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LLNL-TR-415801

Towards a Metric to Estimate Atomic Number from Backscattered Photons

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August 19, 2009

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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September 10, 2009

Abstract

An ability to determine the atomic number of a material in a cargo container would be helpful in interdicting smuggled nuclear materials. This paper examines two processes by which high energy photons interact with matter; Compton scattering and pair production. The ratio of the number of photons which originate from the annihilation of positrons resulting from pair production and the number of photons coming from Compton scattering gives a good indication of atomic number. At large angles relative to an incident beam — i.e. backscattered, there is good separation in energy between Compton scattered photons and photons from positron annihilations. This ratio can then be cleanly determined in order to estimate atomic number.

1 INTRODUCTION

High energy photons incident on a material can interact with the material through any of numerous processes a few of which result in secondary photons that can then be detected. Two such processes are Compton scattering and pair production, the photons in the latter case coming from positron annihilations and having an energy equal to the mass of the electron m_{e}.

In this paper, we consider the outgoing photons from these two processes at large angles (i.e. backscattered) to an incident beam of photons from a bremsstrahlung source and construct a metric by which the atomic number Z of the target material might be estimated.

The ultimate design will no doubt include a forward detector which counts the direct shine from the source after passing through the object of interest. The information from this detector was not considered in this study.

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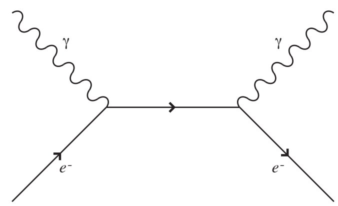

Figure 1: One of the Feynman diagrams for Compton scattering.

2 COMPTON SCATTERING

Compton scattering is the inelastic scattering of a photon from an electron, as shown in Fig. 1. The electron recoils while the scattered photon emerges with correspondingly less energy. The differential cross section for Compton scattering from a single free electron through an angle θ_{S} can be calculated from the well-known Klein-Nishina formula,

$$\frac{d\sigma_{\mathrm{KN}}}{d\Omega} = Zr_{\mathrm{e}}^2 \frac{1}{\left[1+\frac{E_0}{m_{\mathrm{e}}}(1-\theta_{\mathrm{S}})\right]^2} \left[1+\cos^2\theta_{\mathrm{S}} + \frac{\left(\frac{E_0}{m_{\mathrm{e}}}\right)^2 (1-\cos\theta_{\mathrm{S}})^2}{1+\frac{E_0}{m_{\mathrm{e}}}(1-\cos\theta_{\mathrm{S}})}\right] \quad (1)$$

By integrating this over the solid angle $d\Omega$, one obtains the Klein-Nishina total cross section for Compton scattering from a single free electron,

$$\begin{aligned}\sigma_{\mathrm{KN}} = 2\pi Zr_{\mathrm{e}}^2 \Bigg\{ & \frac{1+\frac{E_0}{m_{\mathrm{e}}}}{\left(\frac{E_0}{m_{\mathrm{e}}}\right)^2} \left[\frac{2\left(1+\frac{E_0}{m_{\mathrm{e}}}\right)}{1+2\frac{E_0}{m_{\mathrm{e}}}} - \frac{m_e}{E}\log\left(1+2\frac{E_0}{m_{\mathrm{e}}}\right)\right] \\ & + \frac{m_e}{2E}\log\left(1+2\frac{E_0}{m_{\mathrm{e}}}\right) - \frac{1+3\frac{E_0}{m_{\mathrm{e}}}}{\left(1+2\frac{E_0}{m_{\mathrm{e}}}\right)^2}\Bigg\} \end{aligned} \quad (2)$$

The energy of the scattered photon is

$$E_{\mathrm{S}} = \frac{E_0}{1+\frac{E_0}{m_{\mathrm{e}}}(1-\cos\theta_{\mathrm{S}})} \quad (3)$$

At photon energies below ~ 1 MeV, scattering from a free electron is somewhat different from scattering from an electron that is bound in an atom because the binding energy of the electron is no longer negligible compared to the photon energy. The net effect is that the cross section turns over and then falls off as photon energy decreases. However, in the low energy limit, the Klein-Nishina

