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Pulsed Spheromak Reactor with Adiabatic Compression

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Abstract

Extrapolating from the Pulsed Spheromak reactor and the LINUS concept, we consider ignition achieved by injecting a conducting liquid into the flux conserver to compress a low temperature spheromak created by gun injection and ohmic heating. The required energy to achieve ignition and high gain by compression is comparable to that required for ohmic ignition and the timescale is similar so that the mechanical power to ignite by compression is comparable to the electrical power to ignite ohmically. Potential advantages and problems are discussed. Like the High Beta scenario achieved by rapid fueling of an ohmically ignited plasma, compression must occur on timescales faster than Taylor relaxation.

1. Introduction

In the Pulsed Spheromak reactor concept [1], a spheromak is created by slow gun injection into a flux conserver, requiring a gun and flux conserver similar to SSPX [2] but operated for extended pulse lengths (about 1 GW for 250 ms in the reactor). The SSPX should reach temperatures of 100's of eV in a few milliseconds, and extending the pulse is expected to increase the temperature in proportion to the stored magnetic energy (roughly constant beta), which grows steadily at the power injection rate [3].

Alternatively, injection times of a few milliseconds, as in SSPX, would produce a spheromak that could be ignited by adiabatic compression. As we shall see, like the High Beta scenario of Reference [4], compressional heating requires timescales faster than Taylor relaxation, both to access stable states at high beta and also to avoid excessive heat loss due to magnetic turbulence. However, it may be easier to outrun relaxation by compression, even for the slow compression mode considered here.

Slow compression by injecting a conducting liquid was employed in the LINUS concept [5], with the goal -- as is our goal -- of achieving liner compression in a reactor with reusable parts. The spheromak may be especially attractive for this concept, due to its inherent magnetic stability that might avoid the necessity of rotational stabilization featured in LINUS. As in the LINUS studies, we find that interesting regimes require a liquid less compressible than lithium, the LiPb eutectic considered in those studies being adequate here, or perhaps LiSn with a lower vapor pressure. Adequate liquid injection rates can be obtained with external pressures < 5 bar, and maximum pressures and impulsive loads during the burn do not exceed the tensile strength of high quality steel.

2. Optimum Gain

We consider compression of a spheromak by injection of a liquid into a flux conserver of radius R_o by action of a piston in one or more cylinders, as shown at the top of Figure 1. We wish to calculate the fusion gain G given approximately by:

$$G = \varepsilon\, 300\, f_B = \varepsilon\, 300\, (1/2\, n\, \tau\, \sigma v) = \varepsilon\, n\tau\, 6 \times 10^{-20} \quad , \qquad (1)$$

with MKS units. The factor 300 assumes compressional heating to T = 10 KeV in order to ignite, while in calculating the burnup fraction f_B we take $\sigma v = 4.2 \times 10^{-22}$ corresponding to T = 20 KeV during the burn. Here τ is the burn duration (dwell time) near stagnation of the compression and ε is the efficiency of compression given by:

$$\varepsilon = E_p / (E_p + E_c) \quad , \qquad (2)$$

where E_p is the energy required to compress the spheromak plasma configuration and E_c is the compressional energy stored in the liquid.

To minimize E_c, we restrict the implosion speed $v << c_s = (B_i/\rho)^{1/2}$ (the sound speed), B_i and ρ being the bulk modulus and density of the liquid. In this limit the energy of compression is [6]:

$$E_c = \int dV\, 1/2\, p^2 / B_i \quad . \qquad (3)$$

We can calculate the pressure by assuming incompressibility, the error being order v^2/c_s^2 consistent with the ordering in Eq. (3). Then, in terms of the implosion speed v at the inner surface at radius R, the speed of the liquid at any other location r is $v(R/r)^2$, which follows from div $\mathbf{v} = 0$ in spherical geometry. Other useful scalings for spherical compression are, for the final compressed state [7]:

$$T \propto B \propto C^{2/3} , n \propto C, R \propto \beta \propto C^{1/3} \quad , \qquad (4)$$

where $C = (R_o/R)^3$ is the volume compression ratio and we neglect magnetic diffusion into the liquid (see Section 5). Introducing the above expression for the velocity into the momentum equation and integrating in r gives:

