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Title: INPAINTING WITH SPARSE LINEAR COMBINATIONS OF EXEMPLARS

Author(s): BRENDT WOHLBERG

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Los Alamos NATIONAL LABORATORY EST. 1943

INPAINTING WITH SPARSE LINEAR COMBINATIONS OF EXEMPLARS

*Brendt Wohlberg**

T-7 Mathematical Modeling and Analysis
Los Alamos National Laboratory
Los Alamos, NM 87545, USA

ABSTRACT

We introduce a new exemplar-based inpainting algorithm based on representing the region to be inpainted as a sparse linear combination of blocks extracted from similar parts of the image being inpainted. This method is conceptually simple, being computed by functional minimization, and avoids the complexity of correctly ordering the filling in of missing regions of other exemplar-based methods. Initial performance comparisons on small inpainting regions indicate that this method provides similar or better performance than other recent methods.

Index Terms— inpainting

1. INTRODUCTION

Exemplar based methods are becoming increasingly popular for problems such as denoising [1], superresolution [2, 3, 4], texture synthesis [5], and inpainting [6, 7]. In the case of texture synthesis and inpainting [7], the approach is usually to replace missing regions with the best matching parts of the same image, carefully choosing the order in which the missing region is filled to avoid. Instead, we propose and inpainting method that represents missing regions as a linear combination of other regions in the same image (in contrast to [8], in which sparse representations on standard dictionaries, such as wavelets, are employed).

2. SPARSE LINEAR COMBINATIONS OF EXEMPLARS

The proposed approach depends on overlapped tilings of the region to be inpainted (together with a margin of known pixels) by image blocks. Each block is represented as a linear combination of other image blocks with no unknown pixels, and the inpainted image is obtained by minimizing a global functional which

1. penalizes the ℓ^1 norm of the linear combination coefficients to encourage a sparse, low complexity, solution,
2. penalizes (or constrains) the mismatch between solution blocks and known pixels, and
3. penalizes (or constrains) the mismatch between overlapping parts of solution blocks.

In order to discuss this approach in more detail, we need to establish some notation. Denote the image to be inpainted by vector $\mathbf{s}$, the inpainting region mask by $\mathbf{r}$, and the inpainted result by $\mathbf{u}$. We use overlapping blocks to reduce blocking artifacts, and, more importantly, so that the term penalizing (or constraining) the mismatch between overlapping parts of blocks allows information to propagate into the interior of the inpainting region, to blocks that have no overlap with the exterior of the region of unknown pixels. To reduce the complexity of computing the mismatch between overlapping regions of blocks, the blocks are arranged in indexed grids, with the overlap being produced by an offset of the entire grid, as indicated in Fig. 1. Each block is indexed by the number of its grid and its number within that grid.

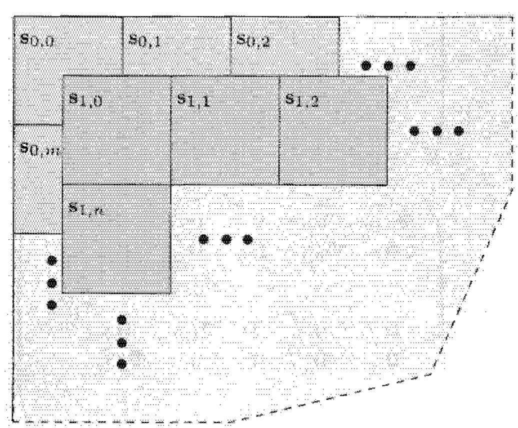

Fig. 1. Block grid

Denote the operator that extracts block k, l from an image as $B_{k,l}$ and the operator that "inserts" the same block back into the image as $B_{k,l}^T$, and make the definitions

$$R = \text{diag}(\mathbf{r}) \;\; \mathbf{r}_{k,l} = B_{k,l}\mathbf{r} \;\; R_{k,l} = \text{diag}(\mathbf{r}_{k,l}) \;\; \mathbf{s}_{k,l} = R_{k,l}B_{k,l}\mathbf{s} \; .$$

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We also define $\Phi_{k,l}$ as the dictionary for block k, l, but for the work reported here all blocks share a common dictionary Φ, so $\Phi_{k,l} = \Phi \quad \forall k, l$. Each block is expressed as a linear combination on its dictionary

$$\mathbf{u}_{k,l} = \Phi_{k,l} \boldsymbol{\alpha}_{k,l} \; .$$

Now, for each block k, l, we wish to minimize or constrain (either equal to zero, or some upper bound) the following terms:

