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SAND2009-0403
Unlimited Release
Printed February 2009

CTH Reference Manual: Composite Capability and Technologies

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National Nuclear Security Administration under Contract DE-AC04-94AL85000.

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Abstract

The composite material research and development performed over the last year has greatly enhanced the capabilities of CTH for non-isotropic materials. The enhancements provide the users and developers with greatly enhanced capabilities to address non-isotropic materials and their constitutive model development. The enhancements to CTH are intended to address various composite material applications such as armor systems, rocket motor cases, etc. A new method for inserting non-isotropic materials was developed using Diatom capabilities. This new insertion method makes it possible to add a layering capability to a shock physics hydrocode. This allows users to explicitly model each lamina of a composite without the overhead of modeling each lamina as a separate material to represent a laminate composite. This capability is designed for computational speed and modeling efficiency when studying composite material applications. In addition, the layering capability also allows a user to model interlaminar mechanisms. Finally, non-isotropic coupling methods have been investigated. The coupling methods are specific to shock physics where the Equation of State (EOS) is used with a non-isotropic constitutive model. This capability elastically corrects the EOS pressure (typically isotropic) for deviatoric pressure coupling for non-isotropic materials.

Acknowledgment

We would like to acknowledge Todd Bjerke, Brian Love (a.k.a Dr. Love) and Dan Casem from the Army Research Laboratory. We thank them for their participation, guidance and passion for composite armor systems.

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1. INTRODUCTION

The composite material modeling enhancements incorporated into CTH have added much needed capability for modeling non-isotropic materials inside of an Eulerian shock physics hydrocode. The first capability added is the ability to insert layered materials and their directionality through Diatom. This capability is independent of a given material model and has been incorporated as a general non-isotropic material capability for object insertion. The second item added to CTH is the ability to couple the Equation of State (EOS) with the Multicontinuum Technology (MCT) model. Two options are added, a method written by C.E. Anderson et. al (1994) and another method by A. A. Lukonov (2006, 2008). Both methods are designed for orthotropic orientated materials and their coupling to a separate EOS model. The third capability added is an interstitial layering capability for modeling composite non-uniformities. The composite non-uniformities can range from resin rich regions to tow joints inside of polymer matrix composite materials. The additional capabilities and technologies will allow a user to model and design directional composite materials in CTH for various applications such as armor or blast performance.

The general non-isotropic material insertion capability in CTH gives the user and developer a greater degree of freedom. In the past, non-isotropic material directions were controlled through the material model itself. The Transverse Isotropic material model rotated the material using material constants, and the general method rotates a material independent of a material model. The material rotations from the material reference frame to the global reference frame were performed using constitutive model material constants. The new capability allows the user to insert an object and apply the material rotations to the object rather than prescribing the orientations through the material model to the object. From the developer point of view, one simply needs to rotate the material constants or the stress and strain based on the object orientation and deformation. The general material insertion provided the foundation for developing a layering capability for an Eulerian hydrocode. The capability allows the user to insert their object and apply the layer (lamina) thickness and orientation of each layer (lamina). The object is then inserted into the mesh, where the number of layers and orientation per cell is tracked using state variables internally. The cell strain is applied to each layer and the resulting stress is volume averaged and returned back to CTH as the cell stress response. The intention of the capability is to increase accuracy while maintaining solution speed of directional composite simulations.

The ability to model non-isotropic materials with minimal assumptions in a shock physics hydrocode is addressed by the second CTH upgrade. For an isotropic material the calculation of the pressure and deviatoric stress can easily be performed based on the fact that the strain can be separated into uncoupled volumetric and deviatoric components. This is the traditional process for most isotropic metals and typical polymers. The pressure is calculated based on energy and density (EOS model) and the deviatoric stress is computed from the strain field (strength model). However, non-isotropic materials can not be handled in the same manner since the equation of state and strength models are coupled based on simple material geometry. In a non-isotropic material, a hydrostatic pressure response will develop a non-uniform stress field much different than that of isotropic materials. C.E. Anderson et. al (2004) and A.A. Lukonov (2006, 2008) have researched this issue for orthotropic materials and have developed corrections to the EOS based on the strength response. In turn, the pressure from the EOS is incorporated into the full stress response to control items such as damage and/or failure. The end result is a consistent orthotropic constitutive material model.

The third capability incorporates an interstitial layer to simulate interlaminar mechanisms in composite materials. The non-uniformities typically come from material processing or handling during the assembly period. For polymer matrix composites, this is typically resin rich regions from processing or tow beginning and ending inside of a laminate. When utilized, this capability modifies the layered cell capability by making every other layer an inelastic interstitial layer when turned on. The user has control over the thickness and the material properties of these layers. Currently, the interstitial material model is an elastic plastic model with bilinear hardening. Since data is very difficult to obtain for such anomalies, this capability is designed to be user friendly and easily tailored to a specific application.

The added composite capabilities and technologies described above are designed to enhance the users ability to model non-isotropic materials in a Eulerian shock physics hydrocode. The intentions of the authors are to address technology gaps and to address user capability for composite materials in a shock physics environment.

2. GENERAL COMPOSITE CAPABILITY ENHANCEMENTS

The Eulerian shock physics code CTH has underwent various enhancements to improve the usability of importing and modeling directional, layered composite materials. The first enhancement allows for the general purpose insertion of directional dependent materials. The user has the ability to define a user defined coordinate system (UDCS) that then allows rotations for proper insertion into the CTH mesh. The UDCS is independent of material model and therefore may be applied to current and future anisotropic models. The second enhancement is a layering capability for directional laminated composite materials. A composite laminate consists of multiple laminas. The layering capability allows the user to easily define composite directional effects in a laminate on a ply-by-ply basis. Traditionally smeared (homogenized) approaches have been used in the past for composite materials. However, the newly incorporated layering capabilities offer greater accuracy along with computational and modeling efficiency.

The general purpose insertion of directional materials is designed to be independent of a chosen material model. The concept inserts a material into a CTH mesh based on user input and then passes a computed rotation tensor to a material model. A material model will use the rotation tensor to rotate the inserted material to the local coordinate frame for computations. Typically, the strain field is rotated from the global to material coordinate frame, but the material constants, etc. could be rotated from the material frame to the global coordinate frame also. After completing all necessary computations, the stress field and related state variables are rotated into the global coordinate frame.

