

Quark-Hadron Duality and $\gamma^* p \rightarrow \Delta$ Form Factors

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Abstract

We use local quark-hadron duality to estimate the purely nonperturbative soft contribution to the $\gamma^* p \rightarrow \Delta$ form factors. Our results are in agreement with existing experimental data. We predict that the ratio $G_E^*(Q^2)/G_M^*(Q^2)$ is small for all accessible Q^2 , in contrast to the pQCD expectations that $G_E^*(Q^2) \rightarrow -G_M^*(Q^2)$.

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1 Introduction

Basically, there are two competing explanations of the experimentally observed power-law behaviour of elastic hadronic form factors: hard scattering [1] and the Feynman mechanism [2]. At sufficiently large momentum transfer, the Feynman mechanism contribution is dominated by configurations in which one of the quarks carries almost all the momentum of the hadron. In QCD, this results in an extra $1/Q^2$ suppression compared to the hard scattering term generated by the valence configurations with small transverse sizes and finite light-cone fractions of the total hadron momentum carried by each valence quark. The hard term, which eventually dominates, can be written in a factorized form [3],[4],[5], as a product of the perturbatively calculable hard scattering amplitude and two distribution amplitudes accumulating the necessary nonperturbative information. However, this mechanism involves exchange of virtual gluons, each exchange bringing in a suppression factor $(\alpha_s/\pi) \sim 0.1$. Hence, to describe existing data by the hard contribution alone, one should intentionally increase the magnitude of the hard scattering term by using distribution amplitudes with a peculiar “humped” profile [6]. In this case, passive quarks carry a small fraction of the hadron momentum and, as pointed out in ref.[7], the “hard” scattering subprocess, even at rather large momentum transfers $Q^2 \sim 10 \text{ GeV}^2$, is dominated, in fact, by very small gluon virtualities. This means that the hard scattering scenario heavily relies on the assumption that the asymptotic pQCD expressions are accurate even for momenta smaller than 300 MeV , *i.e.*, in the region strongly affected by finite-size effects, nonperturbative QCD vacuum fluctuations *etc.* Including these effects shifts the hard contributions significantly below the data level even if one uses the humpy distribution amplitudes and other *ad hoc* modifications intended to increase the hard term (see, *e.g.*, [8]).

Furthermore, as argued in ref.[13], the derivation of the humpy distribution amplitudes in [6] implies a rather singular picture (infinite correlation length) of the QCD vacuum fluctuations. Under realistic assumptions, it is impossible to get distribution amplitudes strongly differing from the smooth “asymptotic” forms. The latter are known to produce hard contributions which are too small to describe the data on elastic form factors. Thus, there is an increasing evidence in favour of the alternative scenario, *viz.*, that for experimentally accessible momentum transfers the form factors are still dominated by the purely soft contribution corresponding to the Feynman mechanism.

In the language of the light-cone formalism [5], the soft term is given by the overlap of the soft parts of the hadronic wave functions, *i.e.*, is an essentially nonperturbative object. Among the existing approaches to the nonperturbative effects in QCD, that which is closest to pQCD is the QCD sum rule method [9]. QCD sum rules were originally used to calculate the soft contribution for the pion form factor in the region of moderately large [10],[11] and then small momentum transfers [12]. It should be emphasized that, in the whole region $0 \leq Q^2 \lesssim 3 \text{ GeV}^2$, the results are very close to the experimental data: the Feynman mechanism alone is sufficient to explain the observed behaviour of the pion form factor. For higher Q^2 , the direct QCD sum rule method fails due to increasing contributions from higher condensates. However, a model summation

of the higher terms into nonlocal condensates [13] indicates that the soft term dominates up to $Q^2 \sim 10 \text{ GeV}^2$ [14]. This conclusion is also supported by a recent calculation within the framework of the light-cone sum rules [15].

An important observation made in ref.[11] is that the results of the elaborate QCD sum rule analysis are rather accurately reproduced by a simple local quark-hadron duality prescription. The latter states that one can get an estimate for a hadronic form factor by considering transitions between the free-quark states produced by a local current having the proper quantum numbers, with subsequent averaging of the invariant mass of the quark states over the appropriate duality interval s_0 . The duality interval has a specific value for each hadron, *e.g.*, $s_0^\pi \approx 0.7 \text{ GeV}^2$ for the pion and $s_0^N \approx 2.3 \text{ GeV}^2$ for the nucleon.

The local duality ansatz, equivalent to fixing the form of a soft wave function, was used to estimate the soft contribution in the case of the proton magnetic form factor [16]. The results agree with available data [17], [18] over a wide region, $3 \text{ GeV}^2 \lesssim Q^2 \lesssim 20 \text{ GeV}^2$. Furthermore, the calculation of ref.[16] correctly reproduces (without any adjustable parameter), the observed magnitude of the helicity-nonconservation effects $F_2^p(Q^2)/F_1^p(Q^2) \sim \mu^2/Q^2$ with $\mu^2 \sim 1 \text{ GeV}^2$ [18]. It is difficult to understand the origin of such a large scale within the hard scattering scenario, since possible sources of helicity nonconservation in pQCD include only small scales like quark masses, intrinsic transverse momenta *etc.*, and one would rather expect that $\mu^2 \sim 0.1 \text{ GeV}^2$. Thus, the study of spin-related properties provides a promising way for an unambiguous discrimination between soft and hard scenarios.

