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Modeling the Influence of Polls on Elections: A Population Dynamics Approach

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Abstract

We propose a population dynamics model for quantifying the effects of polling data on the outcome of multi-party elections decided by a majority-rule voting process. We divide the population into two groups: committed voters impervious to polling data, and susceptible voters whose decision to vote is influenced by data, depending on its reliability. This population-based approach to modeling the process sidesteps the problem of upscaling models based upon the choices made by individuals. We find releasing poll data is not advantageous to leading candidates, but it can be exploited by those closely trailing. The analysis identifies the particular type of voting impetus at play in different stages of an election and could help strategists optimize their influence on susceptible voters.

keywords: voting, polling, sequential voting, landslide, head-to-head.

1 Introduction

We create a mathematical model with which to gain insights into how the outcomes of elections in which voters are exposed to exit polls and other voting information might differ from the results of an election in which voters are not exposed to this information. Consideration of the consequences of the public value of information and its role in shaping voter perception in this type of election of the importance of their vote appears in a recent paper by Larcinese (2007). Some countries restrict broadcasting of exit polling data while elections are taking place. With the increasing popularity of early voting and the speed and voracity of news media the question of whether surveys, polls, and forecasts that are made publicly available while voting is taking place affects the outcomes becomes a pressing issue of theoretical importance and practical consequences. How this information affects collective behavior in a typical presidential election, is different from how it affects voters in a sequential voting situation with perfect information. Thus, restricting information and deciding what types of information to suppress would be difficult to justify within the context of fairness in a type of election process that we know something about.

Voters' knowledge of earlier election results can have an effect on the decisions of later voters (Banerjee 1992; Callander 2007; Plumb 1986; Sudman 1986; Grosser et al. 2005; Goidel & Shields 1994). This information affects strategy as well as perceived relative importance of his/her vote on the election outcome. The manner in which voters are affected relates to their voting traits as well. Furthermore, while it is true that there are many voters who are not going to be affected by information, partisanship is not a good indicator of the degree to which a voter may be influenced. Hillygus (2005) analyzed how campaigning can affect voter choice and found that a large number of voters change their minds about whom they plan to vote for immediately prior to election day. She noted that, while independents tend to change their choice more than partisans, it is not uncommon to see partisan voters change their vote at the last minute. Voters' knowledge may also affect voter participation as well as create or modify a surge or "momentum" for candidates that could change the eventual outcome of the election (McAllister & Studlar 1991).

Our non-equilibrium population-dynamics election model is a time-dependent set of voting rate equations for groups of voting populations that cast their votes in response to how information is perceived. We define the *voting period* as the period over which voters cast their votes and *polling data* as the information arriving during the voting period. This includes exit polls, as well as news reports and opinions. The uncertainty in the polling data is included in the model as a stochastic term and it has a direct bearing on voter perception and strategy. Depending upon the election, the voting period ranges from a few hours to many days, as occurs when elections allow early voting.

The outcome of an election is a collective result of the choices of many individuals and leads to a challenging problem in public choice theory: how can our understanding of individual choice translate to understanding the collective outcomes? A population-dynamics model captures the interaction of classes or groups of individuals, rather than of individuals themselves, *i.e.*, it focuses on the scale at which winners and losers of elections are determined. In doing so it sidesteps an unresolved problem with using game-theoretic models of individuals: how to "upscale" or work out its mean-field generalization to capture a collective outcome. The (macroscopic) population dynamics approach is more amenable to analysis than agent-based modeling. The simplicity of the model permits a full mathematical analysis, which in turn leads to an understanding of its outcomes and the determination of the degree to which parameters need to be known with precision in order to make the model predictive. This paper also adds to the mathematical foundation of modeling large-scale dynamics of a complex sequential voting problem with uncertain voting choice information.

The population-dynamics approach glosses over important individual-choice issues, such as the "paradox of voting" and "rational ignorance," yet it sheds light on what should be expected in upscaling microdynamics as well as in suggesting macroscopic social experiments. Thus, it does not replace, but instead complements the empirical and microdynamic approaches.

The general strategy in formulating a population-based model is to divide the population of voters into two time-dependent groups: *committed* voters and *susceptible* voters. The two groups of voters cast their choices during the election period, subjected to polling data with varying degrees of reliability. The model uncertainty is zero when the voters have full confidence in the information, although this does not imply that the information is necessarily perfect. Susceptible voters can be swayed by the information, while the committed voters are not. This division of the voters is fundamentally different from the commonly-used partisanship categorization of voters. It has the following important modeling consequence: the intentions and political affiliation of individual susceptible voters are not required by the model. This division of the population into susceptible and committed voters is congruent with the manner in which political strategists of both parties are thinking about the close 2008 election for the U.S. presidency. The news media refers to the target voters of the strategists in the twilight of the election date, as "undecided voters," who may yet be swayed to change their choice.

The other modeling ingredient is to catalogue all important collective voting strategies or mechanisms. Each voting mechanism has several voting-rate changing modalities which we call *impetus*. Here we codify how voters react in close races, for example, when favoritism involved or when altruism is at play. The two most important collective voting mechanisms that affect the voting rate of the susceptibles are the *head-to-head voting mechanism* and the *landslide voting mechanism*. We use the *head-to-head voting mechanism* to model susceptible voter behavior when exit polling data indicate there is a close race among candidates. Traditionally a landslide is defined as the phenomenon whereby people tend to vote for the person whom they expect to win (Berelson et al. 1954). Here, we use *landslide voting mechanism* to model susceptible voter behavior for the situation in which exit polls indicate that a particular candidate or group of candidates have a substantial lead over all others, and voting rates change as a consequence of this information. In the election of 1980 it is thought that early projections of Reagan's victory by a substantial margin depressed voter turnout for Carter. Other examples of similar landslide victories are the French presidential election of 2002 and the British Columbia general election of 2001. There are landslide victories, however, that are the result of both a significant voter turnout as well as a concentration of voting for a leading candidate. Such would be the case of many populist election winners. The model also recognizes that elections can be used instead by voters to make a statement (deliberate "expressive" voting). This could be the case for committed or susceptible voters who cast their vote for fringe candidates running to make a statement. In recent US history, the iconoclastic fringe candidate is Ralph Nader. Kunce (2001) points out that pre-election perceptions depend directly on how one measures the closeness of the election race. We suggest that the same thing can be said about how susceptible voters react to exit polling data. Both mechanisms have threshold parameters that are meant to represent an election-dependent and population-dependent measure, which when compared to the exit polling data at any given time, determine the degree to which people react.

2 Development of the Model

We focus on multi-party elections, decided by a majority-rule voting process. We sidestep the problematic division of the voting population into partisans and non-partisans by dividing the total population of registered *potential* voters into the *committed* and the *susceptible* groups. The committed voters can be independent or partisan, but are not swayed in their choice. Susceptible voters are influenced by polling data. The model does not require that the party affiliation of the susceptible voters be known. The committed voting group is further subdivided based on candidate choice, irrespective of whether the individual voters have or do not have a party affiliation or what that affiliation might be. Voters are also subcategorized into "zones," which could represent time zones. More generally, however, the zones incorporate categorization of voters, such as by ethnicity, by affinity, or by physical location.

The total population of potential voters at time t is denoted as

$$M(t) = \sum_{k=1}^{K} \sum_{i=1}^{I} M_i^k(t), \tag{1}$$

and those that have voted are denoted as

$$V(t) = \sum_{k=0}^{K} \sum_{i=1}^{I} V_i^k(t). \tag{2}$$

The total number of registered voters is constant in time and is

$$R = V(t) + M(t). \tag{3}$$

Depending on the context, the subscript i identifies the candidate's party affiliation, or voter's party affiliation. I is the number of candidates. Voters are further classified into K zones; the classification index k can refer to a regional classification, time zones, or some other demographic characteristic. The group $k = 0$ is reserved for pre-election absentee voters.

The total population of committed voters is $\mathcal{C} := \theta M(0)$, with $0 \leq \theta \leq 1$; they are not swayed by exit polls and will *all* cast a vote. The group $\mathcal{S} := (1-\theta)M(0)$ represents the total *susceptible* population of potential voters. At $t = 0$, $R = \mathcal{C} + \mathcal{S} + \mathcal{V}$, where $\mathcal{V}$ is the number of absentee voters. If there are several regions, $\mathcal{C} = \sum_{k=1}^{K} \mathcal{C}^k$, $\mathcal{S} = \sum_{k=1}^{K} \mathcal{S}^k$. However, all $\mathcal{V}$ votes are lumped into their own zone or region, $k = 0$.

At any given time t, $C_i^k(t)$ are voters in group k and party i who are not swayed by the exit polls. The initial fraction of committed voters in party i and region k is given by the constant

$$\phi_i^k = \frac{C_i^k(0)}{\mathcal{C}},$$

with the constraint that

$$\sum_{k=1}^{K}\sum_{i=1}^{I} \phi_i^k = 1,$$

A growing percentage of voters in the U.S. are choosing to vote before the election day, as this option has been made available recently; over 22% of voters in the 2004 U.S. presidential election filed absentee votes. These voters are essentially committed voters, by our definition. The mail-in or absentee vote can be allowed to depend on pre-election information in a dynamic way. However, we choose not to model it this way. The absentee votes ($k = 0$) appear as initial conditions to the voting population in region: $V_i^0(0) = \phi_i^0 \mathcal{V}$; for $i = 1, ..., I$. We note that the voting distribution of the absentee ballots could have been taken to be that of the distribution of eventual votes among the committed. However, this becomes an academic issue if outcomes of an election with and without exit polling data are compared, *i.e.*, the same initial data are used in both of cases.

