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AN ALGORITHM FOR DECONVOLUTION WITH SALT AND PEPPER NOISE

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AN ℓ^1-TV ALGORITHM FOR DECONVOLUTION WITH SALT AND PEPPER NOISE

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ABSTRACT

There has recently been considerable interest in applying Total Variation with an ℓ^1 data fidelity term to the denoising of images subject to salt and pepper noise, but the extension of this formulation to more general problems, such as deconvolution, has received little attention, most probably because most efficient algorithms for ℓ^1-TV denoising can not handle more general inverse problems. We apply the Iteratively Reweighted Norm algorithm to this problem, and compare performance with an alternative algorithm based on the Mumford-Shah functional.

Index Terms— deconvolution, total variation, speckle noise

1. INTRODUCTION

The standard Total Variation (TV) regularization functional [1], which we shall refer to as ℓ^2-TV, may be written as

$$T(\mathbf{u}) = \frac{1}{2}\left\| K\mathbf{u} - \mathbf{s} \right\|_2^2 + \lambda \left\| \sqrt{(D_x\mathbf{u})^2 + (D_y\mathbf{u})^2} \right\|_1 ,$$

where K is the linear operator representing the forward problem, and we employ the following notation:

- the p-norm of vector $\mathbf{u}$ is denoted by $\|\mathbf{u}\|_p$,
- scalar operations applied to a vector are considered to be applied element-wise, so that, for example, $\mathbf{u} = \mathbf{v}^2 \Rightarrow u_k = v_k^2$ and $\mathbf{u} = \sqrt{\mathbf{v}} \Rightarrow u_k = \sqrt{v_k}$, and
- horizontal and vertical discrete derivative operators are denoted by D_x and D_y respectively.

This regularization functional has been applied to a wide variety of image restoration problems, including denoising and deconvolution [2, 3] of images subject to Gaussian white noise.

More recently, the ℓ^1-TV functional [4, 5]

$$T(\mathbf{u}) = \left\| K\mathbf{u} - \mathbf{s} \right\|_1 + \lambda \left\| \sqrt{(D_x\mathbf{u})^2 + (D_y\mathbf{u})^2} \right\|_1$$

has attracted attention due to a number of advantages [6], including superior performance with non-Gaussian noise such as salt and pepper noise. While rapid progress has been made on the development of efficient algorithms for minimizing this functional [7, 8, 9, 10], the majority of these methods are restricted to the denoising problem, corresponding to setting K to the identity operator, and, presumably for this reason, application of the ℓ^1-TV functional for more general inverse problems, such as deconvolution, has received little or no attention in the literature.

In this paper we consider the problem of deconvolution subject to salt and pepper noise, comparing ℓ^1-TV deconvolution, computed via the recently introduced Iteratively Reweighted Norm (IRN) approach [11, 12], with an alternative variational approach designed for this problem [13].

2. ITERATIVELY REWEIGHTED NORM APPROACH

The IRN algorithm [11, 12] for minimizing the generalized TV functional

$$T(\mathbf{u}) = \frac{1}{p}\left\| K\mathbf{u} - \mathbf{s} \right\|_p^p + \frac{\lambda}{q} \left\| \sqrt{(D_x\mathbf{u})^2 + (D_y\mathbf{u})^2} \right\|_q^q \quad (1)$$

is motivated by the Iteratively Reweighted Least Squares (IRLS) method [14, 15, 16] for solving the minimum ℓ^p norm problem $\min_{\mathbf{u}} \frac{1}{p}\|K\mathbf{u} - \mathbf{s}\|_p^p$ by solving a sequence of minimum weighted ℓ^2 norm problems. These methods represent the ℓ^p norm of $\mathbf{u}$

$$\frac{1}{p}\|\mathbf{u}\|_p^p = \frac{1}{p}\sum_k |u_k|^p,$$

by the weighted ℓ^2 norm of $\mathbf{u}$

$$\frac{1}{2}\left\| W^{1/2}\mathbf{u} \right\|_2^2 = \frac{1}{2}\mathbf{u}^T W \mathbf{u} = \frac{1}{2}\sum_k w_k u_k^2$$

with diagonal weight matrix $W = (2/p)\,\mathrm{diag}\left(|\mathbf{u}|^{p-2}\right)$. At each iteration of an iterative scheme, the ℓ^p norm is approximated by the weighted ℓ^2 norm using the weights from the

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previous iteration. To simplify somewhat, this approximation may be used to minimize the norm because, for the same choice of W (and $\mathbf{u}$ such that $u_k \neq 0 \; \forall k$) we have

$$\nabla_{\mathbf{u}} \frac{1}{p} \|\mathbf{u}\|_p^p = (p/2) \nabla_{\mathbf{u}} \frac{1}{2} \left\| W^{1/2} \mathbf{u} \right\|_2^2 ,$$

so that both expressions have the same value and tangent direction.