$$p(r) = P - (dv/dt)\,\rho\,(R_o^2/R)(1 - R/r) + 1/2\,\rho v^2\,(1 - R^4/r^4)\,, \qquad (5)$$

where dv/dt is the rate of change of the velocity at the inner surface and P is the pressure of the compressed spheromak. To calculate E_c, we obtain the pressure at stagnation when v = 0 and the acceleration dv/dt should be chosen to make $p(R_o) = 0$, giving:

$$p(r) = P\,(R_o/r - 1)/(R_o/R - 1) \qquad . \qquad (6)$$

Using this in Eq. (3) gives:

$$E_c / E_p = 1/2\,(P / B_i)\,C^{1/3} \qquad , \qquad (7)$$

where $E_p = 4\pi/3\,R^3 P$ and $P = B^2 / 2\mu_o$ is the pressure at the plasma-liquid interface with the scaling of Eq. (4) if we neglect flux compression by the plasma as beta approaches unity.

To calculate the $n\tau$ we equate the kinetic energy near stagnation to the total energy input E:

$$E \equiv E_p + E_c = \int 4\pi r^2 dr\, 1/2\,\rho\, v^2\,(R/r)^4 = 1/2\,\rho\, v^2\, 4\pi R^3\,, \qquad (8)$$

to lowest order in R/R_o with integration limits R and R_o. We neglect piston energy and assume that the liquid provides all of the inertia. We approximate the dwell time as $\tau = 2R/v$ with v given by Eq. (8). We take the density to be $n = \beta P/2T$ with $\beta = \beta_o\,C^{1/3}$ by Eq. (5) and T = 10 KeV to reach ignition as noted earlier.

Introducing these results into Eqs. (1) and (2) gives the efficiency and gain in terms of the input energy E and flux conserver radius R_o:

$$\varepsilon = \{\, 1/2\,[\, 1 + (\, 1 + 2\,E\,C^{4/3} / V_o B_i\,)^{1/2}\,]\,\}^{-1} \qquad (9)$$

$$G = 3.2 \times 10^{-5} \varepsilon^2 \beta_o R_o B_i (2EC/V_o B_i)^{1/2} / c_s \quad . \quad (10)$$

where $V_o = 4\pi/3 \, R_o^3$.

For a given flux conserver radius R_o and input energy E, the gain G has a maximum versus C given by:

$$\varepsilon = 0.77 \quad (11)$$

$$n = 1.87 \times 10^{14} \beta_o B_i \quad (12)$$

$$C = 2.43 (B_i / E)^{3/4} R_o^{9/4} \quad (13)$$

$$G = [2.07 \times 10^{-5} \beta_o B_i^{7/8} / c_s] R_o^{5/8} E^{1/8} \quad , \quad (14)$$

where in Eq. (14) E is in megajoules. Note that the optimum compression efficiency and compressed density are independent of the input energy and flux conserver radius.

3. Examples

We assume a LiPb eutectic compressor liquid with $B_i = 2 \times 10^{10}$ Pa and density ρ = 7800 kg/m^3 giving a sound speed c_s = 1600 m/s [5]. The main spheromak constraint is the initial value of beta achieved by ohmic heating, taken here to be 4% at the magnetic axis giving about 10% at the liquid-plasma interface [8]; hence we take $\beta_o = 0.1$. With these values, Eqs. (11) - (14) give:

$$n = 3.75 \times 10^{23} \text{ m}^{-3}$$

$$C = 4080 \, R_o^{9/4} / E^{3/4}$$

$$G = 7.5 \, R_o^{5/8} E^{1/8}$$

where again E is in megajoules. Thus high gain requires a large device at high energy input and yield. Note that the input energy E includes magnetic energy injected by the gun.

Table 1 gives parameters for two example cases for a gain $G = 20$, together with a similar case for the ohmically ignited Pulsed Reactor taken from Reference [1]. For the $E = 50$ MJ case, the scaled value of $\beta \approx 1$. If scaling had given $\beta > 1$, we should take $\beta = 1$ corresponding to internal hydrodynamic compression of the flux within the spheromak, to achieve $\beta = 1$ at the liquid wall, as in Magnetic Target Fusion calculations [9].