For $t \in [0, T]$, the extent of time in hours the voting stations remain open, the voting process is modeled, in the absence of controls, by

$$\frac{dC_i^k}{dt} = -\sum_{j=1}^{I} \rho_{ji}^k, \tag{4}$$

$$\frac{dS^k}{dt} = -\sum_{i=1}^{I} r_i^k(t), \tag{5}$$

$$\frac{dV_i^k}{dt} = -\frac{dC_i^k}{dt} + r_i^k(t), \tag{6}$$

where ρ_{ij}^k is the rate of voting for party i by committed voters of party j in zone k, and $r_i^k(t)$ is the rate of voting for party i by susceptible voters in zone k.

For simplicity we assume that the entries of the matrix ρ_{ij}^k associated with the committed vote are constant. The initial conditions are

$$C_i^k(0) = \phi_i^k \mathcal{C}, \tag{7}$$

$$S^k(0) = \mathcal{S}^k, \tag{8}$$

$$\text{for } k = 1, 2, ..., K,$$

$$V_i^0(0) = \phi_i^0 \mathcal{V}, \qquad V_i^k(0) = 0, \quad \text{for } k > 0, \tag{9}$$

where $i = 1, ..., I$. The above equations are the continuum analogue of discrete equations for individual voters; for large populations of voters, the continuum approximation has little effect on the outcome of the model when large populations of voters are considered. The constraint on the initial data $\mathcal{S}^k$ for the susceptible equation is that at $t = 0$ the available susceptibles are those not part of the mail-in group or the committed, and of those, a certain portion is assigned to group k. Table 1 summarizes the nomenclature associated with the model.

We make several simplifying assumptions about the model: (1) We assume that no voting irregularities occur and do not including the cost of voting. These two effects could be incorporated by detailed terms later, and instead parameterize their effect into the voting rate rules by approximating these effects in the impetus modalities; (2) we do not track the fluctuations associated with factors such as the time of day; (3) the model does not have a saturating behavior; this assumption is acceptable if less than about 90% of the total registered population is expected to vote. To first order these effects do not change the outcomes of comparisons of the model run with and without exit polling data information.

We assume that we have an estimate for the uncertainty in the polling data. The time at which data are released and generally available to the voters is crucial and it is thus that there is a delay parameter that accounts for the polling data enter the dynamics of (C_i^k, S^k, V_i^k) via the rates r_i^k. In the absence of these, r_i^k is zero. Otherwise, the values and the forms that the r_i^k's assume depend on a numerical comparison of reported polling data to two thresholds, associated with two voting mechanisms.

The *head-to-head* voting mechanism can be triggered when two or more parties are competing closely, as reported by the exit polls. The *landslide* voting mechanism is triggered if the exit polls indicate that one or more candidates are showing a significant lead when compared to all other candidates. Both mechanisms are endowed with a sense of "urgency" which makes the voters perceive that their vote becomes increasingly important as time marches on. The urgency is greatest when the polls indicate that the election is in a dead-heat. Both mechanisms affect the rate of voting (rather than the total number of votes). The rates are constrained to be non-negative. The rate at which susceptibles may vote for some specific candidate may be no higher than the rate at which the most committed voters cast their preference for that candidate ($r_i^k \le \rho_{ii}^k$).

2.1 The Committed Voters

The rate of voting associated with the committed registered voters is the non-negative voting rate matrix ρ. We assume that this voting rate does not depend on time, which is to say that we ignore voter habits over the course of the election period. We decompose the voting rate into three factors

$$\rho_{ij}^k := \nu_{ij}^k \Lambda_{ij}^k C. \tag{10}$$

In this study we assume that all committed voters will cast a vote by time T. The rate of votes per hour for people in party j voting for candidate i in zone k, denoted by Λ_{ij}^k, is equal to ϕ_j^k/T. The coefficients in ν_{ij}^k represent the fraction of registered voters of party j that would vote for party i in zone k. In each zone k, it satisfies the condition

$$\sum_{j=1}^{I} \nu_{ij}^k = 1. \tag{11}$$

Figure 1 illustrates the outcome of an election in which no exit polling data are released. In this case, no susceptibles vote. This example will be used to compare to cases in which exit polling data are made public. For this case, we use one zone ($K = 1$). The total pool of voters (R) is 70.97 million, the total number of parties (I), is five. There are no absentee ballots cast. The ratio of committed to susceptibles is set to $\theta = 0.589$. Because there are no polls, no susceptible voters cast a vote. We use $\phi = [0.3620, 0.3756, 0.2261, 0.0150, 0.0213]$, and $\nu = \mathbb{I}$, a 5×5 identity matrix. After twelve hours of voting, party 2 wins by a reasonable margin.

2.2 The Susceptible Voters

The rate of voting associated with the susceptible group is defined as

$$r_i^k(t) := \mu_i^k(t)\Gamma_i^k(t), \tag{12}$$

where $\Gamma_i^k(t)$ is the rate of votes per hour for susceptibles voting for candidate i in zone k. The structure of μ and Γ depend on the voting mechanism used.

The exit poll results for party i are given by

$$P_i(t_m) = \sum_{k=1}^{K} V_i^k(t_m - t_d) + \text{noise}, \tag{13}$$

where $t_m \in [0, T]$ and $t_m - t_d$ is non-negative. The time t_d is the time at which the poll numbers are obtained, and t_m at which time the polling data are released to the voting public and the noise represents the uncertainty in the poll numbers. For simplicity we do not distinguish polling data by region.

2.3 The Head-to-Head Mechanism

Candidates who have the potential of beating the expected leader, as reflected by the exit polls, may have an upsurge of support. The leading party is also affected by a close race when the perceived impact of voting increases. In the model, the closeness of the race is compared to a threshold associated with the voters' perception of whether or not their vote makes a difference. We assume that the intensity of the effect depends on the time the exit poll is released. We model the rate of voting as a function of (t_m/T), where t_m is the time at which polling data are released to the public. In the head-to-head mechanism the susceptible voters are compelled to vote for a specific party at a rate commensurate with the most committed voters for that party, *i.e.*, at a fraction (possibly 1) of ρ_{ii} for the i^{th} party. That is, there is an inherent impetus of voting that we include as a multiplicative factor in the rate to reflect either pre-election biases or natural efficiencies.

We propose three vote-rate change modalities within the head-to-head mechanism. They are denoted voting *impetus*. The voting impetus, which takes into account contextual information critical to a particular election, modifies the rates r_i^k. We propose three impetus models for the head-to-head mechanism:

The *proportional impetus* modality, which biases the susceptible rate of voting, making it commensurate to the estimated population of registered voters in each party. A voter impetus that is proportional to the normalized density of committed voters for each party may be appropriate in certain elections and in certain regions, where like-minded voters tend to vote along similar lines. The impetus can also be made to be proportional to some other demographic. The voting rate for a party increases in proportion to the rate of voting of the committed group. (The normalized proportionality model can be some other demographic, other than party proportions). For this mechanism the impetus term (normalized ϕ) associated with each party reflects the distribution of voter allegiances. Parties with greater numbers of voters will have greater voter impetus. This situation may occur when the favored winner is doing better than expected as indicated by polls, building momentum in his or her favor by inducing susceptible voters to vote for the perceived potential winner. It could also be a reflection of the regional phenomena wherein like-minded individuals vote in a similar way. The proportional impetus may apply to issues or referenda rather than candidates: in these cases information-sharing within networks of people may have a synergistic effect and thus the demographics that mirror the structure of the network may play a role in the altruistic behavior of the susceptible voters.

The *favored impetus* modality takes into account pre-election common wisdom and the relative asymmetries that ensue when people vote knowing that candidates may be expected winners or underdogs. There are elections in which there is an overwhelming expectation of its outcomes. Exit polling data can be confirming or denying expectations, and to certain degrees, compelling voters to take into account this comparison in their choice of candidate. Candidates who are expected to win will not experience an upsurge in voting if exit polls indicate that they are winning. In a head-to-head situation, a tied or trailing candidate could experience a voting surge if the voters perceive that their votes can upset the chances of victory for the expected winner. We propose this would be significant when a candidate that is not the expected winner is leading the race as reported by exit polls.

The *lesser-of-the-evils* modality represents the less frequent situation in which voters will attempt to use their vote to defeat a candidate that they do not care for, even if it means voting for someone for whom they have a neutral or negative opinion. This modality accounts for the propensity of some voters to vote against a particular candidate or candidates when they perceive, according to exit polling data, that their action has some impact on the election or on themselves. The lesser-of-the-evils is a collective voting phenomenon that is unlike a form of protest voting: this is because this modality applies to the dynamics of the rate equations of the susceptible, rather than of the committed voters. Moreover, it does not necessarily have to be be true that fringe candidates are the sole beneficiaries of the lesser-of-the-evils modality: individual voters responsible for changing the voting rate can be categorized as instrumental or expressive.

All three variants are modeled by first normalizing the fraction of the vote for each candidate

$$p_i(t_m) = \left| \min \left\{ 1, \frac{P_i(t_m)}{\sum_{j=1}^{I} P_j(t_m)} + \eta_i(t_m) \right\} \right|,$$

In what follows we denote by i^* and i^{**} the parties associated with the two highest voting fractions at time t_m. Note that there can be a tie between two or more parties. A possible model for the noise term η is given by a Gaussian distribution with zero mean and uncorrelated entries, with prescribed variance δ_i^2. The normal distribution is mapped to the interval $[-0.5, 0.5]$. If the normalized polling data are below zero, the entries are set to zero.

