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Mesoscopic Modeling of Multi-Physicochemical Transport Phenomena in Porous Media

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6 Abstract

7 We present our recent progress on mesoscopic modeling of multi-physicochemical
8 transport phenomena in porous media based on the lattice Boltzmann method. Simulation
9 examples include injection of CO₂ saturated brine into a limestone rock, two-phase behavior and
10 flooding phenomena in polymer electrolyte fuel cells, and electroosmosis in homogeneously
11 charged porous media. It is shown that the lattice Boltzmann method can account for multiple,
12 coupled physicochemical processes in these systems and can shed some light on the underlying
13 physics occurring at the fundamental scale. Therefore, it can be a potential powerful numerical
14 tool to analyze multi-physicochemical processes in various energy, earth, and environmental
15 systems.

16

17 **1. Introduction**

18 Multi-physicochemical transport phenomena in porous media are ubiquitous, particularly
19 in various energy, earth, and environment systems. One example is the disposal of supercritical
20 CO₂ in geologic formations, the most promising near-term solution to the problem of reducing
21 carbon emissions into the atmosphere [1]. Experimental analyses of the long-term fate of CO₂
22 after injection into the geologic formations are not possible with relatively short-term laboratory
23 experiments. Therefore it is necessary to employ comprehensive numerical models that
24 incorporate multiple physicochemical processes involving interactions between the injected CO₂,
25 the brine in the pore spaces, and the minerals lining the pores. Supercritical CO₂, as a buoyant
26 and slightly miscible fluid, once injected, displaces brine from the pore space in a complex
27 pattern. At the interface with brine, CO₂ dissolves into the brine to form carbonic acid that can
28 react with and dissolve minerals eventually leading to mineral precipitation further along the
29 flow path. Clearly, there are multiple physics processes involved, including hydrodynamics,
30 thermodynamics, chemical dynamics, and electrodynamics (because the surface of most natural
31 media is charged). All these processes are ultimately governed by pore-scale interfacial
32 phenomena, which occur at scales of microns. However, because of the wide disparity in scales
33 ranging from pore to field, a continuum formulation based on spatial averages taken over length
34 scales much larger than typical pore and mineral grain sizes is often utilized, implying the
35 existence of a representative elemental volume (REV) [2]. As a result, spatial heterogeneities at
36 smaller scales are unresolved and the aggregate effects of the porescale (mesoscopic scale)
37 processes are taken into account through various effective constitutive parameters. One of the
38 goals of performing pore-scale simulations is to obtain values for these constitutive parameters
39 through upscaling the pore-scale results. Other goals are to identify key parameters and

40 physicochemical processes that control macroscopic phenomena, and to validate continuum
41 descriptions.

42 Another example is fuel cells, and in particular polymer electrolyte fuel cells (PEFCs). In
43 PEFCs, the catalyst layer (CL) is the host to several competing transport mechanisms involving
44 charge (proton and electron), species (oxygen, nitrogen, water vapor), and liquid water transport.
45 The multi-faceted functionality of a gas diffusion layer (GDL) includes reactant distribution,
46 liquid water transport, electron transport, heat conduction and mechanical support to the
47 membrane-electrode-assembly. Despite tremendous recent progress in enhancing the overall cell
48 performance, a pivotal performance limitation in PEFCs is manifested in terms of mass transport
49 loss originating from suboptimal liquid water transport and resulting flooding in the constituent
50 components [3]. In recent years, several macroscopic computational models for multiple-
51 physicochemical transport processes in PEFCs [4-10] have been developed. These macroscopic
52 models, again are based on the theory of volume averaging and treat the catalyst layer and gas
53 diffusion layer as macro-homogeneous porous layers. Due to their macroscopic nature, the
54 current models fail to resolve the influence of the structural morphology of the CL and GDL on
55 the underlying physics. Mesoscopic modeling is critical to understanding the overall structure-
56 wettability-transport interactions as well as the underlying multi-physicochemical processes in
57 the CL and GDL, and hence is a useful tool for design and optimization of microstructures of
58 electrodes for better performance and durability.

59 In this paper, we review our recent work on mesoscopic modeling of multi-
60 physicochemical processes in porous media, based on the lattice Boltzmann method (LBM), a
61 relatively new numerical method for simulating fluid flows and modeling physics in fluids [11].
62 Originating from the classical statistical physics, LBM is a mesoscopic method based on

63 simplified kinetic equations. In the LBM, the fluid is modeled as a collection of fictitious-
64 particles propagating and colliding over a discrete lattice domain. Mesoscopic continuity and
65 momentum equations can be obtained from this propagation-collision dynamics through a rigorous
66 mathematical analysis. The particulate nature and local dynamics provide advantages for
67 complex boundaries and parallel computation. In addition, the kinetic nature of the LBM makes
68 it easy to account for new physics in the LBM framework, which is especially useful for
69 modeling multi-physicochemical phenomena. In Section 2, the partial differential equations
70 governing fluid flow, transport of reactive species and electric potential, as well as mineral
71 reactions in porous media will be given. In Section 3, the implementation of the LBM to solve
72 these governing equations will be presented. Some simulation examples will be given in Section
73 4 and concluding remarks in Section 5.

74 **2. Governing Equations**

75 **2.1 Continuity and momentum equations**

76 For isothermal incompressible fluid flow, the continuity and momentum equations can be
77 written as [12]

78
$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

79
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \rho \nu \nabla^2 \mathbf{u} + \mathbf{F}, \quad (2)$$

80 where ρ represents the density of the fluid, t the time, \mathbf{u} the velocity vector, p the pressure, ν
81 the kinetic viscosity, and \mathbf{F} the body force density which may include all the effective body
82 forces.

