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Matter Unification in Warped Supersymmetric $SO(10)$

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Abstract

We construct models of warped unification with a bulk $SO(10)$ gauge symmetry and boundary conditions that preserve the $SU(4)_C \times SU(2)_L \times SU(2)_R$ Pati-Salam gauge group (422). In the dual 4D description, these models are 422 gauge theories in which the apparent unification of gauge couplings in the minimal supersymmetric standard model is explained as a consequence of strong coupling in the ultraviolet. The weakness of the gauge couplings at low energies is ensured in this 4D picture by asymptotically non-free contributions from the conformal sector, which are universal due to an approximate $SO(10)$ global symmetry. The 422 gauge symmetry is broken to the standard model group by a simple set of Higgs fields. An advantage of this setup relative to $SU(5)$ models of warped unification is that matter is automatically required to fill out representations of 422, providing an elegant understanding of the quantum numbers of the standard-model quarks and leptons. The models also naturally incorporate the see-saw mechanism for neutrino masses and bottom-tau unification. Finally, they predict a rich spectrum of exotic particles near the TeV scale, including states with different quantum numbers than those that appear in $SU(5)$ models.

1 Introduction

Two successful aspects of grand unification are the unification of gauge couplings and the unification of matter into a smaller number of representations. Both features explain something about nature. The first explains why the running gauge couplings appear to meet at a very high energy in the context of weak scale supersymmetry. The second helps to explain the quark and lepton gauge quantum numbers in the standard model. The apparent unification of couplings can be easily addressed if the unified group is a simple group containing all the standard model gauge interactions. The smallest successful group is $SU(5)$ [1], and the next smallest is $SO(10)$ [2]. Of course, in these theories coupling unification and matter unification are closely related, as the enlarged gauge symmetry requires quarks and leptons to appear in representations of either $SU(5)$ or $SO(10)$.

Based on matter unification alone, however, the simplest approach is arguably $SU(4)_C \times SU(2)_L \times SU(2)_R$ (422) *a la* Pati and Salam [3]. This group provides a very elegant explanation of the quantum numbers of the standard model fermions, with a full generation of quarks and leptons (including a right-handed neutrino) filling out the representations $(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}^*, \mathbf{1}, \mathbf{2})$. The breaking of the 422 group to its standard model subgroup is quite straightforward: it is attained by a simple set of Higgs fields, and this same set of fields is sufficient to break unwanted quark and lepton mass relations (while preserving bottom-tau unification) through non-renormalizable operators. This breaking also almost automatically leads to small neutrino masses through the see-saw mechanism (which is not the case in $SU(5)$). The simplicity of the gauge breaking makes 422 attractive compared with larger unified groups such as $SO(10)$, which require a more complicated structure for realistic gauge breaking.

In this paper we explore the interesting role that the 422 gauge group can play in models of warped supersymmetric unification. In the $SU(5)$ model introduced in [4], the unified gauge symmetry realized in the bulk is explicitly broken by boundary conditions on the Planck brane. This implies that in the 4D dual description the theory does not have an $SU(5)$ gauge symmetry. Rather, $SU(5)$ appears as an approximate *global* symmetry, possessed by the strongly interacting conformal sector that arises as the dual description of the bulk physics. The successful supersymmetric prediction relating the low-energy gauge couplings then follows from the assumption that the theory becomes strongly coupled in the ultraviolet. The role of the $SU(5)$ global symmetry in this context is to ensure that the contributions from the conformal sector to the gauge coupling evolution are $SU(5)$ symmetric, so that the non-universal contributions are given purely by the elementary states, which are identical to the states of the minimal supersymmetric standard model (MSSM). Thus, a simple group still plays an important role in the unification prediction, but in the infrared, rather than the ultraviolet, and as a global symmetry, rather than a gauge symmetry.

This extra global symmetry does not require a complicated breaking mechanism – it is explicitly broken from the beginning – but nor does it require that the matter fields form unified representations. Suppose we introduce quarks and leptons on the Planck brane in the $SU(5)$ model of [4]. Then there is no reason why they must fill out $SU(5)$ representations, and no reason why their hypercharges must satisfy the appropriate quantization condition. This point becomes especially important in the class of models introduced in [5], where the unified symmetry is broken both on the Planck and TeV branes, in which case the quarks and leptons cannot arise from bulk matter alone. We thus clearly need some other ingredient for understanding matter quantum numbers in these setups. We propose to use the 422 gauge group for this purpose. We promote the gauge group in the 4D dual picture to 422 in these models. This in turn requires that we promote the global group of the conformal sector, which is the bulk gauge group in the 5D picture, to $SO(10)$, at least. The correct prediction relating the gauge couplings at low energies then arises through the global $SO(10)$ group, with the assumption of strong coupling. Enlarging the global group to $SO(10)$ has direct consequences for physics at low energies, because it results in light exotics with different quantum numbers than those in the $SU(5)$ models. The resulting phenomenology is, naturally, quite rich.

The organization of the paper is as follows. In section 2 we construct a model based on the structure of the model of [5], so that the bulk $SO(10)$ group is broken both on the Planck and TeV branes. In section 3 we present a model in which the bulk $SO(10)$ is broken only on the Planck brane, similarly to the model of [4]. In both theories the matter fields are located on the Planck brane, but without the problems such a setup produces in the $SU(5)$ models. Moreover, we gain several desirable features coming from 422, including quark-lepton unification (and thus hypercharge quantization), bottom-tau unification, and a natural see-saw mechanism. Some of the group theory in our models shares certain features with $SO(10)$ models in flat space [6], but the physics involved is quite different. Conclusions are given in section 4.

2 Model with TeV-Brane Symmetry Breaking

In this section we construct a model having the properties described in the introduction, and in which the bulk gauge symmetry is reduced at the TeV brane. This model can be viewed as an extension of the 321-321 model of Ref. [5]. The main new ingredient of the model presented here is the unification of matter fields localized on the Planck brane. This leads to an understanding of the quark and lepton quantum numbers, as well as the ratio of the bottom-quark and tau-lepton masses through Yukawa unification. Small neutrino masses also arise quite naturally through the see-saw mechanism. All these features can be accommodated without spoiling the interesting features of the 321-321 model: automatic doublet-triplet splitting, suppression of proton decay,

and a rich phenomenology of superparticles and grand-unified-theoretic (GUT) particles.

2.1 Basic setup

The model is formulated in a 5D warped spacetime with the extra dimension y compactified on an S^1/Z_2 orbifold: $0 \leq y \leq \pi R$. The metric is given by

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where k is the AdS curvature, which is taken to be somewhat (typically a factor of a few) smaller than the 5D Planck scale M_5 . The 4D Planck scale, M_{Pl} , is given by $M_{\text{Pl}}^2 \simeq M_5^3/k$ and we take $k \sim M_5 \sim M_{\text{Pl}}$. We choose $kR \sim 10$ so that the TeV scale is naturally generated by the AdS warp factor: $k' \equiv ke^{-\pi kR} \sim \text{TeV}$ [7].

We choose the bulk gauge group to be $SO(10)$. This bulk $SO(10)$ symmetry is then broken by boundary conditions imposed at the boundaries both at $y = 0$ (Planck brane) and πR (TeV brane). Using the 4D $N = 1$ superfield notation, in which the 5D gauge multiplet is described by a vector superfield $V(A_\mu, \lambda)$ and a chiral superfield $\Sigma(\sigma + iA_5, \lambda')$, the boundary conditions are given by

$$\begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} PV P^{-1} \\ -P \Sigma P^{-1} \end{pmatrix} (x^\mu, y), \quad \begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y') = \begin{pmatrix} PV P^{-1} \\ -P \Sigma P^{-1} \end{pmatrix} (x^\mu, y'), \quad (2)$$

where $y' = y - \pi R$. The matrix P is chosen such that it leaves the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ (422) subgroup of $SO(10)$ invariant. Specifically, in the basis where the generators of $SO(10)$, which are imaginary and antisymmetric 10×10 matrices, are given by $\sigma_0 \otimes A_5$, $\sigma_1 \otimes A_5$, $\sigma_2 \otimes S_5$ and $\sigma_3 \otimes A_5$, the matrix P can be chosen as $P = \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1)$. Here, σ_0 is the 2×2 unit matrix and $\sigma_{1,2,3}$ are the Pauli spin matrices; S_5 and A_5 are 5×5 matrices that are real and symmetric, and imaginary and antisymmetric, respectively. This reduces the gauge group at the Planck and the TeV branes to 422, and leaves the $(\mathbf{15}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{3})$ component of V and the $(\mathbf{6}, \mathbf{2}, \mathbf{2})$ component of Σ as zero modes, where the numbers in parentheses represent quantum numbers under 422. All components of the $SO(10)$ gauge multiplet have Kaluza-Klein (KK) towers with the typical mass scale of $k' \sim \text{TeV}$.