The head-to-head voting mechanism is triggered if the *spread*, defined as

$$\sigma_i := p_{i^*} - p_i, \qquad \text{for any } i \neq i^*,$$

is less than the threshold parameter $\sigma^h \geq 0$. That is,

$$\sigma_i \leq \sigma^h,$$

where $i = 1, .., I$. The parameter σ^h is called the *head-to-head parameter.* (It is in modeling the spread and the parameter σ^h that one can include the effect of the cost of voting). When the head-to-head mechanism affects voting for a particular candidate i, we assume that the susceptible voters will cast votes for party i at a rate equal to or less than the rate $\mu_i^k = \nu_{ii}^k$. The rate term is

$$r_i^k = \gamma_i^k \rho_{ii}^k.$$

The coefficient γ_i^k includes such factors as the time remaining before polls close, the cost of voting, and the voting impetus, which is meant to model specific characteristics associated with the election in question.

The sense of urgency in affecting the outcomes increases as the election period nears its end and could be modeled by making σ^h be larger as time goes by, making the threshold easier to overcome. However we prefer to disengage σ^h from poll times because we conjecture that voters treat the cost and the sense of urgency as two separate issues that affect voting. We model the temporal effect by making γ_i^k proportional to the constant t_m/T, where t_m is the time a poll is made public. This constant will remain unchanged until the next poll is released (t_{m+1}) or until polls close ($t = T$). An alternative weighting rule could be the one suggested by Battaglini et al. (2007): later voters abstain if their votes are not pivotal and the stakes are not high. Early voters, on the other hand, might defer the "decision" to later voters if the stakes are high.

In our model, an exit poll with no delay released at time 0 will not spur susceptibles to vote. At first, this seems counterintuitive, as it is hard not to imagine that the exit poll at time 0 represents some sort of consensus or projection which is an outcome of the heaviest period in the campaign. The model assumes that this collective effect is already reflected in the voting rates of the committed voters. Whenever the poll is made available, the mechanism will change the voting rate of all parties affected.

In the head-to-head mechanism, the spread determines the effect of the data on the voting rate. If the mechanism is triggered for a given party i, then the voting rate becomes proportional to the *head-to-head voting function* given by

$$\alpha_i = \left(1 - \frac{|\sigma_i|}{\sigma^h}\right), \tag{14}$$

for $t_m \leq t \leq t_{m+1}$. This is a two-parameter function of σ_i normalized to unity. We assume that the effects on the leading and second place parties with regard to increases in voting is equal. We use (14), but a smoother alternative using a Gaussian function of σ_i instead of a triangle is given by

$$\alpha_i = \exp\left[-\left(\frac{\sigma_i}{w\sigma^h}\right)^2\right], \tag{15}$$

where the parameter w may be tuned so that α_i is insignificant for $|\sigma_i| \geq \sigma^h$. Figure 2 shows plots of these two voting-rate function alternatives.

In summary,

$$\gamma_i^k = \mathcal{A}_i \frac{t_m}{T} \alpha_i \tag{16}$$

where the parameter $0 \leq \mathcal{A}_i \leq 1$ is the *voting impetus* of the head-to-head mechanism. We allow for regional dependence and thus $\mathcal{A}$ depends on k. This allows for the possibility of reflecting the common notion that "politics are local."

2.3.1 Voter Impetus $\mathcal{A}_i$

The effect of early results in favor of a given candidate may depend on performance relative to expectations. In a head-to-head situation, it is possible that the expected underdog may achieve a greater number of votes than projected, motivating increased voter impetus. For example, the 1984 Democratic primaries featuring Gary Hart and Walter Mondale saw Hart obtain an unexpectedly high 17% of the votes. Mondale won with 50%, but Hart gained momentum to win the primaries in New Hampshire and Vermont (Bartels 1988). This effect, however, may or may not be strong enough to engage a substantial portion of the population to energize the underdog with the eventual outcome of winning the election. Our model captures these possible scenarios through the favored head-to-head impetus. Alternatively, if the candidate expected to win performs as well or better than expected, this may cause him or her to gain momentum and spur undecided voters to vote for the perceived winner. This effect is modeled by the proportional head-to-head mechanism described below. The total voter impetus is a non-negative value $\mathcal{A}_i = a_f A_i^f + a_p A_i^p + a_l A_i^l$, where a_x is the normalization factor with relative strengths for each of the three efficiencies, so that $\mathcal{A} \leq 1$.

The voting impetus introduces a voter-confidence dependence on how the susceptibles vote in the following way:

- *Favored Head-to-Head Impetus A^f*: There are elections in which a ranking for election outcomes is prominent. These winner-odds affect the voting impetus by boosting the voting rate of parties that are doing better than predicted as reflected by the polls. If a candidate is doing better than expected, the voting impetus of his or her party is boosted. This provides momentum for the party, giving voters who might not otherwise vote incentive to do so by bringing this contender closer to the leader with the possibility of taking the lead.

The candidate with poll results not exceeding expectations does not have an added surge in voting with one exception: if the expected winner falls behind, that candidate gets a boost in voting regardless of how the poll results compare to the expectations.

The odds of winning are given by the chances c_i, with $\sum_{i=1}^{I} c_i = 1$, for some region k. The candidate with the greatest odds of winning (or any one of the parties with the highest odds) will be denoted with the index $i^{\#}$. The candidate with index $i^{\#}$ and called the "leader" if $c_{i^{\#}}$ is significantly greater than the median $\hat{c}$. All other candidates we denote as the "underdogs." (For example, if $c_{i^{\#}}$ is greater than $\hat{c}$ by, say 15%, most people would agree that the odds are significantly larger and would accept separating candidates loosely in terms of a leader and underdogs). If we release a poll at some time t_m and the head-to-head mechanism is triggered ($\sigma_i \leq \sigma_h$), then the impetus for $A_i^f = 1$, if $i = i^{\#}$ and $i \neq i^*$. In this case the leader is not winning and thus the voting impetus is set high. On the other hand, if the leader falls behind in the polls due to momentum in underdog voting, it will be given an impetus of 1 regardless of how the poll compares to the expectation.

The manner in which underdogs are affected depends on whether there is a clear leader. Suppose the poll results at time t_m are given by p_i. We define $D_i := p_i - c_i$ by the difference between the poll outcome and the expectations or chances of winning. The difference is normalized ($d_i = D_i / \max_{i=1}^{I} D_i$) and the favored impetus is given by $A_i^f = (1 + \max\{0, d_i\})/2$. Hence a party not exceeding expectations is assigned an impetus value of 0.5, the party most exceeding expectations will have impetus of 1, and the other parties in head-to-head contention who are exceeding expectations will have an impetus between 0.5 and 1, in proportion to the amount by which they exceed expectations. A smooth voting impetus function that has these properties is

$$A_i^f = \{-\frac{1}{6}(1 + \tanh[l_f(c_i - \hat{c} - 0.5c^l)])(1 + \tanh[q_f d_i]) + \frac{1}{2}(1 + \max\{0, d_i\})\}, \tag{17}$$

in region k, for i such that $\sigma_i \leq \sigma^h$. Here $l_f(c^l)$ and c^l are inter-related parameters associated with the rate which the effect rises if c_i exceeds the median $\hat{c}$. The threshold relative to $\hat{c}$ beyond which a candidate is considered a leader is $0 \leq c^l \leq 1$. The parameter

$$l_f = \frac{1}{c^l} \tanh^{-1}(Q),$$

where $Q = 0.99$, for example, is the percentage over which some candidate may have a near certain odds of winning. The parameter q_f is fixed by qualitative considerations, independent of the type of election. Figure 3 plots the function A^f as a function of c and d. Note that there is a smooth but fast transition as c becomes greater than $\hat{c} = 0.5$ when the poll outcome is less than expected, $d < 0$. When the poll results exceed the expected value for a candidate ($d > 0$) then A^f is relatively insensitive to c and increases with d. The voting impetus makes it possible for a candidate leading in the poll to lose its first place position due to intense competition and voters' impressions that their individual votes do not make much of a difference.

Figure 4a shows the outcome of a simulated election with poll times at three, five, and eight hours with an hour delay in reporting, using the favored head-to-head mechanism. The threshold for the landslide mechanism is set to $\sigma^b = 0.15$ (see ahead, "The Landslide Mechanism" Section) and thus does not influence the outcomes. The threshold for the head-to-head mechanism is set to 0.05. This can be compared to Figure 1 to see how the results change due to the poll influence. We also ran this simulation with no reporting delay in the polls and found that the candidate running for party 2 won the election. The favored mechanism using (17) has parameters as described in Figure 3, except that $c^l = 0.05$. The odds were set to $c = [0.3, 0.28, 0.21, 0.12, 0.08]$. The expected winner is party 2, and this situation is reflected in the polls. However, party 1 wins at the last minute due to a larger contribution from the susceptibles. Party 1 builds enough momentum due to the poll outcomes to win.

- *Proportional Head-to-Head Impetus* A^p:

Voter allegiances are encoded in the fraction of committed voters, ϕ. The voter impetus function could take into account the level of support for each party. In the non-poll situation this is already encoded, to a certain extent, via ϕ_i^k and ν_{ij}^k. This means that this effect is not as crucial in A^p as one might think. The effect is limited to susceptibles that are exposed to polling data and thus are more likely to vote simply because there are more voters (or less likely because there are less). As a result, the proportional impetus model increases the intensity of voting in a head-to-head situation. The intensity increases more for the leading party, possibly because the candidate exceeded expectations causing an increase in votes for the leader. We model these effects in the proportional head-to-head variant by defining $A_i^p := \overline{\phi}_i / \max_{1 \leq i \leq I} \overline{\phi}_i$. The average $\overline{\phi}_i = 1/K \sum_{k=1}^{K} \phi_i^k$.

