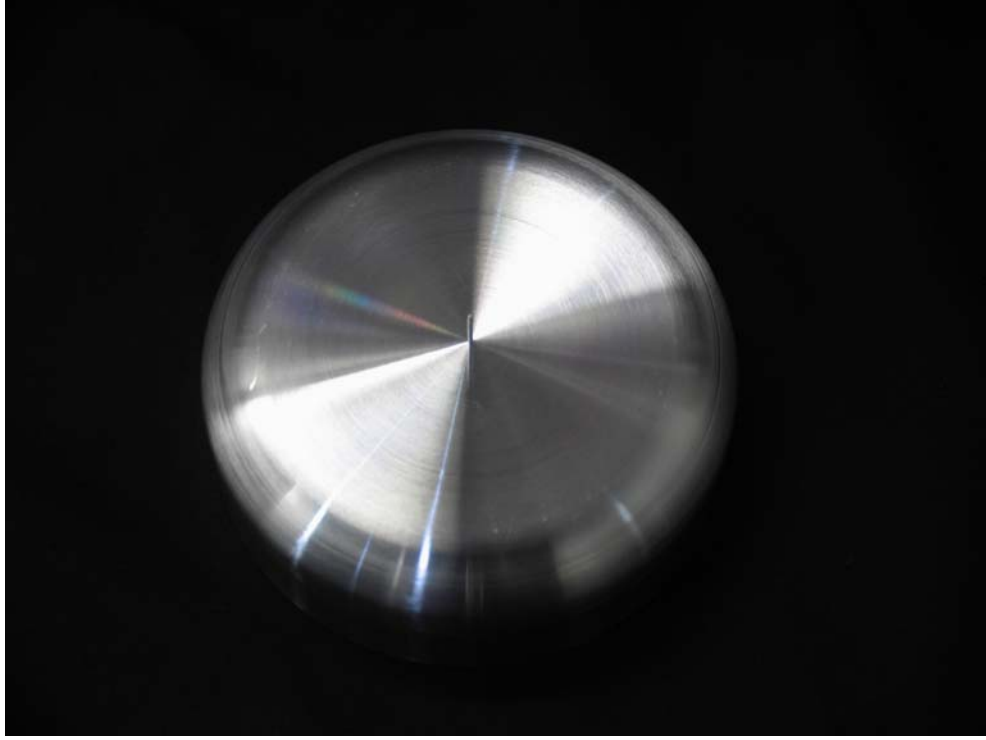


Figure 125. The sharp pin on the electrode initiates the discharge. The streamer must travel through a minimum field, the streamer sustaining field, to cross the gap and cause current to flow.

Note that for a very wide gap $h \ll g$ we have $E_z^{tot}(0, g) \rightarrow E_0$. The derivation of this can be found in the subsection below. The experiment consists of measuring the voltage at which breakdown occurs and calculating the minimum field in the gap. This minimum field can be used as an estimate for the sustaining field. Alternatively the average field in the gap

$$\langle E \rangle = \frac{V_0}{g - h} \quad (335)$$

is also often used to define sustaining field.

The electrodes are parallel plane with a diameter of 9.7 cm (3.8 inches) and spaced a maximum of 3.75 cm apart. The grounded electrode has a pin located at its center and is made from a single piece of stainless steel stock. The pin is 10 mm in height and 1 mm in diameter with a radius of 0.5 mm. The enhanced electrode is shown in Figures 125 and 126, and the other planar electrode has the same profile and dimensions but without the pin. The installed electrode set is shown in Figure 127. This is an open shutter photograph of the fixture in the light (as is customarily taken during experiments in the dark).

The experimental arrangement is shown in Figure 128 with the equipment and chamber shown in Figures 130 and 131. The data is acquired on a single timebase so that each signal is relative to a common datum. Relative timing is controlled with a master trigger generator. A rectangular pulse is generated with a TG-70 with maximum voltage amplitude of 65 kV. Once charged, the master trigger generator initiates the PT55 trigger generator which sends a high voltage (~ 50 kV) pulse to trigger the main unit, a TG-70. The TG 70 produces a rectangular pulse whose amplitude is varied with the charge voltage. The output of the TG-70 is shown in Figure 129.

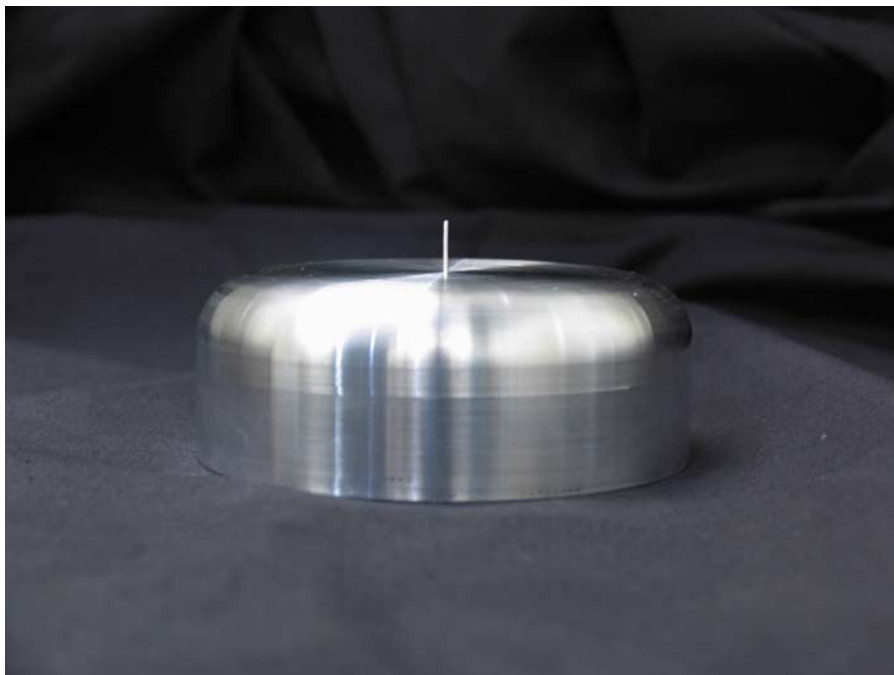


Figure 126. Another view of the electrode with the pin.



Figure 127. The test gap. The electrodes are identical with the exception of the pin.

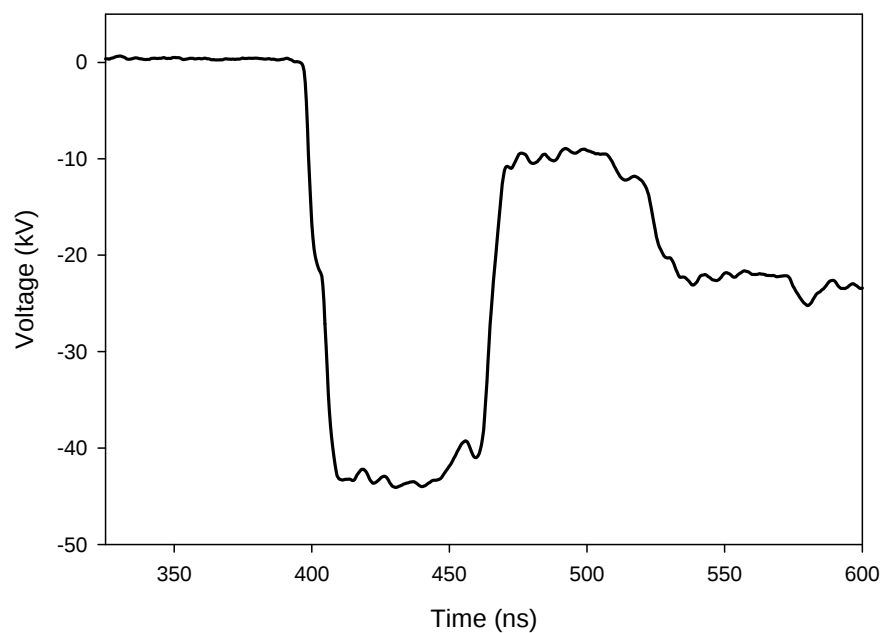


Figure 129. The excitation waveform for the sustaining field experiments.

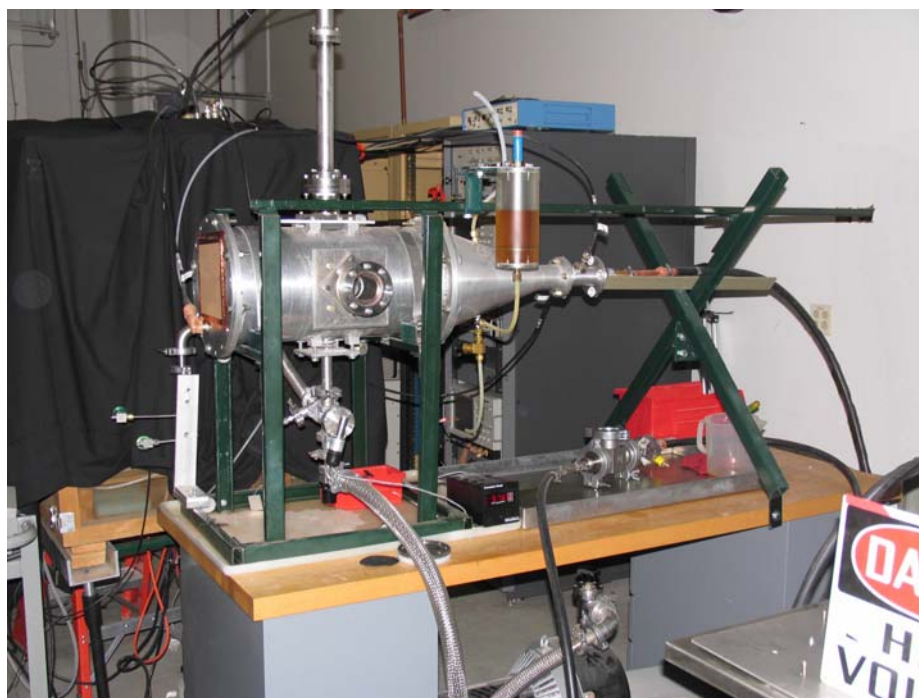


Figure 130. The chamber for studying gas discharges is suitable for pulses of nanosecond duration because it is well matched in impedance.



Figure 131. The experimental layout.

spatial and resistive variations. The load is designed to prevent reflections according to the equation

$$Z_0 = R(z) + Z(z) \quad (336)$$

where z is the direction of propagation, Z_0 is 52 ohms and the term $R(z)$ is the resistance of the inner conductor,

$$R(z) = R_0 z \quad (337)$$

R_0 is the resistance per unit length, and $Z(z)$ is the transmission line impedance variation with geometry,

$$Z(z) = \frac{60 \text{ ohms}}{\sqrt{\epsilon_r}} \ln \left[\frac{b}{a(z)} \right] \quad (338)$$

where b is the outer conductor radius and $a(z)$ is the radius of the inner conductor, which is allowed to vary in the direction of propagation, z . Substituting and solving for $a(z)$,

$$a(z) = b \exp \left[\frac{\sqrt{\epsilon_r}}{-60 \text{ ohms}} (52 \text{ ohms} - R_0 z) \right] \quad (339)$$

The resultant load geometry approximates the exponential shape in five series segments. The resistivity of the inner conductor is obtained by 22 carbon composition resistors connected in parallel.

The gap is formed by an interruption in the center conductor of the 52 ohm coaxial test chamber. The gap initially has a small capacitance between the electrodes before conduction occurs and the displacement current can be observed in the output current monitor. From the transmission line model, when the excitation pulse arrives at the gap, it is reflected until breakdown occurs. This incident voltage is recorded with a calibrated capacitive probe (D-dot) located in the return conductor. A resistive probe is used to accurately reproduce the discharge current and voltage at the load, located 4.5 ns from the input monitor.