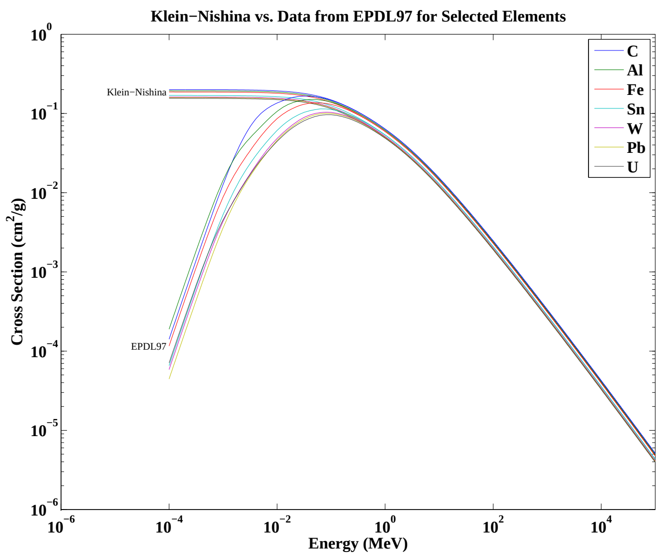

Figure 2: Comparison of the Klein-Nishina total cross section for Compton scattering, Eq. 2, to data from the EPDL97 database for selected elements.

formula converges to

$$\lim_{E_0 \lll m_e} \sigma_{\mathrm{KN}} \quad \rightarrow \quad 2\pi Z r_{\mathrm{e}}^2 \left(\frac{4}{3} - \frac{8}{3} \frac{E_0}{m_{\mathrm{e}}} \right) \tag{4}$$

to first order in E_0/m_{e}, as can be seen by expanding each of the terms in Eq. 2 in a power series. The difference in the behavior of the Klein-Nishina formula at low energy from the measured Compton scattering cross sections is illustrated in Fig. 2.

To compute a reasonable estimate for the differential cross section for Compton scattering, it is necessary to multiply Eq. 1 by a correction factor,

$$\frac{d\sigma_{\mathrm{C}}}{d\Omega} \quad = \quad \left(\frac{\sigma_{\mathrm{EPDL97}}}{\sigma_{\mathrm{KN}}} \right) \frac{d\sigma_{\mathrm{KN}}}{d\Omega} \tag{5}$$

where σ_{EPDL97} is interpolated from entries for Compton scattering in the EPDL97 database and σ_{KN} is computed from Eq. 2.

3 PAIR PRODUCTION

For photon energies above $2m_{\mathrm{e}}$, the photon can create an e^+e^- pair in the electromagnetic field of a nucleus or other charged particle, as shown in Fig. 3.

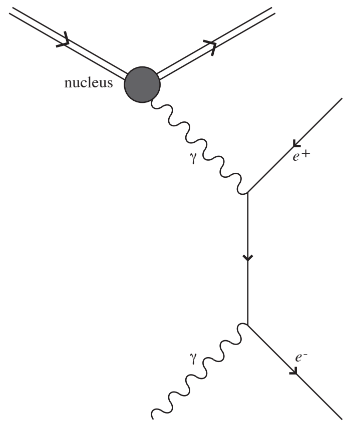

Figure 3: The most significant Feynman diagram for pair production.

The nucleus (or other charged particle) recoils in order to conserve both energy and momentum.

The cross section for pair production can be calculated from the very cumbersome Bethe-Heitler formula [1], or more empirically by interpolating between the data points in the EPDL97 database which includes contributions from both the nucleus and the orbiting electrons. This is shown in Fig. 4. The latter method ignores the angular dependences in the Bethe-Heitler formula, but is nonetheless sufficient here because we are ultimately interested not in the e^+e^- pair itself but in the photons of energy m_{e} created by the annihilation of the e^+. The process of e^+e^- annihilation has no angular dependence and the outgoing photons can be taken as evenly distributed in 4π. Thus, the differential cross section for a photon resulting from pair production going into solid angle $d\Omega$ is

$$\frac{d\sigma_{\mathrm{pp}}}{d\Omega} = \frac{\sigma_{\mathrm{pp}}}{4\pi} \tag{6}$$