The Higgs fields are introduced in the bulk as a hypermultiplet transforming as $\mathbf{10}$ of $SO(10)$, which is described by two chiral superfields as $\{H, H^c\}$ in the 4D $N = 1$ superfield notation. They obey the boundary conditions

$$\begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} -PH \\ PH^c \end{pmatrix} (x^\mu, y), \quad \begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y') = \begin{pmatrix} -PH \\ PH^c \end{pmatrix} (x^\mu, y'), \quad (3)$$

which leave the $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ component of H and the $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ component of H^c as zero modes.¹ The Higgs multiplet can have a mass parameter in the bulk, which we parameterize as $c_H k$. The

¹Alternatively, we could introduce the Higgs field H on the Planck brane in the $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ representation of

parameter c_H then controls the wavefunction profiles for the zero modes arising from $\{H, H^c\}$. As in the 321-321 model of [5], the unwanted zero modes from Σ and H^c obtain masses when supersymmetry is broken.

Matter fields are introduced on the Planck brane as chiral superfields in the $\Psi(\mathbf{4}, \mathbf{2}, \mathbf{1}) + \bar{\Psi}(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ representation of 422 for each generation, which contain our quarks and leptons (and a right-handed neutrino) as $\Psi = \{Q, L\}$ and $\bar{\Psi} = \{U, D, E, N\}$. Since the gauge group on the Planck brane is non-Abelian, the charges of the matter fields are quantized. This setup also requires the existence of right-handed neutrinos, which is an important ingredient for the see-saw mechanism.

To reproduce successful phenomenology at low energies, the low-energy gauge group must be reduced to $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321). To this end, we break the 422 group by the Higgs mechanism on the Planck brane. The simplest possibility is to introduce chiral superfields on the Planck brane in the $\chi(\mathbf{4}, \mathbf{1}, \mathbf{2}) + \bar{\chi}(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ representation of 422, and give appropriate vacuum expectation values for them.² The expectation values are easily induced, for example, by introducing the superpotential interaction $\int d^2\theta S(\chi\bar{\chi} - v_\chi^2) + \text{h.c.}$ on the Planck brane, where S is a singlet chiral superfield. Here we take the expectation values $\langle\chi\rangle = \langle\bar{\chi}\rangle = v_\chi$ to be of order k , which is a natural scale on the Planck brane. This then breaks the gauge group on the Planck brane to 321 at the scale k , and gives masses to the 422/321 component of V , which would otherwise be massless.

Quark and lepton masses are generated on the Planck brane through the following operators:

$$S = \int d^4x \int_0^{\pi R} dy \, 2\delta(y) \left[\int d^2\theta \left(y \Psi \bar{\Psi} H_D + \frac{\lambda}{M_*} (\chi \bar{\Psi})^2 \right) + \text{h.c.} \right], \quad (4)$$

where H_D represents the $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ component of H under the 422 decomposition, and we have omitted generation indices. M_* is the cutoff scale of order M_5 . The first term gives Yukawa couplings for quarks and leptons while the second term gives Majorana masses for right-handed neutrinos of order $\langle\chi\rangle^2/M_*$, which generates small masses for the observed neutrinos through the see-saw mechanism. The unwanted mass relations arising from Eq. (4) for the first-two generation quarks and leptons are broken by higher dimensional operators involving $\langle\chi\rangle$ and $\langle\bar{\chi}\rangle$, allowing for realistic quark and lepton masses and mixings.³ For relatively suppressed 422-breaking expectation values $\langle\chi\rangle = \langle\bar{\chi}\rangle \approx k \lesssim M_*$, the Yukawa couplings for the third generation fermions are approximately unified at the scale k , which gives a successful m_b/m_τ prediction

422. Another possibility is to put the Higgs fields in the bulk and impose the boundary conditions of Eq. (3) with an extra minus sign in the right-hand side of the second equation (i.e. flipping the TeV-brane boundary conditions) [5].

²We could also introduce $\chi'(\mathbf{4}, \mathbf{2}, \mathbf{1}) + \bar{\chi}'(\mathbf{4}^*, \mathbf{2}, \mathbf{1})$ fields on the brane and make the breaking of left-right symmetry entirely spontaneous.

³The higher dimensional operators that allow for realistic fermion masses have the same form as those employed in 4D 422 theories, see e.g. [8].

at low energies. It also leads the theory to the large $\tan\beta$ region, $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle \approx 50$, where H_u and H_d ($\subset H_D$) are the Higgs fields giving masses to the up-type and down-type quarks, respectively. The couplings in Eq. (4) respect a $U(1)_R$ symmetry with the charges given by $V(0), \Sigma(0), H(0), H^c(2), \Psi(1), \bar{\Psi}(1), \chi(0), \bar{\chi}(0)$. This symmetry, when imposed on the theory, forbids potentially dangerous operators, such as $\int d^2\theta H_D^2 + \text{h.c.}$, on the Planck brane. As in the $SU(5)$ models, proton decay is not a problem for the Planck-brane localized matter because all potentially dangerous gauge bosons (and their KK towers) have wavefunctions strongly peaked towards the TeV brane.

2.2 Prediction for gauge couplings

The $SO(10)$ generators in the present model are naturally divided into three classes: (i) the 321 generators, (ii) the generators belonging to 422/321, which we call PS, and (iii) the generators belonging to $SO(10)/422$, which we call XY. The spectrum of the gauge sector is then given as follows. The 321 gauge multiplet has a zero mode V^{321} and a KK tower, which consists of V^{321} and Σ^{321} at each KK level, with the masses m_n given by the solutions of

$$\frac{J_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_{0,a}^2} m_n J_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_{0,a}^2} m_n Y_1\left(\frac{m_n}{k}\right)} = \frac{J_0\left(\frac{m_n}{k'}\right)}{Y_0\left(\frac{m_n}{k'}\right)}, \quad (5)$$

where $J_n(x)$ and $Y_n(x)$ are the Bessel functions of order n , and $a = 1, 2, 3$ represents $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively; g_B is the bulk $SO(10)$ gauge coupling and $\tilde{g}_{0,a}^2$ are the Planck-brane gauge couplings appropriately renormalized at the scale k' (for more precise definitions and our assumptions regarding the ultraviolet values of these parameters, see [5]). The PS gauge multiplet does not have a zero-mass mode, and its KK tower consists of V^{PS} and Σ^{PS} at each KK level with the masses approximately given by⁴

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_1\left(\frac{m_n}{k'}\right)}{Y_1\left(\frac{m_n}{k'}\right)}. \quad (6)$$

Finally, the XY gauge multiplet has a zero mode Σ^{XY} , and its KK tower $\{V^{\text{XY}}, \Sigma^{\text{XY}}\}$ has masses given by

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_1\left(\frac{m_n}{k'}\right)}{Y_1\left(\frac{m_n}{k'}\right)}. \quad (7)$$

⁴More precisely, the left-hand side of Eq. (6) is given by $\{J_0(m_n/k) - (Cg_B^2 v_\chi^2/m_n)J_1(m_n/k)\}/\{Y_0(m_n/k) - (Cg_B^2 v_\chi^2/m_n)Y_1(m_n/k)\}$, where C is an $O(1)$ coefficient depending on the gauge component. For $v_\chi \gg k'$, however, the denominator is well approximated by $\approx -(Cg_B^2 v_\chi^2/m_n)Y_1(m_n/k)$, which is enough to guarantee that the mass eigenvalues are almost given by the zeros of the right-hand side of Eq. (6). The expression for the left-hand side further simplifies for $v_\chi \approx k$ and $g_B^2 k \gg 1$ to $J_1(m_n/k)/Y_1(m_n/k)$, which we have used in Eq. (6). However, this is not essential because the solutions are quite insensitive to the detailed expression of the left-hand side for $v_\chi \gg k'$. The same comment applies also to the expressions involving the PS gauginos, e.g. Eq. (14).