83 **2.2 Transport equations for aqueous species and electrical potential**

84 For the i th ion species in the solute, the mass conservation equation describing transport
85 and reaction can be written in the general form [13]:

86
$$\frac{\partial C_i}{\partial t} + \nabla \cdot \mathbf{J}_i + \lambda_i C_i = R_i, \quad (3)$$

87 where C_i denotes the ionic concentration, \mathbf{J}_i the species flux, λ_i a radioactive decay constant,
88 and R_i the rate at which the i th species is produced or consumed by chemical reactions. The flux
89 \mathbf{J}_i , consisting of contributions from advection, diffusion, and electrochemical migration terms,
90 has the form [13]

91
$$\mathbf{J}_i = -\frac{ez_i D_i}{kT} C_i \nabla \Psi - D_i (\nabla C_i + C_i \nabla \ln \gamma_i) + C_i \mathbf{u}, \quad (4)$$

92 where the first term on the right refers to electrochemical migration, the second term to aqueous
93 diffusion, and the last term to advective transport. Here z_i , D_i and γ_i denote the ion algebraic
94 valence, the diffusivity and the activity coefficient of the i th species, respectively; and e , k , and
95 T denote the absolute charge of electron, the Boltzmann constant and the absolute temperature,
96 respectively. The quantity Ψ represents the local electrical potential caused by the ionic
97 distribution which is governed by the Poisson equation

98
$$\nabla \cdot (\varepsilon \varepsilon_0 \nabla \Psi) = -\rho_e = -\sum_i ez_i C_i, \quad (5)$$

99 where ε is the local dimensionless fluid dielectric constant, ε_0 the permittivity of a vacuum,
100 and ρ_e the net charge density. Assuming no radiation and constant activity coefficient and
101 substituting Eq. (5) into (3), we have

103
$$\frac{\partial C_i}{\partial t} + \mathbf{u} \cdot \nabla C_i = D_i \nabla^2 C_i + \frac{ez_i D_i}{kT} \nabla \cdot (C_i \nabla \Psi). \quad (6)$$

104 This is the Nernst-Planck equation [14], where \mathbf{F} can be any kind of effective body force. In this
 105 contribution we only consider the static electrical force from an external electric field. The
 106 general form of electrical force in electrokinetic fluids can be expressed as:

107
$$\mathbf{F}_e = -\rho_e \nabla \Psi_{ext}, \quad (7)$$

108 where Ψ_{ext} is the external electrical potential field.

109 When the ionic convection is negligible and the electric potential is continuously

110 derivable, Eq. (6) has a simple steady-state solution for dilute electrolyte solutions:

111
$$C_i = C_{i,\infty} e^{-\frac{ez_i \Psi}{kT}}. \quad (8)$$

112 Substituting Eq. (8) into Eq. (5) yields the nonlinear PB equation [15]

113
$$\nabla^2 \Psi = -\frac{1}{\epsilon_r \epsilon_0} \sum_i ez_i C_{i,\infty} \exp\left(-\frac{ez_i}{kT} \Psi\right). \quad (9)$$

114 **2.3 Equations for mineral reaction rates**

115 Heterogeneous reactions between aqueous species and minerals at the pore-mineral

116 interface are given by [16]

117
$$AD \frac{\partial \Psi}{\partial n} \Big|_s = \sum_s \nu_{js} A_s \hat{I}_s(s), \quad (10)$$

118 where n denotes the unit normal perpendicular to the fluid-mineral interface pointing toward the
 119 fluid phase, D denotes the aqueous diffusion coefficient assumed to be the same for all species

120 for simplicity, and $\hat{I}_s(s)$ denotes the reaction flux of the s th mineral at its surface, taken as

121 positive for precipitation and negative for dissolution. The total surface area A across which

122 diffusion takes place equal to the sum of individual mineral surface areas A_s

123 $A = \sum_s A_s$ (11)

124 **3. Lattice Boltzmann Model Implementation**

125 **3.1 Incompressible lattice Boltzmann model for single phase flow**

126 In order to eliminate compressible effects in the conventional LBM, here we use an
 127 incompressible LB model constructed by Guo et al. [17]. The pore-scale flow of a single aqueous
 128 fluid phase is simulated by the following evolution equation:

129 $f_\alpha(\mathbf{x} + \mathbf{e}_\alpha \delta_i, t + \delta_i) = f_\alpha(\mathbf{x}, t) - \frac{f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)}{\tau}.$ (12)

130 In the above equation, δ_i is the time increment, f_α the distribution function along the α
 131 direction in velocity space, f_α^{eq} the corresponding equilibrium distribution function, and τ the
 132 dimensionless relaxation time. For the commonly used two-dimensional, nine-speed LB model
 133 (D2Q9), the discrete velocities \mathbf{e}_α have the following form:

134 $\mathbf{e}_\alpha = \begin{cases} (0,0) & \alpha = 0, \\ (\cos \theta_\alpha, \sin \theta_\alpha) c, \quad \theta_\alpha = (\alpha - 1) \pi / 2 & \alpha = 1 - 4, \\ \sqrt{2} (\cos \theta_\alpha, \sin \theta_\alpha) c, \quad \theta_\alpha = (\alpha - 5) \pi / 2 + \pi / 4 & \alpha = 5 - 8. \end{cases}$ (13)

135 For the incompressible LB model, the equilibrium distribution is defined by Guo et al.
 136 [17]:

137 $f_\alpha^{eq} = \begin{cases} -4\sigma \frac{p}{\rho c^2} + s_\alpha(\mathbf{u}), & \alpha = 0, \\ \lambda \frac{p}{\rho c^2} + s_\alpha(\mathbf{u}), & \alpha = 1 - 4, \\ \gamma \frac{p}{\rho c^2} + s_\alpha(\mathbf{u}), & \alpha = 5 - 8, \end{cases}$ (14)

138 where σ , λ and γ are the parameters satisfying

139

$$140 \quad \begin{aligned} \lambda + \gamma &= \sigma \\ \lambda + 2\gamma &= \frac{1}{2}, \end{aligned} \quad (15)$$

141 and

$$142 \quad s_\alpha(\mathbf{u}) = \omega_\alpha \left[3 \frac{\mathbf{e}_\alpha \cdot \mathbf{u}}{c^2} + 9 \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c^4} - \frac{3\mathbf{u}^2}{2c^2} \right]. \quad (16)$$

143 In these equations, $c = \delta x / \delta t$, where δx is the space increment, and p and \mathbf{u} are the
144 pressure and velocity of the fluid, respectively. The corresponding weight coefficients are

$$145 \quad \omega_\alpha = \begin{cases} 4/9, & \alpha = 0, \\ 1/9, & \alpha = 1-4, \\ 1/36, & \alpha = 5-8. \end{cases} \quad (17)$$

146 Eq. (12) has been shown to recover Eqs. (1, 2) [17], with the velocity and pressure given
147 by

$$148 \quad \mathbf{u} = \sum_{\alpha=1}^8 \mathbf{e}_\alpha f_\alpha, \quad (18)$$

149 and

$$150 \quad \frac{p}{\rho} = \frac{c^2}{4\sigma} \left[\sum_{\alpha=1}^8 f_\alpha + s_0(\mathbf{u}) \right], \quad (19)$$

151 respectively.