4 BREMSSTRAHLUNG SOURCE

An approximate analytic formula for the bremsstrahlung spectrum was developed on the basis of theoretical arguments by Y. S. Tsai and Van Whitis [2]. Their formula is known to be satisfactory for photon energies down to 20% of the incident electron's energy, $E_0 \geq 0.2\, E_{\mathrm{e}}$, but tends to overestimate for smaller values of E_0 and diverges in the infrared limit. Multiplying their formula by an

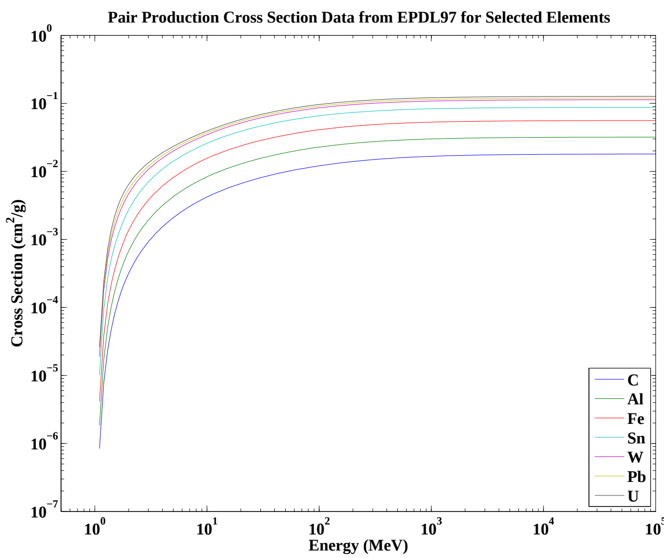

Figure 4: Cross section for pair production as a function of energy from the EPDL97 database for selected elements.

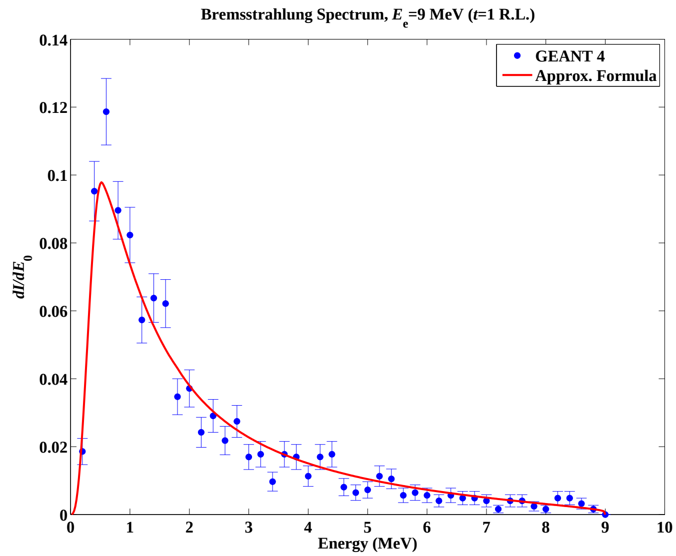

Figure 5: Comparison of the bremsstrahlung spectrum formula of Eqs. 7-10 for an endpoint energy $E_{\rm e} = 9$ MeV to a simulation from GEANT 4. Both assumed a target thickness $t = 1$ radiation length and energy bins $dE_0 = 200$ keV.

empirically-based correction factor $\mathcal{C}$,

$$dI(t, E_{\rm e}, E_0) \quad = \quad \frac{dE_0}{E_0} \frac{\left(1 - \frac{E_0}{E_{\rm e}}\right)^{\frac{4}{3}t} - e^{-\frac{7}{9}t}}{\frac{7}{9} + \frac{4}{3}\ln\left(1 - \frac{E_0}{E_{\rm e}}\right)} \times \mathcal{C} \tag{7}$$

where $E_{\rm e}$ is the energy of the incident electron, E_0 is the energy of the photon, and t is the target thickness, gives reasonably good agreement over all energies, as illustrated in Fig. 5 which shows the specific case where $E_{\rm e} = 9$ MeV. The functional form of the correction factor $\mathcal{C}$ [3] is

