LLNL-TR-415482

LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

Estimation of confidence intervals for multiplication and efficiency

Verbeke, Jerome

August 11, 2009

Disclaimer

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Estimation of confidence intervals for multiplication and efficiency[1]

Jerome M. Verbeke

Introduction:

Helium-3 tubes are used to detect thermal neutrons by charge collection using the $^{3}He(n,p)$ reaction. By analyzing the time sequence of neutrons detected by these tubes, one can determine important features about the constitution of a measured object: Some materials such as Cf-252 emit several neutrons simultaneously, while others such as uranium and plutonium isotopes multiply the number of neutrons to form bursts. This translates into unmistakable signatures. To determine the type of materials measured, one compares the measured count distribution with the one generated by a theoretical fission chain model. When the neutron background is negligible, the theoretical count distributions can be completely characterized by a pair of parameters, the multiplication M and the detection efficiency ε.

While the optimal pair of M and ε can be determined by existing codes such as BigFit, the uncertainty on these parameters has not yet been fully studied. The purpose of this work is to precisely compute the uncertainties on the parameters M and ε, given the uncertainties in the count distribution. By considering different lengths of time tagged data, we will determine how the uncertainties on M and ε vary with the different count distributions.

Object used in this study:

Using the code SourceSim that simulates fission chains for any given set of parameters, we generated a sequence of time tags for a Pu ball. We set the spontaneous fission neutron rate so as to correspond to 270 grams of ^{240}Pu. The multiplication of the object was set to 4.25, and the detection efficiency to 6.15%. The detection time constant λ^{-1} was set to 40 μsecs. Time tags in the ^{3}He tubes were generated with these parameters. The sequence of time tags was analyzed using random time gate count distributions. Five sequences of different lengths were analyzed, the longest sequence was 5120 secs long, the second longest 512 secs, all the way down to the shortest of 0.512 secs.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Methodology:

Each time tag sequence was used to generate 512 different random time gate count distributions, the shortest one being 1 μs long, in increments of 1 μs up to the longest time gate of 512 μs. The count distribution for 51.2 secs in shown in Fig.1.

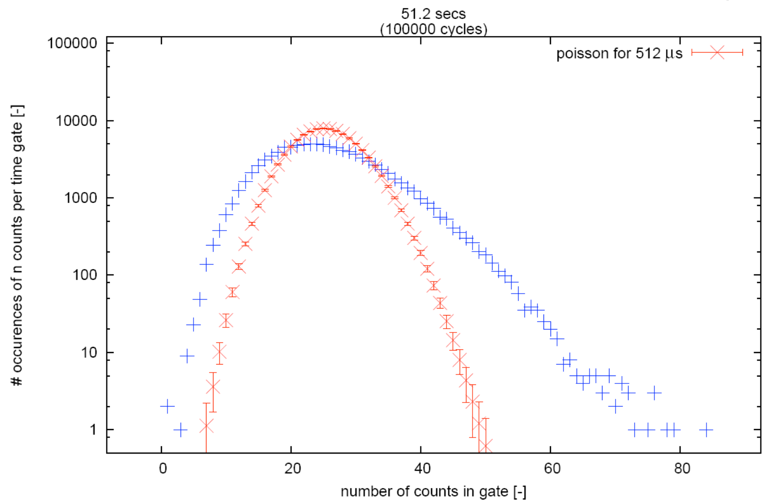

Fig. 1: Random time gate distribution for the 51.2 sec long neutron time tag sequence.

Moments of these count distributions were computed, the Feynman moment for the 51.2 seconds time tag sequence is shown in Fig.2.

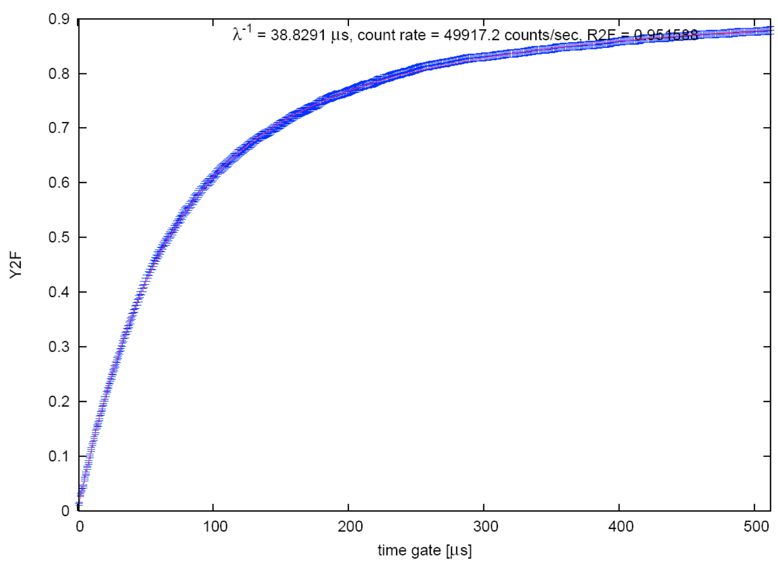

Fig. 2: Feynman moment of the 51.2 sec long time tag sequence.