In Figure 4b we illustrate the same election scenario as in Figure 4a, now using the *proportional* head-to-head mechanism. Note that after the first poll the voting for party 2 becomes more intense because its impetus is higher. We also see that the closeness in the polls in every instance leads to a boost in the voting rate of the parties involved in the head-to-head competition, namely parties 1 and 2. The proportional head-to-head mechanism could eventually trigger the landslide mechanism (explained below), if the efficiencies in the parties are radically different and the outcome leads to a large gap between the leading parties.

– *The Lesser-of-the-Evil Impetus* A^l: A voter may be compelled to switch his/her voting preference if the polling data suggests that a much disliked candidate has a clear shot at winning an election, or if there is a perception that voting against him or her will make a difference. The strategy is to throw the support to an alternative candidate that has some competitive viability when compared to the disliked candidate. There is anecdotical evidence of this occurring but even if there is data to suggest switches in the actual casting of the vote, it is hard to trace if this is the motive, unless voters are specifically asked to describe their reasoning when casting a vote.

In many respects, this particular voting mechanism is already encompassed in the favored head-to-head mechanism. A minor modification is that if the mechanism is efficient, *i.e.*, if for some reason there is a confluence of purpose among voters and they do not just vote for alternative candidates to the "evil" one, but instead vote for the next viable candidate, the favored head-to-head mechanism can be changed to reflect this. The rule is as follows: Let i^b be the index identifying the "evil" candidate. For parties i such that $\sigma_i \leq \sigma^h$, and $i^b = i^*$, set $A_i^l = 1$ for $i = i^b$ and for $i = i^{**}$. Otherwise, $A_i^l = 0.5$.

The cost-benefit landslide impetus model (CLI) is invoked when the spread between the winner and any other candidate is greater than some threshold $\sigma^b \geq 0$. If this cost exceeds the benefit, *i.e.*, the potential to affect the outcome, the voter will simply decide to abstain. Voters perceive that their vote will not make a difference in the outcome.

Marketing researchers bring in notions of cognitive dissonance/consonance to explain a bandwagon in voter behavior. Inspired by this and applied to a landslide situation it could be described as follows: voters compare their expectations to the exit poll numbers, which are perceived as being accurate unless the error is glaringly large, and will either adjust their voting preferences to conform to that data or will not be compelled to adjust when the data vindicates their expectations. We call this landslide variant the marketing-landslide impetus model (MLI).

Although CLI and MLI affect individual voting behavior, they do not change the winner of the election. The landslide effect is engaged when the spread between the leading parties and the rest of the candidates is greater than or equal to the threshold parameter $\sigma^b \geq 0$. The landslide is triggered when $\sigma_{i^*} \geq \sigma^b$, *i.e.*, when the spread between a single leading party and a next-nearest leading party is greater than some threshold. As in the head-to-head mechanism, once CLI or MLI is engaged its effect does not change until either the new polling data is available or the polling stations close.

There are candidates which are *immune* to the landslide effect. In the U.S., many third party candidates are in this category. The candidates for these parties are not running with the expectation of winning, but to make a statement. Voters for these candidates also understand this very well and the cost-benefit argument is moot for them. We exclude these candidates from the landslide effect, as we assume that poll numbers do not differ greatly from expectations. We group these candidates and voters in the set $\mathcal{I}$ and these are deemed immune to both landslide effects. For both the CLI and MLI, members outside of $\mathcal{I}$ parties get a decreased rate of voting overall, due to the sentiment that the individual voter will perceive to make little to no difference in the final outcome. Moreover, with the MLI, there is an increased rate of voting for the perceived winner.

Variations in the landslide mechanism are not as dramatic. The cost-benefit landslide impetus (CLI), encodes the depressing effect in the voting rate when exit polls indicate the emergence of clear leaders. The other variant, the market-based landslide impetus (MLI), models susceptible voters changing their vote from a trailing candidate to one of the leaders.

2.4.1 The CLI

This is the situation in which the voter perceives the cost of voting higher than the benefit (winning) and is thus dissuaded from voting. The situation translates into one in which the voting rate of the susceptibles gets depressed or even driven down to zero. (Again, the cost of voting through physical hardship, loss of work, etc are important factors that influence the model for σ^b).

If $\sigma_{i^*} \geq \sigma^b$, for $i \notin \mathcal{I}$ and $t_m \leq t < t_{m+1}$ and $0 \leq t_m, t_{m+1} \leq T$, then

$$r_i^k(t) = \beta_i^{cle} \rho_{ii}^k,$$

where

$$\beta_i^{cle} = -B_i \frac{t_m}{T} \sigma_i,$$

with $B_i \geq 0$. To prevent the rate from dropping below 0, we impose the constraint:

$$r_i^k(t = t_m) + r_i^k(t_m < t \leq t_{m+1}) \geq 0, \tag{18}$$

for $0 \leq t_m, t_{m+1} \leq T$.

In most every respect the CLI and MLI are different in the intensity of the outcome rather than in the outcome itself. Note that if no susceptibles are voting, there is nothing to suppress and the CLI will have no effect.

2.4.2 The MLI

In the MLI voters are driven by a desire to vote for the winning candidate. For $i \notin \mathcal{I}$, and if $\sigma_{i^*} \geq \sigma^b$, the MLI is triggered. In this case, the CLI gets triggered for $i \neq i^*$ and $i = i^*$. Additionally,

$$r_{i^*}^k(t) = \beta_{i^*}^{mle} \rho_{i^* i^*}^k,$$

where

$$\beta_{i^*}^{mle} = B_{i^*} \frac{t_m}{T} \left| 1 - \frac{\sigma_{i^*}}{\sigma^b} \right|.$$

The constraint (18), with $i = i^*$, applies here.

In Figure 5 we show an election time history in which the MLI takes place. The polling data are released at three, five and eight hours, and is taken an hour prior to its release. The candidate for party 2 wins the election and in fact gets a small but significant surge of susceptible support. The outcome is thus that the 'rich get richer,' which in this graph can be seen by the relative decrease in percentage of total votes for party 1.

3 Analysis and Discussion

The effect of the exit polls on model outcomes is exclusively tied to the contribution of susceptible voters to the total vote count. This contribution is found by subtracting the model with exit poll effects from the model without exit polls, thus obtaining the equation for the susceptible population:

$$\begin{aligned} dS^k/dt &= -r^k(i, t_m, h2h, ls, pars, \eta) \\ S^k(0) &= \mathcal{S}^k. \end{aligned} \tag{19}$$

Here r^k is a piecewise constant function, that depends on i, the poll time t_m, the head-to-head parameters symbolized by $h2h$, the parameters relevant to the landslide mechanism, ls, and the noise η.

The fraction of susceptible voters in party i and region k is given by

$$\psi_i^k(t) = \frac{S_i^k(t)}{\mathcal{S}},$$

with the constraint that

$$\sum_{k=1}^{K} \sum_{i=1}^{I} \psi_i^k(t) \leq 1.$$

Also at any time $0 \leq t \leq T$,

$$\sum_{i=1}^{I} S_i^k(t) \leq \mathcal{S}^k.$$

This means that if at time t^* the number of available susceptible voters is exhausted in zone k, all $r_i^k(t \geq t^*)$ are set to zero. We can determine the party affiliation and region of susceptible voters after the voting occurs:

$$\frac{dS_i^k}{dt} = -r_i^k(t),$$

for $i = 1, 2, ..., I$.

If the noise is ignored, the model is fully deterministic and the outcomes (V, C, S) can be solved for uniquely. When the noise is non-zero the model outcomes are easily found by numerical means.

3.1 Sensitivity to the Susceptible-Vote Mechanisms

With regard to the two proposed voting surge mechanisms, the role played by the susceptibles can be decisive in a head-to-head competition, whereas it merely alters the outcome of an election in the landslide case. This is because the head-to-head is triggered when disparities among candidates, as reported by the exit polls, are small and thus this surge inserts a certain degree of uncertainty in the outcome: a small group of susceptibles could be decisive in a majority vote election.

The favored head-to-head model is more appropriate when expectations on outcomes are overwhelming, and these can potentially generate momentum surges. Favored voting rates change when exit polls are compared to expectations and the outcome of this comparison might change during the election period. In the proportional head-to-head model, the voting rate surges are fixed in intensity. Exit polls may compel susceptibles to vote at a rate commensurate with the population ratios chosen to model the impetus.

If a favored head-to-head model is appropriate, publishing exit polls could be beneficial to either leaders or the underdogs: leaders could get an increase in voting if expectations are matching the exit poll numbers. The underdogs would get sn increase if exit polls exceed expectations by a slight amount.

In the proportional head-to-head model, pre-election campaigning should optimize the distribution of demographic criteria that is used to define impetus. In this study, we used the distribution of committed voters (ϕ), for specificity. In the model runs presented in this article, a head-to-head proportional competition will never lead to a trailing candidate overcoming a leading candidate, *i.e.*, the rich get richer is the only possible outcome.

This is not because of the impetus model itself, but rather because we chose to model the committed vote so simply; the committed vote at any given time has been fixed to respect their proportions. Populist candidates would favor allowing exit polls to be known if the appropriate impetus model is proportional and the impetus mirrored demographics.