$$\mathcal{C} \quad = \quad \begin{cases} \mathcal{A} \times \mathcal{B} & E_0 \leq \frac{3}{2}E' \\ \mathcal{B} & \frac{3}{2}E' < E_0 < 0.2\, E_{\rm e} \\ 1 & E_0 \geq 0.2\, E_{\rm e} \end{cases} \tag{8}$$

where

$$\mathcal{A} = \frac{1}{2}\left[1-\cos\left(\frac{2\pi E_0}{3E'}\right)\right] \tag{9}$$

$$\mathcal{B} = \left[\frac{1-e^{-\frac{E_0}{E'}}}{1-\exp\left(\frac{E_0-2\times 0.2\,E_e}{E'}\right)}\right]^2 \tag{10}$$

$$E' = 0.04\times E_e \tag{11}$$

For $E_0 \geq 0.2\,E_e$, the formula returns the result of Y. S. Tsai and Van Whitis.

5 BACKSCATTERED FLUX

Consider a slab of arbitrary thickness composed of an element with atomic number Z: An incident photon of energy E_0 from a bremsstrahlung source penetrates this slab to a certain depth x, and between x and $x+dx$ interacts via either the Compton scattering or pair production processes. The resulting photon with energy E_S emerges from the slab on the same side from which it entered and gets detected by a finite size detector located at an angle δ relative to the incident beam and subtending a solid angle of $d\Omega$ as viewed from the interaction point. Because independent probabilities multiply, the probability for this to happen is the product of the following probabilities:

- dP_{src} = probability for the source to emit a γ between E_0 and E_0+dE_0
- P_{in} = probability for the γ to go a depth x in the material
- dP_{int} = probability for the γ to interact between x and $x+dx$ and for the scattered γ to be headed in the direction of a detector that subtends a solid angle $d\Omega$
- P_{out} = probability that the scattered γ exits the material

The probability dP_{src} depends on the energy spectrum of the source and is equal to dI from Eqs. 7-10.

The probability $P_{in} = e^{-\rho\sigma(E_0)x}$ depends on the distance x that the photon must travel through the material to get to the interaction point and on the total interaction cross section $\sigma(E_0)$ corresponding to the incident photon's energy. The probability $P_{out} = e^{-\rho\sigma(E_S)x'}$ depends on the distance $x' = x/\cos\delta$ the scattered photon must travel through the material to get out to the detector and on the total interaction cross section $\sigma(E_S)$ corresponding to the scattered photon's energy. The total cross section vs. energy for selected Z is shown in Fig. 6.

For Compton scattering, the probability $dP_{int} = \rho\; d\sigma_C(E_0)\,dx$ where the scattered photon emerges such that the scattering angle θ_S in Eq. 1 corresponds to the direction of the detector. To calculate the flux of photons from Compton scattering that reaches the detector, we must integrate the product of these

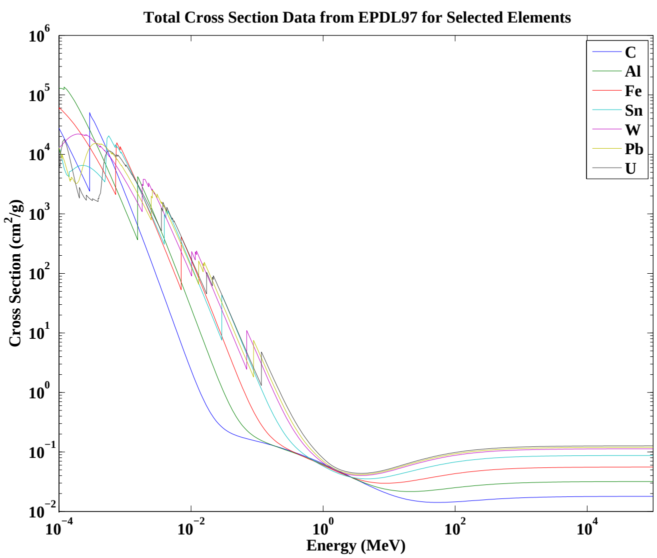

Figure 6: Total interaction cross section as a function of energy from the EPDL97 database for selected elements.