Of a particular interest there is the $\gamma^* p \rightarrow \Delta$ transition. A special attention to this process was raised by the results [19] of the analysis of inclusive SLAC data which indicated that the relevant form factor drops faster than predicted by the quark counting rules. The relevant hard scattering contribution was originally considered in ref.[20], where it was observed that the hard scattering amplitude in this case has an extra suppression due to cancellation between symmetric and antisymmetric parts of the nucleon distribution amplitude, and it was conjectured that the faster fall-off found in [19] can be explained by the dominance of some non-asymptotic contribution. Later, it was claimed [21] that, by appropriately choosing the distribution amplitudes, one can get a leading-twist hard term comparable in magnitude with the data. Furthermore, the results of a recent reanalysis [22] of the inclusive SLAC data are rather consistent with the $1/Q^4$ behaviour, and this revived the hope that the $\gamma^* p \rightarrow \Delta$ form factor can be still described by pQCD.

However, the important result of the pQCD calculation [20] is that the lowest-twist hard contribution has the property $G_E^{*hard}(Q^2) \approx -G_M^{*hard}(Q^2)$. Experimentally, the ratio $G_E^*(Q^2)/G_M^*(Q^2)$ is rather small [23, 24], which indicates that the leading-twist pQCD term is irrelevant in the region $Q^2 \lesssim 3 \text{ GeV}^2$. In the present paper, we use the local quark-hadron duality to estimate the soft contribution for the $G_E^*(Q^2)$ and $G_M^*(Q^2)$ form factors of the $\gamma^* p \rightarrow \Delta$ transition to study whether the soft contribution is large enough to describe the data and whether the relative smallness of the electric form factor persists in the region of moderately large momentum transfers $3 \lesssim Q^2 \lesssim 15 \text{ GeV}^2$.

2 Three-point function and form factors

The starting object for a QCD sum rule analysis of the $\gamma^* p \rightarrow \Delta$ transition is the 3-point correlator:

$$T_{\mu\nu}(p, q) = \int \langle 0 | T\{\eta_\mu(x) J_\nu(y) \bar{\eta}(0)\} | 0 \rangle e^{ipx - iqy} d^4x d^4y \quad (1)$$

of the electromagnetic current

$$J_\nu = e_u \bar{u} \gamma_\nu u + e_d \bar{d} \gamma_\nu d \quad (2)$$

and two Ioffe currents [25]

$$\eta = \varepsilon^{abc} (u^a \mathcal{C} \gamma_\rho u^b) \gamma_\rho \gamma_5 d^c, \quad \eta_\mu = \varepsilon^{abc} (2 (u^a \mathcal{C} \gamma_\mu d^b) u^c + (u^a \mathcal{C} \gamma_\mu u^b) d^c) . \quad (3)$$

We use the following parameterization for the projections of η and η_μ onto the nucleon and Δ -isobar states, respectively:

$$\langle 0 | \eta | N \rangle = \frac{l_N}{(2\pi)^2} v, \quad \langle 0 | \eta_\mu | \Delta \rangle = \frac{l_\Delta}{(2\pi)^2} \psi_\mu . \quad (4)$$

Here, v is the Dirac spinor of the nucleon while ψ_μ is the spin-3/2 Rarita-Schwinger wave function for the Δ -isobar, *i.e.*, $(\hat{p} - \hat{q} - m)v = 0$, $(\hat{p} - M)\psi_\mu = 0$, $p_\mu \psi_\mu = 0$, $\gamma_\mu \psi_\mu = 0$; with m being the nucleon mass and M that of Δ . We use the notation $\hat{a} \equiv a_\alpha \gamma_\alpha$.

On the hadronic level, the $\gamma^* p \rightarrow \Delta$ transition makes the following contribution to the correlator (1):

$$T_{\mu\nu}^{\gamma^* p \rightarrow \Delta} = \frac{l_N l_\Delta}{(2\pi)^4} \frac{X_{\mu\alpha}(p)}{p^2 - M^2} \Gamma_{\alpha\nu}(p, q) \gamma_5 \frac{\hat{p} - \hat{q} + m}{(p - q)^2 - m^2} , \quad (5)$$

where $\Gamma_{\alpha\nu}(p, q)$ is the $\gamma^* p \rightarrow \Delta$ vertex function

$$\begin{aligned} \Gamma_{\alpha\nu}(p, q) = & G_1(q^2) (q_\alpha \gamma_\nu - g_{\alpha\nu} \hat{q}) + G_2(q^2) (q_\alpha P_\nu - g_{\alpha\nu}(qP)) \\ & + G_3(q^2) (q_\alpha q_\nu - g_{\alpha\nu} q^2) \end{aligned} \quad (6)$$

($P \equiv p - q/2$) and $X_{\mu\alpha}(p)$ the projector onto the isobar state

$$X_{\mu\alpha}(p) = \left(g_{\mu\alpha} - \frac{1}{3} \gamma_\mu \gamma_\alpha + \frac{1}{3M} (p_\mu \gamma_\alpha - p_\alpha \gamma_\mu) - \frac{2}{3M^2} p_\mu p_\alpha \right) (\hat{p} + M) . \quad (7)$$