As can be seen from the table, parameters for compressional heating compare favorably with those for ohmic ignition, the main difference being a trade of compressional energy for energy supplied by the gun. Also, for the compression cases, the thickness of the liquid liner between the plasma and the flux conserver is much greater (≥ 1.5 m) at the time of release of fusion neutrons so that the flux conserver conducting shell can be mounted directly on the solid structure, whereas in the Pulsed Spheromak conceptual design the flux conserver is a struturally-supported thin shell with only a thin protective layer of Flibe in front of it and a thick layer behind to do the breeding. Thus, as in the original LINUS concept [5], the liquid liner serves also as the breeding blanket and neutron shield. The liquid also protects the spheromak gun, as in the Pulsed Spheromak [1].

Because the timescale is slow for our examples, it may be possible to operate two compression cells together, as shown at the bottom of Figure 1. This might allow direct utilization of mechanical expansion energy from one cell to drive compression in the other cell, thereby avoiding the cost and further inefficiency of an electrical or other compression power source. Also, though the gun power is high in order to overcome ohmic losses during injection, the injected electrical energy is much less, allowing perhaps the use of a d.c. capacitor bank rather than the flywheel a.c. generator considered for the Pulsed Spheromak [1].

Note that we have not taken into account corrections for communication of compressive pressure at the sound speed considered in the LINUS studies [5], and we have not corrected for resistive diffusion of magnetic flux into the liquid, more important for the slower $E = 50$ MJ case.

4. Energy Losses

Estimates show that conduction processes such as gyroBohm diffusion and collisional heat conduction included in code calculations for the Pulsed Spheromak reactor scenarios, and also Brehmstrahlung and synchrotorn radiation, are unimportant throughout compressional heating and burn for the cases above. However, conduction due to magnetic turbulence overwhelms compressional heating unless heating outruns Taylor relaxation.

Using the scalings of Eq. (5), the compression heating rate is:

$$d(3nVT)/dt = 4(3nVT)/\tau \quad , \quad (16)$$

where as before $\tau = 2R/v$ at any time at which the inner radius of the imploding liquid is R and V is the volume within this radius. We wish to compare this heating rate with the Rechester-Rosenbluth loss rate if Taylor relaxation were present, for which the thermal diffusivity is [8]:

$$\chi = v_e R (\langle \delta B^2 \rangle / B^2) \quad (17)$$

with electron thermal speed v_e. Following Reference [8] we estimate the time-averaged fluctuation level by matching the rate of change of the field energy during compression to the approximate Poynting flux necessary to maintain a Taylor state:

$$dB^2/dt = 8 B^2/\tau = v_A/R \langle \delta B^2 \rangle \quad , \quad (18)$$

where v_A is the Alfven speed. In the first step we again use the scalings of Eq. (4), the coefficients 4 in Eq. (16) and 8 in Eq. (18) reflecting the powers of C in respective scalings. Using these results we can calculate the ratio of compressional heating to Rechester-Rosenbluth losses if Taylor relaxation were active:

$$\text{Heating/Loss} = [4(3nVT)/\tau]/[(3nVT)\chi/R^2] = 1/2\, v_A/v_e = 0.01\beta^{-1/2} . \quad (19)$$

Thus we see that, if Taylor relaxation holds, balancing compressional heating and Rechester-Rosenbluth losses would result in a constant, low value of β. Similar considerations predict a low beta during gun injection in the ohmically heated spheromak, quickly rising to 10% at the wall when the gun is turned off, as assumed in Section 3 [8].

These considerations do not hold if compressional heating outruns Taylor relaxation. An analogous situation was considered in Reference [4] concerning runaway to high beta achieved by rapid fueling. While it is surely true that the rate of Taylor relaxation is finite [3], the rate is not well known. We can say, however, that, for our examples, from the outset compressional heating rates are faster than the plasma resistive L/R rate that underlies Taylor relaxation processes, and the heating rate increases rapidly as compression proceeds, scaling $\propto$ C, while the resistive L/R rate actually decreases, like $C^{-1/3}$ if compressional heating occurs.

5. Stability and Magnetic Diffusion

During compression the ohmically-heated, low beta spheromak is compressed to a high value of beta of order unity. It has been shown that, outrunning Taylor relaxation, such states can be stable since the compression of flux at high beta increases the magnetic shear just so as to maintain stability by the Mercier criterion (not yet analyzed for low mode number internal kinks) [4]. These calculations assume a rigid, perfectly conducting flux conserver shaped to eliminate tilt/shift modes.