This *landslide mechanism* is triggered when one or more candidates display a sufficiently large lead as reported by exit polls. The potential for these leading parties to win the election increases. The interesting dynamic is in the distribution of votes for the trailing candidates and perhaps how these compare to fringe candidates (candidates who run to make a political statement rather than to win the election). Presuming that the group of susceptibles is smaller than the committed group, one might argue that losing by a landslide is due to the failure to secure enough committed voters. We find that when no susceptibles are voting before the CLI is triggered, the landslide has no effect on the outcome. This is a feasible outcome and is not demonstrated by the MLI. In the MLI, susceptibles contribute to increasing the chances for the leaders to win.

3.2 Sensitivity to Model Parameters

The sensitivity analysis focuses on determining the relative importance of the parameters. This information could be used, for example, to determine what conditions need to prevail to ensure a win when exit poll results are released during voting and a substantial number of susceptibles are available.

Consider first a two-party competition with no mail-in vote contribution for simplicity. We focus on the susceptible vote in what follows. If party 1 draws in more susceptibles to vote than party 2 then

$$V_1(T) - V_2(T) = \sum_{m=1}^{M} (r_1(t_m) - r_2(t_m))(t_m - t_{m-1}) > 0.$$

Here $r_i(t_m)$ is the voting rate triggered in time interval $t_m - t_{m-1}$ for party i. Here $\sum_{m=1}^{M} t_m - t_{m-1} = T$. In the event that the rates are not piece-wise constants the above equation would be

$$V_1(T) - V_2(T) = \int_0^T (r_1(t) - r_2(t))dt > 0.$$

Focusing on the head-to-head case: since $r_i(t_m) = \gamma_i \rho_{ii}$ then $r_i(t_m) = \mathcal{A}_i(t_m/T)\alpha_i \rho_{ii}$. Hence, the condition for party 1 to win over party 2 is that

$$\sum_{m=1}^{M} r_1(t_m) > \sum_{m=1}^{M} r_2(t_m),$$

or

$$\frac{\sum_{m=1}^{M}[\mathcal{A}_2(t_m)\alpha_2(t_m)]}{\sum_{m=1}^{M}[\mathcal{A}_1(t_m)\alpha_1(t_m)]} < \frac{\rho_{11}}{\rho_{22}} \equiv \rho.$$

We have assumed that ρ_{ii} are constant, hence ρ is constant. is symmetric and thus it is the ratio of the sums of $\mathcal{A}(t_m)$ that matters. Thus, according to our model, campaigning should focus on identifying and cultivating voter impetus.

The relative importance of each parameter can be estimated by how strongly it affects the change in the final vote tally. A convenient equation to use for this purpose is the equation for susceptibles. Since (19) is a rate equation with piece-wise constant rate values, it can be formally integrated. The integral, with $r_i^k \geq 0$ for $1 \leq i \leq I$, is

$$S^k(t) = S^k(t_m) - (t - t_{m-1}) \sum_{i=1}^{I} r_i^k(t_m, \cdot),$$

where $t_m \leq t \leq t_{m+1}$, and $t_{m+1} \leq T$, $t_0 \geq 0$. Hence,

$$S^k(t) = \mathcal{S}^k - (t - t_{m-1}) \sum_{i=1}^{I} r_i^k(t_m, \cdot) - \sum_{l=1}^{m-1} \sum_{i=1}^{I} (t_{m-l} - t_{m-l-1}) r_i^k(t_{m-l}, \cdot).$$

The sensitivity vector is $dS^k/d\mathbf{y}$, the derivative with respect to $\mathbf{y}$, a vector formed by the parameters in the problem. Comparison of the relative sizes of the entries of the sensitivity vector provides a quantitative assessment of the relative importance of the parameters in the outcome of the election at any $0 \leq t \leq T$.

The above sensitivity analysis is practical when the uncertainty in the exit polls has a specific history (and thus exit polls are effectively deterministic) or when there is no uncertainty. However, when exit poll uncertainty is significant, the sensitivity of the outcomes to it can be ascertained by a standard parameter sweep. (Note that the zero noise limit does not represent the statistical mean behavior of the model itself). A parameter sweep is possible because the model has few parameters. Figure 6a shows the history of election results of a three-party contest ($K = 1$, $I = 3$), with no exit polls made public. The ratio of committed to susceptibles is $\theta = 0.589$. The total time of voting is 12 hours. The electorate makeup is set to $\phi = [0.44, 0.45, 0.11]$. $\nu = \mathbb{I}$ a 3×3 identity matrix. The total registered population R is set to 70.97 million, and we assume zero absentee votes.

Next we examine how the results change when exit polls are taken into consideration: we will only engage exit polling data once, at time $t = 7$, and this exit polling data are actually taken from one hour prior. Consideration is first given to a no-noise favored impetus simulation, with the same parameters as in Figure 6. Figure 7a shows the voting history and the number of susceptibles that modify the outcome from the no-exit-poll case. The odds or chances of winning were set to $c = [0.4, 0.48, 0.12]$. The candidates are in a head-to-head competition. The outcome for the elections using the *proportional* head-to-head strategy appears in Figure 7b. Here we set the proportional impetus to be equal to the normalized ϕ. As expected, exit polls affect the outcomes by intensifying the voting, and thus, if a party is ahead, it will remain so after the recruitment of susceptibles to its ranks. It is noted that party 2 won in both cases, but in the favored case, it would have been entirely possible for party 1 to win had the exit polling data or the efficiencies been different. This could have never been the case in the proportional head-to-head strategy: party 2 would win, regardless.

Figures 8-10 summarize the sensitivity of the model to parameter changes by comparing the difference between the votes in an election with exit polls and an election without polls. The votes are normalized to 1 over the range of the particular parameter being swept. All parameters are the same as in Figure 7, except that the exit polling data has noise with variance of 0.01. The figures are obtained by running the model numerically 100 times, drawing different random numbers for the noise term, and averaging the histories for each parameter value. To generate Figure 8a, $c = [0.4, A_2^f, 0.12]$. The parameter A_2^f is swept between 0 and 1. Figure 8b shows the sensitivity of the outcomes to the threshold parameter σ^h, in the favored head-to-head strategy; the $c = [0.4, 0.48, 0.12]$.

Figure 8a shows a switch between the most affected candidates, and it happens when $A_2^f = 0.4$, approximately. This is not surprising, since candidate 1 has $c_1 = 0.4$. Below $A^f = 0.4$ the difference in voting between the poll and no-poll case decays steadily. The switch occurs, then the difference between the poll and no-poll case for candidate 2 is relatively constant; however, the difference between the poll and no-poll case for candidate 1 grows. Figure 8c shows that the increases in voting rates due to this parameter are proportional to its value.

The difference in voting for candidate 1 is relatively unchanged whereas the difference in voting for candidate 2 grows. Figure 8b and d show that the parameter σ^h has a drastic effect on voting differences between the poll and no-poll case for low values, but saturates at around 0.1. We can interpret this as saying that large variations in the voting can result when voters' perception of making a difference in the outcome of the election is high. Party 3 starts getting susceptibles when σ^h is large enough. We found that for the proportional impetus model the threshold is not as significant as it is in a favored head-to-head case. To get variations due to the landslide for this particular three-party example you need unrealistic parameter values. Nevertheless, we can still do a sensitivity study and obtain meaningful conclusions.

Figure 9a illustrates the dependence of the outcomes on σ^b in the MLI case. Clearly σ^b needs to be exceedingly small to trigger a landslide. The sensitivity is greatest when exit polls slightly exceed σ^b. Figure 9b indicates that the changes in election outcomes in a landslide are proportional to B. This parameter is robust and easily determined by data. Moreover, qualitative fidelity may be attained by using some average value. The outcomes for the CLI case are similar to those in the MLI and are thus not shown.

Finally, we look at how the voting outcomes vary due to exit poll uncertainty. Figure 10 shows how uncertainty in the exit polls may affect the voting in head-to-head and MLI situations. In these runs the parameters are the same as those in Figure 8. The noise variance on the exit poll is increased. We fix the parameter $\sigma^h = 0.05$ for all runs and show the mean normalized voting tally for each parameter value. We averaged 500 runs to compute an average history for each noise variance.

Increasing exit poll uncertainty leads to a greater number of parties taking part in the head-to-head competition. In a large noise situation the susceptible portion of the vote will mirror the vote of the committed, scaled by the weighting in t_m/T, hence in the large noise case the differences in voting rates for the poll and no-poll cases becomes small. (This is not a new insight into what seems to happen in actual elections, rather, it is a statement of the fact that the model incorporates this feature). Shown in Figure 10a is the case when the favored head-to-head model is used. The simulations for the proportional head-to-head model will have similar qualitative outcomes: large variations for small noise and the variations as well as the difference between the exit poll and no-exit-poll case drop as the noise is larger. The reason both mechanisms deliver similar qualitative outcomes is because what is responsible for the shape of the curves is not the head-to-head mechanism itself, but the trigger for the mechanism: for a fixed σ^h the low noise case has greater chances of triggering a head-to-head voter upswell. The larger the noise level is, the less of a chance that the exit poll values will be within the threshold. This explains the decay. The shape of the curve itself comes from the noise model used. Since we are using a modified Gaussian noise model the curve decays exponentially. If a uniform distribution were invoked, the curve would decay linearly. Hence, the fact that the outcome is less sensitive as the noise gets larger is a universal result. However, the manner in which it decays should not be taken as a general feature. This means that an appropriate noise parameterization needs to be determined based on field data. In Figure 10b we get a result that has a clear interpretation: if the noise is large enough, landslides are common and the number of susceptibles affected grows as the noise grows. In landslide situations, increases in voting uncertainty lead to larger disparities.