The form factors G_1, G_2, G_3 are related to a more convenient set G_E^*, G_M^*, G_C^* by

$$\begin{aligned} G_M^*(Q^2) = & \frac{m}{3(M+m)} \left(((3M+m)(M+m) + Q^2) \frac{G_1(Q^2)}{M} \right. \\ & \left. + (M^2 - m^2) G_2(Q^2) - 2Q^2 G_3(Q^2) \right) , \end{aligned} \quad (8)$$

$$G_E^*(Q^2) = \frac{m}{3(M+m)} \left((M^2 - m^2 - Q^2) \frac{G_1(Q^2)}{M} + (M^2 - m^2) G_2(Q^2) - 2Q^2 G_3(Q^2) \right) , \quad (9)$$

$$G_C^*(Q^2) = \frac{2m}{3(M+m)} \left(2MG_1(Q^2) + \frac{1}{2}(3M^2 + m^2 + Q^2)G_2(Q^2) + (M^2 - m^2 - Q^2)G_3(Q^2) \right) , \quad (10)$$

(see, *e.g.*, [26]). One should not confuse the magnetic form factor $G_M^*(Q^2)$ given by eq.(8) with the effective form factor mentioned in refs.[20],[27]. In particular, the form factor $G_T(Q^2)$ defined by eq.(6.2) of ref.[27] can be written in terms of $G_M^*(Q^2)$ and $G_E^*(Q^2)$ (defined by eqs.(8),(9) above) as

$$|G_M^*|^2 + 3|G_E^*|^2 = \frac{Q^2}{Q^2 + \nu^2} \left(1 + \frac{Q^2}{(M+m)^2} \right) |G_T|^2 , \quad (11)$$

where $\nu = (M^2 - m^2 + Q^2)/2m$ is the energy of the virtual photon in the proton rest frame. Note that, for large Q^2 , our $G_M^*(Q^2)$ and $G_T(Q^2)$ of eq.(6.2) of ref.[27] have the same power behaviour.

3 Local quark-hadron duality

Multiplying all the factors in eq.(5) explicitly, one ends up with a rather long sum of different structures $a_{\mu\nu}^i$ accompanied by the relevant invariant amplitudes T_i , each of which is a combination of the three independent transition form factors listed above. To incorporate the local quark-hadron duality, we write down the dispersion relation for each of the invariant amplitudes:

$$T_i(p_1^2, p_2^2, Q^2) = \frac{1}{\pi^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho_i(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} + \text{"subtractions"} , \quad (12)$$

where $p_1^2 = (p - q)^2$, $p_2^2 = p^2$. The perturbative contributions to the amplitudes $T_i(p_1^2, p_2^2, Q^2)$ can also be written in the form of eq.(12). Evidently, the physical perturbative spectral densities $\rho_i(s_1, s_2, Q^2)$ differ from their perturbative analogues, the difference being most pronounced in the resonance region, *i.e.*, for small s_1 and s_2 values. In particular, $\rho_i(s_1, s_2, Q^2)$ contains the double δ -function term corresponding to the $\gamma^* p \rightarrow \Delta$ transition:

$$\rho_i(s_1, s_2, Q^2) \sim l_N l_\Delta F_i(s_1, s_2, Q^2) \delta(s_1 - m^2) \delta(s_2 - M^2) , \quad (13)$$

while the perturbative spectral densities $\rho_i^{pert}(s_1, s_2, Q^2)$ are smooth functions of s_1 and s_2 . The local quark-hadron duality assumes, however, that the two spectral densities are in fact dual to each other:

$$\int_0^{s_0} ds_1 \int_0^{S_0} \rho_i^{pert.}(s_1, s_2, Q^2) ds_2 = \int_0^{s_0} ds_1 \int_0^{S_0} \rho_i(s_1, s_2, Q^2) ds_2 , \quad (14)$$

i.e., they give the same result when integrated over the appropriate duality intervals s_0, S_0 . The latter characterize the effective thresholds for higher states with the nucleon or, respectively, Δ -isobar quantum numbers. As noted in ref.[16], the local duality prescription can be treated as a model for the soft wave functions:

$$\Psi_N(\{x\}, \{k_\perp\}) \sim \theta \left(\sum_{i=1}^3 \frac{k_{\perp i}^2}{x_i} \leq s_0 \right); \quad \Psi_\Delta(\{x\}, \{k_\perp\}) \sim \theta \left(\sum_{i=1}^3 \frac{k_{\perp i}^2}{x_i} \leq S_0 \right),$$

i.e., s_0 and S_0 also set the scale for the width of the transverse momentum distribution. Using this model, we can obtain a good estimate for the overlap of the soft wave functions only in the intermediate Q^2 -region where the soft contribution is sensitive mainly to the k_\perp -widths of the quark distributions rather than to their detailed forms. From experience with the proton form factor calculations, we expect that local duality will work in the region between 3 GeV^2 and 20 GeV^2 . The low- Q^2 region $Q^2 \lesssim 1 \text{ GeV}^2$, in which there appear large nonperturbative contributions due to the long-distance propagation in the Q^2 -channel, can be analyzed within a full-framed QCD sum rule approach supplemented by the formalism of induced condensates [28] or bilocal operators [29].

Applying the local quark-hadron duality to the two-point correlators of η - or, respectively, η_μ -currents, we obtain simple relations between the duality intervals s_0, S_0 and the residues l_N, l_Δ of the Ioffe currents:

$$l_N^2 = \frac{s_0^3}{12} \quad ; \quad l_\Delta^2 = \frac{S_0^3}{10}. \quad (15)$$

After the duality intervals are fixed (e.g., from the QCD sum rule analysis of the relevant two-point function [30]), the local duality estimates for the form factors do not have any free parameters.