152 **3.2 Lattice Boltzmann model for two phase flow**

153 The interaction-potential model, originally proposed by Shan and Chen [18], and
154 henceforth referred to as the S-C model, introduces k distribution functions for a fluid mixture
155 comprising of k components. Each distribution function represents a fluid component and
156 satisfies the evolution equation. The non-local interaction between particles at neighboring lattice

157 sites is included in the kinetics through a set of potentials. The LB equation for the k th
 158 component can be written as:

159

$$f_i^k(\mathbf{x} + \mathbf{e}_i \delta_i, t + \delta_i) - f_i^k(\mathbf{x}, t) = -\frac{f_i^k(\mathbf{x}, t) - f_i^{k(eq)}(\mathbf{x}, t)}{\tau_k}, \quad (20)$$

160 where $f_i^k(\mathbf{x}, t)$ is the number density distribution function for the k th component in the i th
 161 velocity direction at position \mathbf{x} and time t , and δ_i is the time increment. In the term on the right-
 162 hand side, τ_k is the relaxation time of the k th component in lattice unit, and $f_i^{k(eq)}(\mathbf{x}, t)$ is the
 163 corresponding equilibrium distribution function.

164 The phase separation between different fluid phases, the wettability of a particular fluid
 165 phase to the solid, and the body force, are taken into account by modifying the velocity used to
 166 calculate the equilibrium distribution function. An extra component-specific velocity due to
 167 interparticle interaction is added on top of a common velocity for each component. Interparticle
 168 interaction is realized through the total force, \mathbf{F}_k , acting on the k th component, including
 169 fluid/fluid interaction, fluid/solid interaction, and external force. More details can be found in
 170 [19, 20].

171 The continuity and momentum equations can be obtained for the fluid mixture as a single
 172 fluid using Chapman-Enskog expansion procedure in the nearly incompressible limit:

173

$$\left. \begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\ \rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] &= -\nabla p + \nabla \cdot [\rho \nu (\nabla \mathbf{u} + \mathbf{u} \nabla)] + \rho \mathbf{g} \end{aligned} \right\} \quad (21)$$

174 where the total density and velocity of the fluid mixture are given, respectively, by:

175

$$\left. \begin{aligned} \rho &= \sum_k \rho_k \\ \rho \mathbf{u} &= \sum_k \rho_k \mathbf{u}_k + \frac{1}{2} \sum_k \mathbf{F}_k \end{aligned} \right\} \quad (22)$$

176 with a non-ideal gas equation of state given by [21]:

177 **3.3 Lattice Boltzmann model for transport of reactive solutes**

178 In a previous article, Kang et al. [22] have derived the following LB equation for the total
179 primary species concentrations for chemical systems with reactions written in canonical
180 form:

$$181 G_{\alpha j}(\mathbf{x} + \mathbf{e}_\alpha \delta_i, t + \delta_i) = G_{\alpha j}(\mathbf{x}, t) - \frac{G_{\alpha j}(\mathbf{x}, t) - G_{\alpha j}^{eq}(\Psi_j, \mathbf{u})}{\tau_{\alpha j}}, (j = 1, \dots, N_C), \quad (23)$$

182

183 where N_C is the number of primary species, Ψ_j is the total concentration of the j th primary
184 species, $G_{\alpha j}$ is its distribution function along the α direction, $G_{\alpha j}^{eq}$ is the corresponding equilibrium
185 distribution function, \mathbf{e}_α are velocity vectors, δ_i is the time increment, and $\tau_{\alpha j}$ is the dimensionless
186 relaxation time.

187 It has been shown that the above equation can recover the following pore-scale
188 advection-diffusion equation for Ψ_j [23]:

$$189 \frac{\partial \Psi_j}{\partial t} + (\mathbf{u} \cdot \nabla) \Psi_j = \nabla \cdot (D \nabla \Psi_j). \quad (24)$$

190 This equation is the same as Eq. (6) except that here the electrochemical migration is
191 neglected. Assuming the homogeneous reactions are in instantaneous equilibrium, we have the
192 following mass action equation [24, 25]:

$$193 C_i = (\gamma_i)^{-1} K_i \prod_{j=1}^{N_C} (\gamma_j C_j)^{\nu_j}, \quad (25)$$

194 where ν_{ji} are the stoichiometric coefficients, K_i is the equilibrium constant of the i th
 195 homogeneous reaction, γ_i is the activity coefficient of the i th secondary species, and C_j and C_i
 196 are solute concentrations for primary and secondary species, respectively. They are related to by

$$197 \quad \Psi_j = C_j + \sum_{i=1}^{N_R} \nu_{ji} C_i, \quad (26)$$

198 where N_R is the number of independent homogeneous reactions, or, equivalently, secondary
 199 species.

200 More details on the heterogeneous reactions between aqueous species and minerals at the
 201 pore-mineral interface described by Eq. (10), and on the update of solid phase can be found in
 202 [22, 26]

203 **3.4 Lattice Poisson Method**

204 To solve the Poisson equation with strong nonlinearity, Eq. (9), we adopt the lattice
 205 Poisson method (LPM) developed previously [27, 28], which tracks the electrical potential
 206 distribution transporting on the discrete lattices. By expanding Eq. (9) into the time-dependent
 207 form

$$208 \quad \frac{\partial \psi}{\partial t} = \nabla^2 \psi + g_{rhs}(\mathbf{r}, \psi, t)$$

209 with $g_{rhs} = \frac{1}{\epsilon \epsilon_0} \sum_i z_i e n_{i,\infty} \exp\left(-\frac{z_i e}{k_b T} \psi\right)$ representing the *negative* right hand side (RHS) term of

210 the original Eq. (9), we get the discrete evolution equation for the electrical potential distribution

$$211 \quad g_\alpha(\mathbf{r} + \Delta\mathbf{r}, t + \delta_{t,g}) - g_\alpha(\mathbf{r}, t) = -\frac{1}{\tau_g} [g_\alpha(\mathbf{r}, t) - g_\alpha^{eq}(\mathbf{r}, t)] + (1 - \frac{0.5}{\tau_g}) \delta_{t,g} \omega_\alpha g_{rhs}, \quad (27)$$

212 where g_α^{eq} is the equilibrium distribution of the electric potential evolution variable. The time
213 step for the electrical potential evolution is

214
$$\delta_{t,g} = \frac{\delta_x}{c'}, \quad (28)$$

215 where c' is a *pseudo* sound speed in the potential field. After evolving on the discrete lattices,
216 the mesoscopic electrical potential can be calculated using

217
$$\psi = \sum_\alpha (g_\alpha + 0.5\delta_{t,g} g_{rhs} \omega_\alpha). \quad (29)$$

218 Although the electrical potential evolution equations are in an unsteady form, only the
219 steady state result is realistic, because the electromagnetic susceptibility has not been considered.
220 Although the lattice evolution method for nonlinear Poisson equation is not as efficient as the
221 multi-grid solutions due to its long wavelength limit, it has the advantages of suitability for
222 geometrical complexity and parallel computing.