The outcome of a race in which polling data are released is frequently typified by the results displayed in Figure 11a-b. This is the same three-party election. However in this instance, we release exit polling data every hour, with no delay in the reporting and no exit poll uncertainty. The simulation corresponds to a head-to-head/proportional model run. In Figure 11a we show the evolution of available susceptibles, relative to the total available. In Figure 11b we show the exit polling data and at T, the election results. The head-to-head mechanism is triggered, driving one of the parties to assume a significant lead. The number of susceptibles changes in a nonlinear fashion until $t = 6$, at which point the election no longer is in a head-to-head competition. The example is typical of the frequent exit poll situation (for either form of the head-to-head model) when candidates are in a close competition; the frequent reporting of the poll leads to frequent head-to-head competition which forces the parties to eventually separate. With a strong caveat, one could conclude that if reporting the outcome of elections is unavoidable, that it should be done frequently, rather than occasionally. Figures 11c-d show the effect of large delays in reporting in the same competition using the same voting model. The delay in the exit poll release is four hours and polls are then released on the hour. At $t = 4$ the number of total susceptibles available starts declining. It assumes a certain rate of decline for two hours, then it exhibits a smaller decline because the candidates have developed some distance. The distance becomes smaller and at $t = 9$ it adopts yet another decline in the rate which is maintained since the parties fall out of the head-to-head competition for the remaining of the election time. It appears, then, that large delays increase uncertainties and thus potentially have an impact on the fairness of the election.

Controlling the uncertainty in the information has strategic value. According to the model, in a head-to-head (or lesser-of-the evils) situation injecting uncertainty has little effect on modifying the outcomes of the election. If an information strategy is to be used to change the outcomes, it is important to decrease uncertainty of information on all candidates involved in the head-to-head competition. (Though only parenthetically related, McCain's supporters attempted to instill a lack of confidence in susceptible voters siding with Obama in the ending days of the US presidential election of 2008. One might argue that the strategy of promulgating outlandish claims about Obama had no effect on changing the contest.).

4 Conclusions

The effect of releasing exit polls, and other election information while the election itself is still taking place, is frequently debated by political pundits, media, polling and marketing organizations. This issue has influenced voting procedures worldwide, yet it is unclear which types of information affect election results. With the growing popularity of early voting, this issue is bound to increase the debate, both with regard to the issue of the fairness of voting outcomes as well as with regard to campaign strategy procedures and rules.

To address these questions, we proposed a population-dynamics model by casting the problem as a non-equilibrium, time-dependent, imperfectly-sequential voting problem in which groups of voters react to possibly uncertain and delayed information during the voting period. We focused on majority-rule elections and categorized voters depending on their potential to becoming influenced by the polls. Partisanship plays a role in determining voting rates of groups and thus individual party affiliation and voter intent play a lesser role.

Our model incorporates both subjective and rational responses to polling information, which has its own inherent uncertainty. The three fundamental modeling guiding principles were: (1) to divide the population in ways that do not lead to difficulties in definition or in observational quantification; (2) to identify and model what we think are the more important population-level voting mechanisms; (3) to incorporate subjective as well as rational responses to polling information, which has its own inherent uncertainty.

The uncertainty in the exit polls is accounted for by including a noise term in the reporting of exit polls. In our benchmark simulations, we verified that when the exit polls are deemed unreliable, then they have little to no effect on election results. On the other hand, as the data become more reliable, the voting results become more sensitive to changes in the exit poll uncertainty.

The voting rate equations are simple, in the sense that there are only two groups of voters and the number of free parameters is relatively small. The model's simplicity allows it to be mathematically analyzed and the analysis indicates that only a few of the free parameters need to be estimated with great precision. These equations allow the susceptible voters to be influenced by either the *head-to-head* or the *landslide* voting mechanisms.

The head-to-head mechanism is associated with surges in voting when parties appear in dead heats, the intensity of these becoming greater as the voting stations closing time nears. In the landslide mechanism, on the other hand, voters will perceive their vote as having little to no impact on the election outcome. This leads to a further solidification of the leader's position among all of the candidates. The head-to-head mechanism is more subtle, and sensitive to the model parameters, with regard to its effect on election outcomes. The mechanisms themselves are endowed with momentum-changing effects, including rate changes proportional to voter partisanship, the asymmetries of pre-election odds, the influence-changing subjective value of the act of voting, the importance of altruistic motives, and strategic voting for candidates other than the perceived winner. These momentum-changing modalities we denote as the "voting impetuses". It is within the modalities that complexity in the model is hidden, and for which empirical and theoretical knowledge are useful in constraining its free parameters. The focus of attention should be placed on the impetuses in order to improve the model's predictive powers.

When the head-to-head mechanism is triggered among a subset of well matched candidates, the susceptible voters will react in one of three ways, depending on the voting impetus model. In the favored impetus model, the voters react dynamically as comparisons between changing poll numbers and expectations are evaluated. In the proportional impetus model, voters react to a direct comparison of exit polls to a threshold via a rate that is proportional to some demographic aspect of the voting population. The proportional impetus is appropriate during referendum elections. In the lesser-of-the-evils impetus, voters will attempt to defeat the "evil" candidate by bolstering the tally of another candidate.

In a tight race, the head-to-head mechanism becomes important, if exit polling data are released. In these dead-heat situations a candidate exceeding expectations would welcome the release of exit poll information. Determining what voting impetus model is appropriate for a particular election and exploiting its positive and negative aspects could be part of a campaign strategy. Exit polls that put expectations in question may lead to greater uncertainty and thus, if the favored impetus is most appropriate, the strategist might not welcome the release of the exit polling data. If the proportional impetus is appropriate, the exit polls act more as a trigger rather than as a way to radically upset the outcome of the election. Pre-election campaigning should thus focus on identifying and optimizing the demographic that defines the proportional impetus itself.

Landslides occurs less often than the head-to-head situations and reduce outcome uncertainties. The cost-benefit variant suppresses voting by susceptibles when exit polls indicate that a candidate has a significant lead. The marketing-based variant spurs susceptibles to vote for the leaders. The case of Reagan vs. Carter is often used as a classic example in which a landslide occurred and polls had an effect on the outcome. Early on in the election the polls indicated Reagan winning by a large margin and many have the opinion that that this caused Carter voters to stay away from the voting stations. The result was that Reagan won by an even larger margin. This is disputed by others who instead claim that the Iran Crisis had a greater effect on the outcome.

An important voting situation, typified by cases like Perón in Argentina, Chávez in Venezuela, Morales in Bolivia, Huey Long in Louisiana, is the election of populist candidates. They manage to circumvent the perception of irrelevance of a large disenfranchised population. In our model populists win through the committed voters (presuming that susceptibles are a smaller group). The exit polls in this type of election situation make little to no difference and the election is effectively won before the election period. Third-party candidates (as opposed to fringe party candidates) should make their committed voting base as large as possible. Voters tend to give third party candidates a longer time to establish their viability and thus the momentum garnered in one election is sometimes available to them in future elections. This is not always the case for the established party candidates.

In the 2000 U.S. presidential election it was thought that the strong third party presence (strong was 2%) could have been enough to cause Gore's defeat. One might argue that exit polls, of which there were many during the day of the election, should have depressed vote increases and voted for Nader. The model could forecast this possibility. However, the model can also predict another explanation, which is perhaps a more likely one: Nader had only committed voters and no susceptibles that would have otherwise voted for Gore voted for Nader. This conclusion cannot be extended to a scenario in which Nader had not participated in the first place.

In conclusion, a population dynamics model can quantify the effects of polling data on the outcome of multi-party elections. Our model, representing collective groups of people, provides a complementary approach to the existing micro-scale models as well as empirical approaches. Endowing the model with a full complement of collective voting rate strategies is not difficult and in this sense the approach advocated here is powerful. The challenge shifts to building into these voting rate terms the right phenomenology and pinning down the values of the various parameters. That the model is readily analyzed, however, makes the process of model development efficient and the inclusion of empirical facts and other knowledge testable, at the population level.