probabilities over x for the thickness of the material and over the spectrum of incident photon energies E_0,

$$\begin{aligned} P_{\mathrm{C}} &= \int_0^{E_{\mathrm{e}}} \int_0^{x_{\max}} \frac{dP_{\mathrm{src}}}{dE_0} \times P_{\mathrm{in}} \times \frac{dP_{\mathrm{int}}}{dx} \times P_{\mathrm{out}}\, dx\, dE_0 \\ &= \int_0^{E_{\mathrm{e}}} \int_0^{x_{\max}} \frac{dI}{dE_0}\, e^{-\rho\sigma(E_0)x}\, \rho\, d\sigma_{\mathrm{C}}\left(E_0\right)\, dx\, e^{-\rho\sigma(E_{\mathrm{S}})\frac{x}{\cos\delta}}\, dE_0 \end{aligned} \tag{12}$$

For pair production, the probability $dP_{\mathrm{int}} = \rho\, d\sigma_{\mathrm{pp}}\left(E_0\right) dx$ where one of the two resulting photons from the positron annihilation ends up heading in the direction of the detector. All emerging photons have energy m_{e} and it is assumed that none of the e^+ exit the material. The expression for the flux of photons resulting from pair production is analogously

$$P_{\mathrm{pp}} = \int_0^{E_{\mathrm{e}}} \int_0^{x_{\max}} \frac{dI}{dE_0}\, e^{-\rho\sigma(E_0)x}\, \rho\, d\sigma_{\mathrm{pp}}\left(E_0\right) dx\, e^{-\rho\sigma(m_{\mathrm{e}})\frac{x}{\cos\delta}}\, dE_0 \tag{13}$$

Because all the photons resulting from pair production come from the annihilation of the e^+, the corresponding spectrum is simply a δ-function at m_{e}. The spectrum for the Compton-scattered photons can be derived by considering

$$\frac{dP_{\mathrm{C}}}{dE_0} = \int_0^{x_{\max}} \frac{dI}{dE_0}\, e^{-\rho\sigma(E_0)x}\, \rho\, d\sigma_{\mathrm{C}}\left(E_0\right)\, dx\, e^{-\rho\sigma(E_{\mathrm{S}})\frac{x}{\cos\delta}} \tag{14}$$

Eq. 3 can be solved to express the energy E_0 of the incident photon in terms of the energy E_{S} of the scattered photon,

$$E_0 = \frac{m_{\mathrm{e}} E_{\mathrm{S}}}{m_{\mathrm{e}} - E_{\mathrm{S}}\left(1 - \cos\theta_{\mathrm{S}}\right)} \tag{15}$$

to yield an expression for the spectrum of detected photons,

$$\frac{dP_{\mathrm{C}}}{dE_{\mathrm{S}}} = \int_0^{x_{\max}} \frac{dI}{dE_0}\, \frac{dE_0}{dE_{\mathrm{S}}}\, e^{-\rho\sigma[E_0(E_{\mathrm{S}})]x}\, \rho\, d\sigma_{\mathrm{C}}\left[E_0\left(E_{\mathrm{S}}\right)\right]\, dx\, e^{-\rho\sigma(E_{\mathrm{S}})\frac{x}{\cos\delta}} \tag{16}$$

The spectrum of photons from a 9 MeV bremsstrahlung source reaching a detector placed at 120° to the incident beam is shown for selected elements in Fig. 7.

These calculations are in reality somewhat more complex than what is suggested here: A realistic beam would have some angular size and would interact with the material inside a voxel of finite volume. This volume must also be integrated over. The scattering angle θ_{S} is in reality a function of where in this voxel the interaction takes place. If the detector is of finite size, the solid angle $d\Omega$ would need to be integrated over as well. These complications are not particularly challenging to overcome numerically.