4 Invariant amplitudes

Now, choosing a particular Lorentz structure $a_{\mu\nu}^i$, one can get the local duality estimate for the relevant combination of the form factors. One should remember, however, that not all the invariant amplitudes are equally reliable. To compare the contributions related to different structures, one should specify a reference frame. In our case, the most relevant is the infinite momentum frame where $p^\mu \equiv p_\parallel^\mu \rightarrow \infty$ while $q^\mu \equiv q_\perp^\mu$ is fixed. So, *a priori*, the structures with the maximal number of the “large” factors p^μ are more reliable than those in which p^μ is substituted by the “small” parameter q^μ or by $g_{\mu\nu}$. However, dealing with the η_μ -current in the Δ -channel, one should realize that η_μ has also a nonzero projection onto the spin-1/2 isospin-3/2 states $|\Delta^*(p)\rangle$:

$$\langle 0 | \eta_\mu | \Delta^*(p) \rangle = \lambda^* (m\gamma_\mu - 4p_\mu) v^*(p), \quad (16)$$

where λ^* is a constant, m^* is the mass of the spin-1/2 state $|\Delta^*(p)\rangle$ and $v^*(p)$ the relevant Dirac spinor satisfying $(\hat{p} - m^*)v^*(p) = 0$.

From eq.(16), it follows that any amplitude containing the p^μ -factor, is “contaminated” by the transitions into the spin-1/2 states. These states lie higher than the Δ -isobar and, in principle, one

can treat them as a part of the continuum. Then, however, there will be strong reasons to expect that the duality interval S_0 for the “contaminated” invariant amplitudes deviates from that for the amplitudes containing only the spin-3/2 contributions in the η_μ -channel. Another possibility is to project out the amplitudes which have contributions due to the transitions into spin-1/2 isospin-3/2 states. To achieve this, it is convenient to use the basis in which γ_μ is always placed at the leftmost position. Then, according to eqs.(5-7), the invariant amplitudes corresponding to the structures with q_μ and $g_{\mu\nu}$ are free from the contributions due to the spin-1/2 isospin-3/2 states. In this basis, keeping only the terms with q_μ and $g_{\mu\nu}$ in eq.(5), we get

$$\begin{aligned} T_{\mu\nu}^{\gamma^* p \rightarrow \Delta}(p, q) &= \frac{l_N l_\Delta}{(2\pi)^4 (p^2 - M^2)((p-q)^2 - m^2)} \\ &\times \left(g_{\mu\nu}[\hat{p}, \hat{q}] \frac{3(M+m)}{8m} (G_M^*(Q^2) + G_E^*(Q^2)) \right. \\ &+ \frac{q_\mu}{2} (m[\gamma_\nu, \hat{p}] + M[\gamma_\nu, (\hat{p} - \hat{q})]) G_1(Q^2) \\ &- p_\nu[\hat{p}, \hat{q}] G_2(Q^2) - q_\nu[\hat{p}, \hat{q}] \left(G_3(Q^2) - \frac{1}{2} G_2(Q^2) \right) + \dots \left. \right) . \end{aligned} \quad (17)$$

Hence, from the invariant amplitudes related to the structures proportional to q_μ , we can get the local duality estimates for the form factors G_1, G_2, G_3 . Similarly, extracting the structure $g_{\mu\nu}[\hat{p}, \hat{q}]$, we get an expression for $(G_M^* + G_E^*)$. Counting the powers of q , we expect that results for G_2 are less reliable than those for G_1 and $G_M^* + G_E^*$, while results for G_3 are less reliable than those for G_2 .

The number of independent amplitudes can be diminished by taking some explicit projection of the original amplitude $T_{\mu\nu}(p, q)$. In particular, if one multiplies $T_{\mu\nu}$ by p_ν , the invariant amplitude corresponding to the structure $q_\mu[\hat{q}, \hat{p}]$ is proportional to the quadrupole form factor $G_C^*(Q^2)$:

$$p_\nu T_{\mu\nu}^{\gamma^* p \rightarrow \Delta}(p, q) = \frac{l_N l_\Delta}{(2\pi)^4 (p_1^2 - m^2)(p_2^2 - M^2)} \left\{ \frac{3}{8} \frac{M+m}{m} q_\mu[\hat{q}, \hat{p}] G_C^*(Q^2) + \dots \right\} . \quad (18)$$

Another possibility is to take the trace of $T_{\mu\nu}^{\gamma^* p \rightarrow \Delta}$. The result is proportional to the magnetic form factor $G_M^*(Q^2)$:

$$\text{Tr}(T_{\mu\nu}) = \frac{l_N l_\Delta}{(2\pi)^4 (p_1^2 - m^2)(p_2^2 - M^2)} \frac{M+m}{2m} (4i\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta) G_M^*(Q^2) . \quad (19)$$

However, one should remember that the trace of $T_{\mu\nu}$ is not free from contributions due to spin-1/2 isospin-3/2 states.