223 **4. Simulation Examples**

224 **4.1 Injection of CO_2 into a limestone rock**

225 We first present some modeling results on the injection of a fluid saturated with 170 bars
226 $CO_2(g)$ into a limestone rock at the pore scale. The pore structure was derived from a digitized
227 image of a limestone rock thin section with 640×480 pixels (figure 1). We reduced the original
228 resolution to save computational time (figure 2). The chemical system of Na^+ - Ca^{2+} - Mg^{2+} - H^+ -
229 SO_4^{2-} - Cl^- - CO_2 with the reaction of calcite to form dolomite and gypsum is considered. Secondary
230 species included in the simulation are: OH^- , HSO_4^- , $H_2SO_4(aq)$, CO_3^{2-} , HCO_3^- , $CaCO_3(aq)$,
231 $CaHCO_3^+$, $CaOH^+$, $CaSO_4(aq)$, $MgCO_3(aq)$, $MgHCO_3^+$, $MgSO_4(aq)$, $MgOH^+$, $NaCl(aq)$,
232 $NaHCO_3(aq)$, $NaOH(aq)$. Initial fluid composition is pH 7.75 and 2.69 m NaCl brine,

233 equilibrium with minerals calcite, dolomite and gypsum at 25 °C. Initial rock composition is
234 calcite. Secondary minerals include dolomite and gypsum. For boundary conditions, a constant
235 pressure gradient is imposed across the domain for flow. When flow reaches steady state, a fluid
236 with a pH of 3.87 and in equilibrium with 179 bars CO₂(g) and minerals dolomite and gypsum is
237 introduced at the inlet. Zero gradient boundary conditions are imposed at the outlet. Two
238 different cases are considered with different mineral reaction rates to show their effects on
239 solution concentration, mineral deposition and change in geometry.

240 Resulting geometries at time=15625 seconds for two different mineral reaction rate
241 constants are plotted in figure 3. Damkohler is 7.375 for calcite and gypsum and 0.7375 for
242 dolomite for the faster mineral reactions and 7.375×10^{-2} for calcite and gypsum and 7.375×10^{-3}
243 for dolomite for slower reactions. Concentration distribution of total Ca²⁺, Mg²⁺, and SO₄²⁻, pH,
244 and reaction rates for calcite, dolomite, and gypsum for the slower reactions are plotted in figure
245 4. As can be seen from the figures, as the reaction rate constants decrease, the deposition of
246 dolomite becomes more uniform surrounding the dissolving calcite grains. Only a small amount
247 of gypsum forms on top of dolomite. At some point in the simulation, the major pores for flow
248 become blocked halting further fluid flow through the medium. The pH is uniform over the
249 entire pore space. All reaction rates have finite values at the mineral surface in the whole
250 domain, outlining the solid geometry. The reaction rate is negative for calcite and positive for
251 dolomite and gypsum, confirming that calcite is dissolving while dolomite and gypsum are
252 precipitating.

253 ***4.2 Two-phase behavior and flooding phenomena in polymer electrolyte fuel cells***

254 In this Section, we present some results for two-phase flow through the porous CL and
255 the fibrous GDL in a PEFC. Details can be found in [29]. Figure 5 displays the steady state

256 invading liquid water fronts corresponding to increasing capillary pressures from the primary
257 drainage simulation in the reconstructed CL microstructure characterized by slightly
258 hydrophobic wetting characteristics with a static contact angle of 100° . At lower capillary
259 pressures, the liquid water saturation front exhibits finger like pattern, similar to the
260 displacement pattern observed typically in the capillary fingering regime. The displacing liquid
261 water phase penetrates into the body of the resident wetting phase (i.e. air) in the shape of fingers
262 owing to the surface tension driven capillary force. However, at high saturation levels, the
263 invading non-wetting phase tends to exhibit a somewhat flat advancing front. This observation,
264 as highlighted in figure 5(b), indicates that with increasing capillary pressure, even at very low
265 capillary number (Ca), several penetrating saturation fronts tend to merge and form a stable
266 front. The invasion pattern transitions from the capillary fingering regime to the stable
267 displacement regime and potentially lies in the transition zone in between. In an operating fuel
268 cell, the resulting liquid water displacement pattern pertaining to the underlying pore-
269 morphology and wetting characteristics would play a vital role in the transport of the liquid water
270 and hence the overall flooding behavior.

271 Figure 6 shows the liquid water distribution as well as the invasion pattern from the
272 primary drainage simulation with increasing capillary pressure in the initially air-saturated
273 reconstructed carbon paper GDL characterized by hydrophobic wetting characteristics with a
274 static contact angle of 140° . The reconstructed GDL structure used in the two-phase simulation
275 consists of $100 \times 100 \times 100$ lattice points in order to manage the computational overhead to a
276 reasonable level. Physically, this scenario corresponds to the transport of liquid water generated
277 from the electrochemical reaction in a hydrophobic CL into the otherwise air-occupied GDL in
278 an operating fuel cell. At the initially very low capillary pressure, the invading front overcomes

279 the barrier pressure only at some preferential locations depending upon the pore size along with
280 the emergence of droplets owing to strong hydrophobicity. As the capillary pressure increases,
281 several liquid water fronts start to penetrate into the air occupied domain. Further increase in
282 capillary pressure exhibits growth of droplets at two invasion fronts, followed by the coalescence
283 of the drops and collapsing into a single front. This newly formed front then invades into the less
284 tortuous in-plane direction. Additionally, emergence of tiny droplets and subsequent growth can
285 be observed in the constricted pores in the vicinity of the inlet region primarily due to strong wall
286 adhesion forces from interactions with highly hydrophobic fibers with the increasing capillary
287 pressure. One of the several invading fronts finally reaches the air reservoir, physically the
288 GDL/channel interface, at a preferential location corresponding to the capillary pressure and is
289 also referred to as the bubble point. It is important to note that the mesoscopic LB simulations
290 provide fundamental insight into the pore-scale liquid water transport through different GDL
291 structures and would likely enable novel GDL design with better water removal and flooding
292 mitigation.

293 ***4.3 Electroosmosis in homogeneously charged micro- and nano-scale porous media***

294 In this section, we briefly present some simulation results on electroosmotic flows
295 (EOFs) in charged micro porous media using the lattice Poisson-Boltzmann method (LPBM),
296 with geometry effects, solution and surface charge effects considered. More details can be found
297 in [28]. We focus on a cubic system of which each side is 1 micron long. A $60 \times 60 \times 60$ uniform
298 grid is used. We change microstructure geometries of porous media by varying the porosity ε
299 from 0.1 to 0.9. The average characteristic length of particles varies from 20 to 150 nm. Figure 7
300 shows Schematics of the generated porous structures for porosity 0.3 and 0.6. The bulk ionic
301 concentration n_∞ varies from 10^{-6} to 10^{-3} M and the surface zeta potential from 0 to -100 mV.