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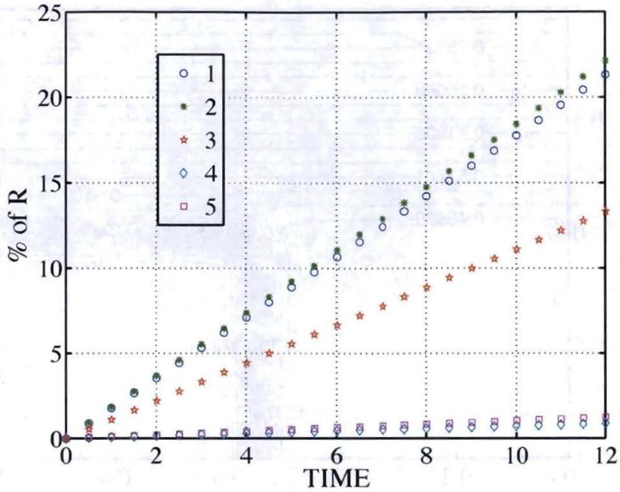
Table 1 *Model constants and other parameters defined, given, or derived.*

symbol	range	units	definition
i	$[1, I]$	1	index of candidate party
i^*	$[1, I]$	1	index of party of a winning candidate
i^{**}	$[1, I]$	1	index of second place candidate (or tied winner)
$i^{\#}$	$[1, I]$	1	index of leader
$i^{\flat}$	$[1, I]$	1	index of evil candidate
j	$[1, I]$	1	index of voter party
k	$[0, K]$	1	index of demographic group or zone; $k = 0$ represents mail-in voters
I	≥ 0	1	total number of candidates
K	≥ 0	1	total number of zones
$M(t)$	≥ 0	people	total population of potential voters
$M_i^k(t)$	≥ 0	people	potential voters in party i, zone k
$V(t)$	≥ 0	people	total votes
$V_i^k(t)$	≥ 0	people	total votes for candidate i in zone k
R	≥ 0	people	total registered voters; $R = M(t) + V(t)$
θ	$[0, 1]$	1	fraction of committed voters
$\mathcal{C}$	≥ 0	people	committed voters at $t = 0$
$\mathcal{S}$	≥ 0	people	susceptible potential voters at $t = 0$
$\mathcal{V}$	≥ 0	people	total number of absentee votes
$\mathcal{I}$	-	-	set of candidates immune to landslide effect
S_i	$[0, M_i]$	people	susceptible potential voters for party i
C_i^k	$[0, M_i]$	people	committed potential voters in party i, zone k
ρ_{ij}^k	≥ 0	$1/hr$	rate of voting for party i by committed voters of party j in zone k
ϕ_i^k	$[0, 1]$	1	initial fraction of committed voters in zone k voting for party i
$\psi_i^k(t)$	$[0, 1]$	1	fraction of susceptible voters in zone k voting for party i
ν_{ij}^k	$[0, 1]$	1	fraction of registered voters of party j in zone k voting for party i
$r_i^k(t)$	≥ 0	$1/hr$	rate of voting for party i by susceptible voters in zone k
Λ_{ij}^k	≥ 0	$1/hr$	votes per hour for committed group
μ_i^k	$[0, 1]$	1	fraction of voters in party i, zone k voting for party i; $\mu_i^k = \nu_{ii}^k$
$\Gamma_i^k(t)$	≥ 0	$1/hr$	rate term associated with r_i^k
T	≥ 0	hr	number of hours voting polls remain open
t	≥ 0	hr	time at which a poll is taken
Δt_n	≥ 0	hr	time between two polls
t_m	$[0, T]$	hr	time at which a poll is released to the public
t_d	$[0, T]$	hr	time at which a poll is taken
P_i	$P_i \leq V_i$	people	exit poll votes for party i
p_i	$0 \leq p_i \leq 1 + \lvert\omega\rvert$	1	exit poll votes for party i, normalized by all votes
c_i	$[0, 1]$	1	odds of party i winning election
η_i	$(-\infty, \infty)$	1	poll noise on results for party i
δ_i^2	≥ 0	1	noise associated with uncertainty in p_i
σ_i	≥ 0	1	spread for party i; $\sigma_i = p_{i^*} - p_i$, $i \neq i^*$
σ_{i^*}	≥ 0	1	spread for party i^* ($\sigma_{i^*} = \sigma_{i^{**}}$)
σ^h	$[0, 1]$	1	head-to-head threshold
σ^b	$[0, 1]$	1	landslide threshold
α_i	-	1	voting rate function
β_i^{cle}	-	1	coefficient in rate term associated with landslide
β_i^{mle}	-	1	coefficient in rate term associated with landslide
A_i^f	≥ 0	1	favored head-to-head impetus
A_i^p	≥ 0	1	proportional head-to-head impetus
A_i^l	≥ 0	1	lesser-of-the-evil head-to-head impetus
a^x	≥ 0	1	relative strengths of efficiencies ($x = f, p$, or l)
$\mathcal{A}_i$	≥ 0	1	total head-to-head impetus; $\mathcal{A}_i = a^f A_i^f + a^p A_i^p + a^l A_i^l$
B_i^k	≥ 0	1	landslide strategy intensity
w	≥ 0	1	parameter associated with voting rate function
γ	≥ 0	1	coefficient in rate term r_i^k associated with head-to-head
$\hat{c}$	≥ 0	1	median of the odds
c^l	$[0, 1]$	1	leader threshold parameter
q_f	≥ 0	1	parameter associated with A^f
Q	$[0, 1]$	1	parameter associated with A^f
l_f	≥ 0	1	$1/c^l \tanh^{-1}(Q)$



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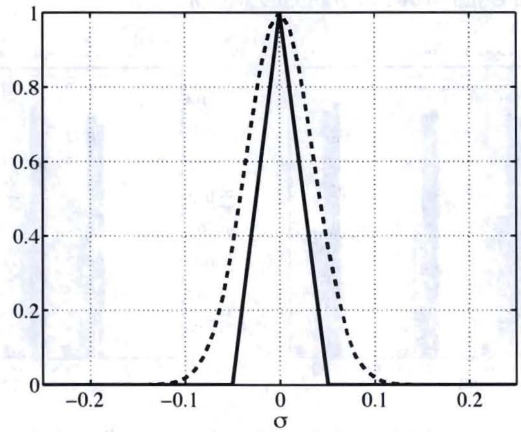

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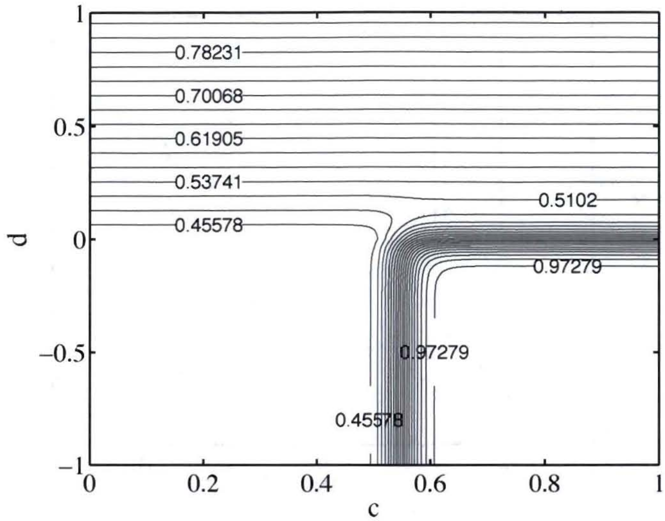

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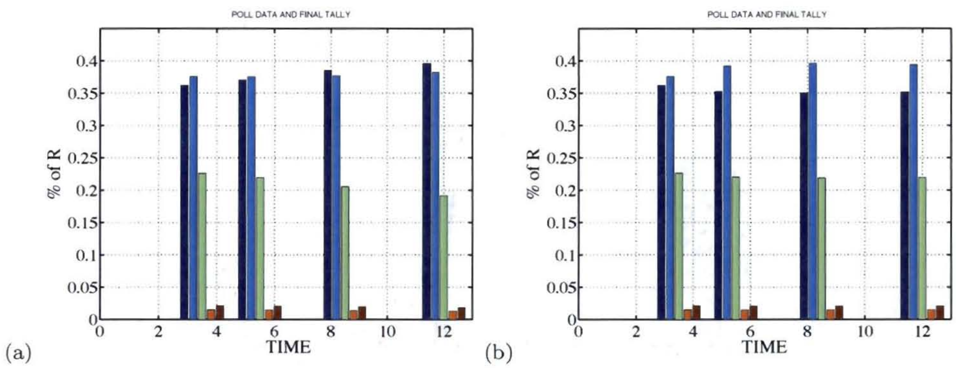

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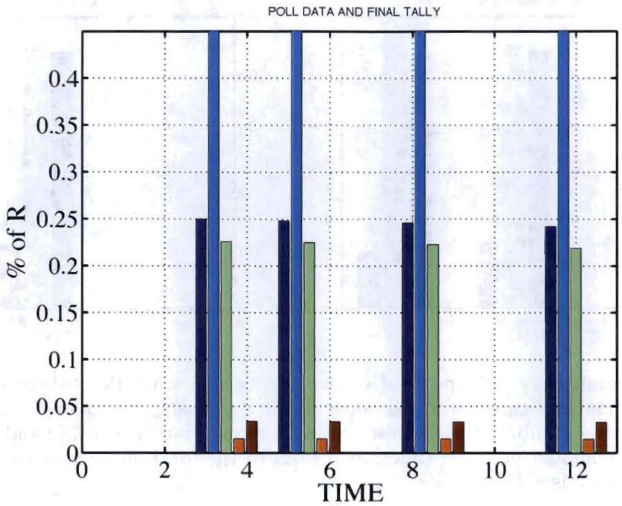

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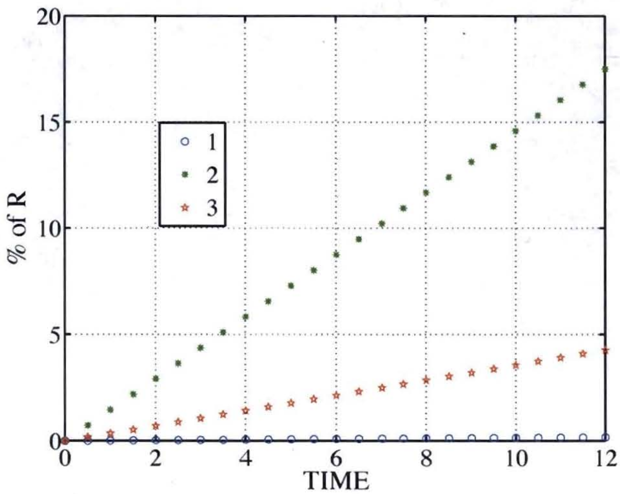