5 Estimates for the $\gamma^* p \rightarrow \Delta$ form factors

Though the invariant amplitude related to the trace of $T_{\mu\nu}$ is contaminated by the transitions into spin-1/2 isospin-3/2 states, it makes sense to consider this amplitude because it has the simplest

perturbative spectral density:

$$\frac{1}{\pi^2} \rho_M^{pert.}(s_1, s_2, Q^2) = \frac{Q^2}{8\kappa^3} (\kappa - (s_1 + s_2 + Q^2))^2 (2\kappa + s_1 + s_2 + Q^2) , \quad (20)$$

where

$$\kappa = \sqrt{(s_1 + s_2 + Q^2)^2 - 4s_1 s_2} . \quad (21)$$

Imposing the local duality prescription, we get

$$\begin{aligned} G_M^*(Q^2) &= \frac{2m}{l_N l_\Delta (M + m)} \int_0^{s_0} ds_1 \int_0^{S_0} \frac{\rho_M^{pert.}(s_1, s_2, Q^2)}{\pi^2} ds_2 \\ &= \frac{6m}{(M + m)} F(s_0, S_0, Q^2) , \end{aligned} \quad (22)$$

where $F(s_0, S_0, Q^2)$ is a universal function

$$F(s_0, S_0, Q^2) = \frac{s_0^3 S_0^3}{9l_N l_\Delta (Q^2 + s_0 + S_0)^3 (1 - 3\sigma + (1 - \sigma)\sqrt{1 - 4\sigma})} \quad (23)$$

and $\sigma = s_0 S_0 / (Q^2 + s_0 + S_0)^2$. As we will see, the results for other invariant amplitudes can be conveniently expressed through $F(s_0, S_0, Q^2)$.

The function $F(s_0, S_0, Q^2)$ depends on the duality intervals s_0 and S_0 . We fix the nucleon duality interval s_0 at the standard value $s_0 = 2.3 \text{ GeV}^2$ extracted from the analysis of the two-point function and used earlier in the nucleon form factor calculations. The results of the existing two-point function analysis for the Δ -isobar [30] are compatible with the Δ duality interval S_0 in the range 3.2 to 4.0 GeV^2 . To fine-tune the S_0 value, we consider two independent sum rules for the G_1 form factor

$$\begin{aligned} mG_1(Q^2) &= 2 \left(3 + Q^2 \frac{d}{dQ^2} \right) F(s_0, S_0, Q^2) \\ &\quad - 2Q^2 \left(\frac{d}{dQ^2} \right)^2 \int_0^{S_0} F(s_0, s_2, Q^2) ds_2 \end{aligned} \quad (24)$$

and

$$MG_1(Q^2) = \frac{3}{2} Q^2 \left(\frac{d}{dQ^2} \right)^2 \int_0^{S_0} F(s_0, s_2, Q^2) ds_2 \quad (25)$$

extracted from the invariant amplitudes corresponding to the structures $q_\mu[\gamma_\nu, \hat{p}]$ and $q_\mu[\gamma_\nu, (\hat{p} - \hat{q})]$, respectively (recall that $p - q$ is the proton's momentum and p is that of Δ). Taking the ratio of these two relations, one can investigate their mutual consistency and test the overall reliability of the quark-hadron duality estimates.

Indeed, on the “hadronic” side, we have the ratio M/m of the isobar and nucleon masses, while on the “quark” side we have the ratio of two explicit and non-trivially related functions.

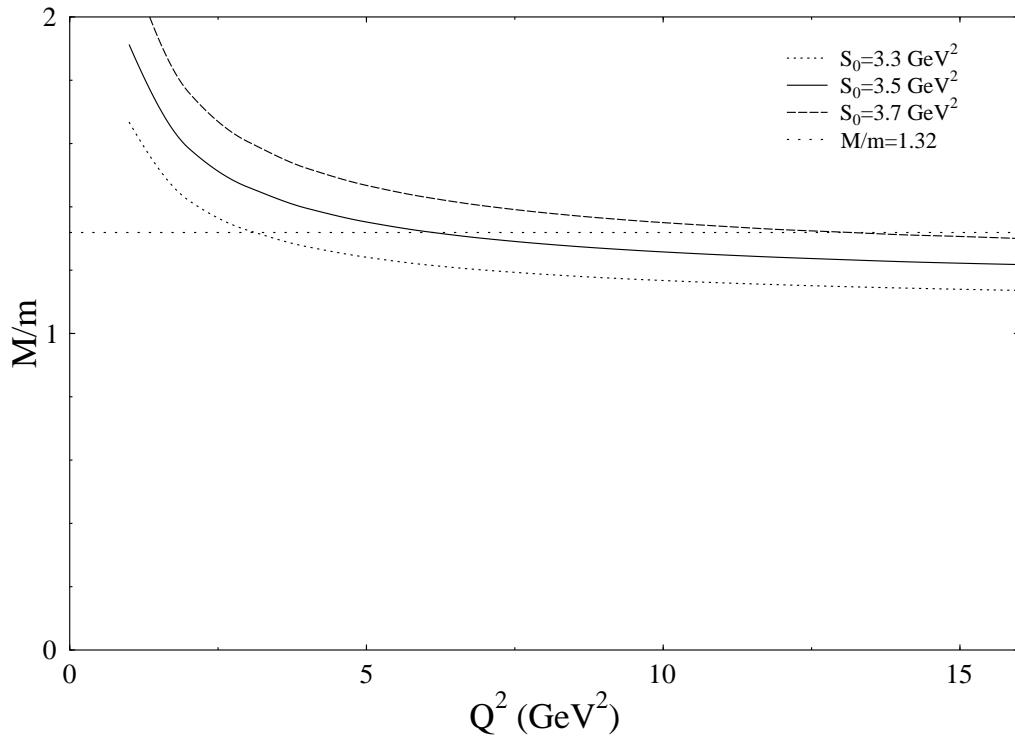


Figure 1: Isobar to proton mass ratio from the sum rules (24),(25).