This spectrum can be summarized, for $k' \ll k$, as

$$\left\{ \begin{array}{ll} V^{321}, \Sigma^{XY} : & m_0 = 0, \\ \{V^{321}, \Sigma^{321}\} + \{V^{\text{PS}}, \Sigma^{\text{PS}}\} : & m_n \simeq (n - \frac{1}{4})\pi k', \\ \{V^{XY}, \Sigma^{XY}\} : & m_n \simeq (n + \frac{1}{4})\pi k', \end{array} \right. \quad (8)$$

where $n = 1, 2, \dots$. The transformation properties of these fields under the 321 gauge group are given by $(\mathbf{8}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{3})_0 + (\mathbf{1}, \mathbf{1})_0$ for V^{321} and Σ^{321} , $(\mathbf{3}, \mathbf{1})_{2/3} + (\mathbf{3}^*, \mathbf{1})_{-2/3} + (\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1} + (\mathbf{1}, \mathbf{1})_0$ for V^{PS} and Σ^{PS} , and $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6} + (\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}^*, \mathbf{2})_{-1/6}$ for V^{XY} and Σ^{XY} . A schematic depiction of the spectrum is given in Fig. 1a.

With the gauge spectrum of Eq. (8) and the Higgs fields of Eq. (3), the MSSM prediction for the low-energy gauge couplings is preserved. Specifically, the low-energy 321 gauge couplings are given by setting $(T_1, T_2, T_3)(V_{++}) = (0, 2, 3)$, $(T_1, T_2, T_3)(V_{+-}) = (0, 0, 0)$, $(T_1, T_2, T_3)(V_{-+}) = (14/5, 0, 1)$ and $(T_1, T_2, T_3)(V_{--}) = (26/5, 6, 4)$ in Eq. (9) of Ref. [4] and adding the Higgs contribution, which is identical to that of the 321-321 model (the spectrum of the PS gauge multiplet is effectively reproduced by imposing the $(-, +)$ and $(+, -)$ boundary conditions on V^{PS} and Σ^{PS} , respectively, and the Higgs spectrum is identical to the $SU(5)$ Higgs spectrum of [5] with $c_H = c_{H(\mathbf{5})} = c_{\bar{H}(\mathbf{5}^*)}$). We then find that for $c_H \geq 1/2$ the prediction for low-energy 321 gauge couplings g_a is given by

$$\frac{1}{g_a^2(k')} \simeq (\text{universal}) + \frac{1}{8\pi^2} \Delta^a, \quad (9)$$

where

$$\begin{pmatrix} \Delta^1 \\ \Delta^2 \\ \Delta^3 \end{pmatrix} \simeq \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \ln \left(\frac{k}{k'} \right), \quad (10)$$

hence reproducing the successful MSSM prediction.⁵ In a suitable renormalization scheme, the logarithmic contribution of Eq. (10) arises entirely from the Planck brane couplings $\tilde{g}_{0,a}$, implying that we should use $1/\tilde{g}_{0,a}^2 \simeq \Delta^a/8\pi^2$ in Eq. (5). The result of Eqs. (9, 10) can also be understood in the 4D dual picture as follows. In the 4D picture the theory between $v_\chi \approx k$ and the TeV scale is described by an $N = 1$ supersymmetric gauge theory with the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times G$ with the G sector possessing a global $SO(10)$ symmetry, where G represents some gauge interaction whose coupling evolves very slowly over this energy interval. The quark, lepton and Higgs doublets are interpreted as elementary fields, while various GUT states are regarded as composite states arising from the non-trivial infrared (of order TeV) dynamics of G . Since the contribution from the G sector is universal due to the global $SO(10)$, the differences among the low-energy 321 gauge couplings arise entirely from the contribution

⁵Successful gauge coupling unification in warped unified theories was anticipated in [9] based on a heuristic argument, and was shown explicitly in [4]. Techniques for calculating gauge coupling evolution in warped space were developed in Refs. [10].

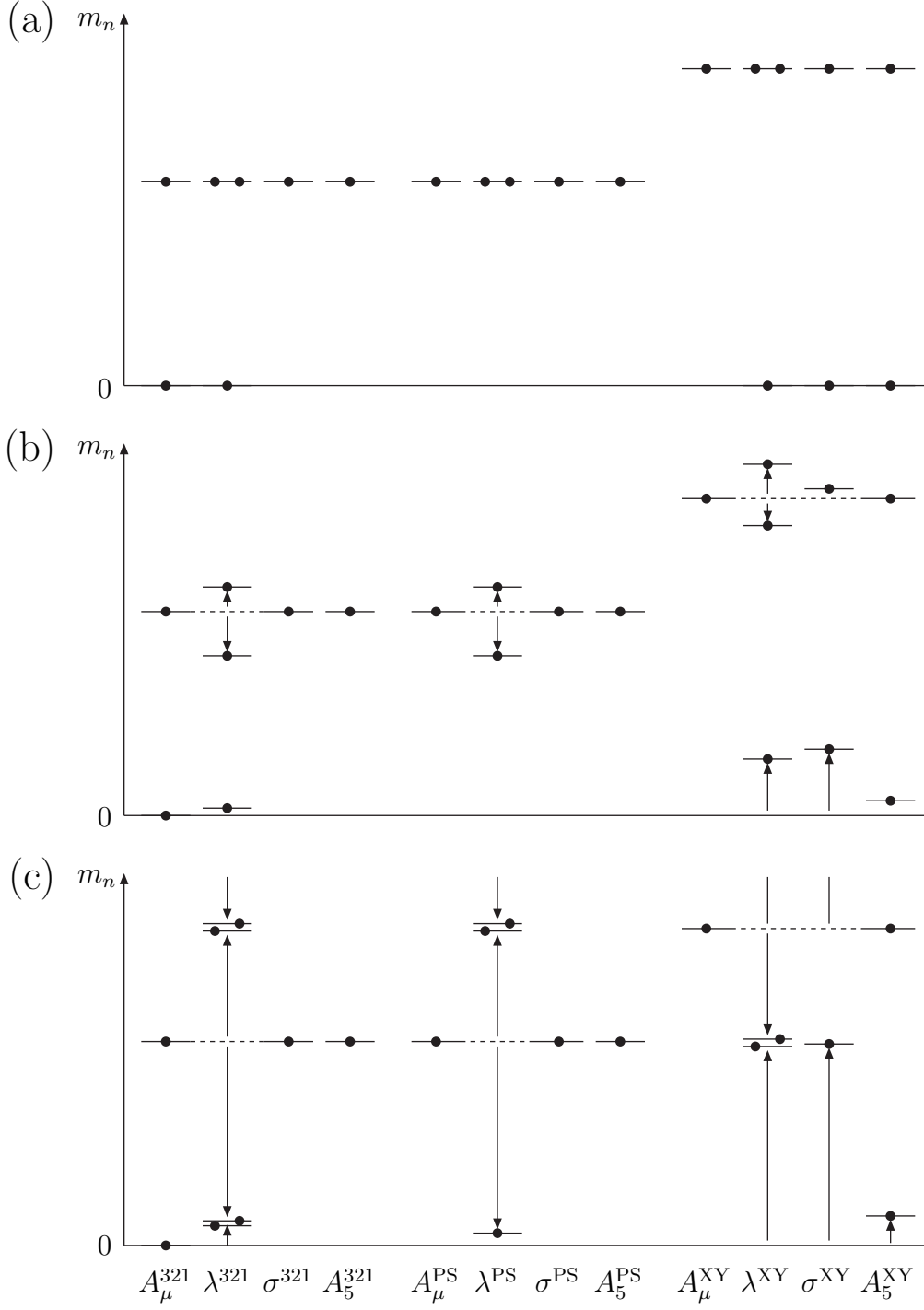


Figure 1: Schematic depiction for the lowest-lying masses for the gauge multiplet. The three figures represent the spectrum (a) in the supersymmetric limit, (b) for small supersymmetry breaking and (c) for large supersymmetry breaking. Each bullet for λ^{321} (λ^{PS} and λ^{XY}) represents a Majorana (Dirac) degree of freedom.

of the elementary fields (assuming strong coupling at ultraviolet, see [4]). This gives the desired MSSM prediction because the elementary sector is identical to the MSSM.