2.1. Layering Capability

The first step to utilizing the layering capability is to input the solid object through Diatom. The insert function is used to insert a solid object into Diatom. Currently the layering capability can accommodate cylinders and plates. Other objects can be included based on customer needs. In addition to the Diatom *insert*, CTH needs to be told to apply the layering capability to the solid object inserted. This is performed by adding either of the following keywords, *cmp_rotate* for rectangular objects and *cmp_wrap* for cylindrical objects.

```
diatom
    package 'composite'
      material 1
       insert box
        p1 = -4.5, -4.5, 0.1
        p2 = 4.5, 4.5, 1.3
      endinsert
      cmp_rotate
   endpackage
enddiatom
```

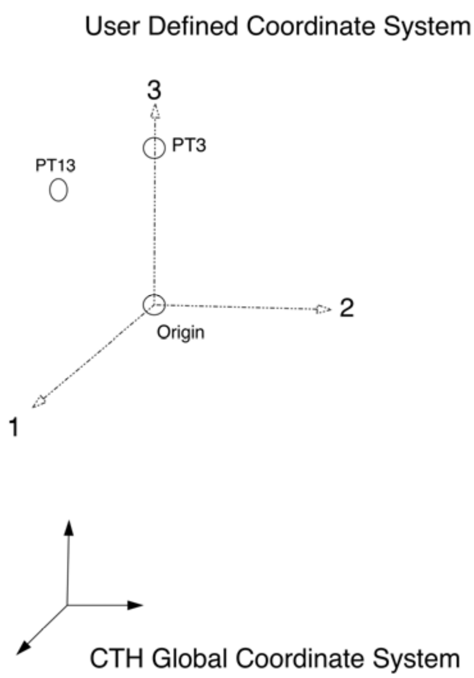
Figure 2.1 CTH composite rectangle insertion through Diatom.

```
diatom
    package 'composite'
      material 1
      insert cylinder
        ce1    5.5  5.5 0.0
        ce2    5.5  5.5 10.0
      radius 2
      endinsert
     cmp_wrap
    endpackage
enddiatom
```

Figure 2.2 CTH composite cylinder insertion through Diatom.

The second step is to setup the user defined coordinate system (UDCS) by three points: an origin, a point along the 3 axis and a point in the 1-3 plane. Figure 2.1 illustrates the required points.

NOTE: The UDCS needs to align with the bottom of the solid object and the 3 axis is through the thickness (ply stacking direction) of the rectangle. For the cylindrical insertion, the UDCS needs to align with the bottom of the cylinder and the 3 axis needs to align with the cylinder axis. Suggestions for a rectangular plate: align the UDCS with a lower corner of the rectangle.

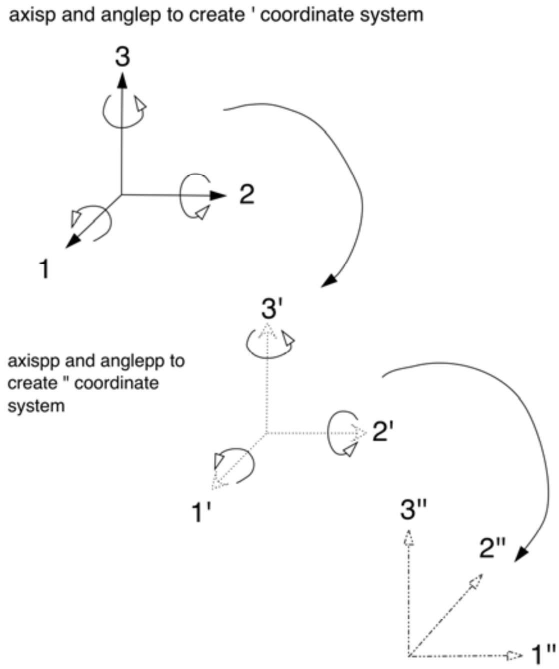


CTH input example

origin = 0.0, 0.0, 0.1
pt3 = 0.0, 0.0, 1.3
pt13 = 1.0, 0.0, 0.1

Figure 2.3 CTH user defined coordinate system declaration and input.

Once the UDCS is defined the rotations of the individual lamina may be added. The rotations are performed relative to the UDCS. A figure showing the rotations and CTH input are provided in Figure 2.4. In this figure the initial rotation is a 0° rotation (*anglep*) about the 2 axis (*axisp*). The second rotation is a 90° rotation (*angledp*) about the 3' axis (*axisdp*).



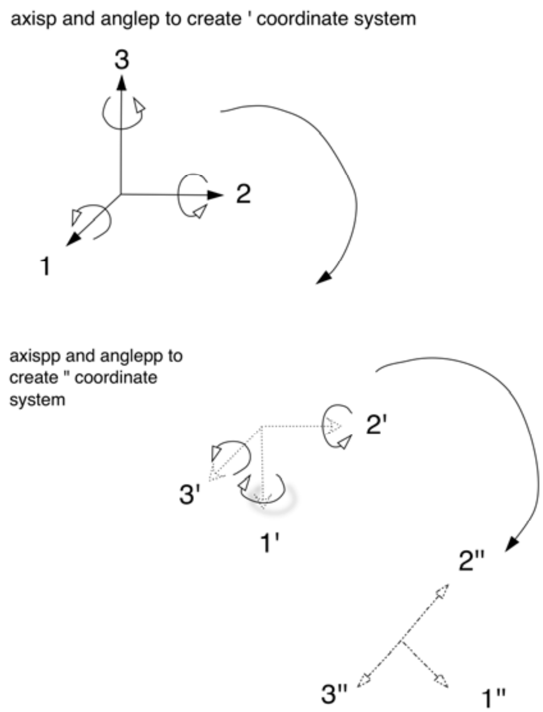
CTH input example

lamina = 1
axisp = 2
anglep = 0
axisdp = 3
angledp = 90

Figure 2.4 CTH rotation about the UDCS and user input example.

An additional example (Figure 2.5) is shown below where the lamina is first rotated 90˚ (*anglep*) about the 2 axis (*axisp*), thus creating the “single prime” coordinate system. The second rotation rotates the lamina in the “single prime” coordinate system to the “double prime” coordinate system, by rotating 45˚ (*axisdp*) about the 3’ axis (*angledp*).

NOTE: If rotations are not desired the user can choose either of the 1, 2 or 3 axes and input 0˚ for the rotation angle.