The consistency requires, first, that the ratio of these functions must be close to a constant and, second, that this constant must be close to the experimental value for the ratio of the isobar and nucleon masses: $(M/m)^{exp} \approx 1.32$. On Fig.1, we plot the Q^2 -dependence for the ratio of the right hand sides of eqs.(24) and (25) for the standard value $s_0 = 2.3 \text{ GeV}^2$ of the nucleon duality interval and three different values of S_0 . One can see that one should not rely on local duality estimates below $Q^2 \sim 3 \text{ GeV}^2$. However, above $Q^2 \sim 3 \text{ GeV}^2$, the ratio is pretty constant for all three values of S_0 , and rather close to 1.3. The best agreement is reached for $S_0 = 3.5 \text{ GeV}^2$, and we will use this value as the basic isobar duality interval in further calculations. In particular, the l_Δ parameter will be fixed by $l_\Delta^2 = \frac{1}{10}(3.5 \text{ GeV}^2)^3$ (cf. eq.(15)).

From eqs.(8) and (9), it follows that G_1 is proportional to the difference of the magnetic G_M^* and electric G_E^* transition form factors:

$$G^{(-)}(Q^2) \equiv G_M^*(Q^2) - G_E^*(Q^2) = \frac{2m}{3M(M+m)} \left((M+m)^2 + Q^2 \right) G_1(Q^2). \quad (26)$$

According to eq.(17), the sum $G^{(+)}(Q^2) \equiv G_M^*(Q^2) + G_E^*(Q^2)$ of these form factors can be obtained from the invariant amplitude corresponding to the structure $g_{\mu\nu}[\hat{p}, \hat{q}]$. Applying the local

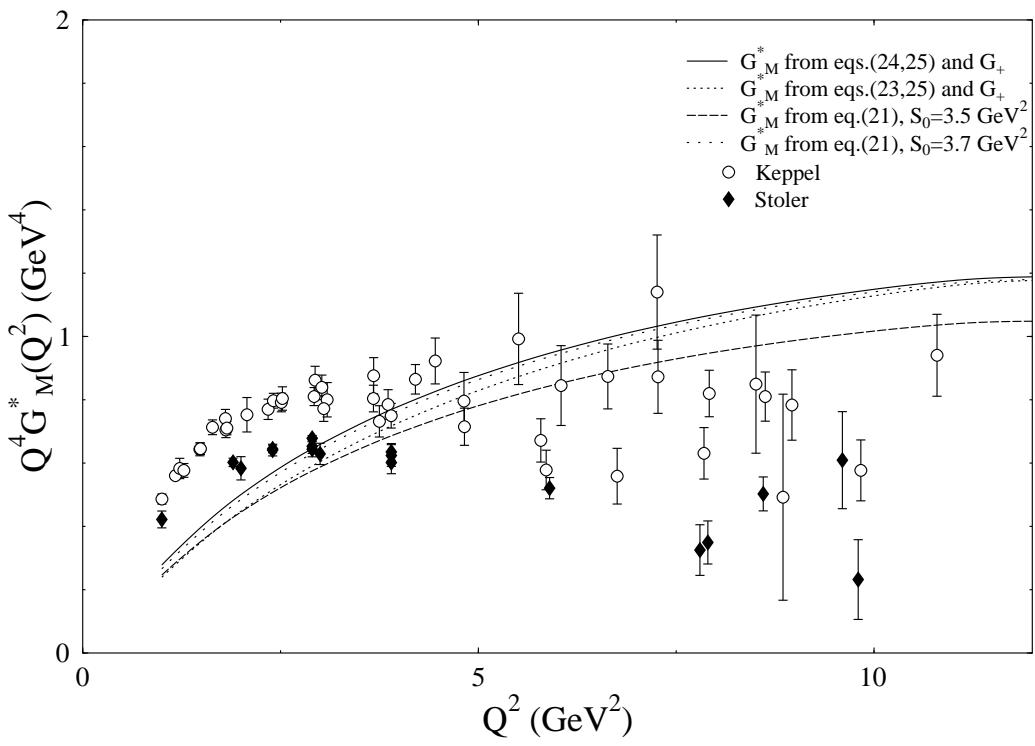


Figure 2: Form factor $G_M^*(Q^2)$.

duality prescription, we obtain

$$G^{(+)}(Q^2) = \frac{8m}{M+m} \left[F(s_0, S_0, Q^2) - \frac{Q^2}{12} \left(\frac{d}{dQ^2} \right)^2 \int_0^{s_0} F(s_1, S_0, Q^2) ds_1 \right]. \quad (27)$$

Now, having expressions both for $G^{(+)}(Q^2)$ and $G^{(-)}(Q^2)$, we can calculate $G_M^*(Q^2)$ and $G_E^*(Q^2)$. The results for the combinations $Q^4 G_M^*(Q^2)$ and $G_E^*(Q^2)/G_M^*(Q^2)$ are shown on Figs.2 and 3, respectively. It should be noted that, though $F(s_0, S_0, Q^2)$ has the $1/Q^6$ asymptotics for large Q^2 (see eq.(23)), the local duality results are fairly consistent with the $1/Q^4$ -behaviour in the wide range $5 \text{ GeV}^2 \lesssim Q^2 \lesssim 20 \text{ GeV}^2$.