The weighted ℓ^2 equivalent of (1) may be written (the reader is referred to [11, 12] for full details of the derivation) as

$$T(\mathbf{u}) = \frac{1}{2} \left\| W_F^{1/2} (K\mathbf{u} - \mathbf{s}) \right\|_2^2 + \frac{\lambda}{2} \left\| \tilde{W_R}^{1/2} D\mathbf{u} \right\|_2^2 ,$$

where

$$W_F = \operatorname{diag}\left(\frac{2}{p} f_F(K\mathbf{u} - \mathbf{s}) \right)$$

$$W_R = \operatorname{diag}\left(\frac{2}{q} f_R\left((D_x \mathbf{u})^2 + (D_y \mathbf{v})^2 \right) \right)$$

$$D = \begin{pmatrix} D_x \\ D_y \end{pmatrix} \quad \tilde{W_R} = \begin{pmatrix} W_R & 0 \\ 0 & W_R \end{pmatrix} ,$$

and functions (with corresponding threshold parameters ϵ_F and ϵ_R)

$$f_F(x) = \begin{cases} |x|^{p-2} & \text{if } |x| > \epsilon_F \\ \epsilon_F^{p-2} & \text{if } |x| \leq \epsilon_F, \end{cases}$$

and

$$f_R(x) = \begin{cases} |x|^{(q-2)/2} & \text{if } |x| > \epsilon_R \\ 0 & \text{if } |x| \leq \epsilon_R, \end{cases}$$

are required to avoid the possibility of infinite weights. The minimum of this functional is

$$\mathbf{u} = \left(K^T W_F K + \lambda D^T \tilde{W_R} D \right)^{-1} K^T W_F \mathbf{s}, \qquad (2)$$

and the resulting algorithm consists of the following steps:

Initialize

$$\mathbf{u}_0 = \left(K^T K + \lambda D^T D \right)^{-1} K^T \mathbf{s}$$

Iterate

$$W_{F,k} = \operatorname{diag}\left(\frac{2}{p} f_F(K\mathbf{u}_{k-1} - \mathbf{s}) \right)$$

$$W_{R,k} = \operatorname{diag}\left(\frac{2}{q} f_R\left((D_x \mathbf{u}_{k-1})^2 + (D_y \mathbf{u}_{k-1})^2 \right) \right)$$

$$\mathbf{u}_k = \left(K^T W_{F,k} K + \lambda D_x^T W_{R,k} D_x + \lambda D_y^T W_{R,k} D_y \right)^{-1} K^T W_{F,k} \mathbf{s}$$

The matrix inversion is achieved using the Conjugate Gradient (CG) method. We have found that a significant speed improvement may be achieved by starting with a high CG tolerance which is decreased with each main iteration until the final desired value is reached.

3. RESULTS

We compare the performance of ℓ^1-TV deconvolution with that of an alternative variational approach [13] (which we shall refer to as the BKS method) making use of the regularization term of the Mumford-Shah functional [17]. The first test uses the 236 × 236 pixel "Einstein" image (see Fig. 1), convolved with a 3 × 3 pillbox kernel and subjected to salt and pepper noise. This example is identical to one of those set up by Bar *et. al.* [13], allowing us to use their parameter choices, for their method, to provide a fair comparison. The second test uses the 512 × 512 pixel "Boat" image (see Fig. 2), convolved with a 7 × 7 Gaussian kernel of standard deviation 2.0, and subjected to salt and pepper noise (in this case we made our own choice of parameters for the BKS method).

Reconstruction SNR values and computation times are compared in Table 1, and noisy and reconstructed images are displayed in Figs. 3 to 5. For comparable computation times, the IRN algorithm for ℓ^1-TV deconvolution gives signicantly better results, both in terms of SNR and visual quality.