To evaluate the sustaining fields, a rectangular voltage pulse was applied to the planar electrode. The pulse was reflected once it arrived at the open circuited gap. Once breakdown had initiated, the voltage across the gap collapsed and the voltage was read from the derivative voltage sensor located in the outer conductor near the electrode. This procedure was done for a number of gap lengths in both voltage polarities. The electrode containing the rod remained on the grounded side of the test gap. The breakdown level, along with the gap length was used to calculate the minimum field using the expression above.

10.2 Air Results

Atmospheric plasmas, especially atmospheric air plasmas, have undergone a renaissance since the 1980s for a variety of applications including novel plasma processing applications and remediation of gaseous pollutants. Air breakdown has been of interest to a wide body of researchers in different specialties starting when gas discharges were initially studied in the 1880s. There exists a large volume of research into long gap breakdown in air, notably by the Les Renardieres group. The physical processes that take place in air are relatively well understood and it has been possible to produce fairly reliable models to represent breakdown in air under a variety of conditions. The procedure described above was used to determine the sustaining field for dry (synthetic) air at gap lengths, $g = 1.674, 2.29$ and 2.75 cm. Atmospheric air has also been studied extensively under small gap conditions and is discussed extensively in Meek and Craggs and Raizer. Literature describing experiments to measure the streamer sustaining field in gases is sparse at best. Tables 26 and 27 give both the minimum field on the plane, from equation 333, and the average field from the pin tip to the plane, from equation 335 (because of the large pin this is not the same as the external field level in equation 334).

gap length (mm)	Minimum Field (kV/cm)	Average Electric Field (=V/(g-h)) (kV/cm)
16.74	25	49.2
22.9	24-2	40.1
27.5	26	42.1

Table 26. Cathode-directed (Positive Charging Voltage) in air.

gap length (mm)	Minimum Field (kV/cm)	Average Electric Field
16.74	26	50.9
22.9	26	41.5
27.5	27	41.8

Table 27. Anode-directed Streamers (Negative Charging Voltage) in air.

gap length (mm)	Minimum Field (kV/cm)	Average Electric Field
14.54	51	103.4
16.74	55	108.8

Table 28. Cathode-directed (Positive Charging Voltage) in SF6.

10.3 SF6 Results

Sulfur hexafluoride is a gas that has been studied extensively for use in the power industry because of its electronegative properties and relatively high electrical strength. The concept of the sustaining field was introduced in conjunction with breakdown in long air gaps and remains controversial in SF6. Sulfur hexafluoride is well known to be susceptible to local enhancements. Experiments with rough electrode surface show a marked decrease in breakdown strength. In practical applications then, the sustaining field is particularly important for non-uniform geometries where it is entirely plausible that a streamer could be initiated but then travel through a region of much lower field strength. Tables 28 and 29 give both the minimum field on the plane, from equation 333, and the average field from the pin tip to the plane, from equation 335.

Typical data for the sustaining field experiments are shown in Figure 132 for a gap size of 1.454 cm, showing the current rise as the voltage begins to fall. The doubling of the incident voltage pulse can be seen in Figure 132 until the breakdown voltage has been reached, and current flows through the gap. We are limited in our voltage capability so the minimum field is calculated by means of equation 333 instead of its limiting value for wide gaps, equation 334.

In SF6, the repeatability of the data is remarkable. In extracting the voltage values, the numerical values of the voltage were within 0.1 kV, independent of time. This is illustrated in Figure 133 where an overlay of the data indicates when the breakdown field level was reached. This repeatability is seen in both polarities, as shown in Figure 134 for the positive polarity. The detail is indicated in Figure 135.

In our experiments, when the gap does not breakdown, corona is observed on the tip of the pin electrode. The corona in SF6 is much brighter than in air. It also shows up equally bright with polarity changes in SF6 but not in air.

Use of voltage collapse to determine sustaining field as we have done here is somewhat controversial. Photography of streamer propagation and length might be more definitive. Voltage collapse requires eventual channel thermalization, which may require several generations of avalanches and streamers. Also our gaps were limited in length (partly due to source constraints), where electrode effects may have played a role (Townsend mechanism) and where stored energy in the streamer head could result in the streamer crossing very low field regions of limited spatial extent. Meek and Craggs estimate the boundary for a fully formed streamer in atmospheric air to be on the order of 5 cm, or $pd > 4000$ Torr-cm. In discussing the avalanche-to-streamer transition, Raizer remarks that “it is likely that certain intermediate forms exist in the boundary range $pd \sim 200 - 5000$ Torr-cm. Our gap spacings are in the range $g = 1.674 - 2.75$ cm, and at local pressure of 620 Torr, $pd \sim 1040 - 1708$ Torr-cm and we can assume the breakdown mechanism is in the transition region. Future work will investigate larger gaps.

10.4 Rod-Plane Problem

We now give the approximate solution for the field off the tip of a rod in a rod-plane gap. We modify techniques from [32] for uniform field excitation of the rod on a plane, then we approximately add in the other plane by inserting an image. The geometry is shown in Figure 136. The cylindrical radius is r in this

gap length (mm)	Minimum Field (kV/cm)	Average Electric Field
14.54	51	103.1
16.74	55	108.8

Table 29. Anode-directed Streamers (Negative Charging Voltage) in SF6.

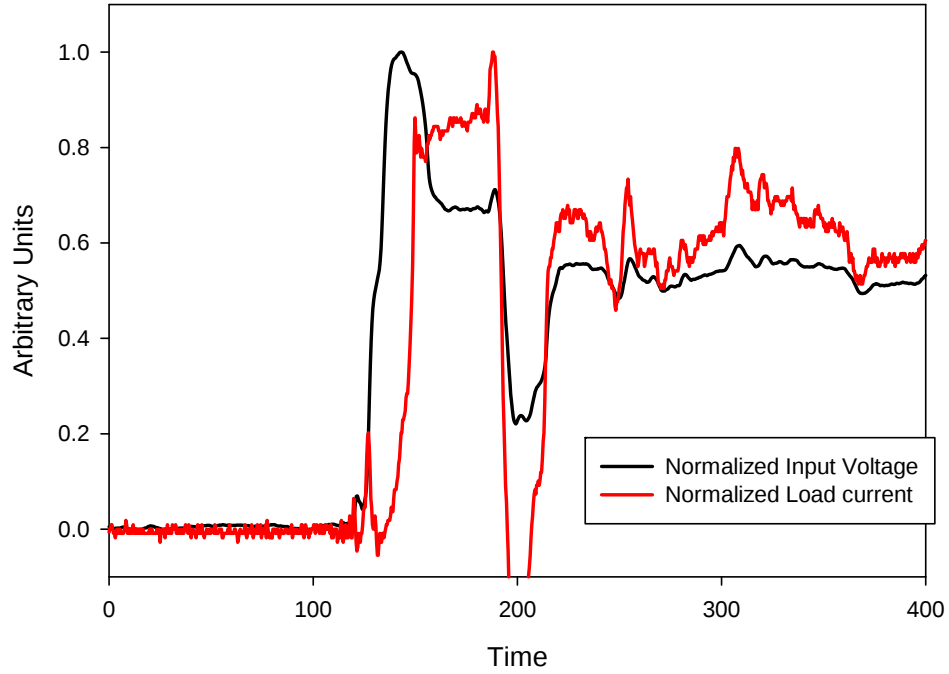


Figure 132. Overlays of the normalized waveforms of the input voltage and load current showing their relative timing. The current waveform is shift 4.5 ns to account for physical distance between the monitors. The first current bump is the capacitance of the gap.

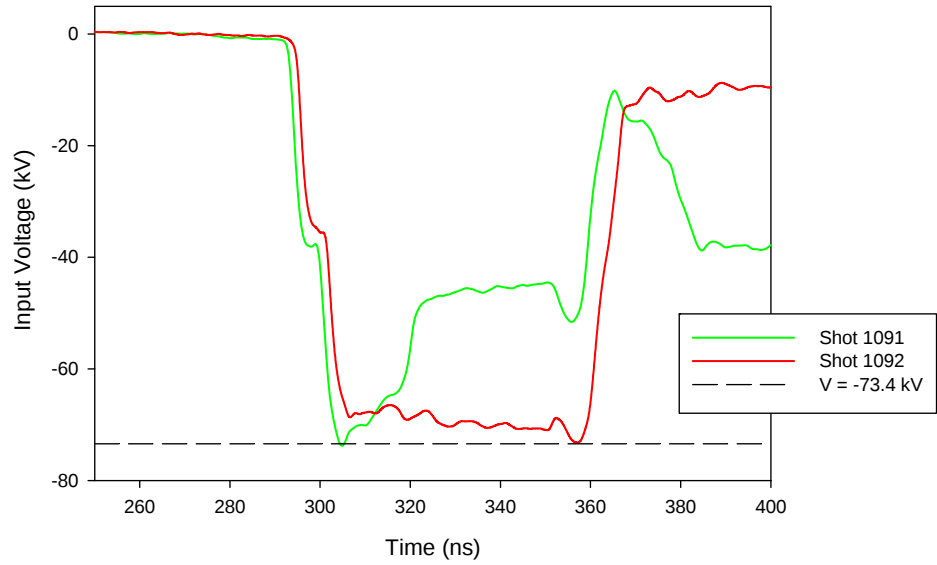


Figure 133. The negative voltage waveforms for SF6 at 1 atm during the sustaining field calculations.

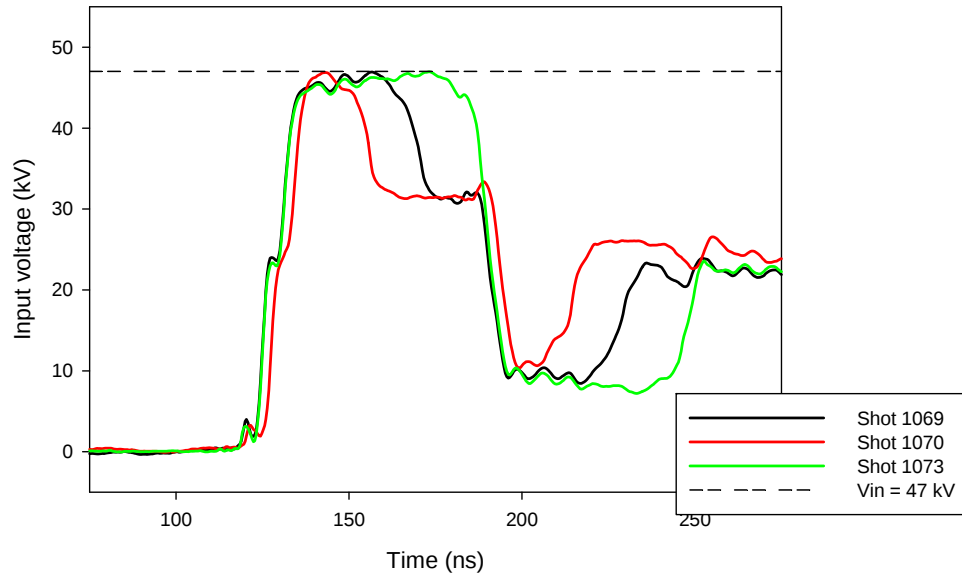


Figure 134. The gap does not breakdown until the value at the tip of the pin exceeds the critical value. The experiment shows very repeatable results.