An important observation is that $G_E^*(Q^2)$ is predicted to be much smaller than $G_M^*(Q^2)$ (see Fig.3). It should be noted that if the $\gamma^* p \rightarrow \Delta$ transition form factors are calculated in a purely pQCD approach (in which only the $O((\alpha_s/\pi)^2)$ double-gluon-exchange diagrams are taken into account), the sum of electric and magnetic form factors $G_M^*(Q^2) + G_E^*(Q^2)$ is suppressed for asymptotically large Q^2 by a power of $1/Q^2$ [20]. This is because the matrix element

$$\langle 3/2|\Gamma|1, -1/2 \rangle \sim (G_M^* + G_E^*) \quad (28)$$

violates the helicity conservation requirement for the hard subprocess amplitude. In other words, the pQCD prediction is that $(G_M^* + G_E^*)$ should behave asymptotically like $1/Q^6$, while each of G_M^*

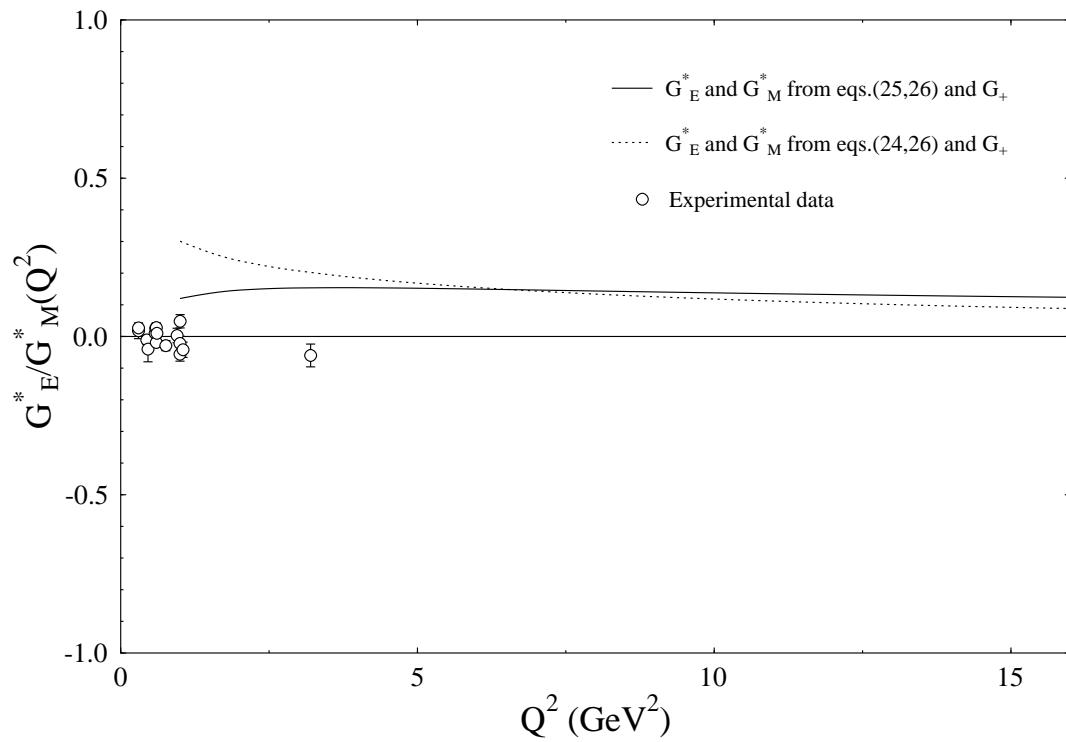


Figure 3: Ratio of form factors $G_E^*(Q^2)$ and $G_M^*(Q^2)$ as calculated from eqs. (24)-(27). Experimental data were taken from ref.[23] and the point at 3.2 GeV^2 from ref.[24].

and G_E^* behaves like $1/Q^4$. As a result, asymptotically $G_E^* \sim -G_M^*$. However, we consider here only the soft contribution generated by the Feynman mechanism for which the helicity conservation arguments are not applicable. Thus, for the soft term, there are no *a priori* grounds to expect that $G_E^* \sim -G_M^*$.

The smallness of $G_E^*(Q^2)/G_M^*(Q^2)$ dictated by local duality, strongly contrasts with the pQCD-based expectation that $G_E^*(Q^2) \sim -G_M^*(Q^2)$, and this allows for an experimental discrimination between the two competing mechanisms. One should realize, however, that $G_E^*(Q^2)$ is obtained in our calculation as a small difference between two large combinations $G^{(+)}(Q^2)$ and $G^{(-)}(Q^2)$, both dominated by $G_M^*(Q^2)$. Hence, even a relatively small uncertainty in either of these combinations (which is always there, since the local duality gives only approximate estimates) can produce a rather large relative uncertainty in the values of G_E^* . In this situation, we restrict ourselves to a conservative statement that the electric form factor $G_E^*(Q^2)$ is small compared to $G_M^*(Q^2)$ in the whole experimentally accessible region without insisting on a specific curve for $G_E^*(Q^2)$.

Experimental points for G_M^* shown in Fig.2 were taken from the results for the $G_T(Q^2)$ form factor obtained from analysis of inclusive data [19], [22]. Since our results give a very small value for the ratio $(G_E^*/G_M^*)^2$, the G_E^* term in eq.(11) can be neglected. One can see that, in the

