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Microstrip superconducting quantum interference device radio-frequency amplifier: Scattering parameters and input coupling

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Abstract

The scattering parameters of an amplifier based on a dc Superconducting QUantum Interference Device (SQUID) are directly measured at 4.2 K. The results can be described using an equivalent circuit model of the fundamental resonance of the microstrip resonator which forms the input of the amplifier. The circuit model is used to determine the series capacitance required for critical coupling of the microstrip to the input circuit.

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A growing number of applications including a search for dark-matter axions[1], the readout of flux qubits[2] and a postamplifier for the radio-frequency single electron transistor (RFSET[3]) require high-gain, low-noise amplifiers at frequencies of the order of 1 GHz. Although the high electron mobility transistor (HEMT)[1] has long been the amplifier of choice, the microstrip SQUID (Supercondcuting QUantum Interference Device) amplifier (MSA)[4], cooled to millikelvin temperatures, offers both a gain in excess of 25 dB and a noise temperature within a factor of two[5] of the standard quantum limit[6]-typically 50 times lower than that of a HEMT. The MSA consists of a superconducting square washer[7] interrupted by two resistively shunted tunnel junctions. A thin dielectric layer covers the washer and an n-turn square coil is deposited on top of it, forming a microstrip resonator with the washer. Near the fundamental resonance, there is substantial magnetic coupling between the magnetic field of the microstrip mode and the SQUID. With appropriate SQUID current and flux biases, the flux Φ coupled into the SQUID is converted to a voltage V with a flux-to-voltage transfer function $V_\Phi = \partial V/\partial\Phi$. To obtain the optimum performance from the MSA, it is essential to know the (generally complex) input impedance in order to optimize the input circuit. The magnitude of this impedance was measured previously.[8] In this Letter, we report measurements of the real and imaginary parts of this impedance at 4.2 K, including their dependence on the SQUID flux bias and on the capacitance of a varactor diode used to tune the frequency at which the maximum gain occurs,[9] and of the forward gain.

Our MSA consists of a square Nb washer with inner and outer dimensions of 0.2 mm and 1.0 mm, corresponding to an estimated inductance L of 450 pH. Each Nb-AlOx-Nb tunnel junction had a critical current I_0 of 2 μA, an estimated capacitance C of 0.2 pF and an external shunt resistance of 20 Ω. These parameters lead to $\beta_L \equiv 2LI_0/\Phi_0 = 0.9$ and $\beta_C \equiv 2\pi I_0 R^2 C/\Phi_0 \approx 0.2$; $\Phi_0 \equiv h/2e$ is the flux quantum. The 11-turn Nb coil had a length ℓ of 15 mm, a width w of 5 μm and was separated from the washer by a 400 nm-thick layer of SiO with a dielectric constant ϵ of about 5.5.

A 2-port microwave network is conveniently described with a scattering matrix that relates the voltage incident on one part with that reflected from a second port.12 The scattering parameter is defined as $S_{ij} = V_i^-/V_j^+$, where S_{11} is the input reflection coefficient with the output port terminated and S_{21} is the forward transmission (gain). We measured S_{11} using an Agilent 4396B vector network analyzer (VNA). Figure 1 shows the configuration for a reflection measurement of S_{11}; the use of a cold directional coupler prevents noise from the VNA from saturating the MSA. We calibrated the VNA and the experimental probe for the reflection measurements by replacing the input of the MSA in turn with an open-circuit, a short-circuit and a 50-Ω resistor. Figure 2 shows S_{11}, converted to input impedance, versus measurement frequency. For a low-loss transmission line, these resonance curves can be described[11] by the input impedance

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma \ell)}{Z_0 + Z_L \tanh(\gamma \ell)} \tag{1}$$

where Z_0 is the characteristic impedance, Z_L is the terminating impedance and γ is the complex propagation constant. With $Z_L = \infty$, the impedance can be equivalently described the equivalent parallel $R_e L_e C_e$ (resistance-inductance-capacitance) circuit shown in Figure 1(a). The series capacitance represents the dc capacitance of the line. As discussed earlier[8] for the MSA, this model gives a direct measure of the parameters of the circuit. We performed a series of measurements in which we varied the flux applied to the SQUID, at each value adjusting the current bias to produce the maximum gain. Figure 1(b) shows the variation of the resonant frequency and characteristic impedance with bias flux. From the microstrip geometry, we estimate Z_0 to be 16 Ω and the velocity of light to be 0.33 c, where c is the velocity of light in vacuum.[10] The measured Z_0 of 14 ± 2 is consistent with the estimate, but the frequency is a factor of 6-7 below the prediction of 3 GHz. This is due to the inductive loading of the microstrip coil by the transformed SQUID inductance. Since the effect on f_r is much greater than the effect on Z_0, we conclude that this loading is in the form of a lumped inductance as opposed to distributed. Figures 1(b) and (c) show the resulting variation of R_e, L_e and C_e. We observe that R_e is approximately at 350 ± 20 Ω as the flux is varied, except near $3\Phi_0/4$. On the other hand, L_e varies from about 2.5 nH near $\Phi = 0$, to about 3.5 nH at $\Phi = \Phi_0/2$, while the C_e is roughly 30 pF, except near $\Phi = \Phi_0/4$ where it rises to 40 pF .