Here we comment on calculability in this model. In a theory of warped supersymmetric unification, the size of the bulk gauge coupling g_B is related to the 4D gauge coupling as $1/g_4^2 = \pi R/g_B^2$, where g_4 represents the unified gauge coupling in conventional 4D supersymmetric unification, $g_4 \simeq 0.7$. Defining M_* to be the scale where the 5D theory becomes strongly coupled, we obtain $1/g_B^2 \simeq CM_*/L$, where C and L are the group-theoretic and 5D-loop factors, respectively [11]. For the $SO(10)$ theory, $C \simeq 8$. Using the 5D-loop factor of $L \simeq 24\pi^3$ [12] and $kR \sim 10$, we obtain $M_*/\pi k \simeq 2$. This implies that the infrared cutoff of the theory, $M'_* \equiv M_* e^{-\pi k R}$, is close to the scale of the first KK excitation, $\pi k'$: $M'_*/\pi k' \simeq 2$. This strongly restricts calculability — we generically expect errors of order $(\pi k'/M'_*)^n$ in various predictions, where n depends on the quantity (errors for the masses of the lightest 321 gauginos could be suppressed further by $1/\ln(k/k')$).⁶ The equations that follow should thus be interpreted with care: their precision is not very high and the results for higher KK towers are not meaningful. In general, this is the case for any warped theory with a large bulk gauge symmetry, so the same comment also applies to the model presented in the next section. We stress, however, that the main features of the model, such as its unified understanding of matter quantum numbers and the qualitative aspects of its spectrum, are not affected by these numerical limitations.

2.3 Supersymmetry breaking

We now consider the effects of supersymmetry breaking in the present model. Here we follow the notation of Ref. [5]. Supersymmetry breaking is introduced on the TeV brane through the following potential [13]:

$$S = \int d^4x \int_0^{\pi R} dy \, 2\delta(y - \pi R) \left[e^{-2\pi k R} \int d^4\theta Z^\dagger Z + \left\{ e^{-3\pi k R} \int d^2\theta \Lambda^2 Z + \text{h.c.} \right\} \right], \quad (11)$$

where Z is a singlet chiral superfield and Λ is a mass parameter of order $M_* \sim M_5$. This potential gives the vacuum expectation value $\langle Z \rangle = -e^{-\pi k R} \Lambda^{*2} \theta^2$, breaking supersymmetry and the $U(1)_R$ symmetry (to the $Z_{2,R}$ subgroup). This breaking does not destroy the successful prediction relating the low-energy gauge couplings, although it causes a distortion of the spectrum.

The masses for the 321 and PS gauginos, λ^{321} and λ^{PS} , are generated through the operators on the TeV brane of the form $\int d^2\theta Z \text{Tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha] + \text{h.c.}$ Since the gauge symmetry on the TeV brane is 422, we have three independent coefficients ζ_C , ζ_L and ζ_R for these operators, corresponding to the $SU(4)_C$, $SU(2)_L$ and $SU(2)_R$ factors, respectively. Specifically, the operators are given

⁶The situation is somewhat better in theories with the bulk $SU(5)$ symmetry because of the smaller value of C : $C \simeq 5$.

by

$$S = \int d^4x \int_0^{\pi R} dy 2\delta(y - \pi R) \sum_{A=C,L,R} \left[- \int d^2\theta \frac{\zeta_A}{2M_*} Z \text{Tr}[\mathcal{W}_A^\alpha \mathcal{W}_{A\alpha}] + \text{h.c.} \right], \quad (12)$$

where $A = C, L, R$ denotes $SU(4)_C$, $SU(2)_L$ and $SU(2)_R$. This implies that the masses for the 321 gauginos λ_a^{321} ($a = 1, 2, 3$) and the PS gauginos λ_U^{PS} , λ_E^{PS} and λ_S^{PS} are all determined in terms of four parameters $\zeta_C M'$, $\zeta_L M'$, $\zeta_R M'$ and M'/k' , where $M' \equiv e^{-\pi k R} \Lambda^{*2}/M_*$. Here λ_U^{PS} , λ_E^{PS} , and λ_S^{PS} transform under 321 as $(\mathbf{3}, \mathbf{1})_{2/3} + (\mathbf{3}^*, \mathbf{1})_{-2/3}$, $(\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1}$, and $(\mathbf{1}, \mathbf{1})_0$, respectively. Moreover, if left-right symmetry is unbroken on the TeV brane, which is natural if left-right symmetry is broken spontaneously only on the Planck brane, we have an additional relation $\zeta_L = \zeta_R$. In this case the masses for the above gauginos are all determined by the three parameters $\zeta_C M'$, $\zeta_L M'$ and M'/k' .

Solving the equations of motion in 5D, we find that the masses for the $SU(3)_C$ and $SU(2)_L$ gauginos ($a = 3$ and 2 respectively) are given by

$$\frac{J_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{g_{0,a}^2} m_n J_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{g_{0,a}^2} m_n Y_1\left(\frac{m_n}{k}\right)} = \frac{J_0\left(\frac{m_n}{k'}\right) + g_B^2 M_{\lambda,a} J_1\left(\frac{m_n}{k'}\right)}{Y_0\left(\frac{m_n}{k'}\right) + g_B^2 M_{\lambda,a} Y_1\left(\frac{m_n}{k'}\right)}, \quad (13)$$

where $M_{\lambda,3} = \zeta_C \Lambda^{*2}/M_*$ and $M_{\lambda,2} = \zeta_L \Lambda^{*2}/M_*$. The masses for the $SU(3)_C$ and $SU(2)_L$ gaugino towers are given as the solutions to this equation, which can be $m_n < 0$ as well as $m_n > 0$ (the physical masses are given by $|m_n|$). The masses for the PS gauginos λ_U^{PS} and λ_E^{PS} are similarly given by

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_0\left(\frac{m_n}{k'}\right) - g_B^2 M_{\lambda,A} J_1\left(\frac{m_n}{k'}\right)}{Y_0\left(\frac{m_n}{k'}\right) - g_B^2 M_{\lambda,A} Y_1\left(\frac{m_n}{k'}\right)}, \quad (14)$$

where A takes U and E for λ_U^{PS} and λ_E^{PS} respectively, and $M_{\lambda,U} = \zeta_C \Lambda^{*2}/M_*$ and $M_{\lambda,E} = \zeta_R \Lambda^{*2}/M_*$. An interesting point is that for large supersymmetry breaking (i.e. large Λ) the lowest λ_3^{321} and λ_2^{321} gauginos both become pseudo-Dirac states with the masses $\approx (2/\pi k R)^{1/2} k' \simeq k'/4$, while the lowest λ_U^{PS} and λ_E^{PS} modes become very light with the masses given by $\approx (2/g_B^2 M_{\lambda,U}) k'$ and $\approx (2/g_B^2 M_{\lambda,E}) k'$, respectively. This can be easily understood by noticing that the form of Eq. (13) and Eq. (14) are identical, respectively, to the equations determining the masses of the 321 and $SU(5)/321$ gauginos in the model of [4, 14]. For small supersymmetry breaking, the lowest modes of λ_3^{321} and λ_2^{321} (i.e. the MSSM gauginos) have masses much smaller than k' , while λ_U^{PS} and λ_E^{PS} do not have a mode lighter than k' .