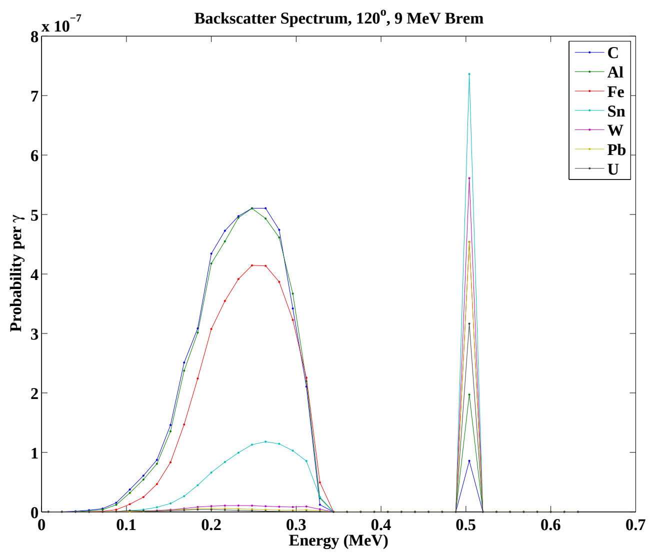

Figure 7: Spectrum of photons from Compton scattering and from pair production. The broad peak at low energies results from Compton scattering. The sharp peak at 0.511 MeV are the photons produced by the pair production process. The calculation underlying this spectrum assumed a 9 MeV bremsstrahlung source evenly illuminating a 5 cm diameter voxel 2 m away with a 3 inch diameter detector placed at 120° to the incident beam at 2 m from the voxel.

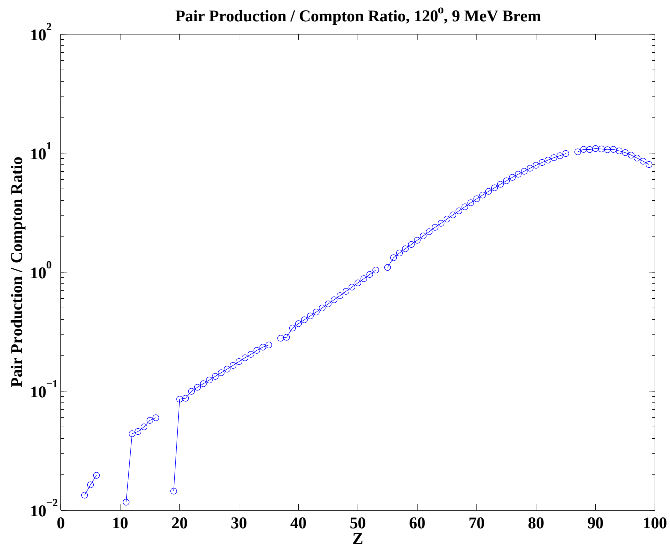

Figure 8: Ratio of the flux from pair production to Compton scattering for a 9 MeV bremsstrahlung source and a detector placed at 120° to the incident beam.

6 ESTIMATING ATOMIC NUMBER

It can be seen in Fig. 7 that at 120° to the incident beam, there is good separation between the Compton scattered photons and the photons resulting from pair production. An estimate of atomic number Z can be made by considering the ratio of these two fluxes. The ratio of the photon flux from pair production (Eq. 13) to photons from Compton scattering (Eq. 12) is shown in Fig. 8 (the calculations underlying this plot assumed a 9 MeV bremsstrahlung source evenly illuminating a 5 cm diameter voxel 2 m away with a 3 inch diameter detector placed at 120° to the incident beam at 2 m from the voxel). Aside from gases and group 1A alkali metals, this ratio rises monotonically with Z until roughly $Z = 90$.

7 FURTHER CONSIDERATIONS

This study has only considered a slab of elemental material in a vacuum. If the material in question were embedded inside other materials, it is expected that the direct-shine detector would become necessary to assist in backing out the effects of those embedding materials.

One important source of photons which has not been considered here is bremsstrahlung photons from the Compton scattered electrons and from the e^+e^- pair. The direction and energy of the Compton electron are easily determined; the e^+ and e^- from pair production could be distributed in 4π. A natural next step would then be to attempt to estimate the flux from bremsstrahlung using the formula of Eq. 7 and to then refine the above metric on that basis.

References

[1] H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) A146, 83 (1934).

[2] Y. S. Tsai and Van Whitis, Phys. Rev. 149, 1248–1257 (1966), Eq. 25.

[3] The functional form for the correction factor was originally suggested by Ed Wishnow and then fine-tuned by this author.