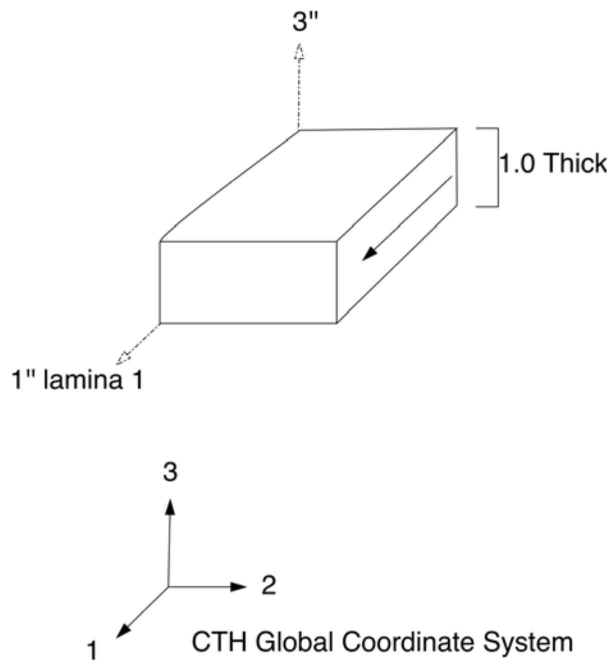


<u>CTH input example</u>

lamina = 1
axisp = 2
anglep = 90
axisdp = 3
angledp = 45

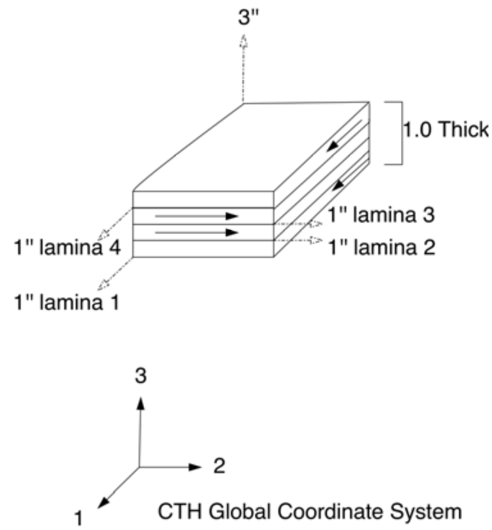
Figure 2.5 CTH rotation about UDCS with only one rotation.

Once UDCS and the rotations are defined the user must next assign which material(s) to apply the rotations to and define the number of layers in the particular material to be inserted through diatom. The composite routine is called by CTH with the keywords *composite* and *endcomposite*. A complete input section is shown below in Figure 2.6 and 2.7 for a 1 ply and 4 ply laminate, respectively.




```
composite
  compmat = 1
  layers = 1
  origin = 0.0, 0.0, 0.1
  pt3 = 0.0, 0.0, 1.3
  pt13 = 1.0, 0.0, 0.1
  lamina = 1
  axisp = 2
  anglep = 0
  axisdp = 3
  angledp = 0
  thickness = 1.0
 endcomposite
```

Figure 2.6 CTH user input for 1 lamina representing the laminate.




```
composite
  compmat = 1
  layers = 4
  origin = 0.0, 0.0, 0.1
  pt3 = 0.0, 0.0, 1.3
  pt13 = 1.0, 0.0, 0.1
  lamina = 1
  axisp = 2
  anglep = 0
  axisdp = 3
  angledp = 0
  thickness = 0.25
  lamina = 2
  axisp = 2
  anglep = 0
  axisdp = 3
  angledp = 90
  thickness = 0.25
  lamina = 3
  axisp = 2
  anglep = 0
  axisdp = 3
  angledp = 90
  thickness = 0.25
  lamina = 4
  axisp = 2
  anglep = 0
  axisdp = 3
  angledp = 0
  thickness = 0.25
 endcomposite
```

Figure 2.7 CTH user input for 4 lamina representing the laminate.

The layering capability within CTH is designed to reduce computational times for composite materials by allowing the user to analyze each lamina individually using a volume averaging approximation instead of modeling each discrete layer. Modeling each discrete layer for 4 lamina in a laminate would require approximately be 20 cells (using 5 cells through the thickness as a rule of thumb) across the thickness of the laminate. However, using the layering capability this is reduced to just 5 cells across the thickness of entire laminate. Currently 20 layers is the maximum number of layers allowed in any one material. Therefore, at the upper limit of 20 plies, the required number of cells is reduced from 100 (5 cells per lamina) to 5. Considering the other two directions, the cell reduction is quite significant. The assumptions for the layering capability are currently the following:

1. Limited to 20 layers for one inserted material.
2. Each layer within a cell receives the same strain field and the resulting lamina stresses are volume averaged to represent the cell level material stress response.
3. Failure is also volume averaged across each lamina in a cell to represent material fracture at the cell level.

2.2. Interstitial Model

An isotropic interstitial layering capability has been added to mimic nonlinear composite behavior that may be due to inter-laminar shear or other nonlinear effects in laminated composite materials. The strength model used for this lamia is an elastic-plastic model with bilinear kinematic and/or isotropic hardening, called simple elastic-plastic (SEP) model. The interstitial layering capability works only with the layering capability combined with either the Transverse Isotropic model or the Multicontinuum (MCT) model. When the interstitial layering capability is turned on, every other layer in the laminated structure is inserted as an interstitial layer, as shown in Figure 2.8.

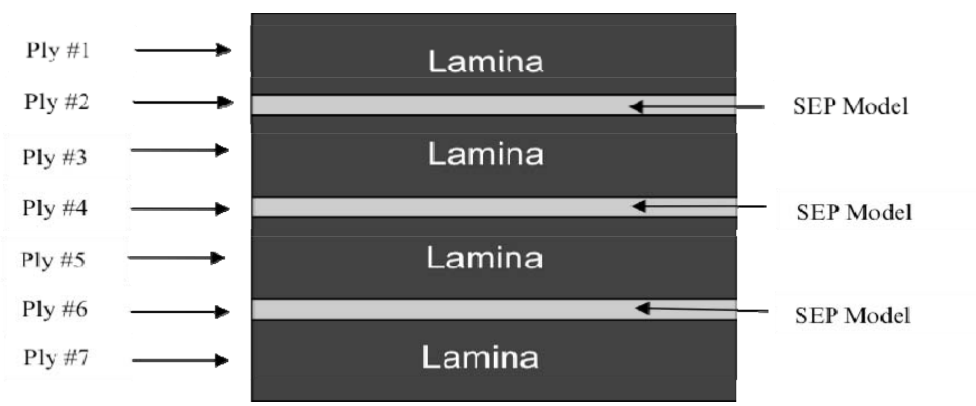


Figure 2.8 Interstitial layering capability.