The masses for the $U(1)_Y$ gaugino, λ_1^{321} , and the PS gaugino λ_S^{PS} obey a somewhat more complicated equation, since they generically mix with each other. The gauginos λ_1^{321} and λ_S^{PS} are associated with two $U(1)$ factors arising from $SU(4)_C \times SU(2)_R$: $U(1)_Y \subset 321$ and $U(1)_\chi$, respectively. Taking the two $U(1)$'s to be orthogonal, $U(1)_\chi$ is the “fiveness” charge arising as $U(1)_\chi \subset SO(10)/SU(5)$ in the standard GUT embedding. The generators for $U(1)_Y$ and $U(1)_\chi$,

Y and Q_S , normalized in the $SO(10)$ covariant manner are given by $Y = \sqrt{2/5} T_{15}^C + \sqrt{3/5} T_3^R$ and $Q_S = -\sqrt{3/5} T_{15}^C + \sqrt{2/5} T_3^R$, where T_{15}^C (T_3^R) is a generator of $SU(4)_C$ ($SU(2)_R$) that commutes with 321. The masses for these gauginos are then given by the equation

$$\begin{aligned} & \left(J_Y\left(\frac{m_n}{k'}\right) - \frac{\tilde{J}_0\left(\frac{m_n}{k}\right)}{\tilde{Y}_0\left(\frac{m_n}{k}\right)} Y_Y\left(\frac{m_n}{k'}\right) \right) \left(J_S\left(\frac{m_n}{k'}\right) - \frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} Y_S\left(\frac{m_n}{k'}\right) \right) \\ & - \left(J_M\left(\frac{m_n}{k'}\right) - \frac{\tilde{J}_0\left(\frac{m_n}{k}\right)}{\tilde{Y}_0\left(\frac{m_n}{k}\right)} Y_M\left(\frac{m_n}{k'}\right) \right) \left(J_M\left(\frac{m_n}{k'}\right) - \frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} Y_M\left(\frac{m_n}{k'}\right) \right) = 0, \end{aligned} \quad (15)$$

where $\tilde{J}_0(m_n/k)$, $J_Y(m_n/k')$, $J_S(m_n/k')$ and $J_M(m_n/k')$ are defined by

$$\begin{aligned} \tilde{J}_0\left(\frac{m_n}{k}\right) & \equiv J_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{g_{0,1}^2} m_n J_1\left(\frac{m_n}{k}\right), \\ J_Y\left(\frac{m_n}{k'}\right) & \equiv J_0\left(\frac{m_n}{k'}\right) + g_B^2 M_{\lambda,1} J_1\left(\frac{m_n}{k'}\right), \\ J_S\left(\frac{m_n}{k'}\right) & \equiv J_0\left(\frac{m_n}{k'}\right) + g_B^2 M_{\lambda,S} J_1\left(\frac{m_n}{k'}\right), \\ J_M\left(\frac{m_n}{k'}\right) & \equiv g_B^2 M_{\lambda,M} J_1\left(\frac{m_n}{k'}\right), \end{aligned} \quad (16)$$

and similarly for $\tilde{Y}_0(m_n/k)$, $Y_Y(m_n/k')$, $Y_S(m_n/k')$ and $Y_M(m_n/k')$. Here M_λ 's are given by $M_{\lambda,1} = ((2/5)\zeta_C + (3/5)\zeta_R)\Lambda^{*2}/M_*$, $M_{\lambda,S} = ((3/5)\zeta_C + (2/5)\zeta_R)\Lambda^{*2}/M_*$ and $M_{\lambda,M} = (-\sqrt{6}/5)\zeta_C + (\sqrt{6}/5)\zeta_R)\Lambda^{*2}/M_*$. In the supersymmetric limit, Eq. (15) gives two decoupled KK towers for each of λ_1^{321} and λ_S^{PS} , reproducing the spectrum given in Eq. (8). When supersymmetry is broken the two towers mix, but for small supersymmetry breaking the resulting tower can still be effectively described by the sum of the two independent towers for λ_1^{321} and λ_S^{PS} . The lightest state is almost purely λ_1^{321} with the mass given as the lowest solution of Eq. (13) with $a = 1$, and all the other states are heavier than k' (the mixings are not negligible for the excited states). With an increased strength for supersymmetry breaking, the mixing among the states becomes more important, giving, for example, a non-negligible effect on the mass of the lightest state. For very large supersymmetry breaking, the lowest state is a Majorana fermion with the mass given by $\simeq (2k'/g_B^2)|M_{\lambda,1}/(M_{\lambda,S}M_{\lambda,1} - M_{\lambda,M}^2)| = 2((3/5)\zeta_C^{-1} + (2/5)\zeta_R^{-1})(M_*/g_B^2\Lambda^{*2})k'$; the next state is a pseudo-Dirac fermion with the mass $\simeq g_1 k' (2/g_B^2 k)^{1/2} \approx 0.2 k'$.

The effects of supersymmetry breaking on the XY states are similar to those on the $SU(5)/321$ states in the model of [5]. Before supersymmetry breaking, the massless XY states consist of two Dirac fermions λ_X^{XY} and λ_Q^{XY} , and four sets of real scalars σ_X^{XY} , σ_Q^{XY} , $A_{5,X}^{\text{XY}}$ and $A_{5,Q}^{\text{XY}}$, where the subscripts X and Q represent the $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6}$ and $(\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}^*, \mathbf{2})_{-1/6}$ components under the 321 decomposition. The masses for λ_X^{XY} and λ_Q^{XY} are generated through the operator

$$S = \int d^4x \int_0^{\pi R} dy 2\delta(y - \pi R) \left[e^{-2\pi k R} \int d^4\theta \frac{\eta}{2M_*} Z^\dagger \text{Tr}[\mathcal{P}[\mathcal{A}]\mathcal{P}[\mathcal{A}]] + \text{h.c.} \right], \quad (17)$$

where

$$\mathcal{A} \equiv e^{-V}(\partial_y e^V) + (\partial_y e^V)e^{-V} - \sqrt{2}e^V \Sigma e^{-V} - \sqrt{2}e^{-V} \Sigma^\dagger e^V. \quad (18)$$

Here, the trace is taken over $SO(10)$ space and $\mathcal{P}[\mathcal{X}]$ is a projection operator: with $\mathcal{X}$ an adjoint of $SO(10)$, $\mathcal{P}[\mathcal{X}]$ extracts the $(\mathbf{6}, \mathbf{2}, \mathbf{2})$ component of $\mathcal{X}$ under the decomposition to 422. The coefficient η is a dimensionless parameter. Since the X and Q components are embedded in a single 422 multiplet $(\mathbf{6}, \mathbf{2}, \mathbf{2})$, the masses of their fermionic components are determined by this single coefficient. The equation determining the λ_X^{XY} and λ_Q^{XY} masses is given by

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_1\left(\frac{m_n}{k'}\right) - g_B^2 M_{\lambda, X} J_0\left(\frac{m_n}{k'}\right)}{Y_1\left(\frac{m_n}{k'}\right) - g_B^2 M_{\lambda, X} Y_0\left(\frac{m_n}{k'}\right)}, \quad (19)$$

where $M_{\lambda, X} \equiv \eta \Lambda^2 / M_*$. Note that the masses for the λ_X^{XY} and λ_Q^{XY} towers are degenerate at tree level, although they split at loop level through the 321 gauge interactions (these splittings are finite and calculable as the XY towers are localized to the TeV brane while the breaking of 422 resides at the Planck brane). The masses for σ_X^{XY} and σ_Q^{XY} are generated through the operator

$$S = \int d^4x \int_0^{\pi R} dy 2\delta(y - \pi R) \left[-e^{-2\pi k R} \int d^4\theta \frac{\rho}{4M_*^2} Z^\dagger Z \text{Tr}[\mathcal{P}[\mathcal{A}]\mathcal{P}[\mathcal{A}]] \right], \quad (20)$$

where the consistency of the effective theory requires ρ to take the form $\rho = -8g_B^2 |\eta|^2 \delta(0) + \rho'$, where ρ' is a dimensionless parameter [5]. The equation determining the σ_X^{XY} and σ_Q^{XY} masses is then given by

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_1\left(\frac{m_n}{k'}\right) - \frac{g_B^2 M_{\sigma, X}^2 k'}{m_n k} J_0\left(\frac{m_n}{k'}\right)}{Y_1\left(\frac{m_n}{k'}\right) - \frac{g_B^2 M_{\sigma, X}^2 k'}{m_n k} Y_0\left(\frac{m_n}{k'}\right)}, \quad (21)$$

where $M_{\sigma, X}^2 \equiv \rho' |\Lambda|^4 / M_*^2$. Again the masses for the σ_X^{XY} and σ_Q^{XY} towers are degenerate at tree level, although they split at loop level. The masses of $A_{5, X}^{\text{XY}}$ and $A_{5, Q}^{\text{XY}}$ are not generated by the operators in Eqs. (17, 20). In fact, the 5D gauge invariance forbids any local operator giving these masses (in the 4D dual picture the zero modes of $A_{5, X}^{\text{XY}}$ and $A_{5, Q}^{\text{XY}}$ are pseudo-Goldstone bosons associated with the $SO(10) \rightarrow 422$ breaking at the TeV scale, which encodes the TeV-brane gauge breaking in 5D). These masses, however, are generated at loop level, picking up the effects of both Planck-brane and TeV-brane breakings. The resulting masses are finite and approximately given by

$$m_{A_5^{\text{XY}}}^2 \simeq \frac{g^2 C}{\pi^4} m_{\lambda^{\text{XY}}}^2 \ln \frac{\pi k'}{m_{\lambda^{\text{XY}}}}, \quad (22)$$

where g represents a 4D gauge coupling and C the group theoretical factor. The masses for $A_{5, X}^{\text{XY}}$ and $A_{5, Q}^{\text{XY}}$ are different, because they have the different 321 quantum numbers and thus different values of C s. This difference, however, will be small, since the quantum numbers for $A_{5, X}^{\text{XY}}$ and $A_{5, Q}^{\text{XY}}$ are the same under $SU(3)_C$ and $SU(2)_L$ so that the mass difference only comes from the $U(1)_Y$ part, which is expected to give a mass splitting of $O(10\%)$.