- Solution sparsity
$$\|\boldsymbol{\alpha}_{k,l}\|_1$$
- Mismatch with known pixels in s
$$\|R_{k,l}\Phi_{k,l}\boldsymbol{\alpha}_{k,l} - R_{k,l}B_{k,l}\mathbf{s}\|_2$$
- Mismatch between overlapping blocks
$$\|\Phi_{k,l}\boldsymbol{\alpha}_{k,l} - B_{k,l}\sum_m B^T_{n,m}\Phi_{n,m}\boldsymbol{\alpha}_{n,m}\|_2 \quad n \neq k$$

We now combine these per-block objectives/constraints into combined terms

$$\boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\alpha}_{0,0} \\ \boldsymbol{\alpha}_{0,1} \\ \vdots \\ \boldsymbol{\alpha}_{1,0} \\ \boldsymbol{\alpha}_{1,1} \\ \vdots \end{pmatrix} \qquad \tilde{\mathbf{s}} = \begin{pmatrix} \mathbf{s}_{0,0} \\ \mathbf{s}_{0,1} \\ \vdots \\ \mathbf{s}_{1,0} \\ \mathbf{s}_{1,1} \\ \vdots \end{pmatrix}$$

$$R = \begin{pmatrix} R_{0,0} & 0 & 0 & 0 & 0 & \cdots \\ 0 & R_{0,1} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ 0 & 0 & 0 & R_{1,0} & 0 & \cdots \\ 0 & 0 & 0 & 0 & R_{1,1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \Phi_{0,0} & 0 & 0 & 0 & 0 & \cdots \\ 0 & \Phi_{0,1} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ 0 & 0 & 0 & \Phi_{1,0} & 0 & \cdots \\ 0 & 0 & 0 & 0 & \Phi_{1,1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$B = \begin{pmatrix} B_{0,0} & 0 & 0 & 0 & 0 & \cdots \\ 0 & B_{0,1} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ 0 & 0 & 0 & B_{1,0} & 0 & \cdots \\ 0 & 0 & 0 & 0 & B_{1,1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & \cdots & B^T_{1,0} & B^T_{1,1} & \cdots \\ 0 & 0 & \cdots & B^T_{1,0} & B^T_{1,1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B^T_{0,0} & B^T_{0,1} & \cdots & 0 & 0 & \cdots \\ B^T_{0,0} & B^T_{0,1} & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Our global objectives/constraints may then be written as

- Solution sparsity
$$\|\boldsymbol{\alpha}\|_1$$
- Mismatch with known pixels in s
$$\|R\Phi\boldsymbol{\alpha} - \tilde{\mathbf{s}}\|_2$$
- Mismatch between overlapping blocks
$$\|(I - BC)\Phi\boldsymbol{\alpha}\|_2$$

We would like to solve

$$\min \|\boldsymbol{\alpha}\|_1 \text{ such that } \|R\Phi\boldsymbol{\alpha} - \tilde{\mathbf{s}}\|_2 \leq \sigma_0 \text{ and } \|(I - BC)\Phi\boldsymbol{\alpha}\|_2 \leq \sigma_1 \; ,$$

but in the absence of a readily available solver for this problem, we instead define

$$A = \begin{pmatrix} \gamma_0 R\Phi \\ \gamma_1(I - BC)\Phi \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} \gamma_0\tilde{\mathbf{s}} \\ \mathbf{0} \end{pmatrix}$$

and solve the problem

$$\min \|\boldsymbol{\alpha}\|_1 \;\text{ such that }\; \|A\boldsymbol{\alpha} - \mathbf{b}\| \leq \sigma \; ,$$

using SPGL1 [9, 10].

3. RESULTS

We illustrate the performance of the proposed method using two small test regions extracted from the well-known Lena and Barbara test images. Results for the first, extracted from the hat in the Lena image, are displayed in Figure 2, and the results for the second, extracted from the textured pants in the Barbara image, are displayed in Figure 3, and a comparison for SNR values is displayed in Table 1. In each case, a subregion of the image was extracted for training (i.e. as a source for the image blocks populating the dictionary), and the smaller subregion actually displayed as tiled by the overlapping block grids and used for minimization of the functional. (For a fair comparison, the large regions were presented to the competing algorithms for which results are provided here.) For the results presented here, we set $\gamma_0 = 1.0$, $\gamma_1 = 0.01$, and $\sigma = 40.0$. The comparison results for the Field of Experts method [11] and the method of Criminisi *et. al.* [7] were computed using the publicly available software [12, 13].