In most applications, it is desirable to tune the frequency of maximum gain of the MSA. We accomplish this by means of a varactor diode connected in series with a capacitor across the free end of the microstrip;[9] by varying the reverse voltage bias of the diode, we can control its capacitance and hence the effective length of the microstrip. The inset to Fig. 4 shows the $\lambda/2$ resonant frequency versus measured varactor diode capacitance, together with the prediction from the transmission line model (λ is the wavelength on the microstrip). Figure 4 shows the real and imaginary parts of the input impedance for a varactor capacitance of 5.4 pF. The solid lines are predictions using parameters from the transmission line model measured with the microstrip open circuited, with the addition of a fitting parameter that corresponds to a resistance that effectively appears in series with the varactor diode capacitance. This resistance varies with frequency from 12 to 40 Ω. In the fit, we used an average value of 16 Ω.

The forward scattering parameter S_{21} is essentially the gain. We were able to make this measurement with the same setup used for S_{11}. As shown in Figure 1, to measure S_{21}, the output of the SQUID was connected to the room temperature postamplifier and measured with the VNA. We calibrated the measurement circuitry by placing a shorting wire across the MSA. The real and imaginary parts of S_{21} for our 11-turn MSA are shown in Figs. 5(b and c). From the circuit model in Fig. 5(a), S_{21} is predicted to be $MV_\Phi i_L/V_i$ where M is the mutual inductance between the coil and the SQUID, i_L is the rf current in the microstrip and V_i is the input voltage. This circuit model is the same as before with the addition of a voltage source with an impedance of 50 Ω. The solid lines are predictions from the expression for S_{21} using the values $C_1 = 3.2$ pF, $L_e = 2.4$ nH, $C_e = 40.9$ pF, and $R_e = 400$ Ω from the measurement of S_{11}, and fitting an overall scale factor corresponding to $MV_\Phi =$ 23.0. Using the measured value $V_\Phi = 40\mu V/\Phi_0$, gives a value of $M = 1.9$ nH, roughly half the measured dc value of 3.5 nH.

The central goal of these measurements is to determine the coupling of the input circuit to the MSA required to optimize the gain and noise temperature. The S_{11} results indicate that the intrinsic quality factor Q of the MSA at 4.2 K is typically 40 to 80. The Q values found from the S_{21} measurements with the input loaded with a 50-Ω source impedance are typically 5 to 20, implying that the source impedance significantly damps the resonator. Decreasing the coupling between the source and the MSA would increase Q, but at the same time reduce the signal coupled to the resonator. This is a familiar problem in coupling a resonator to a real source impedance, and is solved by implementing critical coupling, that is, matching the real impedance at resonance to the source impedance. Critical coupling for a microstrip resonator is commonly achieved by introducing a small gap between the source and the microstrip. A gap is well represented by a π-network of capacitors. In most cases, however, the two shunting capacitances are so small compared with the series capacitance that they can be neglected, and the gap replaced with a single-series capacitor. The circuit model resulting from the S_{11} measurement can be used to estimate the required series coupling capacitance[11] $C_c = (\pi/2\omega_o^3 R_e^3 C_e)^{1/2}$, where $\omega_o/2\pi$ is the resonant frequency. For the circuit parameters of a 9-turn MSA, $\omega_o/2\pi = 0.816$ GHz, $R_e = 714\ \Omega$ and $C_e = 4.4$ pF, we find $C_c \approx 1.4$ pF. The measured gain with three values of C_c are shown in Fig. 6. The maximum gain occurs for $C_c = 2.2$ pF. The resonator is clearly overcoupled when C_c = 10 pF and undercoupled with $C_c = 0.5$ pF. These results are in good agreement with the predictions from the equivalent circuit model.

These measurements demonstrate that many of the properties of the MSA can be represented by a low-loss transmission line leading to an equivalent circuit model. We have shown that by measuring the input impedance with the coil open-circuited, one can predict the maximum gain and frequency response even with a varactor diode loading the input microstrip. One can also design the input circuit to give maximum gain by critically coupling the source to the microstrip resonator. The parameters of the model are of course, strongly dependent on the properties of the SQUID (I_o, β_C, β_L, R_n) which will change with temperature. We have already seen that the Q goes up significantly as the MSA is cooled to mK temperatures. This suggests that the measurements should ultimately be performed at the desired operating temperature. In the future, we hope to be able to relate the equivalent circuit parameters to the SQUID parameters and microstrip geometry.

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FIG. 1: Schematic of the experimental setup to measure both S_{11} and S_{21} of the MSA at 4.2 K. The signal generator is part of the Agilent 4396B VNA. The cold 3dB attenuator ensures that the output of the MSA is matched to 50 Ω during the S_{11} measurement.

FIG. 2: Fit (solid lines) of a low-loss transmission line model to the measured S11 data converted to impedance (dots).

FIG. 3: (a) Equivalent circuit model of the low-loss transmission line. C_1 represents the dc capacitance, while the RLC circuit represents the lowest resonance. Variation of the equivalent (b) capacitance, (c) inductance, and (d) resistance with flux bias.

FIG. 4: Measured (dots) and prediction (line) of the real and imaginary input impedance with the microstrip terminated with a 5.4 pF in series with 16 Ω. The inset shows the measured resonant frequency (dots) versus capacitance along with the transmission line model prediction (line).

FIG. 5: (a) Equivalent circuit model for S_{21}. The model is the same as for S_{11} with the addition of a voltage source with impedance R_s (normally 50 Ω) and a coupling capacitor C_c. (b) Prediction of the circuit model (solid line) versus frequency along with the measured values (dots) of S_{21}. (c) Prediction (line) and measured for the imaginary part of S_{21}.

FIG. 6: Measured gain versus frequency for a 9-turn MSA with three different input coupling capacitors. With the 0.5 pF, 2.2 pF and 10 pF coupling capacitors, the microstrip resonator is respectively undercoupled, critically coupled, and overcoupled.