The Higgs spectrum is identical to the 321-321 model, except that the two $SU(5)$ multiplets, $\mathbf{5}$ and $\mathbf{5}^*$, of the 321-321 model are now unified into a single multiplet, $\mathbf{10}$ of $SO(10)$. As in [5], the unwanted zero modes in H^c acquire TeV-scale masses through their tree-level couplings to the supersymmetry breaking on the TeV brane. The Higgs spectrum of our model can be obtained from the expressions given in [5] by setting $c_H = c_{\bar{H}}$, $\eta_{H_D} = \eta_{\bar{H}_D}$ and $\eta_{H_T} = \eta_{\bar{H}_T}$. Note, however, that the Planck-brane kinetic terms can still be different for the up-type and down-type Higgs fields, i.e. $z_H \neq z_{\bar{H}}$, due to the gauge breaking on the Planck brane. This could be important for obtaining the correct electroweak symmetry breaking vacuum, depending on the details of the Higgs sector.

It is useful here to consider a limit of small supersymmetry breaking $\Lambda \ll M_*$ and to see what the spectrum looks like. In this limit we can expand Eqs. (13, 14, 15, 19, 21) in powers of Λ/M_* . We then find that the PS gauginos do not have any light mode with the mass smaller than k' , while the 321 gauginos, λ_a^{321} ($a = 1, 2, 3$), and the XY gauginos and scalars, λ_Z^{XY} and σ_Z^{XY} ($Z = X, Q$), do with the masses approximately given by

$$m_{\lambda_a^{321}} = g_a^2 M'_{\lambda,a}, \quad (23)$$

$$m_{\lambda_Z^{XY}} = 2g_B^2 k M'_{\lambda,X}, \quad (24)$$

$$m_{\sigma_Z^{XY}}^2 = 2g_B^2 k M_{\sigma,X}^{\prime 2}, \quad (25)$$

where $M'_{\lambda,a} \equiv M_{\lambda,a} e^{-\pi k R}$, $M'_{\lambda,X} \equiv M_{\lambda,X} e^{-\pi k R}$ and $M'_{\sigma,X} \equiv M_{\sigma,X} e^{-\pi k R}$ are parameters of order TeV, and $g_a \equiv (\pi R/g_B^2 + 1/\tilde{g}_{0,a}^2)^{-1/2}$ are the 4D gauge couplings. Considering $g_a = O(1)$ and $g_B^2 k = O(\pi k R)$, we expect that the XY states are generically heavier than the 321 gauginos. In fact, in the case of $M_{\lambda,a} \simeq M_{\lambda,X} \simeq M_{\sigma,X}/4\pi$, as suggested by naive dimensional analysis, the ratios of the masses are roughly given by $m_{\lambda_a^{321}} : m_{\lambda_Z^{XY}} : m_{\sigma_Z^{XY}} \simeq 1 : \pi k R : \pi k R$. The masses for $A_{5,Z}^{XY}$ are given by Eq. (22) so they could be somewhat lighter than λ_Z^{XY} and σ_Z^{XY} . Similarly in the Higgs sector, the colored-triplet states are generically heavier than the doublet states. These little mass hierarchies among the TeV states arise because the wavefunctions for the exotic states are localized to the TeV brane, where supersymmetry breaking occurs, while those of the MSSM states are not. This effect is therefore related to the model's successful prediction relating the low-energy gauge couplings, which crucially relies on the fact that all the exotic states are strongly localized to the TeV brane [5]. The spectrum for small supersymmetry breaking is depicted in Fig. 1b for the gauge sector.

In the small supersymmetry breaking limit, the masses of the 321 gauginos, λ_a^{321} , are given by

$$M_1 = g_1^2 \left(\frac{2}{5} \zeta_C + \frac{3}{5} \zeta_R \right) M', \quad M_2 = g_2^2 \zeta_L M', \quad M_3 = g_3^2 \zeta_C M', \quad (26)$$

where $M_1 \equiv m_{\lambda_1^{321}}$, $M_2 \equiv m_{\lambda_2^{321}}$, and $M_3 \equiv m_{\lambda_3^{321}}$ are the bino, wino, and gluino masses, and $M' = e^{-\pi k R} \Lambda^{*2}/M_*$ is a parameter of order TeV. A particularly interesting case is where left-right symmetry is unbroken on the TeV brane. In this case $\zeta_L = \zeta_R$, so that we have a non-trivial relation among the 321 gaugino masses, which can be written as

$$\frac{M_1}{g_1^2} = \frac{2}{5} \frac{M_3}{g_3^2} + \frac{3}{5} \frac{M_2}{g_2^2}. \quad (27)$$

Note that, in contrast to high-scale supersymmetry breaking scenarios, this relation arises from the physics at an energy scale of order TeV as a threshold effect.⁷

We finally discuss the squark and slepton masses. They are generated at one-loop level through the standard model gauge interactions. Because of the geometrical separation between supersymmetry breaking and the place where squarks and sleptons are located, the generated squark and slepton masses are finite and calculable in the effective field theory. Although the remaining gauge symmetry on the Planck brane after the orbifolding is 422, the squarks and sleptons interact effectively only with the 321 gauge multiplet at the scale where their masses are generated (due to the spontaneous breaking of 422 at a high scale). This means that the squark and slepton masses in the present model are given by

$$m_{\tilde{f}}^2 = \frac{1}{2\pi^2} \sum_{a=1,2,3} \mathcal{F}^{\delta_{a1}} C_a^{\tilde{f}} \mathcal{I}_a, \quad (28)$$

where $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$ represents the MSSM squarks and sleptons, and the $C_a^{\tilde{f}}$ are the group theoretical factors given by $(C_1^{\tilde{f}}, C_2^{\tilde{f}}, C_3^{\tilde{f}}) = (1/60, 3/4, 4/3), (4/15, 0, 4/3), (1/15, 0, 4/3), (3/20, 3/4, 0)$ and $(3/5, 0, 0)$ for $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}$ and $\tilde{e}$, respectively. The functions $\mathcal{I}_a$ are defined in Eq. (21) of [14], where we have to use three different gaugino mass parameters $M_{\lambda,a}$ because of the non-universal gaugino masses.⁸ The quantity $\mathcal{F}$, which represents a mixing effect between the $U(1)_Y$ and $U(1)_X$ gaugino towers, is a function of $M_{\lambda,1}$, $M_{\lambda,S}$ and $M_{\lambda,M}$ and takes a value of order 1. Since the mixing effect vanishes for small supersymmetry breaking, $\mathcal{F}$ approaches to 1 for small M_λ 's. Note that because the squark and slepton masses are generated through the gauge interactions, they are flavor universal and the supersymmetric flavor problem is absent. Small mass splittings among different generations arise through the Yukawa couplings at two loop orders, making the masses for the third generation squarks and sleptons slightly lower than those for the first two generation ones. However, they do not generate flavor changing neutral currents at a dangerous level.

⁷The relation Eq. (27) is essentially determined by the symmetry of the theory — 422 is preserved in the G sector and its breaking comes only through the gauge kinetic terms of the gaugino fields. Thus, Eq. (27) holds accurately even for relatively small values of $M'_*/\pi k'$, i.e. it is not subject to errors of $O(\pi k'/M'_*)$ coming from unknown TeV-brane operators (note that M_a here are the running masses and not the physical pole masses).