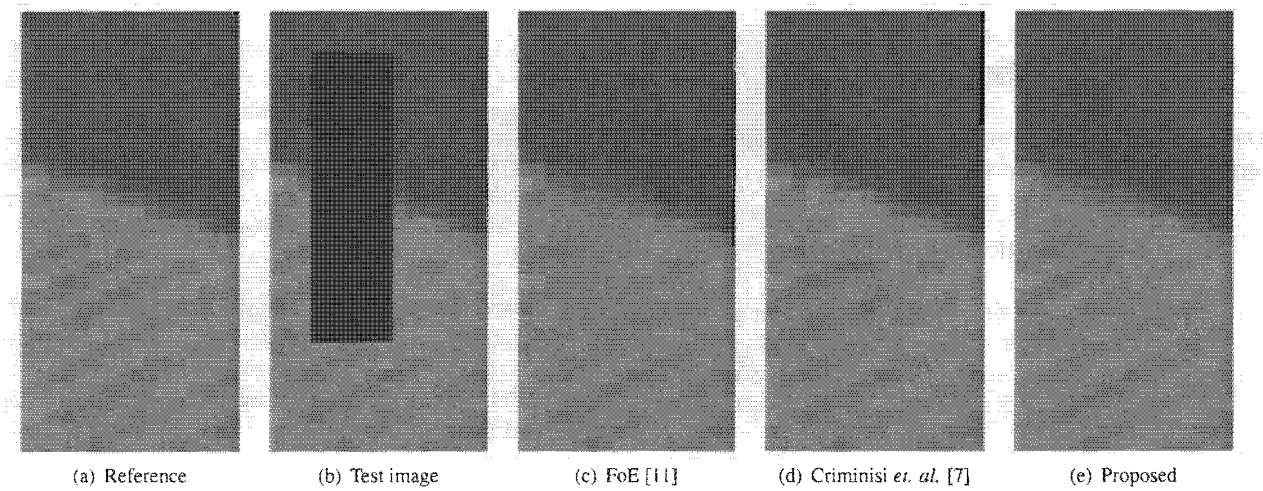

Fig. 2. Inpainting comparison on a subimage cropped from the Lena image. The region to be inpainted is 20×8 pixels in size.

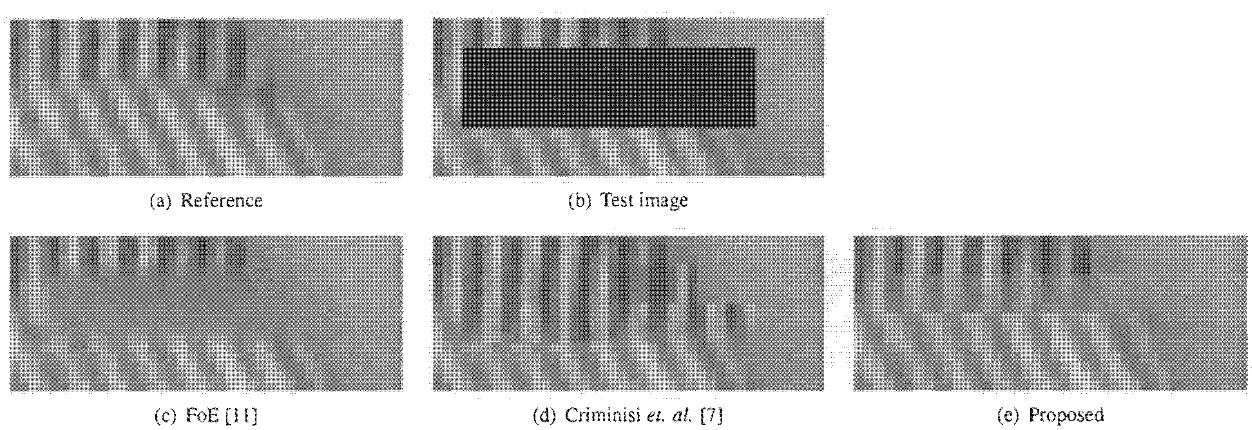

Fig. 3. Inpainting comparison on a subimage cropped from the Barbara image. The region to be inpainted is 8×30 pixels in size.

Image	FoE [11]	Criminisi *et. al.* [7]	Proposed
Lena	15.4dB	15.6dB	17.8dB
Barbara	6.1dB	2.2dB	8.5dB

Table 1. Comparison of inpainting SNR values.

4. CONCLUSIONS

The proposed method provides comparable or superior performance to the alternatives methods.

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