The count rate was computed to be 49,917 neutrons/sec and the detection time constant λ^{-1} was determined to be 38.8 μsecs by fitting the computed Feynman moment in Fig. 2 with an analytical expression from the point-model fission chain theory.

Knowing the count rate and the detection time constant λ^{-1}, we then used the count distribution generator code developed by M. Prasad and N. Snyderman to generate theoretical count distributions for different pairs of M and ε. We compared the distribution shown in Fig. 1 and its theoretical counterpart using a likelihood estimator. One should note that we used a single count distribution for this comparison, the 512 μsecs count distribution. A Markov Chain Monte Carlo method was then used to determine the confidence intervals in the (M,ε) space. Markov chains sample the parameter space using random walks and accept or reject the (M,ε) pairs based on the value of the likelihood at that point. A fundamental property of the Markov Chain method is that the density of points obtained in the (M,ε) space is proportional to the likelihood of the solution to be in that space. Starting with an area bounded by lower and upper limits in the parameter space of M and ε, one can then just determine the probability of a solution to lie within an area or contour by taking the ratio of points within that contour to the total number of points in the full parameter space. Fig. 3 shows the several plots, the upper 2 and the M and ε sampled by the Markov Chain, the middle 2 are the auto-correlation functions for M and ε, The lower left plot shows the points accepted by the Markov Chain algorithm and the lower right plot shows the confidence intervals for 2 contours, the red one corresponds approximately to a 68.3% confidence, the blue one to a 95.4% confidence. These 2 numbers were chosen to correspond to confidence intervals corresponding to 1 sigma and 2 sigma in a Gaussian distribution.