302 The other properties and parameters used in this work are: the fluid density $\rho=999.9 \text{ kg/m}^3$, the
303 dielectric constant $\epsilon, \epsilon_0=6.95 \times 10^{-10} \text{ C}^2/\text{J m}$, the dynamic viscosity $\mu=0.889 \text{ mPa s}$, the
304 temperature $T=273 \text{ K}$ and the external electrical field strength $E=1 \times 10^4 \text{ V/m}$.

305 First, the geometry effects on the electroosmotic permeability in micro porous media are
306 investigated by changing volume fraction and particle size (or number density) of the solid
307 phase. We define the electroosmotic permeability, κ_e , as

308
$$\kappa_e = \frac{\bar{u}}{E}, \quad (30)$$

309 where \bar{u} is the averaged velocity of EOF along the direction of the driving electrical field E .

310 The coefficients of electroosmotic permeability (κ_e) for different porosities (ϵ) of porous media
311 are shown in figure 8. The bulk molar concentration $c_\infty=10^{-4} \text{ M}$, and $\zeta=-50 \text{ mV}$. The
312 electroosmotic permeability increases with the porosity monotonically. The increasing rate rises
313 with the porosity as well which is very low when the porosity is smaller than 0.5 and becomes
314 sharply high when the porosity is larger than 0.7. The predicted electroosmotic permeability is in
315 the order of $10^{-9} \text{ m}^2/\text{s V}$, which is consistent with the existing experimental measurements.

316 Figure 9 shows the predicted electroosmotic permeability versus the bulk ionic
317 concentration of the electrolyte solution. We used $\epsilon=0.38$. The electroosmotic permeability κ_e
318 increase monotonically with the bulk ionic concentration c_∞ as c_∞ varies from 10^{-6} to 10^{-3} M .
319 This result can be explained by the undeveloped electrical potential distributions in narrow
320 channels, whose similar results can be found in Fig. 2 of Ref. [30] and Fig. 1&2 of Ref. [31].
321 When c_∞ is lower than 10^{-4} M , the electroosmotic permeability is nearly proportional to the bulk
322 ionic concentration. When c_∞ is higher, the increasing rate becomes a little smaller.

323 Zeta potential on solid surfaces of porous media affects EOF permeability directly.
324 Simple proportional relationships have been obtained between the electroosmotic permeability
325 and the zeta potential for electrical transports in soils and in polymer composites recently based
326 on the boundary-layer theory. Here we analyze such effects using our numerical methods.

327 Figure 10 shows the calculated electroosmotic permeability versus the zeta potential on
328 solid surfaces of porous media. All surfaces are homogeneously charged with a same value of
329 ζ . The other parameters used are: $c_\infty = 10^{-4}$ M, $c_d = 0.1$ and $\varepsilon = 0.38$. The zeta potential ζ
330 changes from 0 to 100 mV. It shows that the proportionally linear relationship between
331 electroosmotic permeability and zeta potential is accurate only when ζ is very small (<30 mV).
332 The permeability increases much sharper when the zeta potential ζ is larger than 40 mV and
333 then smoother when the zeta potential ζ is larger than 90 mV.

334 **5. Conclusions**

335 We have presented our recent work on mesoscopic modeling of multi-physicochemical
336 processes in porous media, based on the LBM. For the problem of injecting CO₂ saturated brine
337 into a limestone rock, it is shown that the LBM is able to provide detailed information on fluid
338 velocity, solute concentration, mineral composition, and reaction rates, as well as the evolution
339 of the porous media geometry, and therefore can shed some light on the fundamental physics
340 occurring at the pore scale for reactive transport involved in geologic CO₂ sequestration. For
341 two-phase behavior and flooding phenomena in PEFCs, the LBM is a powerful tool to study the
342 influence of the pore structure and surface wettability on liquid water transport and interfacial
343 dynamics in the PEFC catalyst layer and gas diffusion layer. Particularly, the two-phase regime
344 transition phenomenon in the capillary dominated transport in the CL and the influence of the
345 mixed wetting characteristics on the flooding dynamics in the GDL are demonstrated. For

346 electroosmotic flows in charged porous media, the strongly nonlinear governing equations of
347 electroosmosis in three-dimensional porous media are solved using the LPBM. The effects of
348 pore structure, bulk ionic concentration, and the surface charge on electroosmotic permeability
349 are carefully investigated. It is concluded that the LBM is a powerful numerical tool to simulate
350 multi-physicochemical processes in porous media at the pore-scale.

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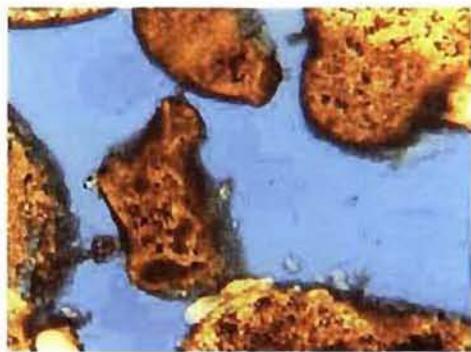


Figure 1 Photographic image of a limestone rock thin section (640x480 pixels).

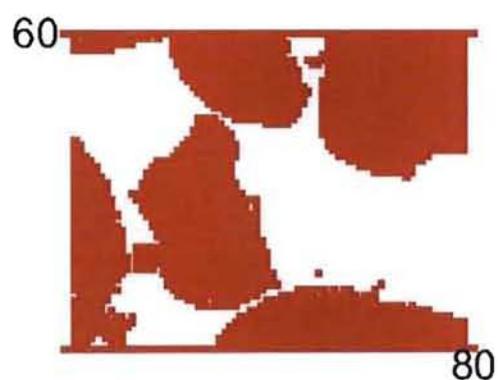


Figure 2 Digitized image with reduced resolution.