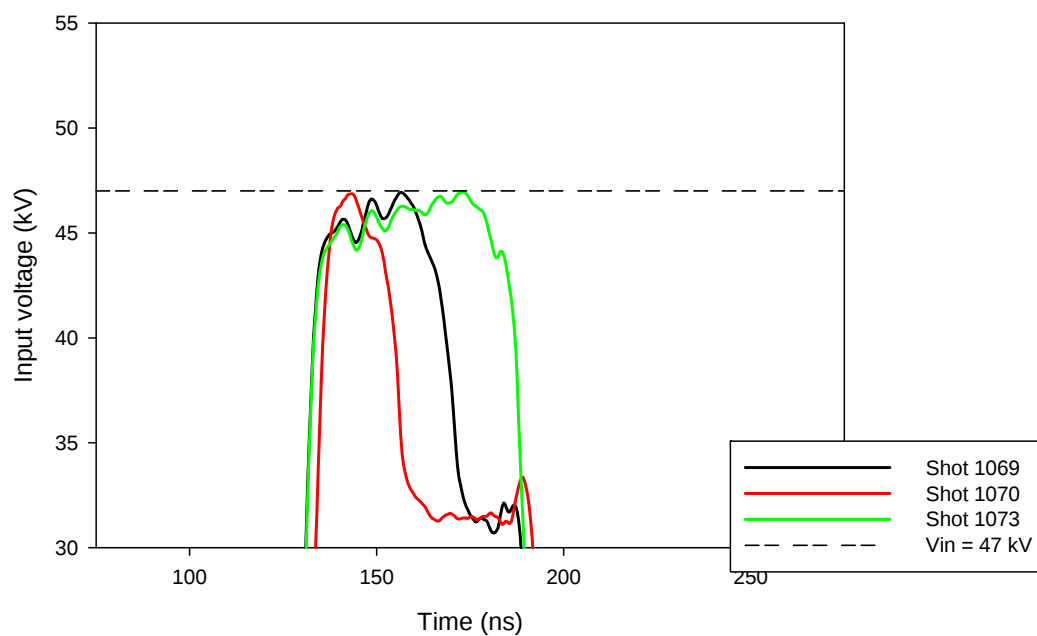


Figure 135. A blowup of three overlaid voltage signals showing the repeatability of the measurement.

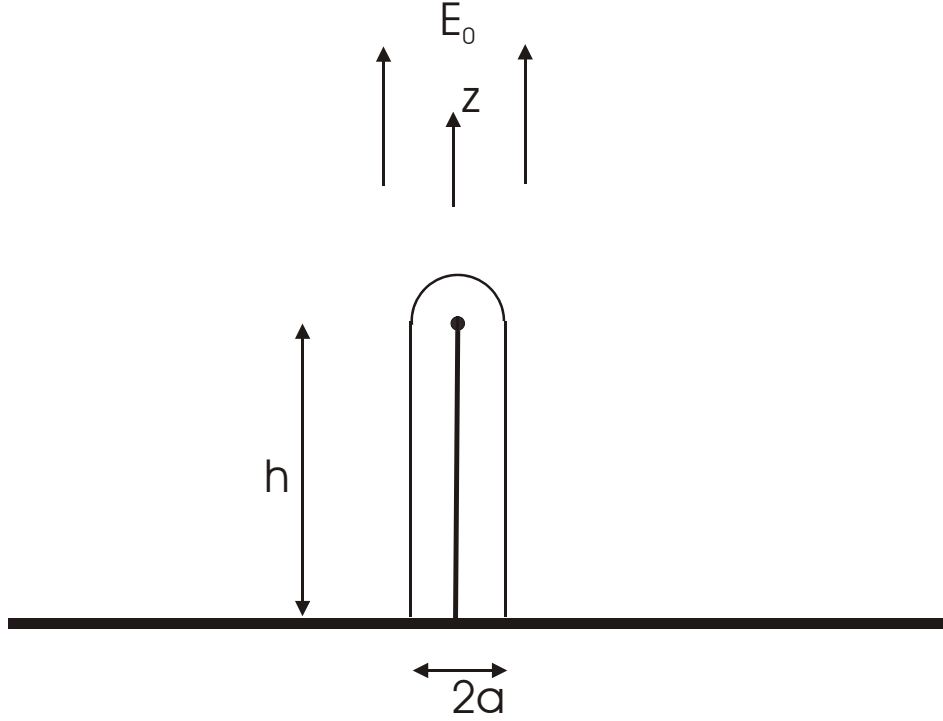


Figure 136. Rod-plane geometry excited by uniform field.

subsection.

The external field is taken as $E_0 \underline{e}_z$ with potential

$$\phi_0 = -E_0 z \quad (340)$$

The total potential is taken as

$$\phi_{tot} = \phi_0 + \phi \quad (341)$$

10.4.1 Line Charge

The rod is treated as a cylinder with line charge density $q(z)$, $0 < z < h$. The potential from this contribution is then imaged about the ground plane at $z = 0$ to give

$$\phi(r, z) = \frac{1}{4\pi\epsilon_0} \int_{-h}^h \frac{q(z')}{\sqrt{r^2 + (z - z')^2}} dz' \quad (342)$$

where $q(-z) = -q(z)$. Enforcing the equipotential boundary condition, which is taken to be zero, gives the integral equation

$$4\pi\epsilon_0 E_0 z = \int_{-h}^h \frac{q(z')}{\sqrt{a^2 + (z - z')^2}} dz' \quad (343)$$

Following the usual iteration technique

$$\begin{aligned}
\int_{-h}^h \frac{dz'}{\sqrt{a^2 + (z - z')^2}} &= \int_{z-h}^{z+h} \frac{du}{\sqrt{a^2 + u^2}} = \text{Arcsinh} \left(\frac{h+z}{a} \right) + \text{Arcsinh} \left(\frac{h-z}{a} \right) \\
&= \ln \left[(h+z)/a + \sqrt{1 + (h+z)^2/a^2} \right] + \ln \left[(h-z)/a + \sqrt{1 + (h-z)^2/a^2} \right] \\
&\sim \ln (1 - z^2/h^2) + \Omega, \quad 0 < z < h
\end{aligned} \tag{344}$$

where the fatness parameter is

$$\Omega = 2 \ln (2h/a) \tag{345}$$

Thus we write

$$\begin{aligned}
4\pi\epsilon_0 E_0 z &\sim \int_{-h}^h \frac{q(z') - q(z)}{|z - z'|} dz' \\
&+ q(z) [\ln (1 - z^2/h^2) + \Omega] \\
&\sim \frac{\Omega}{4\pi\epsilon_0} q(z)
\end{aligned} \tag{346}$$

Therefore the zero order solution for the charge per unit length is

$$q(z) \sim q_0 z = \frac{4\pi\epsilon_0}{\Omega} E_0 z \tag{347}$$

10.4.2 Far Field

Far from the rod tip (but near the axis of the rod) we have

$$\begin{aligned}
\phi(r, z) &\sim \frac{q_0}{4\pi\epsilon_0} \int_{-h}^h \frac{z'}{\sqrt{r^2 + (z - z')^2}} dz' \\
&\sim \frac{q_0}{4\pi\epsilon_0} \left[\sqrt{r^2 + (z - h)^2} - \sqrt{r^2 + (z + h)^2} \right] + \frac{q_0 z}{4\pi\epsilon_0} \int_{-h}^h \frac{dz'}{\sqrt{r^2 + (z - z')^2}} \\
&= \frac{q_0}{4\pi\epsilon_0} \left[\sqrt{r^2 + (z - h)^2} - \sqrt{r^2 + (z + h)^2} \right] \\
&+ \frac{q_0 z}{4\pi\epsilon_0} \left[\text{Arcsinh} \left(\frac{h+z}{r} \right) + \text{Arcsinh} \left(\frac{h-z}{r} \right) \right]
\end{aligned}$$

$$\begin{aligned}
&= \frac{q_0}{4\pi\epsilon_0} \left[\sqrt{r^2 + (z-h)^2} - \sqrt{r^2 + (z+h)^2} \right] \\
&+ \frac{q_0 z}{4\pi\epsilon_0} \ln \left[\frac{(z+h)/r + \sqrt{1 + (z+h)^2/r^2}}{(z-h)/r + \sqrt{1 + (z-h)^2/r^2}} \right]
\end{aligned} \tag{348}$$

On axis $r \rightarrow 0$

$$\begin{aligned}
\phi &\sim -\frac{q_0 h}{2\pi\epsilon_0} \\
&+ \frac{q_0 z}{4\pi\epsilon_0} [\ln(z+h) - \ln(z-h)]
\end{aligned} \tag{349}$$

The scattered axial electric field on axis is thus

$$E_z(0, z) \sim -\frac{q_0}{4\pi\epsilon_0} [\ln(z+h) - \ln(z-h)] - \frac{q_0 z}{4\pi\epsilon_0} \left(\frac{1}{z+h} - \frac{1}{z-h} \right) \tag{350}$$

10.4.3 Line And Point Charges

To determine an approximation to the field which is valid from the tip outward we must include the charge concentrations near the end of the rod. A simple assumption would be

$$q(z') = q_0 z' + Q_0 \delta(h - z') - Q_0 \delta(h + z') \tag{351}$$

The potential is

$$\begin{aligned}
4\pi\epsilon_0\phi(a, z) &= \int_{-h}^h \frac{q(z') dz'}{\sqrt{a^2 + (z - z')^2}} \\
&\sim q(z) \int_{-h}^h \frac{dz'}{\sqrt{a^2 + (z - z')^2}} + \int_{-h}^h \frac{q(z') - q(z)}{|z' - z|} dz' \\
&+ Q_0 \left[\frac{1}{\sqrt{a^2 + (h - z)^2}} - \frac{1}{\sqrt{a^2 + (h + z)^2}} \right] \\
&\sim q_0 z \int_{-h}^h \frac{dz'}{\sqrt{a^2 + (z - z')^2}} + q_0 \int_{-h}^h \text{sgn}(z' - z) dz' \\
&+ Q_0 \left[\frac{1}{\sqrt{a^2 + (h - z)^2}} - \frac{1}{\sqrt{a^2 + (h + z)^2}} \right]
\end{aligned} \tag{352}$$

where the integrals are defined to leave out the end point charges. Then

$$\begin{aligned}
& 4\pi\epsilon_0 E_0 z = \\
& = q_0 z \left[\ln(1 - z^2/h^2) + \Omega \right] + 2q_0 z \\
& + Q_0 \left[\frac{1}{\sqrt{a^2 + (h - z)^2}} - \frac{1}{\sqrt{a^2 + (h + z)^2}} \right]
\end{aligned} \tag{353}$$