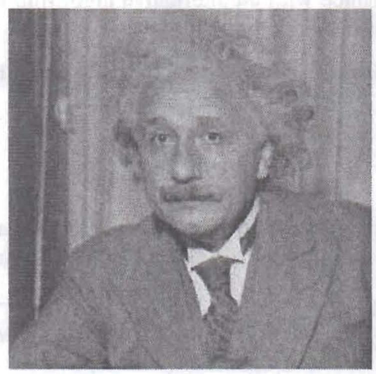
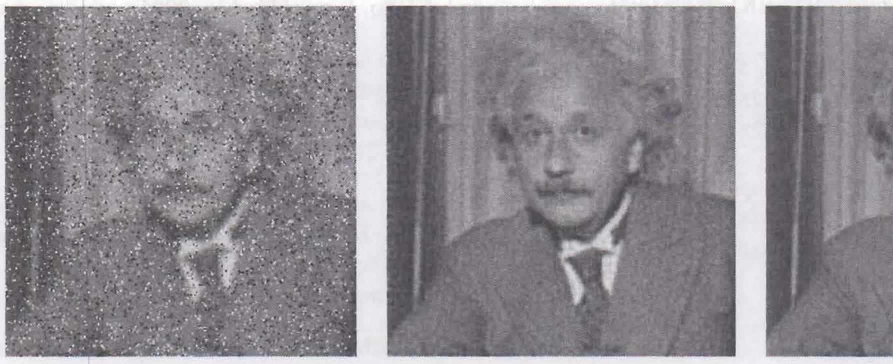


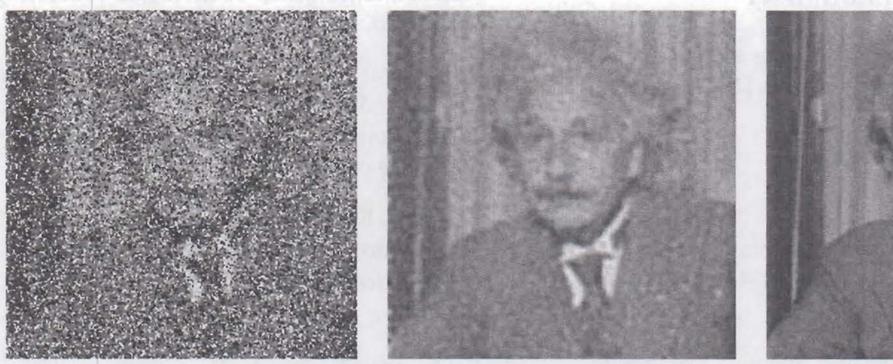
Fig. 1. "Einstein" test image (236 × 236 pixel).

Fig. 2. "Boat" test image (512 × 512 pixel).



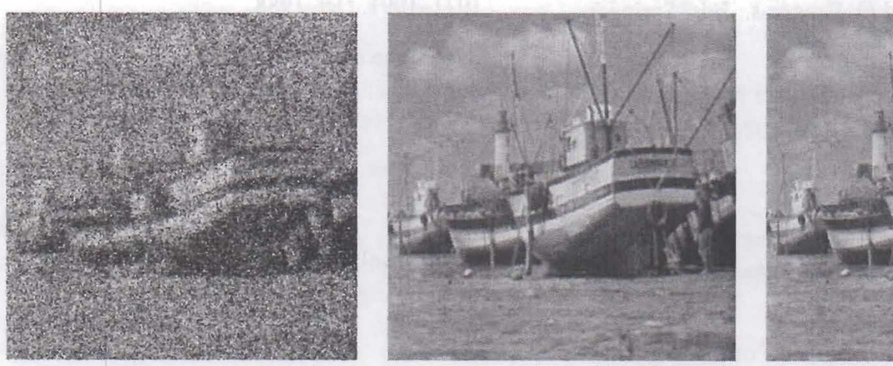
(a) Blur and 10% salt and pepper noise (b) BKS reconstruction (c) ℓ^1-TV IRN reconstruction

Fig. 3. Deconvolution with 10% salt and pepper noise.



(a) Blur and 30% salt and pepper noise (b) BKS reconstruction (c) ℓ^1-TV IRN reconstruction

Fig. 4. Deconvolution with 30% salt and pepper noise.



(a) Blur and 30% salt and pepper noise (b) BKS reconstruction (c) ℓ^1-TV IRN reconstruction

Fig. 5. Deconvolution with 30% salt and pepper noise.

		SNR (db)		Time (s)	
Image	Noise	BKS	ℓ^1-TV	BKS	ℓ^1-TV
Einstein	10%	7.9	20.5	58	55
	30%	2.2	15.8	57	50
Boat	10%	9.3	20.1	282	356
	30%	9.7	16.5	282	289

Table 1. Deconvolution performance comparison between BKS [13] method and ℓ^1-TV, computed via the IRN algorithm, on the "Einstein" and "Boat" test images.

4. CONCLUSIONS

The ℓ^1-TV method gives very good reconstruction quality when applied to deconvolution of images with salt and pepper noise. The IRN algorithm represents a computationally efficient approach to minimizing the ℓ^1-TV functional.

5. ACKNOWLEDGMENT

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