Inter-laminar shear behavior is difficult to characterize from testing; therefore a simplistic model was chosen to represent this type of nonlinear behavior. The interstitial layer with the elastic-plastic model allows slippage of lamina layers and "virtual fracture" representing ultimate failure by delamination.

The SEP model for interstitial layers within MCT is based on the work of Kreig and Key (1976) which utilizes the von Mises yield criterion where yield is defined by:

$$\phi = J_2 - \frac{\sigma_y^2}{3} = 0 \tag{2.1}$$

where J_2 is the second invariant of the deviatoric stress tensor and σ_y is the yield stress. If yielding of the material has occurred the plastic strain increment is calculated as:

$$\varepsilon_{ij}^{P^{n+1}} = \varepsilon_{ij}^{P^n} + \frac{\sqrt{3J_2} - \sigma_y}{3G + \frac{E_{hard} \cdot E_{elas}}{E_{elas} - E_{hard}}} \tag{2.2}$$

where E_{elas} and E_{hard} are the user defined elastic and hardening modulus for the material.

The corresponding return of the stress state to the yield surface is given by:

$$s_{ij}^{n+1} = s_{ij}^{*} - \frac{\left(1 - \frac{\sigma_y}{\sqrt{3J_2}}\right)}{\left(1 + \frac{\frac{E_{hard} \cdot E_{elas}}{E_{elas} - E_{hard}}}{3G}\right)} \xi_{ij}^{*} \tag{2.3}$$

Where s_{ih}^{*} is the trial stress for the current increment and ξ_{ij}^{*} is defined as:

$$\xi_{ij}^{*} = s_{ij}^{*} - \alpha_{ij}, \tag{2.4}$$

where α_{ij} is the center of the yield surface.

Finally the center of the yield surface is update according to the following relationship:

$$\alpha_{ih}^{n+1} = \alpha_{ij}^{n} + \frac{(1-\beta)\left(1 - \frac{\sigma_y}{\sqrt{3J_2}}\right)}{\frac{3G}{\left(\frac{E_{hard} \cdot E_{elas}}{E_{elas} - E_{hard}}\right)} + 1} * s_{ij} \tag{2.5}$$

Where α_{ih}^{n+1} is the center of the yield surface at increment n+1.

2.3. Using the Interstitial Layering Capability

<u>*Multicontinuum Model*</u>

Setting the MCTSL keyword turns the interstitial layering capability on (mctsl=1).
An example of the CTH input is shown below in Figure 2.9.

epdata
matep=1 mct visc_2_dam_f mctsl=1 *turns the interstitical layering capability on
ende

Figure 2.9 Interstitial laying capability input example for MCT

As described above, the interstitial layering capability uses an elastic plastic model with bilinear hardening. The input data is provided at the end of the MCT data file. A description of how the data is added to the MCT model is provided in Appendix A under the ELPLASTIC keyword.

<u>*Transverse Isotropic Model*</u>

The interstitial layering capability is also part of the Transverse Isotropic model (TI model).
This capability is used by setting TISL (tisl=1).

epdata
matep=1 transv grp_uniaxial tisl=1 *turns the interstitical layering capability on
ende

Figure 2.10 Interstitial laying capability input example for the TI model

The input is located in the VP_data file. The keywords are provided below in Table 2.1.

Table 2.1 Interstitial layering capability keywords using TI model.

Keyword	Description
TISEL	Tensile Modulus
TISPS	Poisson Ratio
TISY	Yield Strength
TISHR	Hardening Modulus
TISBT	Beta Beta = 1 Isotropic Hardening Beta = 0 Kinematic Hardening 0 < Beta < 1 Combined Hardening

3. MULTICONTINUUM FOR DIRECTIONAL COMPOSITES

This section describes a Multicontinuum (MCT) constitutive model for both the elastic and damage/failure induced inelastic response of unidirectional (transversely isotropic) and woven fabric (orthotropic) composite materials and its implementation into the shock physics analysis code, CTH (McGlaun, et al. 1990). A viscoelastic MCT constitutive model is also available for the resin constituent of the unidirectional composite microstructure.

MCT provides the user with not only the typical homogenized composite material stress and strain fields, but also the constituent (fiber and resin) stress and strain fields. The constituent stress and strain fields allow for more accurate modeling of intermediate and ultimate failure modes that are inherent in composite materials.

MCT focuses on the definition of a continuum point. A continuum point is defined as a point conceived as occupying no volume, but which retains the properties associated with a finite volume. The continuum hypothesis relies on averaging the properties of interest over a finite volume. For example, the stress tensor at a continuum point is arrived at by volume averaging the stress throughout the region as

$$\underset{\sim}{\sigma} = \frac{1}{V} \int_D \underset{\sim}{\sigma}(\mathbf{x})\, dV \quad . \tag{3.1}$$

The averaging process outlined above for the stress tensor leads to the concept of a multicontinuum. Specifically, a multicontinuum implies that at every continuum point in a structure there exists multiple constituents with different properties. A composite material is an example of a multicontinuum where two distinct and fundamentally different constituents, fiber and resin, exist at a continuum point.

The failure criterion within the MCT model is credited to Mayes (1999, 2001). This criterion is loosely based on the continuous fiber, unidirectional composite failure criterion developed by Hashin (1980). Hashin's failure criterion is a quadratic condition that uses two different and distinct failure modes, namely the matrix and fiber failure modes. However, the overall criterion is based on the composite state of stress rather than the individual constituent's state of stress. The MCT failure criterion is an alteration of Hashin's criterion that utilizes the power of MCT to predict failure based on the constituent state of stress rather than composite state of stress. The MCT failure criterion has also been expanded to handle woven fabric composite materials.

The continuum damage evolution scheme within the MCT model is motivated by the kinetic theory of fracture for polymers (Zhurkov and Kuksenko 1975, Zhurkov 1984, Hansen and Baker-Jarvis 1990). Kinetic theory is centered around predicting the evolution of molecular level bond rupture as a function of the applied stress. The kinetic theory is used as a basis for degrading the matrix properties within the resin constituent based on the amount of matrix damage resulting from sub-microcrack accumulation in the material. The damage model is then related to material property degradation in the matrix, fiber bundles, and the composite. The resulting damage analysis produces a smooth macroscopic response for the composite that reflects observed inelastic material behavior.