Let us enforce the average of this times z over the interval

$$\begin{aligned}
& 4\pi\epsilon_0 E_0 \frac{1}{2h} \int_{-h}^h z^2 dz = \\
& = \frac{1}{2h} \int_{-h}^h q_0 z^2 \left[\ln(1 - z^2/h^2) + \Omega + 2 \right] dz \\
& + Q_0 \frac{1}{2h} \int_{-h}^h \left[\frac{1}{\sqrt{a^2 + (h - z)^2}} - \frac{1}{\sqrt{a^2 + (h + z)^2}} \right] z dz
\end{aligned} \tag{354}$$

or using

$$\begin{aligned}
& \int_{-h}^h \frac{z' dz'}{\sqrt{a^2 + (z - z')^2}} = \sqrt{a^2 + (z - h)^2} - \sqrt{a^2 + (z + h)^2} \\
& + z \ln \left\{ (h + z)/a + \sqrt{1 + (h + z)^2/a^2} \right\} + z \ln \left\{ (h - z)/a + \sqrt{1 + (h - z)^2/a^2} \right\}
\end{aligned} \tag{355}$$

and

$$\begin{aligned}
& \int_{-h}^h \ln(1 - z^2/h^2) z^2 dz \\
& = h^3 \int_{-1}^1 \left[(u - 1)^2 + 2(u - 1) + 1 \right] \ln(1 - u) du + h^3 \int_{-1}^1 \left[(u + 1)^2 - 2(u + 1) + 1 \right] \ln(1 + u) du \\
& = 2h^3 \int_0^2 \left[x^2 - 2x + 1 \right] \ln x dx = 2h^3 \left[\frac{1}{3} x^3 \left(\ln x - \frac{1}{3} \right) - 2 \frac{1}{2} x^2 \left(\ln x - \frac{1}{2} \right) + x (\ln x - 1) \right]_0^2 \\
& = h^3 \frac{4}{3} \left(\ln 2 - \frac{4}{3} \right)
\end{aligned} \tag{356}$$

gives

$$4\pi\epsilon_0 E_0 h^2/3 =$$

$$\begin{aligned}
&= q_0 \left(2 \ln 2 - \frac{2}{3} + \Omega \right) h^2/3 \\
&+ Q_0 \left[(a/h) \left(1 - \sqrt{1 + 4h^2/a^2} \right) + \ln \left\{ 2h/a + \sqrt{1 + 4h^2/a^2} \right\} \right]
\end{aligned} \tag{357}$$

or dropping the quadratic terms in a/h

$$\begin{aligned}
4\pi\epsilon_0 E_0 h^2/3 &= \\
&= q_0 \left(2 \ln 2 - \frac{2}{3} + \Omega \right) h^2/3 \\
&+ Q_0 (a/h + \Omega/2 + \ln 2 - 2)
\end{aligned} \tag{358}$$

The other match condition is taken at the tip

$$\begin{aligned}
4\pi\epsilon_0 \phi(r, z) &= \int_{-h}^h \frac{q(z') dz'}{\sqrt{r^2 + (z - z')^2}} \\
&= q_0 \int_{-h}^h \frac{z' dz'}{\sqrt{r^2 + (z - z')^2}} + Q_0 \left[\frac{1}{\sqrt{r^2 + (h - z)^2}} - \frac{1}{\sqrt{r^2 + (h + z)^2}} \right] \\
&= q_0 \left[\sqrt{r^2 + (z - h)^2} - \sqrt{r^2 + (z + h)^2} \right] \\
&+ q_0 z \ln \left[\frac{(z + h)/r + \sqrt{1 + (z + h)^2/r^2}}{(z - h)/r + \sqrt{1 + (z - h)^2/r^2}} \right] \\
&+ Q_0 \left[\frac{1}{\sqrt{r^2 + (h - z)^2}} - \frac{1}{\sqrt{r^2 + (h + z)^2}} \right]
\end{aligned} \tag{359}$$

On axis $r = 0$ and $z > h$

$$4\pi\epsilon_0 \phi(0, z) = -q_0 2h + q_0 z [\ln(z + h) - \ln(z - h)] + Q_0 \left(\frac{1}{z - h} - \frac{1}{z + h} \right) \tag{360}$$

At the tip $z = h + a$

$$4\pi\epsilon_0 E_0 (h + a) = q_0 [(h + a) \ln(2h/a + 1) - 2h] + Q_0 \left(\frac{1}{a} - \frac{1}{2h + a} \right) \tag{361}$$

10.4.4 On Axis Field

The field on axis is then

$$4\pi\epsilon_0 E_z(0, z) = -q_0 [\ln(z+h) - \ln(z-h)] - q_0 z \left[\frac{1}{z+h} - \frac{1}{z-h} \right] + Q_0 \left[\frac{1}{(z-h)^2} - \frac{1}{(z+h)^2} \right] \quad (362)$$

The total field on axis is

$$E_z^{tot}(0, z) = E_0 - \frac{q_0}{4\pi\epsilon_0} [\ln(z+h) - \ln(z-h)] - \frac{q_0}{4\pi\epsilon_0} z \left[\frac{1}{z+h} - \frac{1}{z-h} \right] + \frac{Q_0}{4\pi\epsilon_0} \left[\frac{1}{(z-h)^2} - \frac{1}{(z+h)^2} \right] \quad (363)$$

10.4.5 Integral Equation

Because of the fundamental nature of the rod-to-plane gap we have also solved the rod problem on a plane with an integral equation method. For numerical purposes it is convenient to replace the thin wire Green's function by the exact elliptic kernel form

$$\begin{aligned} \frac{1}{\sqrt{(z-z')^2 + r^2}} &\Rightarrow \frac{1}{2\pi} \int_0^{2\pi} \frac{d\varphi'}{\sqrt{(z-z')^2 + (r \sin \varphi - r' \sin \varphi')^2 + (r \cos \varphi - r' \cos \varphi')^2}} \\ &= \frac{2/\pi}{\sqrt{(z-z')^2 + (r+r')^2}} \int_0^{\pi/2} \frac{dt}{\sqrt{(1-\xi^2 \sin^2 t)}} \\ &= \frac{\xi}{\pi \sqrt{rr'}} K(\xi) \end{aligned} \quad (364)$$

where the complete elliptic integral of the first kind is

$$K(\xi) = \int_0^{\pi/2} \frac{dt}{\sqrt{1-\xi^2 \sin^2 t}} \quad (365)$$

and

$$\xi = \sqrt{\frac{4rr'}{(z-z')^2 + (r+r')^2}} \quad (366)$$

Inserting the images and the integration over the hemisphere gives the integral equation

$$\begin{aligned}
4\pi\epsilon_0 E_0 z &= \int_0^h q_c(z') \left[\frac{\xi}{\pi\sqrt{ra}} K(\xi) - \frac{\xi^*}{\pi\sqrt{ra}} K(\xi^*) \right] dz' \\
&+ \int_0^{\pi a/2} q_s(s') \left[\frac{\xi}{\pi\sqrt{rr'}} K(\xi) - \frac{\xi^*}{\pi\sqrt{rr'}} K(\xi^*) \right] ds'
\end{aligned} \tag{367}$$

where

$$\xi^* = \sqrt{\frac{4rr'}{(z+z')^2 + (r+r')^2}} \tag{368}$$

Note that in the first integral

$$r' = a \tag{369}$$

and in the second integral

$$a\varphi' = s' \tag{370}$$

$$r' = a \cos \varphi' = a \cos (s'/a) \tag{371}$$

$$z' = h + a \sin \varphi' = h + a \sin (s'/a) \tag{372}$$

The cylindrical surface has

$$r = a \tag{373}$$

and the hemispherical surface has

$$a\varphi = s \tag{374}$$

$$r = a \cos (s/a) \tag{375}$$

$$z = h + a \sin (s/a) \tag{376}$$

We think of $s' = a\varphi'$. Now the solution is found as the basis expansions

$$q_c(z') = \sum_{n=1}^{N_c} q_n p_n(z') \tag{377}$$

$$q_s(s') = \sum_{n=1}^{N_s} q_n p_n(s') \tag{378}$$

$$p_n(z') = 1, \quad z'_{n-1} < z' < z'_n$$

$$= 0 \text{ , otherwise} \quad (379)$$

$$z'_0 = 0 \quad (380)$$

$$z'_{N_c} = h \quad (381)$$

$$s'_0 = 0 \quad (382)$$

$$s'_{N_s} = \pi a/2 \quad (383)$$

$$z_n = (z'_n + z'_{n-1})/2 \text{ , } n = 1, \dots, N_c \quad (384)$$

$$s_n = (s'_n + s'_{n-1})/2 \text{ , } n = 1, \dots, N_s \quad (385)$$

We will take the basis function distributed according to

$$s'_n = (\pi a/2) n/N_s \quad (386)$$

$$z'_n = hn/N_c \quad (387)$$

A comparison of the axial electric field on axis is shown in Figure 137 for $h = 1$ cm and in Figure 138 for $h = 4$ cm with $a = 0.2$ cm.

10.4.6 Field At Plane Opposite From Plane With Rod

The preceding solution is for a rod of radius a and height h (for the line charge model the hemispherical cap is not really present) on a plane immersed in a uniform field E_0 . The geometry here is $a = 0.05$ cm and $h = 1$ cm. The approximate line charge formulas are

$$q_0 = \frac{4\pi\epsilon_0}{\Omega_e} E_0 \quad (388)$$

$$\Omega_e = \Omega + 2(\ln 2 - 7/3) \quad (389)$$

$$\Omega = 2 \ln(2h/a) \quad (390)$$

The scattered axial electric field on axis is thus

$$E_z(0, z) \sim -\frac{q_0}{4\pi\epsilon_0} [\ln(z+h) - \ln(z-h)] - \frac{q_0 z}{4\pi\epsilon_0} \left(\frac{1}{z+h} - \frac{1}{z-h} \right) \quad (391)$$

The following plot in Figure 139 shows the accuracy of the approximate models compared to the numerical

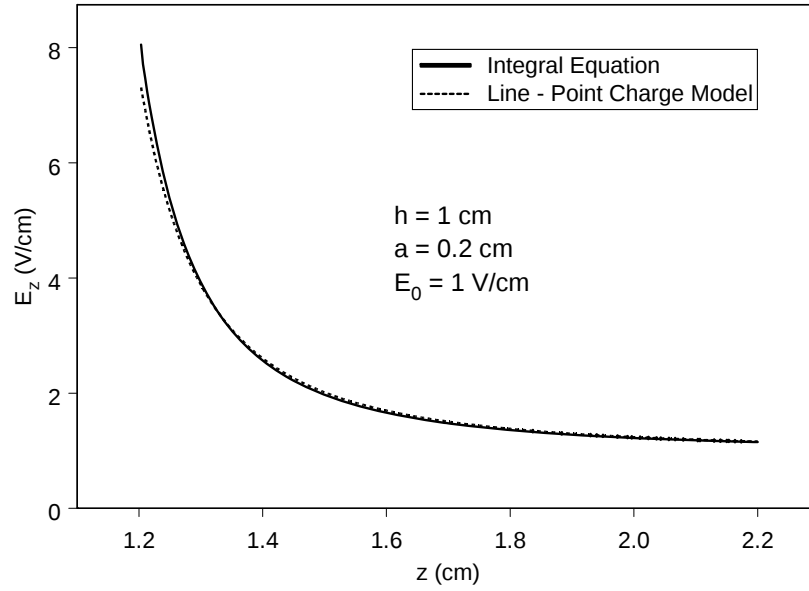


Figure 137. Comparison of axial field in a rod plane geometry. In this case the rod connects to the plane and the excitation is a uniform field in the z direction. The numerical solution of the integral equation and the approximate formula are compared. The rod height is $h = 1$ cm.