It seems likely that the inherent magnetic stiffness of the spheromak, as indicated by its stability properties in a rigid flux conserver, will also aid stability during compression. Penetration of liquid at the weak field region at the poles should actually be stabilizing to tilt modes (analogous to the "bow tie" configuration [10]).

Stability calculations will be the subject of further work. Here we only note that these calculations must take into account the liquid itself. Shercliff has pointed out that, because liquid lithium (also LiPb, LiSn) is a good conductor with low viscosity, it behaves like a plasma, sometimes allowing fast propagation of magnetic disturbances where magnetic fields have resistively diffused inside the liquid [11]. For lithium-based liquid liners, with an electrical conductivity < 10% of copper, magnetic field does diffuse into the liquid during slow compression, perhaps to a depth of order $\leq 0.1\ R_o$. While this does not prevent compression of the field, it does imply that, in the immediate neighborhood of the compressed spheromak, magnetic field exists in the liquid to a depth comparable to the final radius R and this must be taken into account in analyzing equilibrium and stability during the course of the compression process. On the other hand, resistive MHD growth rates are slower than compression rates. Thus the issue is one of stability of the liquid-plasma interface to ideal MHD perturbations of the interface.

6. Summary

We have shown by examples the feasibility in principle of applying the LINUS slow liner compression scheme to the Pulsed Spheromak reactor, thus providing an alternative means of igniting the plasma by adiabatic compression. Matching requirements for ignition to devices of suitable yield and dimensions suitable for tritium breeding and shielding by the liquid compression fluid gives input energy to compress comparable to that required to ignite ohmically. Because of spherical convergence, implosion speeds of the order of 20 % of the liquid sound speed -- below strong shock limits -- can be produced with surface speeds < 10 m/s that can be produced with a few bar of piston pressure ($1/2\ \rho\ v_o^2 < 5 \times 10^5$ Pa). The maximum magnetic pressure during the burn is P < 20 Kbar, giving an impulse of < 50 bar-sec on the structure yielding $1/2\ \rho\ \Delta v^2 < 50$ Kbar, under the tensile strength of 80 Ksi steel. The fusion-generated pressure $Y/V_o < 20$ Kbar. Thus, fatigue, not instantaneous cracking, is the major structural concern.

If all liquid is removed each cycle as assumed here, a large fluid mass flow is required -- 180 tons for the 150 MJ case. However, noting that most of the pressure has been relieved after displacement of only a small fraction of the liquid, the main reason to transfer the bulk of the liquid is to prepare room to create the next spheromak. Ultimate optimization may suggest expanding only part way, say to 1/2 the radius (1/8 of the liquid) and creating the next spheromak in this smaller space, albeit at a higher initial field. For our 150 MJ case, creating the spheromak in a cavity in the liquid of radius 0.9 m (rather than 1.77 m) would require an initial field of 6.4 T and $E_{mag} = B^2R^3$ (MJ) = 30 MJ but only 120 MJ of mechanical compression, still yielding 3000 MJ at a gain of G = 20.