3.1. Elastic Two-Constituent MCT (Unidirectional Composite)

For two-constituent composite materials (unidirectional) consisting of fibers and matrix there exists well known algebraic relations to decompose the composite stress/strain fields to the constituent level. The decomposition first appeared in Hill (1964) who developed the relations in an effort to estimate composite material stiffness properties. In the case of MCT, the relations are the same but the motivation is entirely the opposite. That is, we utilize known composite properties in conjunction with the decomposition of Hill to determine constituent stress/strain fields. We have relied on detailed finite element micromechanics models to compute the composite material mechanical properties.

For a multicontinuum point, the previous definition of continuum stress in Eq. (3.1) is taken down to the constituent level. In particular, for a continuum point representing a two-constituent composite, volume averaged stresses for the fiber and resin constituents may be expressed as:

$$\underset{\sim}{\sigma}_{Fiber} = \frac{1}{V_{Fiber}} \int_{D_{Fiber}} \underset{\sim}{\sigma}(\mathbf{x})\, dV \,, \tag{3.2}$$

and

$$\underset{\sim}{\sigma}_{Matrix} = \frac{1}{V_{Matrix}} \int_{D_{Matrix}} \underset{\sim}{\sigma}(\mathbf{x})\, dV \,, \tag{3.3}$$

where

$$D = D_{Fiber} \cup D_{Matrix} \,.$$

The composite and constituent stress fields defined by Equations (3.1-3) lead directly to

$$\underset{\sim}{\sigma} = \phi_{Fiber} \underset{\sim}{\sigma}_{Fiber} + \phi_{Matrix} \underset{\sim}{\sigma}_{Matrix} \,, \tag{3.4}$$

where ϕ_{Fiber} and ϕ_{Matrix} are the volume fractions of the fiber and matrix constituents, respectively.

Equations (3.1-3) have analogous definitions for the strain tensors. Using these definitions, the composite strain tensor is related to the constituent strain tensors as

$$\underset{\sim}{\varepsilon} = \phi_{Fiber} \underset{\sim}{\varepsilon}_{Fiber} + \phi_{Matrix} \underset{\sim}{\varepsilon}_{Matrix} \,. \tag{3.5}$$

In order to obtain closure of equations (3.4) and (3.5) it is necessary to introduce constitutive relationships for the composite and the constituents. In contracted matrix notation, the linear elastic constitutive laws for the composite and constituents are as follows:

$$\{\sigma\} = [C]\left(\{\varepsilon\} - \{\varepsilon_0\}\right) \ , \tag{3.6}$$

$$\{\sigma_{Fiber}\} = [C_{Fiber}]\left(\{\varepsilon_{Fiber}\} - \{\varepsilon_{Fiber\,0}\}\right) \ , \tag{3.7}$$

$$\{\sigma_{Matrix}\} = [C_{Matrix}]\left(\{\varepsilon_{Matrix}\} - \{\varepsilon_{Matrix0}\}\right) \ . \tag{3.8}$$

In the above, $[C]$, $[C_{Fiber}]$, and $[C_{Matrix}]$ are the material stiffness matrices and $\{\varepsilon_0\}$, $\{\varepsilon_{Fiber0}\}$, and $\{\varepsilon_{Matrix0}\}$ are the thermal strains of the composite and its respective constituents. Substituting equations (3.6), (3.7), and (3.8) into (3.4) and using (3.5) to solve for $\{\varepsilon_{Fiber}\}$ gives

$$\{\varepsilon_{Fiber}\} = [A]\,\{\varepsilon_{Matrix}\} + \frac{\theta}{\phi_{Fiber}}\,\{a\} \ , \tag{3.9}$$

where

$$[A] = -\frac{\phi_{Matrix}}{\phi_{Fiber}}\left([C]-[C_{Fiber}]\right)^{-1}\left([C]-[C_{Matrix}]\right) \ , \tag{3.10}$$

and

$$\{a\} = \left([C]-[C_{Fiber}]\right)^{-1}\left([C]\,\{\eta\} - \phi_{Fiber}\,[C_{Fiber}]\,\{\eta_{Fiber}\} - \phi_{Matrix}\,[C_{Matrix}]\,\{\eta_{Matrix}\}\right) \ . \tag{3.11}$$

In the above, $\{\eta\}$, $\{\eta_{Fiber}\}$, and $\{\eta_{Matrix}\}$ are vectors that contain the thermal expansion coefficients of the composite and constituents, respectively. Also, θ denotes the relative temperature so that we can write

$$\{\varepsilon_0\} = \theta\{\eta\}, \quad \{\varepsilon_{Fiber0}\} = \theta\{\eta_{Fiber}\}, \quad \{\varepsilon_{Matrix0}\} = \theta\{\eta_{Matrix}\} \ . \tag{3.12}$$

In equations (3.10) and (3.11), $[C_{Fiber}]$, $[C_{Matrix}]$, $\{\eta_{Fiber}\}$, and $\{\eta_{Matrix}\}$ are all composed of known material properties whereas $[C]$ and $\{\eta\}$ are composite properties that are obtained from micromechanics analyses using the constituent values as input.

Given the composite strain fields determined in a structural finite element analysis, the constituent strains can now be determined from the composite strains by first substituting equation (3.9) into equation (3.5) and solving for $\{\varepsilon_{Matrix}\}$ as

$$\{\varepsilon_{Matrix}\} = \left(\phi_{Matrix}[I] + \phi_{Fiber}[A]\right)^{-1}\left(\{\varepsilon\} - \theta\{a\}\right) \quad , \tag{3.13}$$

where $[I]$ is the identity matrix. Finally, $\{\varepsilon_{Fiber}\}$ may be determined using equation (3.5) in conjunction with the above, i.e.,

$$\{\varepsilon_{Fiber}\} = \frac{1}{\phi_{Fiber}}\left(\{\varepsilon\} - \phi_{Matrix}\{\varepsilon_{Matrix}\}\right) \quad . \tag{3.14}$$

With these constituent strain values, the constituent stresses can be calculated from the constitutive equations (3.7) and (3.8).