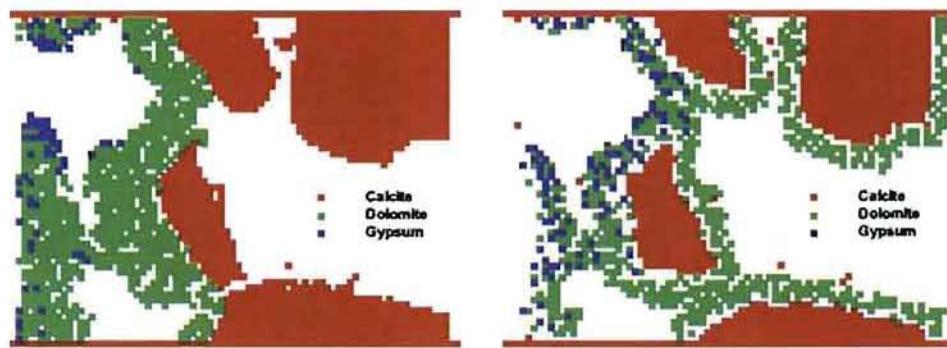


Figure 3 Resulting geometries at time=15625 seconds for two different mineral reaction rate constants: a) large reaction rate constants; b) small reaction rate constants.

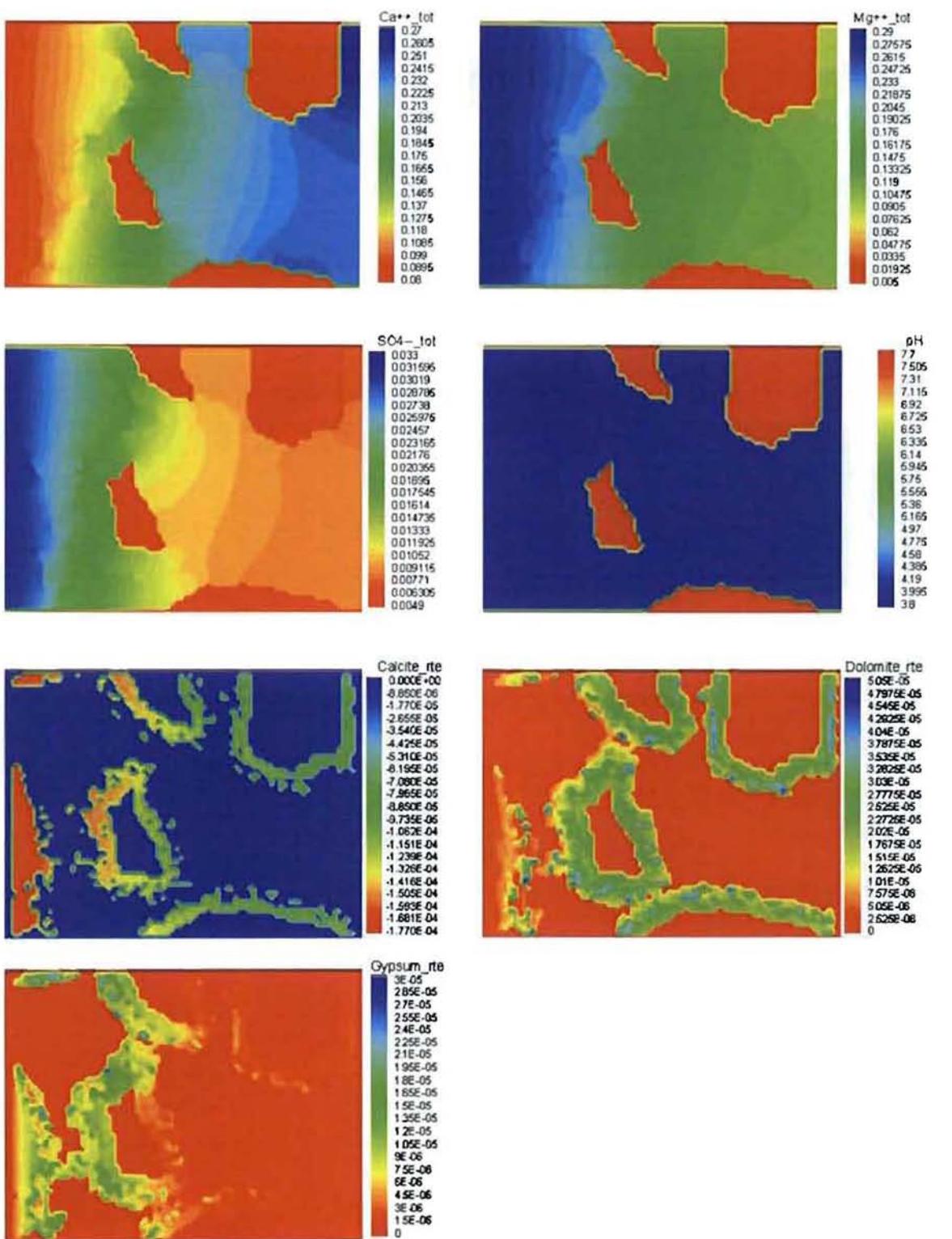


Figure 4 Distribution of solute concentrations, pH value, and reaction rates at time=15625 seconds for small reaction rate constants.