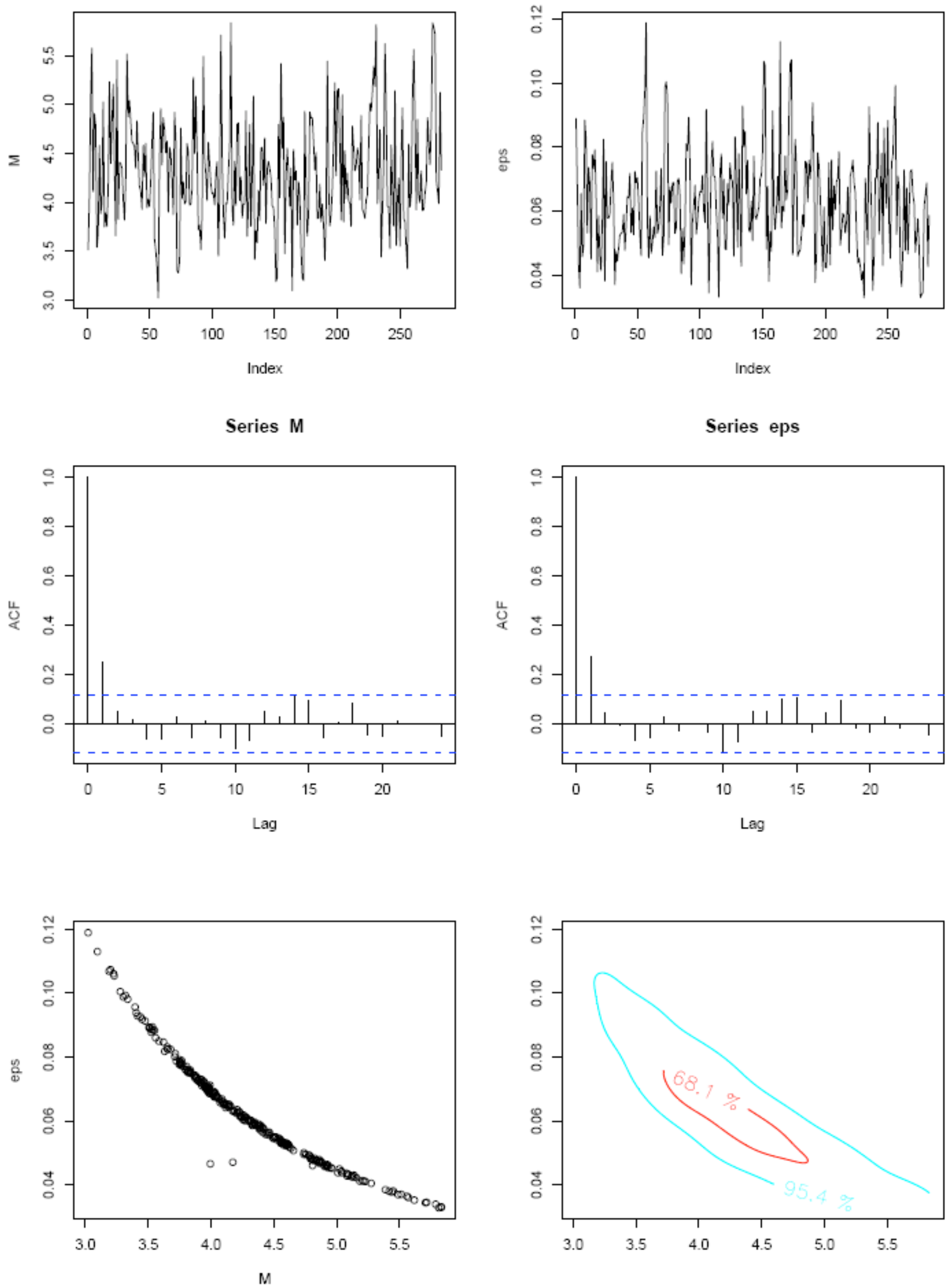

Fig. 3: Detailed results of the Markov-Chain algorithm.

Results:

The confidence intervals for the 5 count times ranging from 5120 seconds downs to 0.512 seconds are shown in Figs. 4 through 8.

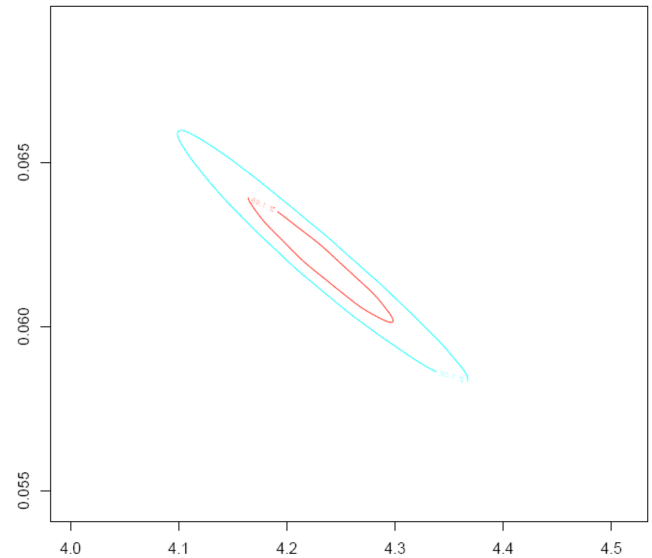

Fig. 4: Confidence interval for count time of 5120 secs (red ~ 68%, turquoise ~ 95%)

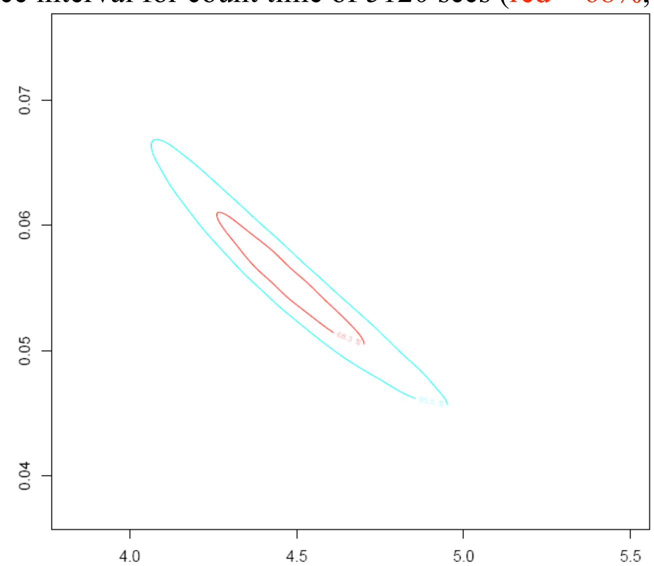

Fig. 5: Confidence interval for count time of 512 secs (red ~ 68%, turquoise ~ 95%).

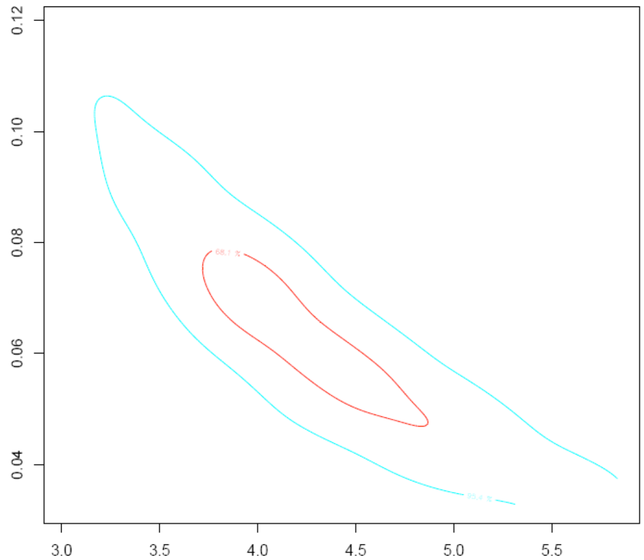

Fig. 6: Confidence interval for count time of 51.2 secs (red ~ 68%, turquoise ~ 95%).