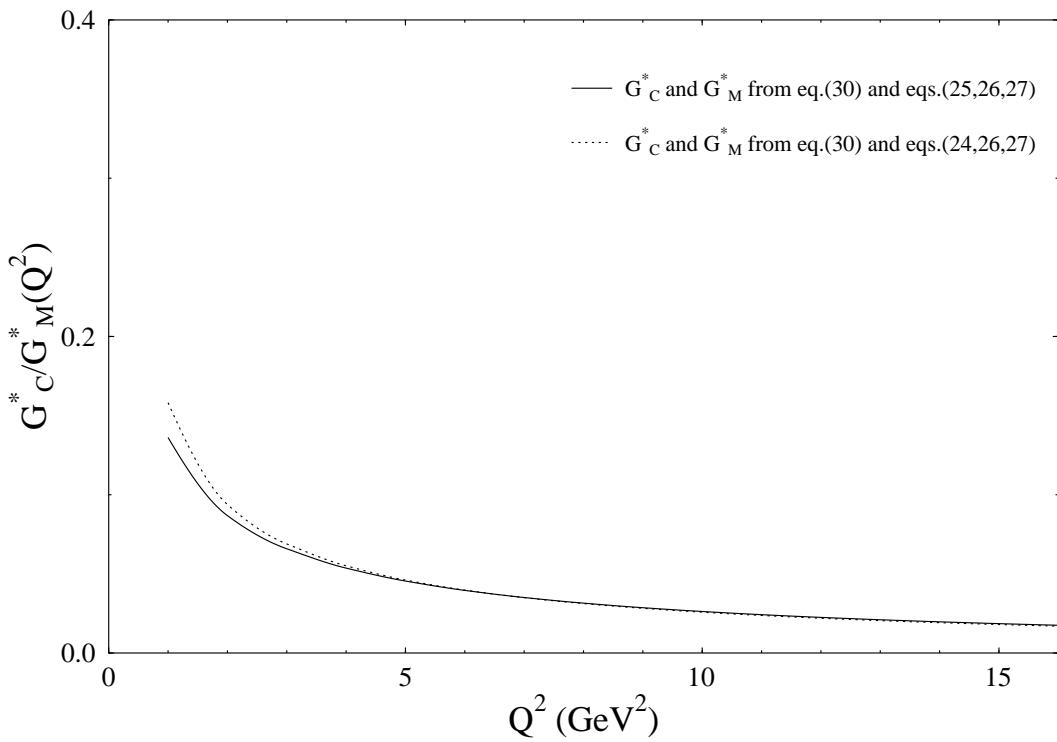


Figure 4: Ratio $G_C^*(Q^2)/G_M^*(Q^2)$ as calculated from eqs. (24)-(27) and (30).

$Q^2 \gtrsim 3 \text{ GeV}^2$ region, the local duality predictions $G_M^*(Q^2)$ are close to the results of the recent analysis [22].

The magnetic form factor $G_M^*(Q^2)$ can also be obtained from eq.(22). If one takes the basic duality interval $S_0 = 3.5 \text{ GeV}^2$, the resulting values of $G_M^*(Q^2)$ (Fig.2) are somewhat smaller than those obtained by combining the results for $G^{(+)}(Q^2)$ and $G^{(-)}(Q^2)$. As emphasized earlier, the spin-1/2 states also contribute to the trace of $T_{\mu\nu}$, and the duality interval in this case can be different from the basic value. In fact, taking $S_0 = 3.7 \text{ GeV}^2$ in eq.(22), we get a curve for $G_M^*(Q^2)$ (Fig.2) essentially coinciding with those obtained from the sum of $G^{(+)}(Q^2)$ and $G^{(-)}(Q^2)$.

The quadrupole (Coulomb) form factor $G_C^*(Q^2)$ can be calculated from the expression (18) for the contracted amplitude $p_\nu T_{\mu\nu}$:

$$\begin{aligned}
 G_C^*(Q^2) = & \frac{8m}{3(M+m)} \left[-\frac{d}{dQ^2} \int_0^{S_0} F(s_0, s_2, Q^2) ds_2 \right. \\
 & - \frac{Q^2}{4} \left(\frac{d}{dQ^2} \right)^2 \int_0^{s_0} F(s_1, S_0, Q^2) ds_1 \\
 & \left. + \frac{1}{2} \left(\frac{d}{dQ^2} \right)^2 \left(1 + \frac{d}{dQ^2} \right) \int_0^{s_0} ds_1 \int_0^{S_0} F(s_1, s_2, Q^2) ds_2 \right] . \tag{29}
 \end{aligned}$$

Again, $G_C^*(Q^2)$ is essentially smaller than $G_M^*(Q^2)$ (see Fig.4). Furthermore, eq.(29) predicts that, for large Q^2 , the quadrupole form factor $G_C^*(Q^2)$ has an extra $1/Q^2$ suppression compared to $G_M^*(Q^2)$. In fact, if the duality intervals are equal, $s_0 = S_0$, the suppression is even stronger, namely, by two powers of $1/Q^2$.

6 Conclusions

We applied the local quark-hadron duality prescription to estimate the soft contribution to the $\gamma^* p \rightarrow \Delta$ transition form factors. We observed a reasonable agreement between the results obtained from different invariant amplitudes. We found that the transition is dominated by the magnetic form factor $G_M^*(Q^2)$ while electric $G_E^*(Q^2)$ and quadrupole $G_C^*(Q^2)$ form factors are small compared to $G_M^*(Q^2)$ for all experimentally accessible momentum transfers. Numerically, our estimates for $G_T(Q^2)$ are close to those obtained from a recent analysis of inclusive data [22]. Hence, there is no need for a sizable hard-scattering contribution to describe the data. Furthermore, if future exclusive measurements at CEBAF would show that the ratio $G_E^*(Q^2)/G_M^*(Q^2)$ is small above $Q^2 \sim 3 \text{ GeV}^2$, this would give an unambiguous experimental proof of the dominance of the soft contribution.

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