3.2. Elastic Three-Constituent MCT (Woven Fabric Composites)

An MCT analysis of a woven fabric composite material treats the composite as a three-constituent microstructure composed of warp fiber bundles (α), fill fiber bundles (β), and pure matrix pockets (γ). The introduction of the third constituent, γ, to the continuum adds the following constitutive relation to the previously described system of equations:

$$\{\sigma_\gamma\} = \left[C_\gamma\right]\left(\{\varepsilon_\gamma\} - \{\varepsilon_{\gamma o}\}\right) \quad . \tag{3.15}$$

It is important to note here that Eqs. (3.2-14) are altered only in the subscripts for the three-constituent MCT derivation. That is, the fiber subscripts are all replaced with an α subscript and the matrix subscripts are all replaced with a β subscript. This is done based on the assumption that the warp (α) and fill (β) bundles are considered constituents for the three-constituent MCT derivation.

With the addition of the third constituent, γ, and noting the previous development, we have introduced one new equation, (3.15), and two unknowns given by $\{\sigma_\gamma\}$ and $\{\varepsilon_\gamma\}$. This leads to a set of equations that is indeterminate. To eliminate the indeterminacy introduced by a third constituent, the treed approach shown in Figure 3.11 is utilized. The extension of the MCT decomposition to woven fabrics is summarized below and may be found in detail in Key (2000) and Key *et al.* (2003).

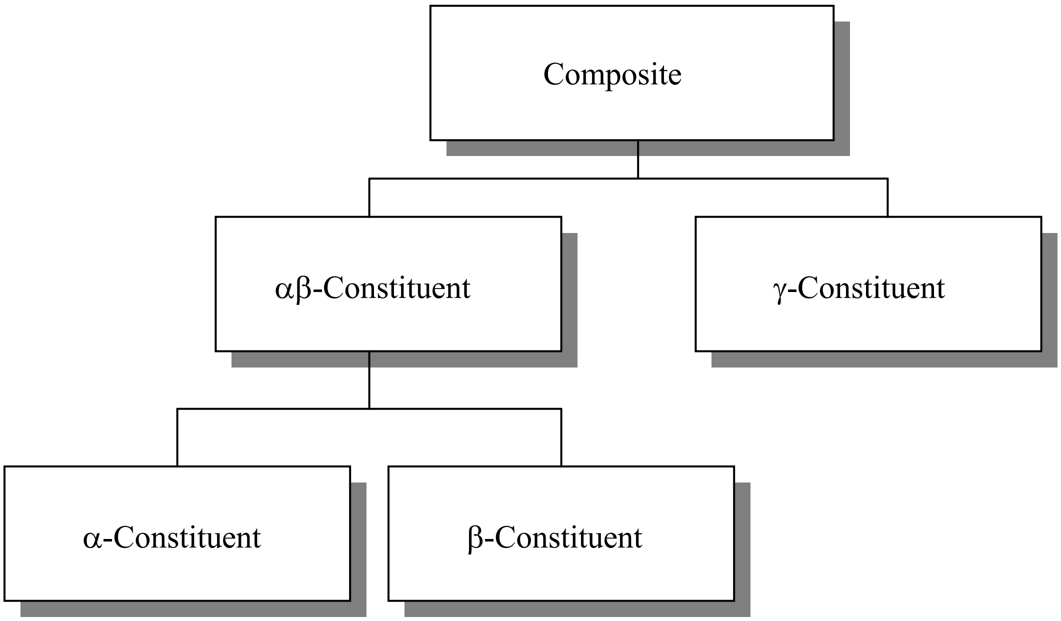


Figure 3.11 Three-constituent MCT decomposition.

In the approach of Figure 3.11, we first combine the warp fiber bundle (α) and fill fiber bundle (β) constituents into a single constituent denoted by $\alpha\beta$. This combination allows the previously indeterminate set of equations to be reduced to a set of branched two-constituent problems, each composed of determinant sets of equations. The first branch of the tree structure consists of constituents $\alpha\beta$ and γ, with unknowns $\{\sigma_{\alpha\beta}\}$, $\{\varepsilon_{\alpha\beta}\}$, $\{\sigma_{\gamma}\}$, and $\{\varepsilon_{\gamma}\}$. For this first branch of the three-constituent theory, Equations (3.13), (3.10), (3.11) and (3.14) are respectively modified as:

$$\{\varepsilon_\gamma\} = \left(\phi_\gamma [I] + \phi_{\alpha\beta}[A]\right)^{-1}\left(\{\varepsilon\} - \theta\{a\}\right) \quad , \tag{3.16}$$

where

$$[A] = -\frac{\phi_\gamma}{\phi_{\alpha\beta}}\left([C]-[C_{\alpha\beta}]\right)^{-1}\left([C]-[C_\gamma]\right) \; , \tag{3.17}$$

$$\{a\} = \left([C]-[C_{\alpha\beta}]\right)^{-1}\left([C]\{\eta\} - \phi_{\alpha\beta}[C_{\alpha\beta}]\{\eta_{\alpha\beta}\} - \phi_\gamma [C_\gamma]\{\eta_\gamma\}\right) \; , \tag{3.18}$$

and

$$\{\varepsilon_{\alpha\beta}\} = \frac{1}{\phi_{\alpha\beta}}\left(\{\varepsilon\} - \phi_\gamma\{\varepsilon_\gamma\}\right) \; . \tag{3.19}$$

The constitutive equation for the $\alpha\beta$ constituent assumes the form

$$\{\sigma_{\alpha\beta}\} = \left[C_{\alpha\beta}\right]\left(\{\varepsilon_{\alpha\beta}\} - \{\varepsilon_{\alpha\beta o}\}\right) \quad . \tag{3.20}$$

Once $\{\sigma_{\alpha\beta}\}$ and $\{\varepsilon_{\alpha\beta}\}$ are calculated in the first branch of the theory, the $\alpha\beta$ constituent can then be viewed as the composite for the second branch of the tree, where α and β are its respective constituents. The fundamental strain relations given in Equations (3.10,3.11,3.13,3.14) are again modified respectively as:

$$\{\varepsilon_\alpha\} = \left(\phi_\alpha [I] + \phi_\beta [A]\right)^{-1}\left(\{\varepsilon_{\alpha\beta}\} - \theta\{a\}\right) \quad , \tag{3.21}$$

where

$$[A] = -\frac{\phi_\alpha}{\phi_\beta}\left([C_{\alpha\beta}] - [C_\beta]\right)^{-1}\left([C_{\alpha\beta}] - [C_\alpha]\right) \quad , \tag{3.22}$$

$$\{a\} = \left([C_{\alpha\beta}] - [C_\beta]\right)^{-1}\left([C_{\alpha\beta}]\{\eta_{\alpha\beta}\} - \phi_\beta [C_\beta]\{\eta_\beta\} - \phi_\alpha [C_\alpha]\{\eta_\alpha\}\right) \quad , \tag{3.23}$$

and

$$\{\varepsilon_\beta\} = \frac{1}{\phi_\beta}\left(\{\varepsilon_{\alpha\beta}\} - \phi_\alpha \{\varepsilon_\alpha\}\right) \quad . \tag{3.24}$$

An important point in Equations (3.21-24) is that the volume fractions ϕ_α and ϕ_β represent the volume of constituents α and β relative to the volume of the $\alpha\beta$ constituent.