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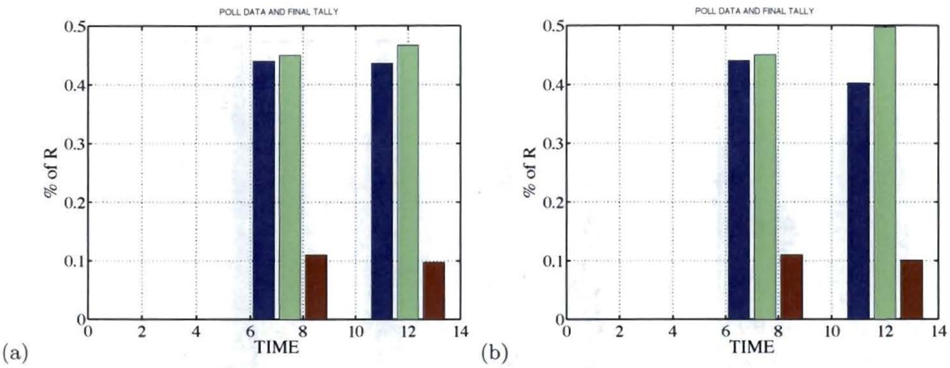

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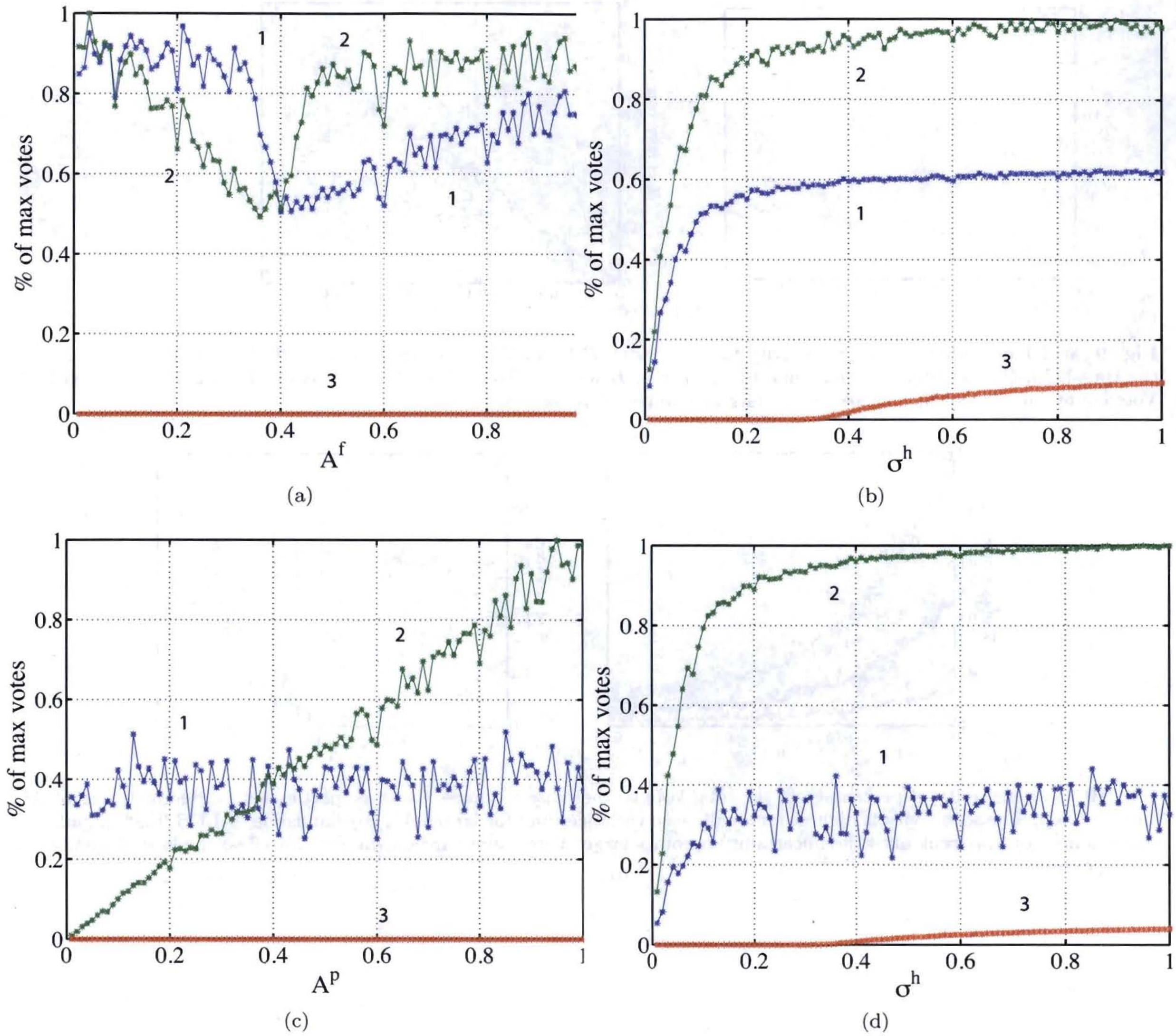

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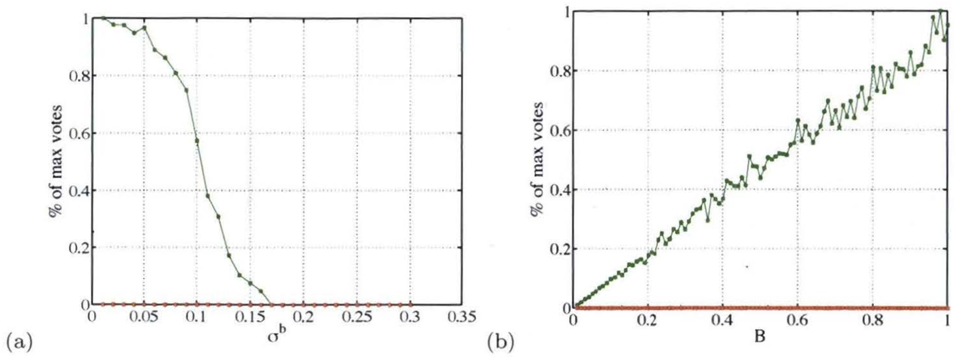

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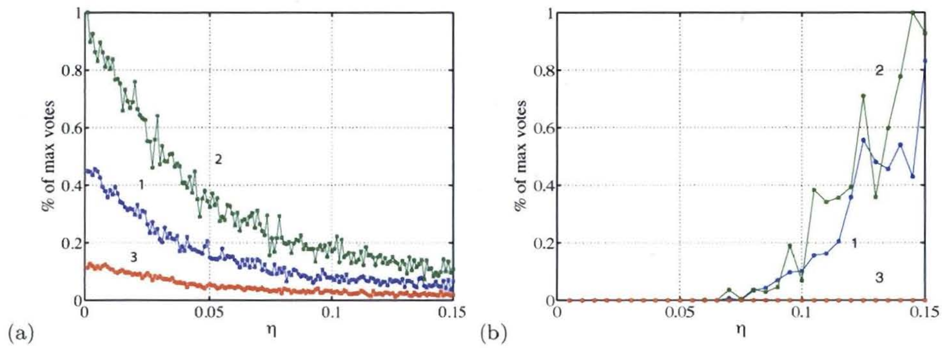

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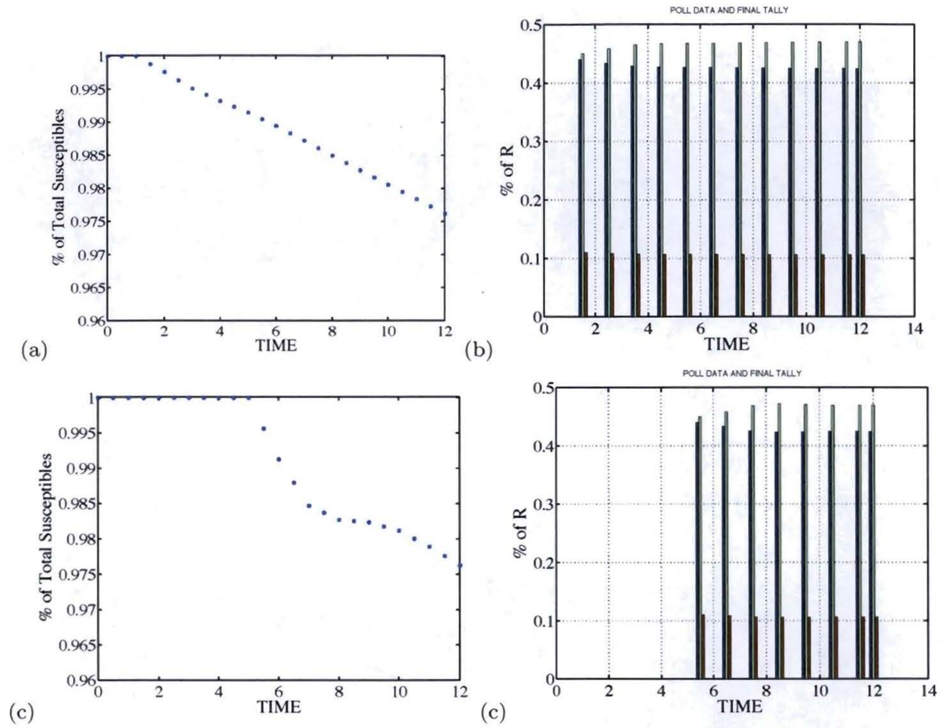

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