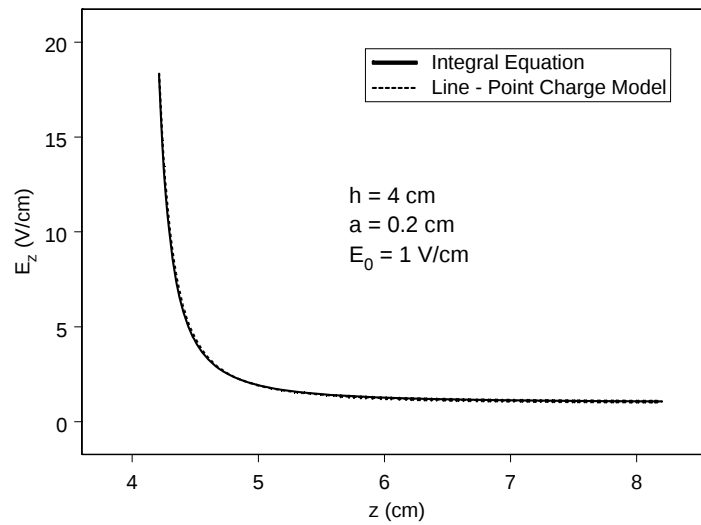


Figure 138. Comparison of axial field in a rod plane geometry. In this case the rod connects to the plane and the excitation is a uniform field in the z direction. The numerical solution of the integral equation and the approximate formula are compared. The rod height is $h = 4$ cm.

solution for the total field (scattered plus E_0). Note that when we approach 1 cm from the rod tip ($z = 2$ cm) the total field is very close to E_0 . This means that for these types of distances we can approximately ignore the effect of the rod on the field and evaluate the field on the plane as the voltage over the plane-to-plane gap. To obtain a formula for the field on the plane including the rod we can image the preceding results in the other plane at $z = g > h$.

$$E_z^{tot}(0, z) \approx E_0 + E_z(0, z) + E_z(0, 2g - z) \quad (392)$$

Using the line charge approximation and setting

$$E_0 = V/g \quad (393)$$

gives

$$E_z^{tot}(0, g) \approx \frac{V}{g} \left[1 - \frac{2}{\Omega_e} \ln \left(\frac{g+h}{g-h} \right) + \frac{2}{\Omega_e} \left(\frac{2gh}{g^2-h^2} \right) \right] \quad (394)$$

If we look at the case where $g = 2h$ and $h/a = 20$

$$E_z^{tot}(0, g) \approx \frac{V}{g} \left[1 + \left(\frac{4}{3} - \ln 3 \right) / (\ln 80 - 7/3) \right] \approx \frac{V}{g} (1.115) \quad (395)$$

For $g = 3h/2$

$$E_z^{tot}(0, g) \approx \frac{V}{g} \left[1 + \left(\frac{12}{5} - \ln 5 \right) / (\ln 80 - 7/3) \right] \approx \frac{V}{g} (1.386) \quad (396)$$

For $g = 1.7h$

$$E_z^{tot}(0, g) \approx \frac{V}{g} (1.22) \quad (397)$$

Line And Point Charges

The more accurate solution using line and point charges determines q_0 and Q_0 by means of the match equations

$$\begin{aligned} 4\pi\epsilon_0 E_0 h^2/3 &= \\ &= q_0 \left(2 \ln 2 - \frac{2}{3} + \Omega \right) h^2/3 \\ &+ Q_0 (a/h + \Omega/2 + \ln 2 - 2) \end{aligned} \quad (398)$$

and

$$4\pi\epsilon_0 E_0 (h+a) = q_0 [(h+a) \ln(2h/a+1) - 2h] + Q_0 \left(\frac{1}{a} - \frac{1}{2h+a} \right) \quad (399)$$

The scattered field on axis is then

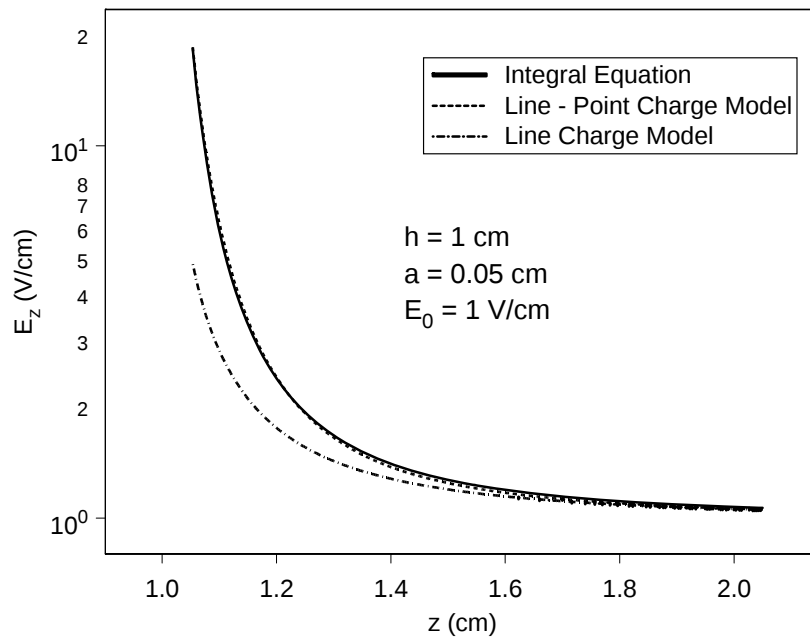


Figure 139. Comparison of line charge and line-point charge models for rod in iniform field with numerical solution.

$$4\pi\epsilon_0 E_z(0, z)$$

$$= -q_0 [\ln(z+h) - \ln(z-h)] - q_0 z \left[\frac{1}{z+h} - \frac{1}{z-h} \right] + Q_0 \left[\frac{1}{(z-h)^2} - \frac{1}{(z+h)^2} \right] \quad (400)$$

The total field on axis with both planes is then approximately

$$E_z^{tot}(0, z) \approx E_0 + E_z(0, z) + E_z(0, 2g - z) \quad (401)$$

The field at the other plane $z = g$ is

$$E_z^{tot}(0, g) \approx E_0 \left[1 - 2q_1 \ln \left(\frac{g+h}{g-h} \right) + 2q_1 \left(\frac{2gh}{g^2 - h^2} \right) + 2Q_1 \frac{4gh^3}{(g^2 - h^2)^2} \right] \quad (402)$$

where

$$q_1 = \frac{q_0/E_0}{4\pi\epsilon_0} \quad (403)$$

$$Q_1 = \frac{Q_0/E_0}{4\pi\epsilon_0 h^2} \quad (404)$$

$$1 = q_1 \left(2 \ln 2 - \frac{2}{3} + \Omega \right) + Q_1 3(a/h + \Omega/2 + \ln 2 - 2) \quad (405)$$

$$1 = q_1 \left[\ln(2h/a + 1) - \frac{2h}{h+a} \right] + Q_1 \frac{h}{h+a} \left(\frac{h}{a} - \frac{h}{2h+a} \right) \quad (406)$$

Taking $h/a = 20$ and solving for q_1 and Q_1 gives

$$1 = q_1 8.097387 + Q_1 7.296080 \quad (407)$$

$$1 = q_1 1.808810 + Q_1 18.58304 \quad (408)$$

or

$$q_1 = 0.0822204 \quad (409)$$

$$Q_1 = 0.04580947 \quad (410)$$

Now for $g = 2h$

$$E_z^{tot}(0, g) \approx E_0 \left[1 + 2q_1 \left(\frac{4}{3} - \ln 3 \right) + 2Q_1 \frac{8}{9} \right] \approx E_0 (1.12) \quad (411)$$

Also for $g = 3h/2$

$$E_z^{tot}(0, g) \approx E_0 \left[1 + 2q_1 \left(\frac{12}{5} - \ln 5 \right) + 2Q_1 \frac{96}{25} \right] \approx E_0 (1.48) \quad (412)$$

Also for $g = 1.7h$

$$E_z^{tot}(0, g) \approx E_0 \left[1 + 2q_1 \left\{ \frac{3.4}{1.7^2 - 1} - \ln \left(\frac{2.7}{0.7} \right) \right\} + 2Q_1 \frac{6.8}{(1.7^2 - 1)^2} \right] \approx E_0 (1.248) \quad (413)$$

which are not too different from the simpler preceding line charge results.

10.4.7 More Exact Approach

If it is desired to allow $g \rightarrow h$ we can apply the analysis of the preceding subsection with a line and point charge to the problem with the two planes (including a summation of images). This should result in a uniformly valid formula, although somewhat more complicated than the preceding one.

11 SUMMARY OF PAPERS ON LEADER BREAKDOWN IN ELECTRONEGATIVE GASES

This section summarizes papers that talk about leader breakdown in electronegative gas, how breakdown mechanisms differ from those in air or nitrogen and what simple models are available to use. Although the preceding sections of the report have focussed on fast streamer phenomena, and gas switch breakdowns have shown rapid development, it should be remembered that the precursor charging phase of such switches is longer, and hence slower processes are also of interest. In other applications of surface flashover, drive durations can certainly be long enough to drive leader development also.

11.1 Chalmers 1972

Chalmers [70] looks experimentally at breakdown in nitrogen, oxygen and SF₆ in a 3 cm, uniform-field gap from primary avalanche to development of filamentary discharge. This paper describes experimental observations and doesn't talk about leaders specifically or about modeling of leaders. It is included in this section because it discusses phenomenology of breakdown in electronegative gases. The experiments were done at 100 – 300 Torr, where streamers are the dominant breakdown mechanism, but some of the phenomena described bear a resemblance to the streamer-leader transition seen in high pressure regimes.

11.1.1 Nitrogen

Initially, in nitrogen at low overvoltage, a series of avalanches are initiated by photons. Next, a luminous front moves from the anode region toward the cathode at 5×10^7 cm/s followed by a second front traveling in the opposite direction and moving faster (2×10^8 cm/s). A diffuse glow discharge fills the gap during which the voltage across the gap falls rapidly to level V_g (half the applied voltage) and after a delay falls again to the arc voltage V_a once the spark is formed. Chalmers thinks of the glow as a Townsend process where multiple generations of avalanches from the cathode build up a positive space charge around the anode. The positive cloud moves toward the cathode which intensifies the field there and gets the cathode to emit electrons. The electrons shift the space charge distribution giving the appearance of a second, anode-directed front. The transitory diffuse glow is missing some expected features and only exhibits a positive column.

The diffuse glow transitions to filamentary channels. These travel slower than the fronts ($3 - 6 \times 10^6$ cm/s), propagate in short steps, and are established by thermal dissociation. This is much like a leader. Chalmers divides the energy needed to dissociate a channel per unit length by the power supplied to the channel (assuming maximum power transfer) to get a velocity of filament (leader) propagation that matches well with experiment.

At high overvoltages, the breakdown proceeded directly from avalanche to filamentary arc. Chalmers doesn't seem to believe that photoionization is necessary for getting the breakdown parameters. He bases this on simulation work by Davies. As with the low overvolted discharge, Chalmers observes filaments that are due to thermal dissociation.

11.1.2 Oxygen

In an electronegative gas such as oxygen at small overvoltages, the diffuse glow phase, described above, never develops. Time lags were repeatable in nitrogen, but not in oxygen at low overvoltage. At high overvoltage Chalmers saw a thermal dissociation in mid-gap that propagated toward each electrode with similar characteristics as in nitrogen. The absence of a glow phase in oxygen is because even at low overvoltages breakdown takes place by a space-charge distortion of the applied field that only takes place in nitrogen at high overvoltages. Oxygen sometimes has long time delays (several hundred ns) between the avalanche and arcing. Nitrogen shows a delay of 10 ns at the most.

11.1.3 SF6

The SF6 experiments were conducted at low pressure (100 Torr) because of difficulties in achieving breakdown voltage levels. Chalmers notes a bimodal time delay that appears as the gap is over-volted by 4% and 8%. As in oxygen, breakdown is preceded by accelerated development of the primary avalanche. Spatially and temporally there are regions of dark space while forming the subsequent filaments. As with oxygen, Chalmers assumes that observed filaments in the gap after the avalanche is due to dissociation of the SF6 molecule and uses the measured velocity and knowledge of the dissociation energies to predict that the important reaction is



11.2 Malik 1978

Malik [71] discusses some basic aspects of SF6 breakdown. He lists an ionization process that leads to α and two attachment processes (direct and disassociation) leading to η . He discusses both the Townsend (pointing out that it is not useful for high voltage design because of uncertainty in γ) and streamer breakdown taking into account the effect of η on the formulas. Formulas for α and η are

$$\begin{aligned} \frac{\alpha}{p} &= 23 \left(\frac{E}{p} \right) - 12.34 & (\text{cm}^{-1} \text{kPa}^{-1}) \\ \frac{\eta}{p} &= -4 \left(\frac{E}{p} \right) + 11.35 & (\text{cm}^{-1} \text{kPa}^{-1}) \\ \frac{\alpha - \eta}{p} &= 27 \left[\left(\frac{E}{p} \right) - 877.5 \right] & (\text{cm}^{-1} \text{kPa}^{-1}) \end{aligned} \quad (415)$$

E is in [kV/cm] and p is pressure referred to a temperature of 20°C . At the time this paper was written, the values α and η were extrapolated from low pressure (< 100 kPa) measurements.