⁸The expressions for squark and slepton masses, Eq. (28), are subject to errors of $O(\pi k'/M'_*)$ arising from the TeV-brane localized 321 gauge kinetic terms (only errors of $O((\pi k'/M'_*)^2)$ are mentioned in [14]).

The spectrum described above provides a rich phenomenology. For example, it gives a variety of possibilities for the next-to-lightest supersymmetric particle (NLSP) (the lightest supersymmetric particle is the gravitino with the mass $\sim k'^2/M_{Pl} \sim 0.01 - 0.1$ eV). Possible patterns for the superparticle spectrum are similar to those discussed in [5]. The model also predicts relatively light GUT particles $A_{5,X}^{XY}$ and $A_{5,Q}^{XY}$, transforming as $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6}$ and $(\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}^*, \mathbf{2})_{-1/6}$ under 321 respectively. The masses for $A_{5,X}^{XY}$ and $A_{5,Q}^{XY}$ are close but expected to have a relative splitting of $O(10\%)$. The lighter one will presumably be $A_{5,Q}^{XY}$, which is stable for collider purposes due to the conservation of $SU(3)_C$ charges and the location of fields [5]. Once $A_{5,Q}^{XY}$ is produced, it hadronizes to either of four fermionic mesons $\hat{T}^0, \hat{T}'^0, \hat{T}^+, \hat{T}'^+$ (and their anti-particles), depending on whether it picks up an up or down quark or anti-quark. All these states are sufficiently long-lived, so that the charged ones would be detectable through highly ionizing tracks. For strong supersymmetry breaking (large Λ), the gauginos of the PS multiplet λ_U^{PS} , λ_E^{PS} and λ_S^{PS} , transforming as $(\mathbf{3}, \mathbf{1})_{2/3} + (\mathbf{3}^*, \mathbf{1})_{-2/3}$, $(\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1}$ and $(\mathbf{1}, \mathbf{1})_0$, also become light (see Fig. 1c for the overall spectrum of the gauge sector for large supersymmetry breaking). Some of these states could also be stable and seen at colliders.

3 Model without TeV-Brane Symmetry Breaking

In this section we construct a model in which the bulk gauge symmetry is not reduced at the TeV brane. The construction closely follows that of the previous section. The model presented here can be viewed as an extension of the $SU(5)$ model of [4] to a larger unified gauge group.

The model is again formulated in the 5D warped spacetime compactified on S^1/Z_2 ($0 \leq y \leq \pi R$), with the metric given by Eq. (1). The parameters M_5 , M_* , k and R take similar values. The gauge group in the bulk is taken to be $SO(10)$, with the boundary conditions given by

$$\begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} PVP^{-1} \\ -P\Sigma P^{-1} \end{pmatrix} (x^\mu, y), \quad \begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y') = \begin{pmatrix} V \\ -\Sigma \end{pmatrix} (x^\mu, y'), \quad (29)$$

where $y' = y - \pi R$. The matrix P is the same as before: $P = \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1)$. This reduces the gauge group at the Planck brane to 422, but leaves the gauge group at the TeV brane to be $SO(10)$. The resulting zero modes are only the $(\mathbf{15}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{3})$ component of V (the 422 $N = 1$ gauge supermultiplet). The excited KK states all have masses of order k' or larger.

The Higgs fields are introduced in the bulk as a hypermultiplet transforming as $\mathbf{10}$ of $SO(10)$, with the boundary conditions given by

$$\begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} -PH \\ PH^c \end{pmatrix} (x^\mu, y), \quad \begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y') = \begin{pmatrix} H \\ -H^c \end{pmatrix} (x^\mu, y'). \quad (30)$$

This leaves the $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ component of H as zero modes. All the other modes have masses of order k' or larger.⁹

The matter and the 422 gauge breaking sectors are identical to those in the model of section 2. The quark and lepton chiral superfields are introduced on the Planck brane in the $\Psi(\mathbf{4}, \mathbf{2}, \mathbf{1}) + \bar{\Psi}(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ representation of 422 for each generation.¹⁰ The 422 gauge breaking is introduced on the Planck brane. The simplest possibility is the Higgs breaking $\langle \chi \rangle = \langle \bar{\chi} \rangle = O(k)$, where χ and $\bar{\chi}$ transform as $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ under 422, respectively. We introduce the Planck brane couplings Eq. (4), which give the Yukawa couplings for the quarks and leptons as well as Majorana masses for the right-handed neutrinos. With suitable non-renormalizable operators realistic quark and lepton masses and mixings are reproduced. For $\langle \chi \rangle, \langle \bar{\chi} \rangle \simeq k \lesssim M_*$, the Yukawa couplings for the third generation fermions are approximately unified at the scale k , giving a successful low-energy prediction for m_b/m_τ . It also leads to $\tan \beta \approx 50$. Small masses for the observed neutrinos are naturally obtained through the see-saw mechanism. The model possesses a $U(1)_R$ symmetry: $V(0), \Sigma(0), H(0), H^c(2), \Psi(1), \bar{\Psi}(1), \chi(0), \bar{\chi}(0)$ (broken to $Z_{2,R}$ at the TeV brane through supersymmetry breaking), which prevents potentially dangerous operators such as $\delta(y) \int d^2\theta H_D^2 + \text{h.c.}$. There is not a proton decay problem for Planck-brane localized matter.

The 422 breaking at the Planck brane gives masses for the 422/321 component of V . The resulting spectrum for the gauge sector is then given by Eq. (5) for the 321 component and by Eq. (6) for the $SO(10)/321$ component (with the comment in footnote 4 applying to 422/321). The spectrum, therefore, can be summarized as

$$\begin{cases} V^{321} : & m_0 = 0, \\ \{V^{321}, \Sigma^{321}\} + \{V^{\text{GUT}}, \Sigma^{\text{GUT}}\} : & m_n \simeq (n - \frac{1}{4})\pi k', \end{cases} \quad (31)$$

where $n = 1, 2, \dots$. Here, the superscript GUT represents the $SO(10)/321$ component, whose transformation properties are $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6} + (\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}^*, \mathbf{2})_{-1/6} + (\mathbf{3}, \mathbf{1})_{2/3} + (\mathbf{3}^*, \mathbf{1})_{-2/3} + (\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1} + (\mathbf{1}, \mathbf{1})_0$ under the 321 decomposition. Note that the spectrum for the excited KK towers are approximately $SO(10)$ symmetric. This is because in the 4D dual picture the

⁹We can alternatively put the Higgs field on the Planck brane in the $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ representation of 422, or in the bulk but with the boundary conditions of Eq. (30) with an extra minus sign in the right-hand side of the second equation (cf. footnote 1). The latter case gives two triplet zero modes from H^c and four relatively light doublet modes from H and H^c (two for each) in the supersymmetric limit (the doublet states are even exponentially lighter than k' for $c_H > 1/2$). A realistic model is then obtained by introducing a mass term of the form $\int d^2\theta H^c H^c + \text{h.c.}$ on the TeV brane. The MSSM prediction for gauge coupling unification is preserved in these cases, too (for $c_H \geq 1/2$ in the case of the bulk Higgs).

¹⁰In the present model, (some of) matter fields could be introduced in the bulk as hypermultiplets transforming as $\mathbf{16}$ of $SO(10)$ (a hypermultiplet transforming as $\mathbf{16}$ of $SO(10)$ yields either Ψ or $\bar{\Psi}$ as a zero mode, depending on the boundary conditions). The prediction for the low-energy gauge couplings is the same as that in the brane matter case if the bulk mass parameters, c_M , for matter fields take the values $c_M \geq 1/2$.

global $SO(10)$ symmetry of G is not dynamically broken so that the composite states of G , which are identified as KK towers in 5D, are approximately $SO(10)$ symmetric.