Any application of the proposed three-constituent decomposition requires one to determine the material stiffness matrix $[C_{\alpha\beta}]$ as well as the coefficients of thermal expansion $\{\eta_{\alpha\beta}\}$. To determine these material properties we rely on the finite element micromechanics models. A judicious selection of mechanical load cases for the composite allows one to induce specific stress states within the $\alpha\beta$ constituent that lead to straightforward calculations of the material properties. Once $[C_{\alpha\beta}]$ is known, $\{\eta_{\alpha\beta}\}$ may be determined by applying a thermal load to the micromechanics model. Upon volume averaging the appropriate strain fields for the thermal load, Equation (3.21) may be used to determine the vector $\{a\}$. Substituting $\{a\}$ into Equation (3.23) allows one to compute $\{\eta_{\alpha\beta}\}$ directly. The reader is referenced to Key (2000) for a more detailed description of the determination of $[C_{\alpha\beta}]$ and $\{\eta_{\alpha\beta}\}$.

The geometry of a balanced plain weave composite presents some difficulties associated with the three-constituent decomposition. In particular, singular matrices are encountered in the second decomposition, thereby preventing the required matrix inversions. In the material model, the traditional decomposition is altered by condensing out appropriate stress/strain terms that produce the singular matrices. Again, the reader is referred to Key (2000) for details of these alterations.

3.3. Linear Viscoelastic Two-Constituent MCT (Unidirectional Composite)

Linear viscoelastic behavior is currently only available for two-constituent MCT within CTH. The viscoelastic behavior is limited to the matrix constituent of the composite while the fibers are modeled as linear elastic. A brief description of the transversely isotropic viscoelastic constitutive law used in MCT is provided in this section of the document. For details of how the viscoelastic behavior is applied to the MCT decomposition algorithm and solution techniques the reader is referenced to Garnich (1996) and Garnich and Hansen (1997).

Linear viscoelasticity may be thought of as the time dependent deformation of a viscous solid. Two types of loading, constant stress or constant strain are used to characterize the uniaxial material behavior in time. Under constant stress, strain is time dependent resulting is creep behavior. Conversely, under constant strain, stress is time dependent resulting in relaxation behavior.

In the case of creep behavior, under constant stress, the constitutive equation is given by

$$\varepsilon(t) = D(t)\sigma \quad , \tag{3.25}$$

where D(t) is the creep compliance and σ is a constant stress. The creep compliance is generally expressed in terms of exponential functions as

$$D(t) = D^0 + D^k e^{\frac{-t}{\Pi^k}} \quad . \tag{3.26}$$

Assuming linearity, a convolution integral is used to determine the time dependent strain for a variable stress history as,

$$\varepsilon(t) = \int_{0^-}^{t} D(t-\tau)\frac{d\sigma(\tau)}{d\tau}d\tau \quad . \tag{3.27}$$

The above constitutive law completely characterizes the linear viscoelastic response of an isotropic material under uniaxial stress.

In the case of an orthotropic material, the constitutive equations become direction dependent. The three-dimensional constitutive law for a constant stress state is given by

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{44} \\ \gamma_{55} \\ \gamma_{66} \end{bmatrix} = \begin{bmatrix} D_{11}(t) & D_{12}(t) & D_{13}(t) & 0 & 0 & 0 \\ & D_{22}(t) & D_{23}(t) & 0 & 0 & 0 \\ & & D_{33}(t) & 0 & 0 & 0 \\ & & & D_{44}(t) & 0 & 0 \\ & \text{sym} & & & D_{55}(t) & 0 \\ & & & & & D_{66}(t) \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{44} \\ \sigma_{55} \\ \sigma_{66} \end{bmatrix} \quad . \tag{3.28}$$

Nine independent creep functions are necessary to fully characterize the material.

Now consider the case of transverse isotropy where the $x_2 - x_3$ is the plane of symmetry. Using material symmetry arguments one can show:

$$D_{22} = D_{33}, \qquad D_{44}=D_{55}, \qquad \text{and} \qquad D_{12}=D_{13}.$$

Furthermore, the shear creep compliance, D_{66}, may be expressed as

$$D_{66} = 2(D_{22} - D_{23}) \ . \tag{3.29}$$

Therefore, a total of five independent creep functions completely characterize the material.

A continuous fiber unidirectional composite is an example of a transversely isotropic material. The creep compliance coefficients $D_{ij}(t)$ for such a material may be determined using the hexagonal packed micromechanics finite element model. Again, a fundamental assumption underlying the entire process is that the mechanical properties for the matrix and fiber are known. Volume averaging the micromechanical stress and strain fields provides the necessary continuum field information for determining the creep functions. For instance consider a constant stress, σ_{11}, with all other $\sigma_{ij} = 0$. Then,

$$D_{11}(t) = \frac{\varepsilon_{11}(t)}{\sigma_{11}} \ , \tag{3.30}$$

and

$$D_{12}(t) = \frac{\varepsilon_{22}(t)}{\sigma_{11}} \ , \tag{3.31}$$

where ε_{11} and ε_{22} are the volume averaged strain fields. Now consider a constant σ_{22}, with all other $\sigma_{ij} = 0$. Then,

$$D_{22}(t) = \frac{\varepsilon_{22}(t)}{\sigma_{22}} \ , \tag{3.32}$$

and

$$D_{23}(t) = \frac{\varepsilon_{33}(t)}{\sigma_{22}} \ . \tag{3.33}$$

Finally the shear creep coefficient D_{44} is determined from a constant σ_{12}, with all other $\sigma_{ij} = 0$. Then,

$$D_{44}(t) = \frac{\varepsilon_{44}(t)}{\sigma_{44}} \quad . \tag{3.34}$$

Finally, for a variable stress history, the three-dimensional linear viscoelastic constitutive law assumes the form

$$\varepsilon_i(t) = \sum_{j=1}^{6} \int_{0^-}^{t} D_{ij}(t-\tau) \frac{d\sigma_j(\tau)}{d\tau} d\tau \quad . \tag{3.35}$$