In high voltage SF6 equipment, the maximum insulation is achieved when uniform or slightly non-uniform field distribution occurs ($E_{average}/E_{max} > 0.2$). The uniform field breakdown values are represented by a Paschen curve. DC data is available for $pd < 300$ kPa-cm. The DC Paschen curve has a minimum of 500 V at 3.5×10^{-2} kPa-cm. Deviations from the Paschen curve have been seen for small gaps near the Paschen minimum and at high pressures. The reasons for these deviations are unknown. AC (50 – 60 Hz) and DC Paschen curves are the same.

In order to calculate the minimum discharge voltage (to get corona inception or actual breakdown), Malik uses the streamer criterion:

$$\int_0^x (\alpha - \eta) dx = \ln(N_c) \quad (416)$$

where N_c is the critical number of ions in the avalanche when streamer formation starts. The streamer criterion is satisfied even for a small increase above the field strength corresponding to $(\alpha - \eta) = 0$. At atmospheric pressure this occurs when $E_{crit} = 87.7$ kV/cm. Malik says this criterion works well for low pressure, but because of lack of knowledge of α and η at high pressures, theory and experiment deviate at high pressure. He discusses various empirical tweaks to get the model and experiment to agree.

11.2.1 Factors that Affect Discharge Voltage in SF6

Any metallic particles in the gas can lower the breakdown voltages considerably. In gas insulated systems a lot of effort goes into moving the particles into low field traps to counteract this effect. Insulating particles like glass or Lucite have little effect on the breakdown voltage. Dust, though, can lower the breakdown voltage 30%. The electrode material has no effect on breakdown voltage at low pressure and a small effect at high pressure. No effect due to electrodes are seen in non-uniform fields up to 400 kPa. A dielectric coating on the electrode increases the breakdown strength. The breakdown voltage decreases with increasing electrode area. This becomes more pronounced at high pressure. It is thought that this is due to the fact that breakdown occurs at defects and the greater the electrode area, the more chance for encountering a defect. Related to this is the fact that electrode roughness decreases the breakdown voltage slightly. Malik has a large bibliography in his paper – 84 entries.

11.3 Niemeyer 1983

The paper by Niemeyer [72] describes a simplified leader model based on measurements of a surface discharge in compressed SF6. In this paper Niemeyer says that leader mechanism of breakdown is dominant for $p > 0.1$ MPa. (Later, he lowers this pressure.) Niemeyer measured breakdown across a surface to simplify recording the breakdown optically (the breakdown doesn't branch and wander as much). The leader consists of an extended luminous zone called an **ionization zone** at the tip of a thin, less-luminous **leader channel** as shown in Figure 140. The leader propagation has two steps: 1) rapid development of the ionization zone. During this step the leader channel becomes illuminated and simultaneously current spikes are observed. 2) the ionization zone transforms to a new section of leader channel of length l . This step takes a significant amount of time. After the second step, a new ionization zone develops and the process repeats.

Niemeyer's discharge model includes: 1) the thermodynamic state and electrical conduction in the leader channel, 2) the ionization zone and how it transitions to a leader channel, 3) the interaction of the discharge with the gap geometry, and 4) the development of the thermodynamic state and electrical conductivity in the leader channel.

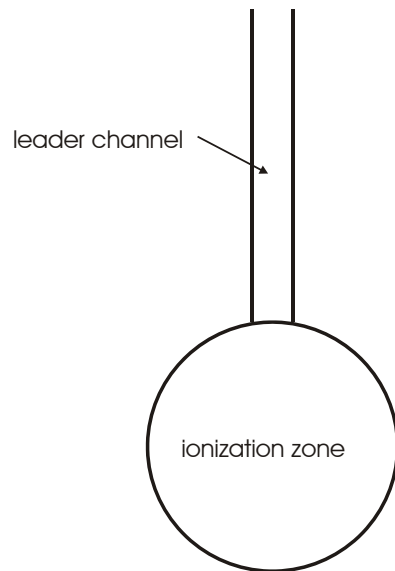


Figure 140. Structure of leader breakdown.

11.3.1 Thermodynamic state of leader channel

Measurements of the current density inside the leader channel show that it is as large as $1 - 10 \text{ A/mm}^2$. This level implies that free electrons must be available in the channel to allow a sufficient conductivity. This, in turn implies that the (E/N) ratio in the channel must be above critical ($\alpha_{eff} > 0$). The problem is that for SF6 at 760 Torr, 293⁰ K, $(E/N)_{cr} = 3.5 \times 10^{-15} \text{ V-cm}^2$ and is ten times the measured channel field. This contradiction disappears if we consider the critical field for SF6 that has a temperature greater than 2000⁰ K. At this temperature SF6 is completely dissociated into atomic fluorine and the critical field drops significantly. We need enough enthalpy, therefore, to disassociate the channel ($6 \times 10^6 \text{ J/kg}$) and heat it to 2100⁰ K ($4 \times 10^6 \text{ J/kg}$). This means we have to add a critical amount of enthalpy:

$$\Delta h_{cr} = (8 \pm 2) \times 10^6 \text{ J/kg} \quad (417)$$

to allow the leader channel to form. In this channel $(E/N)_{cr} = (2 \pm 1) \times 10^{-10} \text{ V-m}^2$ derived from the critical field of atomic fluorine.

11.3.2 Ionization zone and leader propagation

The formation of the ionization zone and transformation into a leader isn't known well enough to do any first-principles modeling. Niemeyer proposes some scaling laws based on some ad-hoc assumptions and measurements. First, he examines the leader step length (l). Figure 141 shows a leader channel with a tip potential of U . Surrounding the tip is a region where $E > E_{cr}$. Avalanches can form there. Once the avalanches form and become streamers, they can propagate into a region outside the avalanche region as long as E is larger than the stability field. As we have seen, the stability field level of SF6 is a matter of some controversy. Niemeyer uses the value $(E/p_0)_{stab} = 40 \pm 10 \text{ V/(m-Pa)}$. Niemeyer appears in this calculation to assume no voltage drop between the leader channel tip and the stability field outer boundary and from that he calculates

$$l = \frac{\gamma}{(E/p_0)_{stab} p_0} U \quad (418)$$

The empirical factor $\gamma = 0.3$ is to make this expression agree with the experiments.

Niemeyer then calculates the voltage drop between the original leader head (radius R_0 and voltage U) and the ionization zone boundary (radius R , and voltage $U - \Delta U$). We find that

$$\Delta U = \left[1 - \frac{R_0}{R} \right] U \quad (419)$$

$$= \varepsilon U \quad (420)$$

The scaling factor ε is estimated to be ~ 1 .

The energy gained by the electrons in the ionization zone is $\Delta W = \Delta Q \Delta U$. This energy is transferred to the neutrals in the new leader channel causing them to heat up. The area of the leader channel is scaled as inversely proportional to pressure based on experimental observation:

$$\pi R_0^2 = \frac{\delta}{p_0} \quad (421)$$

The scaling factor δ is dependent on the polarity of the breakdown. It equals $1.4 \times 10^{-4} \text{ m}^2\text{Pa}$ for a positive point and $5 \times 10^{-4} \text{ m}^2\text{Pa}$ for a negative point.

The time it takes to heat up the new leader channel to where the potential transfers to its tip is τ . This is the time it takes to transfer Δh_{cr} to the channel and is proportional to the molecular number density (N) and inversely proportional to the collision rate between electrons and molecules. The collision rate is

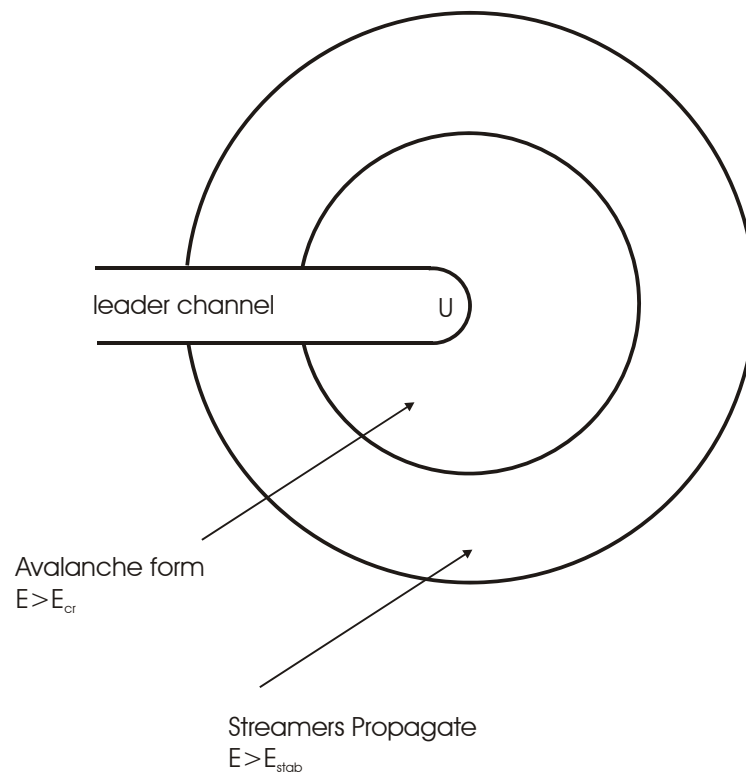


Figure 141. Extent of ionization zone.

proportional to the product of the electron and neutral densities (NN_e).

$$\begin{aligned}\tau &\propto \frac{N}{NN_e} \propto \frac{1}{N_e} \propto \frac{1}{J_e} \propto \frac{\pi R_0^2}{I} \\ &= \frac{\beta\delta}{p_0}\end{aligned}\tag{422}$$

The scaling factor β characterizes the electron-molecule energy transfer process. It is 12 s-m^{-2} for a positive point and 16 s-m^{-2} for a negative point. Experimental data confirms the $1/p_0$ dependence of the time step. Negative discharges have longer time steps than positive discharges, surface discharges have smaller time steps than volumetric. The scaling factor product $\beta\delta = 2.5 \times 10^{-3} \text{ s-Pa}$ for a positive point and $\beta\delta = 8 \times 10^{-3} \text{ s-Pa}$ for a negative point.

11.3.3 Interaction of leader and gap geometry

Every time the leader makes a step of length l , the gap field re-distributes and sends a charge pulse ΔQ along the channel.

$$\Delta Q = U \frac{dC}{dX_L} l \tag{423}$$

where X_L is the length of the leader channel and C is the capacitance of the leader channel. C is determined by the geometry of the gap, the radius of the space charge surrounding the leader channel deposited by the ionization zone and the length of the leader channel. The space charge consists of negative ions and is quasi-stationary during the discharge.