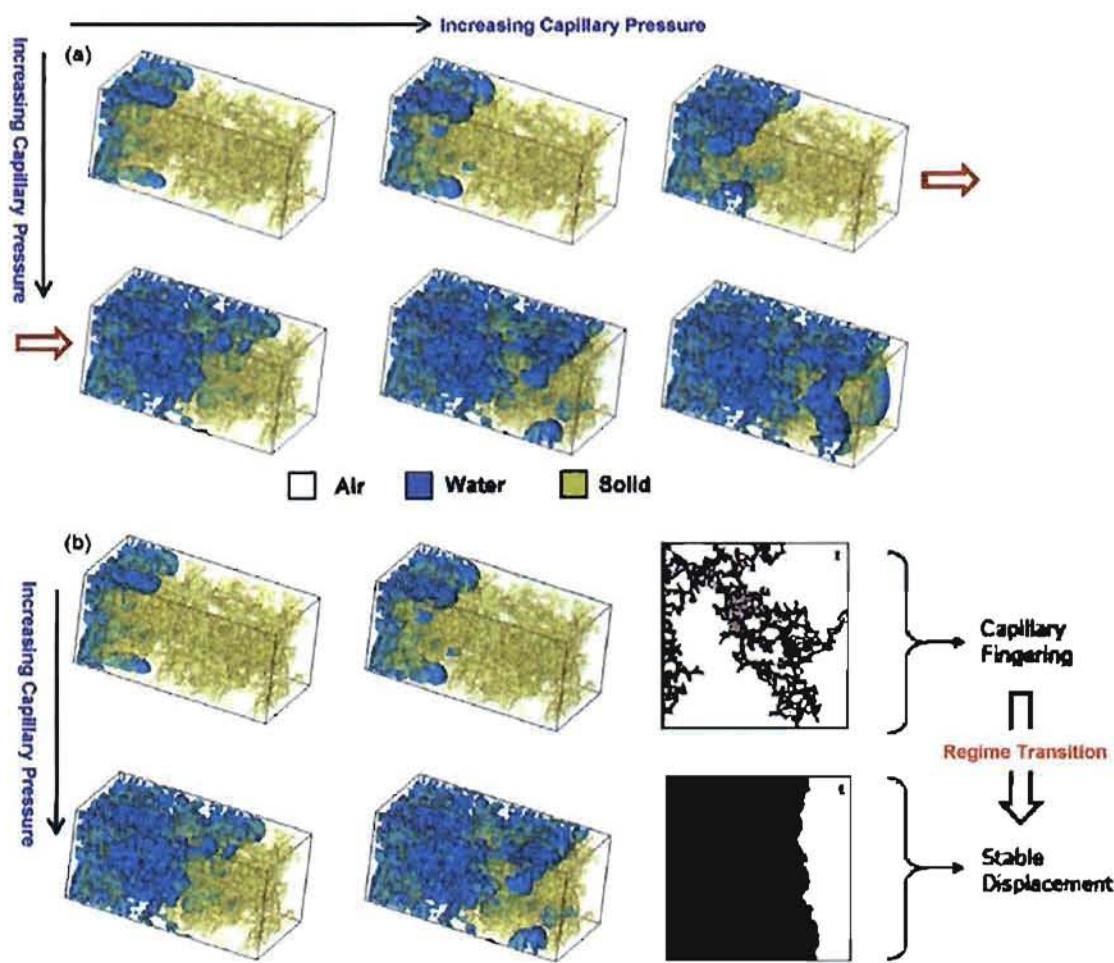


Figure 5 Advancing liquid water front with increasing capillary pressure through the initially air-saturated reconstructed CL microstructure from the primary drainage simulation.

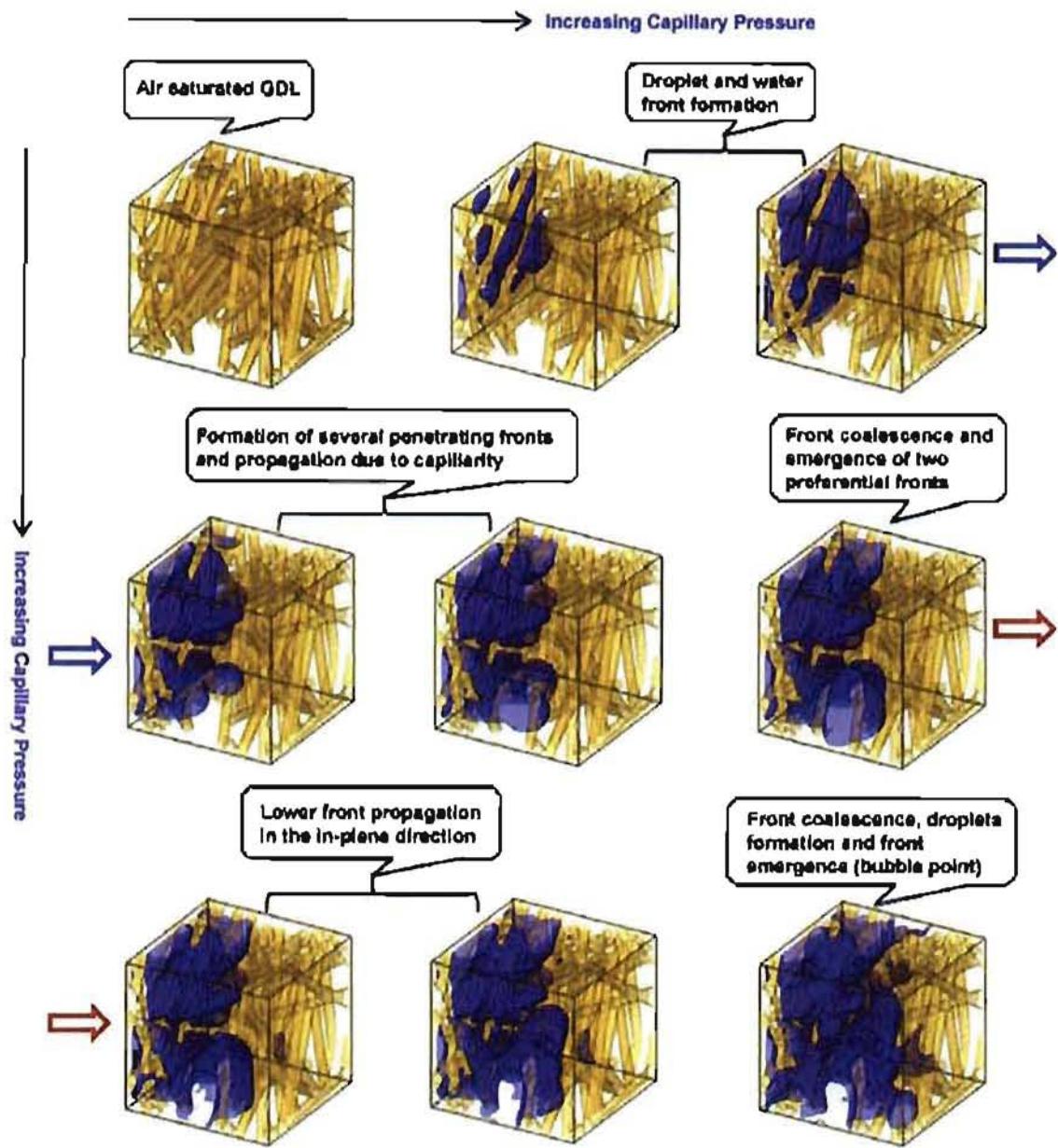


Figure 6 Advancing liquid water front with increasing capillary pressure through the initially air-saturated reconstructed GDL microstructure from the primary drainage simulation.