Acknowledgment

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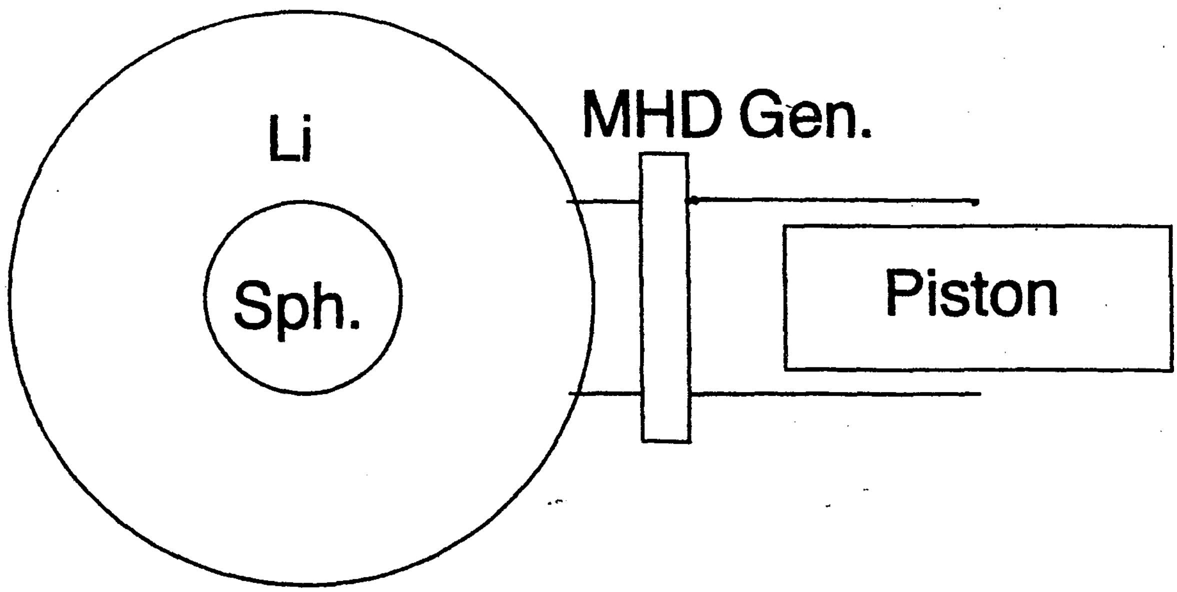
Table 1. Example Parameters

	Ohmically Ignited	Compression	Compression
Chamber			
Diameter (m)	3	3.6	4.4
Flux Cons. R_o (m)	0.6	1.8	2.2
Liquid mass (tons)	24	180	350
Liquid	Flibe	LiPb	LiPb
Output			
Electric (Mwe)	250	250	250
Cycle rate (Hz)	0.4	0.4	1
Injection/Comp. time (s)	0.25	0.17	0.7
Yield (MJ)	3000	3000	1000
Gain G	20	20	20
Input			
E (MJ)	157	150	50
E_{mag} (MJ)	157	9.8	2.1
B_o (T)	27	1.3	0.44
n_o (m^{-3})	10^{22}	1.1×10^{21}	2.9×10^{20}
T_o(KeV)	10	0.2	0.08
At Maximum Compression			
C (volume)		344	1295
$C^{1/3}$(radial)		7.0	10.9
n (m^{-3})		3.7×10^{23}	3.7×10^{23}
τ (ms)		1.2	1.2
B (T)		65	53
β		0.7	1.09
v/c_s		0.27	0.22
P/B_i		0.09	0.06
P (Kbar)		18	12
Y/V_o (Kbar)		1.3	0.2
$1/2\ \rho\ v_o^2$ (bar)		3.1	0.35

PULSED MTF REACTOR



Reciprocating Engine

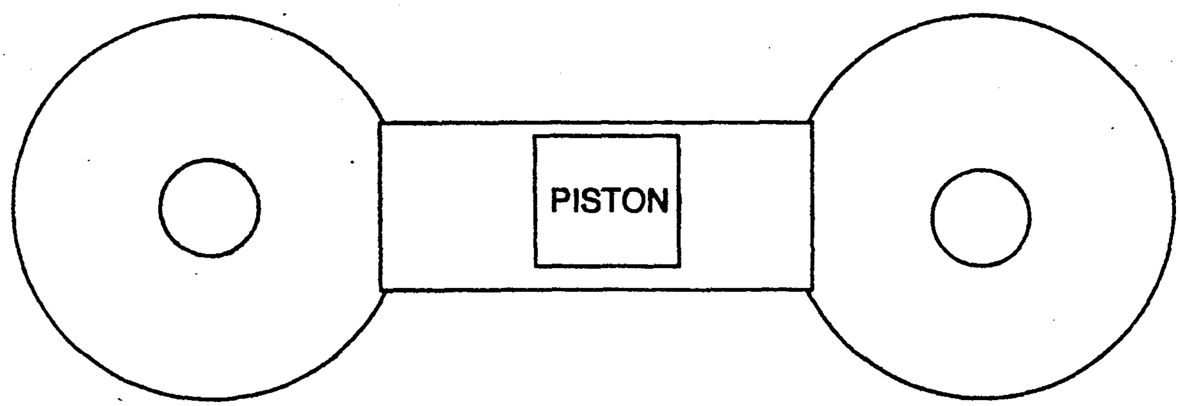


Figure 1. Pulsed Spheromak Reactor with Adiabatic Compression