As in the model of section 2, the MSSM prediction for low-energy gauge couplings is preserved. Specifically, low-energy 321 gauge couplings are given by setting $(T_1, T_2, T_3)(V_{++}) = (0, 2, 3)$, $(T_1, T_2, T_3)(V_{-+}) = (8, 6, 5)$ and $(T_1, T_2, T_3)(V_{+-}) = (T_1, T_2, T_3)(V_{--}) = (0, 0, 0)$ in Eq. (9) of Ref. [4] and adding the Higgs contribution, which is identical to that of the $SU(5)$ model of [4] with $c_H = c_{H(5)} = c_{\bar{H}(5^*)}$. Here, c_H is the dimensionless bulk mass parameter for the Higgs multiplet, $\{H, H^c\}$. Therefore, for $c_H \geq 1/2$ we find that the prediction for low-energy 321 gauge couplings g_a is given by Eq. (9) with Eq. (10), reproducing the successful MSSM prediction.

Supersymmetry breaking is introduced on the TeV brane through the potential Eq. (11). This gives the vacuum expectation value $\langle Z \rangle = -e^{-\pi k R} \Lambda^{*2} \theta^2$, breaking supersymmetry and the $U(1)_R$ symmetry. The breakings are transmitted to various fields through the operator

$$S = \int d^4x \int_0^{\pi R} dy \, 2\delta(y - \pi R) \left[- \int d^2\theta \, \frac{\zeta}{2M_*} Z \text{Tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha] + \text{h.c.} \right], \quad (32)$$

where the trace is taken over $SO(10)$ space. Since the TeV brane respects full $SO(10)$, the coefficient ζ is universal for the entire gauge components. The above operator gives masses for the 321 gaugino zero modes and modifies the spectrum for the gaugino towers of both 321 and $SO(10)/321$ components. The masses for the 321 and $SO(10)/321$ gaugino towers are given, respectively, by Eq. (13) and Eq. (14), but now the gaugino mass parameters M_λ 's take a universal value: $M_{\lambda,a} = M_{\lambda,A} = \zeta \Lambda^{*2}/M_* \equiv M_\lambda$. The values of the Planck-brane gauge couplings renormalized at the TeV scale are the same as in the previous model: $1/\tilde{g}_{0,a}^2 \simeq \Delta^a/8\pi^2$ should be used in Eq. (13).

For the Higgs sector, the only light states are those of the two MSSM Higgs doublets. They obtain supersymmetry-breaking as well as supersymmetry-preserving masses through the couplings to the Z field on the TeV brane. The resulting masses are identical to those in the model of section 2, whose explicit expressions are given in Ref. [5].

Squark and slepton masses are generated at one-loop level through gauge interactions. Despite the 422 symmetry after the orbifolding, the squark and slepton masses come almost entirely from the 321 gauge loops because 422 is broken at a high scale. The resulting masses are finite and calculable, due to the spatial separation between supersymmetry breaking and the matter location, and are given by

$$m_{\tilde{f}}^2 = \frac{1}{2\pi^2} \sum_{a=1,2,3} C_a^{\tilde{f}} \mathcal{I}_a, \quad (33)$$

where $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$, and $C_a^{\tilde{f}}$ are the group theoretical factors given below Eq. (28). The functions $\mathcal{I}_a$ are defined in [14], where we have to use the gaugino mass parameter M_λ for all

$a = 1, 2, 3$. Because of the universality of the operator Eq. (32), the effect of the $U(1)_Y$ - $U(1)_\chi$ mixing is negligible. The supersymmetric flavor problem is absent because the squark and slepton masses are flavor universal (up to small higher-order corrections from the Yukawa couplings).

The phenomenology of the present model is similar to that of the model in [4]. In fact, the masses for all the superparticles as well as the KK towers are all determined in terms of only two parameters M_λ/k and k' , up to parameters associated with the Higgs sector [14]. The main difference is that the $SU(5)/321$ gauge multiplet of [4] is now replaced by the $SO(10)/321$ multiplet. Therefore, the light exotics now consist of five components transforming as $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6}$, $(\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}^*, \mathbf{2})_{-1/6}$, $(\mathbf{3}, \mathbf{1})_{2/3} + (\mathbf{3}^*, \mathbf{1})_{-2/3}$, $(\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1}$, and $(\mathbf{1}, \mathbf{1})_0$ under 321. In particular, in the case of large supersymmetry breaking the gauginos for all these components become light, thus providing the possibility of testing the underlying enlarged group structure of the model at the LHC.¹¹

4 Conclusions

In this paper we have seen that the Pati-Salam (422) gauge group fits extremely naturally into the framework of warped unification. The two models we have constructed both have a bulk $SO(10)$ gauge symmetry broken by boundary conditions to 422 on the Planck brane, corresponding in the 4D picture to a 422 gauge theory with an approximate global $SO(10)$ symmetry. They differ in whether the full $SO(10)$ is realized on the TeV brane, or equivalently, in whether or not the global $SO(10)$ is spontaneously broken in the infrared by the strong dynamics of the conformal sector.

These models incorporate matter unification in a very economical way, with a full generation of quarks and leptons transforming as $(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ under 422. They also require only a very simple Higgs sector for breaking of the 422 gauge symmetry and easily accommodate the see-saw mechanism for neutrino masses, realistic fermion masses, and bottom-tau unification. At the same time they explain the successful unification of gauge couplings in the MSSM, something that Pati-Salam unification by itself does not do. The same prediction relating the low energy couplings given in the MSSM applies here to good approximation under the assumption that the gauge interactions grow strong in the ultraviolet. The $SO(10)$ symmetry plays an important role in this prediction; in the 4D description, it ensures that the contribution to the evolution of the gauge couplings from the conformal sector is universal.¹²

¹¹In the present model, the Z_2 GUT parity of [4] is extended to two Z_2 parities: one which acts non-trivially on the $SO(10)/422$ component of the gauge multiplet and the colored Higgs states and the other which acts non-trivially on the “ $SO(10)/(SU(5) \times U(1)_\chi)$ ” component of the gauge multiplets, i.e. the U and E of PS and Q of XY. These symmetries ensure the quasi-stability for the lightest of λ_X^{XY} and λ_Q^{XY} and of λ_Q^{XY} , λ_U^{PS} and λ_E^{PS} , respectively.

¹²It is worth noting that the $SO(10)$ -symmetric conformal sector can be replaced by extra vector-like states,

The phenomenology of these models is quite different from conventional supersymmetric unification, and different as well from models of warped unification built on $SU(5)$ symmetry. In both models a rich array of exotic particles appears near the TeV scale: these include supermultiplets with 321 quantum numbers $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6}$ as in warped $SU(5)$, but also states with quantum numbers $(\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}^*, \mathbf{2})_{-1/6}$, $(\mathbf{3}, \mathbf{1})_{2/3} + (\mathbf{3}^*, \mathbf{1})_{-2/3}$, and color-neutral states transforming as $(\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1} + (\mathbf{1}, \mathbf{1})_0$. In the model with $SO(10)$ broken to 422 on the TeV brane, some of these states are massless in the supersymmetric limit. The prospects for producing these particles at future colliders such as the LHC depend on the scale k' , the strength of supersymmetry breaking on the TeV brane, and in the model with TeV-brane symmetry breaking, on the free parameters that determine how strongly the pseudo-Goldstone multiplet feels the supersymmetry breaking (η and ρ of Eqs. (17, 20)).

The spectrum of MSSM superparticles differs between the two models. In the model with TeV-brane symmetry breaking, the gaugino mass terms on the TeV brane are non-universal, although there is the possibility of one relation among the 321 gaugino masses if left-right symmetry is unbroken on the TeV brane. One interesting point is that in the TeV-brane symmetry breaking model, there is generally a mixing between the bino and the gaugino associated with the other $U(1)$ factor contained in $SU(4)_C \times SU(2)_R$. The effect of this mixing is small for weak supersymmetry breaking, but becomes significant as the supersymmetry breaking is increased. Because of the non-universality in the gaugino masses, there is a broad range of possibilities for what the gaugino and scalar spectrum will look like, and in particular, there are many possibilities for what the NLSP will be, as discussed in [5]. The model without TeV-brane symmetry breaking, on the other hand, has a more constrained spectrum due to the universal gaugino mass terms on the TeV brane. The spectrum of MSSM particles has similar features to that of [4, 14] in this case, but it should be stressed again that the spectrum of exotic particles is different.

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such as $(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}^*, \mathbf{2}, \mathbf{1})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2}) + (\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ with TeV-scale masses, without spoiling many of the desired features of the model. This provides a class of purely 4D 422 theories with the successful gauge coupling prediction arising from strong coupling in the ultraviolet [15].

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