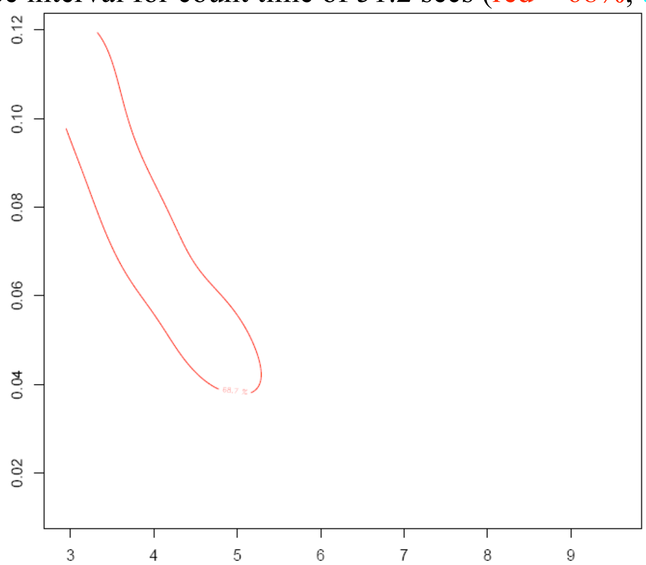

Fig. 7: Confidence interval for count time of 5.12 secs (red ~ 68%).

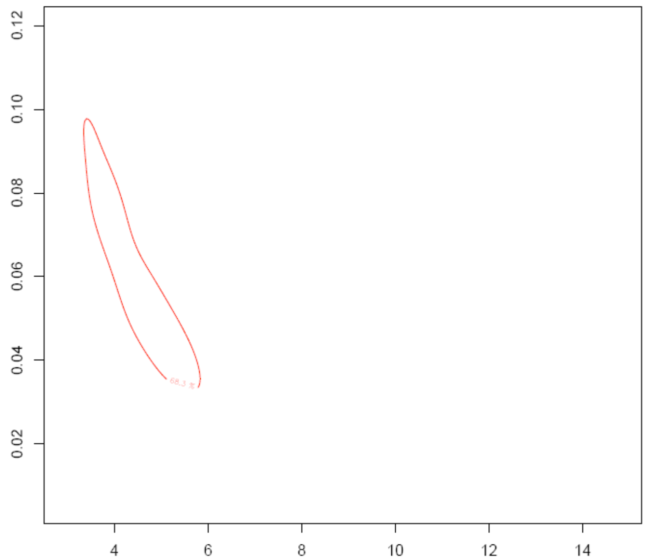

Fig. 8: Confidence interval for count time of 0.512 secs (red ~ 68%).

While the data contained in the 2-dimensional graphs shown in Figs. 4 though 8 cannot be condensed into a single multiplication interval, because this range is dependent on the efficiency, Table 1 takes into consideration the envelopes for the 68.3% and 95.4% confidence intervals and shows the minimum and maximum multiplications for these envelopes as a function of the count time.

Count time in seconds	68.3%	95.4%
0.512	3.0 - 6.0	n/a
5.12	3.0 - 5.3	n/a
51.2	3.75 – 4.9	3.2 – 5.9
512	4.25 – 4.65	4.05 – 5.0
5120	4.15 – 4.3	4.1 – 4.35

Table 1: Range of multiplication M for 68.3% and 95.4% confidence intervals.

Comparison to count time estimator:

Manoj Prasad wrote a code "Count Time Estimator"[2] that computes the range of M and ε based on the count time for different sources. The Count Time Estimator propagates the statistical error bars in Y_{2F} and Y_{3F} through the moments algebra to determine the range of M and efficiency solutions. Since the statistical error bars depend on how long one collects data, the count time estimator gives an estimate of M and efficiency ranges for a given count time.

Using the same parameters as above, we get the Count Time Estimator results in Table 2.

[2] Count Time Estimator is in the LLNL code Gamma Designer. Gamma Designer is an Unclassified Controlled Nuclear Design Information (UCNI) software.

Count time in seconds	M range
0.512	1.4 – 15.
5.12	2.0 – 101.
51.2	2.7 – 9.8
512	3.7 – 5.1
5120	4.1 – 4.5

Table 2: Range of multiplication M using the Count Time Estimator.

The ranges are for one standard deviation. Compared the Count Time Estimator, the Markov Chain Monte Carlo method gives smaller ranges for the multiplication parameter M.