11.3.4 Leader propagation criterion

To propagate the energy gained by the electrons in the ionization zone $\Delta W = \Delta Q \Delta U$ must be greater than the amount needed to disassociate and heat the leader channel. Substituting from Equations 419 and 423 we obtain

$$U^2 \frac{dC}{dX_L} l \varepsilon > \rho_o \pi R_o^2 l \Delta h_{cr} \tag{424}$$

where ρ_o is the mass density of the channel. This should be the density of gas at temperature 2100^0 K and pressure p_o because we haven't started the arc expansion phase. Re-arranging and using Equation 421 we obtain an expression for a critical voltage at the leader tip:

$$U_{cr}^2 = \left(\frac{\rho_o}{p_o} \right) \Delta h_{cr} \left(\frac{\delta}{\varepsilon} \right) \frac{1}{dC/dX_L} \tag{425}$$

The leader head voltage must exceed U_{cr} which depends on the gap geometry through dC/dX_L , the scaling factors (δ and ε) and thermodynamic quantities ($\rho_o/p_o = 6 \times 10^{-5} \text{ kg-m}^{-3}\text{-Pa}^{-1}$) and Δh_{cr} .

By using combinations of the equations already given, Niemeyer calculates the leader velocity, which is dependent on leader head potential and independent of pressure, and energy. The current carried by the leader is dependent on the square of head potential, and dC/dX_L . Niemeyer begins to examine the arc expansion phase that occurs due to energy deposition behind the leader tip. He uses this to get an expression for the expanding arc radius, local electric field in the channel and eventually voltage drop along the channel. He compares all calculated quantities to measured results and gets good agreement.

11.4 Pinnekamp 1983

Pinnekamp states that there has been no consistent model of breakdown in inhomogeneous geometries

because both streamer and leader modes exist in these geometries and confuse the issue [73]. Pinnekamp previously did a series of breakdown experiments over a wide pressure range and came up with some breakdown characteristics, but not a quantitative model. This paper summarizes the general characteristics.

There are two things that affect the characteristics of a discharge: 1) E throughout the discharge gap including space-charge effects and 2) how local discharge mechanisms (streamers and leaders) are affected by E . These are dependent on each other, but Pinnekamp treats them separately for clarity.

11.4.1 E Field

In a point-plane gap there will be a corona around the point which reduces the field around the point and enhances it in the remainder of the gap. There are 3 features: 1) corona occurs according to streamer criterion; 2) the extent of the corona depends on U_0/p_0 where U_0 is the voltage on the point electrode and p_0 is the pressure: the bigger the voltage the larger the corona, the smaller the pressure the larger the corona; and 3) since a corona takes time to form, the faster the risetime of the voltage, the smaller the corona.

A leader will have a voltage drop along it (ΔU_L) that decreases the voltage at the head of the leader from that of the electrode (U_0). Since the leader has propagated part way across the gap, the field in the remainder of the gap increases. The radius of the leader is also critical to determining the field in the gap.

11.4.2 Discharge mechanisms

There are two discharge mechanisms: streamer and leader. Streamers are only able to propagate if the local electric field exceeds the stability field. Pinnekamp uses the value

$$\left(\frac{E}{p_0}\right)_{stab} = 40 - 50 \quad \text{V/(m-Pa)} \quad (426)$$

In order for leaders to propagate, their head potential must be greater than a critical voltage given by Equation 425.

Pinnekamp uses the above criteria to draw a diagram of breakdown in a point-to-plane gap, plotting applied field vs. gas pressure. The diagram gives four limiting curves dividing, for example, pressure/voltage regions where streamers will form from regions where they will not form. Heuristic arguments concerning how corona radius varies with pressure, for example, and how radius influences dC/dX_L which in turn affects U_{cr} shows how the limiting curves should behave with changes of pressure. A conglomeration of all these limiting curves gives a curve that is similar to the diagram put forth by Chalmers in 1984 and is discussed below.

11.5 Chalmers 1984

Chalmers wants to see if leaders form in short, point-plane gaps using time-resolved Schlieren photography [74]. The reason for this interest is that SF6 insulated equipment is strongly influenced by high-field sites from particulate contamination or surface protrusions. Most interesting is Figure 1 in the paper, which shows the conglomeration of the limiting curves discussed by Pinnekamp. At DC (and power frequency AC), below a certain pressure (p_1) breakdown occurs by streamers. Streamers will start at a certain voltage, but the corona discharge (which looks like a glow around the defect) will shield the defect and cause the actual breakdown voltage to be much higher. Breakdown occurs when the field everywhere in the gap exceeds the voltage where streamers will propagate.

Above p_1 breakdown occurs via filamentary looking leaders that coexist with the corona around the

point electrode. The corona still stabilizes the discharge, but the stabilizing effect decreases as pressure increases until at a high enough pressure (p_c) breakdown will occur at streamer onset. In the pressure range between p_1 and p_c there is a point where further increases in pressure actually cause a decrease in the breakdown voltage level. Breakdown occurs when one of the leader filaments propagates all the way across the gap. For short risetime impulse voltages, the stabilizing corona has an effect only up to p_1 . Between p_1 and p_c the stabilizing corona doesn't have time to form and the breakdown voltage is constant (doesn't rise with pressure) until it intersects the streamer onset voltage at p_c .

Since leaders are ten times as hot as streamers, Chalmers used Schlieren to look at small (< 10 mm gaps). He saw the breakdown development occur in steps, but the slow framing rate of his camera gave him problems. The leader would take a few steps (unresolved), then stall for several frames and then cross the remaining gap within one frame.

Chalmers compares his measurements of step length to the step length predicted by Niemeyer in Equation 418. This equation is supposedly good for the range of l between 1 and 30 mm, but Chalmers states that its voltage dependence is not consistent with his measurements. The step length was seen to decrease at elevated pressure as Niemeyer predicted.

11.6 Chalmers 1987

Chalmers extends his 1984 work looking at the effects of gas pressure, impulse voltage and irradiation on SF6 breakdown and draws an analogy between high-pressure, short-gap SF6 and long air gaps [75]. The scaling parameter of importance is the pressure-distance product (pd) in both uniform and non-uniform fields and in both SF6 and air. The corona-leader sequence are the same in air and SF6.

With respect to streamer formation, for SF6 at low pressure ($pd < 0.3$ cm-bar) the sequence of events is similar to air at $pd < 50$ cm-bar. In DC conditions the stability field of air is $4 - 6$ kV/(cm-bar) while the stability field of SF6 is much higher $30 - 40$ kV/(cm-bar). In impulse conditions, the streamer propagation is limited by the critical field rather than the stability field. As pd increases to less than 1 cm-bar, discharges in SF6 develop from mid-gap ionizations that are not understood and have not been observed in air.

With respect to leader formation, $pd > 1$ cm-bar in SF6 corresponds to $pd > 100$ cm-bar in air. In SF6 the leader develops in steps and this doesn't happen in air until the gap becomes several meters long. In air with smaller gaps, leaders propagate continuously. Chalmers goes on to look at various observations of the leader process and gives insight into each. He examines the large scatter in corona inception times, transition from corona to leader channel, the leader step length and charge and the conditions inside the leader channel.

11.7 Wiegart 1988

Wiegart and others wrote a three-part paper that summarizes the state of knowledge on inhomogeneous breakdown in gas insulated systems (GIS) as of 1988 [76]. Its utility is that it places the other, more detailed papers that we have summarized in this section in context of the GIS application and adds some facts that are of interest to this application. Since the gas switch failure has much in common with GIS failure, this paper is worthy of some consideration. In terms of leader modeling, this paper summarizes Niemeyer's work.

One item of interest in these papers was the general statement that much of breakdown theory was developed for air-insulated systems, but since air systems are exposed to the elements, practical systems are

so different from the pristine systems assumed by theory that theory is of little use. GIS is an inherently pristine environment so theory has more use. GIS is designed not to fail unless a defect is introduced and this implies that a theory of highly inhomogeneous breakdown is of most interest.

Another item of interest is in positive point-to-plane breakdown, seed electrons are detached from negative ions. The negative ions are always in the system arising from cosmic rays. Creation rates are on the order of 10 ion pairs/(cm³ s bar). Equilibrium density is 2500 ions pairs /(cm³-bar), equilibrium is achieved 2 cm from the electrodes and has a time constant of 2.5 minutes. The probability of an electron detaching from the negative ion increases sharply (orders of magnitude) as the field exceeds the critical field, so a small volume of high field surrounding a defect will be the source of a corona that leads to breakdown.

There is a good description of Niemeyer's ionization zone - leader channel process in Figure 4 and surrounding text of this paper. A change from the 1983 Niemeyer paper discussed above is that the stability field value seems to have changed to that of the critical field (89 kV/cm-bar), the dissociation of the gas drops this value to 2.5 kV/cm and more information is given concerning the transition from ionization zone to leader channel. After the ionization zone is filled with streamers, "luminous event" starts at the outer boundary of the ionization zone where streamers cease to propagate. This "luminous event" propagates back toward the electrode (or previous leader channel if it has developed). A current pulse occurs and heats the new channel, extending the leader to the edge of the ionization zone. This is mentioned in Part I and discussed qualitatively in Part III. The transition description is the precursor mechanism that Niemeyer develops later in 1989.

11.8 Niemeyer 1989

This paper by Niemeyer [77] is a summary of leader breakdown in electronegative gases for non-uniform fields and fast-rising voltage waveforms. It improves on his previous work. In SF₆, breakdown occurs by leader mechanism for $p > 0.05$ MPa (Note the change from his previous papers.), gaps greater than a few cm, and field enhancement factors greater than 1.5.

11.8.1 Phases of Leader Breakdown

1. When the gap voltage reaches a sufficient level, then there exists a volume (V_{cr}) around the high-field electrode where $E > E_{cr}$. This causes primary electrons to detach from negative ions or (if the electrode is negative) electrons are field-emitted from the electrode. The electrons avalanche. If streamer criteria is satisfied, filaments form in the volume and extend outside into lower field areas. Photoionization triggers streamers in other locations and eventually a "first corona" forms surrounding the electrode. The first corona stops growing when "streamer criteria" is no longer satisfied and the electrons in the streamer immediately attach and a pattern of positive and negative channels remains.

2. In the remains of the corona, two processes occur: a) the streamer channels expand thermally because they have gotten energy from the current flow during the corona propagation phase, b) positive and negative ions drift apart and form space charge filaments. These two processes are the origin of two different types of leaders. First, if enough streamers feed into a common stem, the thermal expansion reduces gas density and reduces E_{cr} . Ionization restarts and creates a leader section. This is the **stem mechanism** for leader inception. The stem leader is a conducting filament that propagates away from the electrode to the corona periphery. Second, the drifting ions can cause the field between them to be enhanced and restarts ionization. The current enhances the field distortion and makes it unstable. This is called the **precursor mechanism** for leader inception and it propagates toward the electrode. Both mechanisms cause a conducting filament that extends the electrode.

3. From the extended electrode, a second corona forms, puts more energy into the stem or precursor

filament. The gas in the filament heats, the channel expands, the density drops, the molecules within disassociate and can't attach electrons and hence the critical field continues to drop. The channel resistance reduces and eventually the heating reduces. The leader steps across the gap until a stem or precursor can't form (then it stops) or it touches the opposite electrode and establishes a breakdown path.

11.8.2 Streamer Properties

In this section Niemeyer integrates α along a field line to reach a streamer inception criterion. Once the streamer is formed, its propagation is controlled by strong attachment in the gas. The field in the streamer tail must exceed $E_{cr} = 89 \text{ V(Pa-m)}^{-1} p_o$ for SF6. This is different than streamers in air where the field in the tail is much less than E_{cr} .