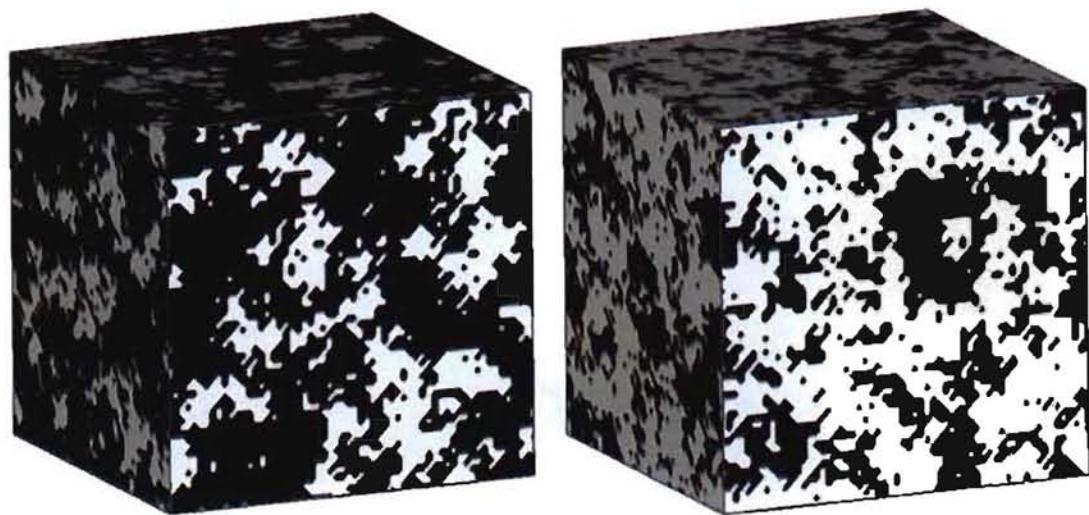


Figure 7 Schematics of the generated porous structures on $60 \times 60 \times 60$ grid systems. The white is solid particles and the dark is fluid: a) porosity=0.6; b) porosity=0.3.

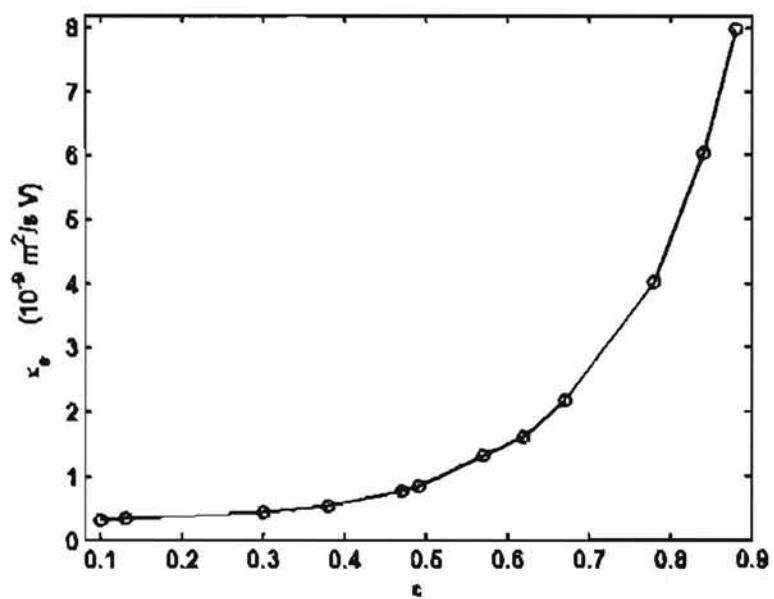


Figure 8 Predicted electroosmotic permeabilities for various porosities of porous media at $c_\infty = 10^{-4} \text{ M}$, $\zeta = -50 \text{ mV}$, $E = 1 \times 10^4 \text{ V/m}$.

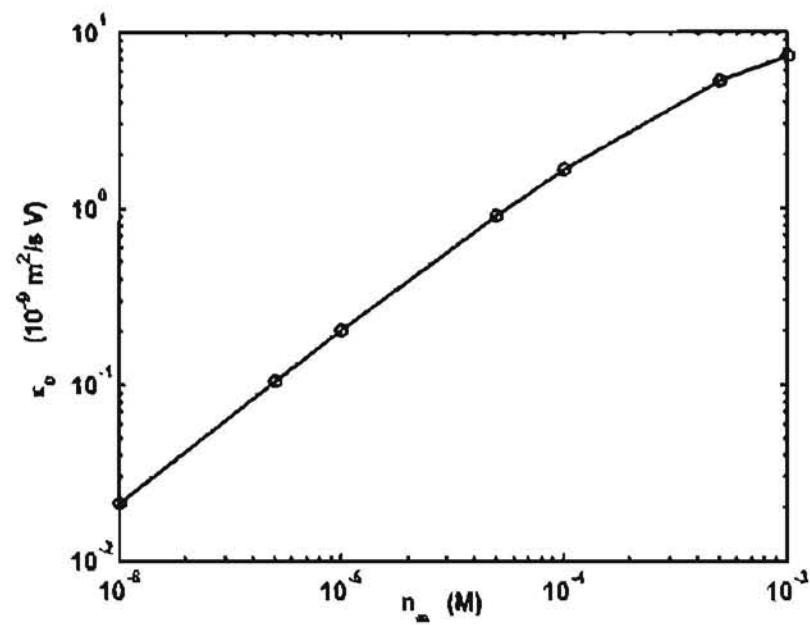


Figure 9 The electroosmotic permeability changing with the bulk ionic concentration for $\epsilon = 0.38$, $\zeta = -50 \text{ mV}$, and $E = 1 \times 10^4 \text{ V/m}$.

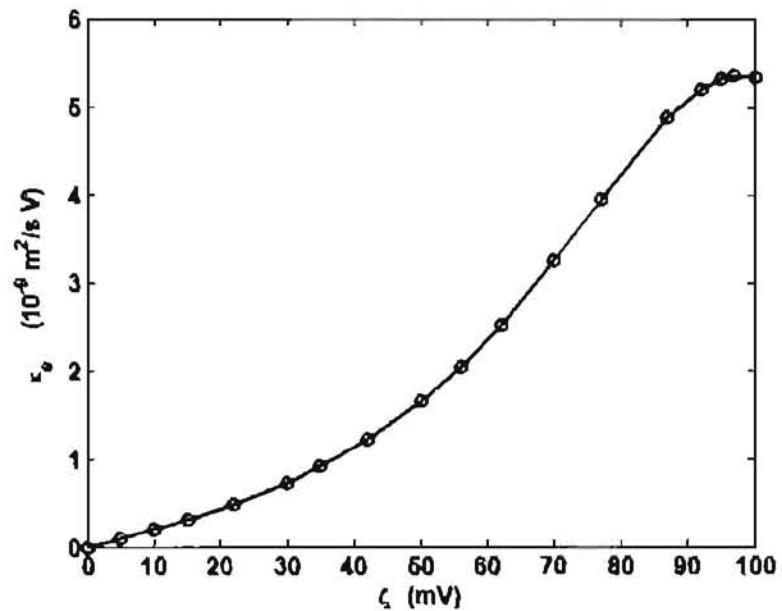


Figure 10 The electroosmotic permeability versus the zeta potential for $\varepsilon=0.38$, $c_\infty=10^4 \text{ M}$, and $E=1\times 10^4 \text{ V/m}$.