The streamer radius is controlled by radial diffusion and photoionization. Cathode-directed streamers are narrower than anode directed streamers because the avalanches converge towards the cathode-directed streamer head. The radius is inversely proportional to pressure and density because of its dependence on diffusion and photoionization. For anode directed $R \approx 5 \text{ (meter-Pa)}/p_0$. For cathode-directed $R \approx 15 \text{ (meter-Pa)}/p_0$.

The streamer doesn't develop exactly along a field line because of avalanche randomness. Niemeyer models the probability of formation can be modeled by

$$p(E) = \begin{cases} k(E - E_{cr}) & \text{if } E > E_{cr} \\ 0 & \text{if } E < E_{cr} \end{cases} \quad (427)$$

which he later uses to model the corona phase.

11.8.3 Streamer Corona

Corona is thought of as a pattern of streamers. All streamers are assumed to propagate simultaneously. Propagation is subject to randomness and space charges from all streamers control the propagation of each streamer. Niemeyer proposes a lattice random growth model. A lattice occupies the space surrounding the electrode and terminates on the electrode surface. Each lattice point is assigned a voltage. The voltages of the points on the electrodes are given. Along a formed discharge channel Niemeyer multiplies channel length by an average field value in the channel. This field is set to be E_{cr} . A Laplace equation is then solved for the remaining points' potentials. For each point along the existing channels, the field is calculated between it and each nearest-neighbor point. Equation 427 is then used to assign a probability for each proposed jump. A random number weighted by the probabilities gives the path and the entire process is repeated. The process stops when the field at each point is below E_{cr} . This is an impressive model, but not necessary because it is just models the corona, which can be modeled more simply.

The corona model can be simplified by modeling it as a spherical shell with a constant radial field (E_{cr}). The space charge density then must vary as

$$\sigma(r) = \frac{2\varepsilon_o E_{cr}}{r} \quad (428)$$

between r_o (the radius of the spherical electrode) and r_c (the outer radius of the spherical shell). An important fact to note is that the electrode has a surface charge density of $\varepsilon_o E_{cr} \delta(r - r_o)$. We can get r_c by integrating the E field from r_o to infinity and setting this equal to the imposed voltage.

$$E(r) = \begin{cases} E_{cr} & r_o < r < r_c \\ \frac{E_{cr} r_c^2}{r^2} & r > r_c \end{cases} \quad (429)$$

So

$$r_c = 0.5 \left(\frac{U_0}{E_{cr}} + r_0 \right) \quad (430)$$

Once we have the radius, we can get the charge in the shell. Equation (8) in the paper seems incorrect, the proper form apparently given by

$$Q_c = \frac{\pi \varepsilon_0 U_0^2}{E_{cr}} \left[(1 + \rho_0)^2 - 4\rho_0^2 \right] \quad (431)$$

where $\rho_0 = r_0 E_{cr} / U_0$. From these equations we can get even simpler equations by looking at electrodes with small radii. We can approximate how far a corona will extend into the gap by dropping the applied voltage U_0 the amount $E_{cr} l$ where l is the unknown extension. The field integrated across the remainder of the gap cannot exceed E_{cr} .

11.8.4 Leader Inception

In SF6, the voltage drop along a leader is 1/100 of that along a streamer. This change occurs because of gas heating, dissociation, pressure rise and density reduction. When the gas is heated, the strongly attaching SF6 molecules dissociate and can't attach as well. E_{cr} , therefore, depends on temperature

$$E_{cr} = \left[\frac{E}{n} \right]_{cr0} n F(T) \quad (432)$$

where n is the particle density of the gas ($n = p/kT$ making the ideal gas approximation). $[E/n]_{cr0}$ is the critical field at $T = 300^0$ K. $F(T)$ is a function that drops precipitously at 1000^0 K, when SF6 begins to dissociate and saturates at 3000^0 K. Leader formation depends on heating the gas up to dissociation temperature and reducing attachment.

If we deposit charge in the channel via a current over a time.

$$q_{cr} > \frac{\Delta h_{diss}}{E_{cr}} \rho_0 \pi R^2 \quad (433)$$

where R is the channel radius, ρ_0 is the channel density, and Δh_{diss} is the dissociation enthalpy. We need enough charge to dissociate the gas. The charge is inversely proportional to the square of pressure and you need 10 times as much charge for negative polarity as positive polarity. Charge gets pumped into the streamer channel by the stem and precursor mechanism to create a leader.

The stem mechanism is not as important as the precursor mechanism in SF6. It only occurs in combination with the precursor and less frequently than the precursor. It only occurs for negative polarity. There are a lot of assumptions in this section of the paper. The charge q , which is a fraction of the total corona charge (Q_c) given by Equation 431, passes through the stem. The time to form the leader is the time needed to expand the channel. If all the energy due to q goes into a pressure increase

$$\Delta p \approx \frac{q E_{cr}}{\pi R^2} \quad (434)$$

from Δp we can get the Mach number and then the velocity of radius expansion, but we don't really know q or R . We are left with stating that the stem leader formation time is proportional to:

$$\tau_{st} \propto \frac{1}{\sqrt{p_0} U_0} \quad (435)$$

The precursor mechanism only exists in an electronegative gas and is dominant in it. The positive and negative ions in the corona streamer drift apart under the influence of E_{cr} . The field at each end of the channel is enhanced and the ionization process restarts sending a current through the channel, which heats and expands. This process of ionization in the channel ends and expansion of the channel is an unstable process and the leader filament is elongated. Given a space charge (q_i), the field at each end of the ion

filament is enhanced by

$$\Delta E \approx \frac{q_i}{4\pi\epsilon_0 R^2} \quad (436)$$

Again q_i is a fraction of the total corona charge (Q_c). The leader formation time is defined as the time needed to drift the ions and expand the channel. The expansion process is the same as in the stem mechanism. The ion drift time is

$$\tau_p \propto \frac{1}{p_0^2 U_0} \quad (437)$$

11.8.5 Summary of Formula

From an experiment

$$\begin{aligned} q_{cr} &= f_{pol} \frac{50 \text{ (coulomb-Pa}^2\text{)}}{p_0^2} \\ \tau &= f_{pol} \frac{4 \times 10^6 \text{ (volt-sec-Pa}^2\text{)}}{U_0 p_0^2} \end{aligned} \quad (438)$$

where $f_{pol} = 1$ for positive polarity and 10 for negative polarity. If the leader step length is taken to scale as the corona radius, given by Equation 430, an average leader propagation velocity is obtained

$$v_L \propto \frac{U_0^2 p_0}{f_{pol}} \quad (439)$$

so the leader velocity strongly depends on gap voltage, increases as pressure increases and is 10 times faster for positive polarity as for negative polarity. These equations lead to a simplified leader inception model. It represents the corona as a conducting sphere electrically connected to the starting electrode. The radius of this sphere is chosen so that the E field is a multiple of E_{cr} . The multiplying factor is empirically determined and a numerical calculation is performed to get the field.

11.8.6 Leader Channel Development

The leader model has to account for ohmic heating, channel expansion, field reduction due to density reduction and dissociation. Niemeyer describes the model inputs and outputs but the details are in another paper. A key quantity is the voltage drop along the channel because this determines the voltage that controls the corona of the next step. Niemeyer observed that the first current pulse after the leader step provides the dominant energy into the channel and heats the gas to dissociation temperature. Subsequent pulses don't add much energy because the channel resistance has dropped. The channel temperature stabilizes at the upper end of the dissociation temperature (2400 – 2800⁰ K). The channel field drops rapidly at first then levels out. After about 0.5 μ s the average channel field is 3 – 5 kV/cm.

11.8.7 Leader Propagation Through the Gap

In this section, Niemeyer proposes using the lattice approach, used previously to model the corona streamer, to model how the leader randomly propagates across the gap. The distance between lattice points is the corona radius given in Equation 430. The voltage drop along the leader is 3 – 5 kV/cm. The field is re-calculated after every step. The path that the leader takes is determined probabilistically using an equation like Equation 427 where E_{cr} is now the leader propagation threshold (E^*), which has to be determined. Niemeyer first used the simplified leader inception model mentioned above to find the leader inception voltage (U_i). He applied U_i to the electrode and calculated the field between the electrode and all surrounding lattice points. The highest field is taken to be E^* . Although the leader and streamer patterns are calculated using the same method, leaders are less dense than streamers because the voltage drop along the leader channel is less.

11.8.8 Conclusions Of Paper

There are two major deficiencies given. The leader inception due to a slowly rising voltage waveform, which is controlled by space charge due to ion drift, is not understood. Second, leader breakdown along the interface between an electronegative gas and a solid or liquid insulator is not understood.

12 CONCLUSIONS

This project was motivated by several application areas: gas switch insulator flashovers at low field levels in SF₆; safety considerations based on breakdown levels in air with dielectric surfaces present, experiments at Texas Tech University (TTU) in nitrogen as well as electronegative gases where the discharge path was diverted from volume to surface. The scope of the project was reduced to a focus on fast ionization processes (ionization waves or streamers) with a brief summary of the literature for slower leader phenomena at the end of this report (precharging of the insulator was also out of scope). The presence of the insulator can in principle lead to what we termed first order effects (voltage and field threshold changes) and second order effects (velocity and path changes). The modeling efforts were focused on first order effects. Because of this focus we first examined thresholds in the TTU experiment and found that the changes in discharge path, following the surface versus going through the volume, could not be explained by a significant drop in breakdown voltage.

The critical field is defined as the field where ionization processes exceed attachment processes. To model surface effects on the critical field, ionization rates in air and SF₆ were first summarized and kinetic simulations were carried out to obtain photoemission rates and secondary electron emission (SEE) rates with a dielectric surface present. These rates indicate that photoemission does not significantly change the critical field level in the gas alone, and with SEE acting to attach electrons to the surface (in this energy range) tends to actually increase the critical field slightly. Because triple points (junctions of insulators, gases, electrodes) and other localized field enhancements could initiate ionization and lead to the start of an ionization wave (or streamer), we are also interested as to what field level is required to keep such a wave propagating through a lower field region (such as the insulator housing in the cascade region of the gas switch); this is called the sustaining field. To model sustaining field in gases and on surfaces we used a 1.5D fluid model (the field is 3D but the fluid equations are solved in 1D). We examined the field levels in air and SF₆ without and with a surface required to keep a streamer going, maintaining the head field level. The model in air predicted anode and cathode directed sustaining field levels consistent with known experimental results (the anode directed is less than critical and the cathode directed is much less than critical). When a surface was introduced there were no significant changes to these levels. The model with SF₆ predicts sustaining fields just over half the critical level in both directions. The presence of a surface decreased the anode directed level and increased the cathode directed level. Because the results here show some dependence on streamer radius (assumed in the 1.5D model), we examined radial expansion of the streamer channel and found that the expansion in air and SF₆ were similar (note that Aleksandrov chose the effective radius in a 1.5D model for air based on this radial expansion). The sustaining field was also examined in SF₆ at two atmospheres and it was found that the sustaining field was increased by less than a factor of two (two would be consistent with pure pressure scaling).

An experimental impedance matched fixture was constructed to study fast discharges in high pressure gases without and with surfaces. Instrumentation included electrical measurements, streak cameras, and a framing camera. Experiments were conducted with uniform fields to ascertain critical field levels and with nonuniform (rod-plane) field arrangements in an attempt to determine sustaining field levels. These were done in air, nitrogen, and SF₆. Threshold (critical) fields without and with a dielectric surface did not significantly differ on virgin samples. There is a small decrease with shot number (damage?) with the

surface present. Velocity values were significantly affected by the surface (although there was evidence of discharge initiation in the center of the gap, depending on voltage polarity and gas). There was also a dependence on surface type. The results for sustaining field in SF₆ were surprisingly close to the model predictions even though the gap dimensions were smaller than desirable.

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