3.4. Stress Based Failure with Binary Degradation

Two-Constituent (Unidirectional Composite)

The MCT two-constituent transversely isotropic failure criterion is loosely based on the continuous fiber, unidirectional composite failure criterion developed by Hashin (1980). Hashin's failure criterion is a quadratic condition that uses two different and distinct failure modes, namely the matrix and fiber failure modes. However, the overall criterion is based on the composite state of stress rather than the individual constituent's state of stress. Mayes (1999, 2001) altered Hashin's criterion and coupled this with the ability of MCT to decompose composite stress fields into constituent stress fields. This allowed the distinct failure modes to be predicted based on constituent continuum stress fields rather than the homogenized composite stress fields.

Since a unidirectional fiber reinforced composite is assumed to be transversely isotropic, the failure state of either constituent within the composite can be expressed in terms of transversely isotropic stress invariants. The form of these transversely isotropic stress invariants used for the MCT failure criterion are given in Hansen *et al.* (1991). The five transversely isotropic stress invariants are:

$$\begin{aligned} I_1 &= \sigma_{11}, \\ I_2 &= \sigma_{22} + \sigma_{33}, \\ I_3 &= \sigma_{22}^2 + \sigma_{33}^2 + 2\sigma_{23}^2, \\ I_4 &= \sigma_{12}^2 + \sigma_{13}^2, \\ I_5 &= \sigma_{22}\sigma_{12}^2 + \sigma_{33}\sigma_{13}^2 + 2\sigma_{12}\sigma_{13}\sigma_{23} \quad . \end{aligned} \tag{3.36}$$

Since Hashin's failure criterion utilizes a quadratic form, the invariant I_5 is eliminated from the failure criterion, as it is cubic in stress. With this simplification, the most general form of Hashin's failure criterion is

$$A_1 I_1 + B_1 I_1^2 + A_2 I_2 + B_2 I_2^2 + C_{12} I_1 I_2 + A_3 I_3 + A_4 I_4 = 1 \quad , \tag{3.37}$$

where the A_i, B_i, and C_i coefficients in equation (3.37) are functions of the ultimate tensile, compressive, and shear strengths of the composite.

Mayes used a failure criterion of the same form as equation (3.37) but related it to the constituent stress information calculated in MCT rather than the composite stress information used by Hashin.

Mayes made several simplifications and assumptions to equation (3.37) according to the investigations of various researchers and himself. The reader is referenced to Mayes (1999, 2001) for a detailed discussion of these simplifications and assumptions. The resulting general form of the stress interactive failure criterion for MCT is given by:

$$A_1 I_1{}^2 + A_2 I_2{}^2 + A_3 I_3 + A_4 I_4 = 1 \ . \tag{3.38}$$

Two-Constituent Fiber Failure

Since most fibers used in composite materials have a much greater transverse strength than the matrix material that bonds them together, one can assume that the transverse failure of a unidirectional composite is controlled by matrix failure. Likewise, longitudinal (along the axis of the fibers) failure of a unidirectional composite will be controlled by fiber failure. Assuming that the fibers run in the x_1 direction, the coefficients A_2 and A_3 in the fiber failure criterion can be set to zero due to their relationship with transverse stresses σ_{22} and σ_{33}. This reduces equation (3.38) to the MCT fiber failure criterion:

$${}^{\pm}A_1^f \left(I_1^f\right)^2 + A_4^f I_4^f = 1 \tag{3.39}$$

The coefficients for the two invariant stress terms can now be determined by considering the load cases of pure in-plane shear, tension, and compression. Considering each of these three cases individually, we find the following relationships for the fiber failure criterion coefficients:

$${}^{\pm}A_1{}^f = \frac{1}{({}^{\pm}S_{11}{}^f)^2} \tag{3.40}$$

and

$${}^{\pm}A_1{}^f = \frac{1}{({}^{\pm}S_{11}{}^f)^2} \tag{3.41}$$

In the above, S_{ij}^{f} are the ultimate strengths for the fiber and the ± symbol indicates either a tensile or compressive loading situation.

Two-Constituent Matrix Failure

For the case of transverse failure in a unidirectional composite, where the matrix material controls failure, A_1 can be set to zero since it is related to the stress invariant that is only a function of the longitudinal stress σ_{11}. Again, from Eq. (3.38) this then yields the MCT matrix failure criterion:

$$ {}^{\pm}A_2^m \left(I_2^m\right)^2 + A_3^m I_3^m + A_4^m I_4^m = 1 \quad . \tag{3.42}$$

Similar to the case for fiber failure, we consider uniaxial loading cases of pure in-plane shear, transverse shear, transverse tension, and transverse compression to determine the coefficients of equation (3.38). Applying these load cases individually, the following results are arrived at for the matrix failure criterion coefficients:

$$ {}^{+}A_2{}^{m} = \frac{1}{({}^{+}S_{22}{}^{22m} + {}^{+}S_{33}{}^{22m})^2}\left(1 - \frac{({}^{+}S_{22}{}^{22m})^2 + ({}^{+}S_{33}{}^{22m})^2}{2(S_{23}{}^{m})^2}\right) \quad , \tag{3.43}$$

$$ {}^{-}A_2{}^{m} = \frac{1}{({}^{-}S_{22}{}^{22m} + {}^{-}S_{33}{}^{22m})^2}\left(1 - \frac{({}^{-}S_{22}{}^{22m})^2 + ({}^{-}S_{33}{}^{22m})^2}{2(S_{23}{}^{m})^2}\right) \quad , \tag{3.44}$$

$$ A_3{}^{m} = \frac{1}{2(S_{23}{}^{m})^2} \tag{3.45}$$

and

$$ A_4{}^{m} = \frac{1}{(S_{12}{}^{m})^2} \quad . \tag{3.46}$$

Note that in the relationships for the coefficients ${}^{\pm}A_2{}^{m}$, the $S_{33}{}^{22m}$ term implies the matrix normal stress in the x_3 direction when the resin has reached its ultimate x_2 strength.

Three-Constituent (Woven Fabric Composite)

In extending the MCT failure criterion generated by Mayes (1999, 2001) to the three- constituent weave model, two key assumptions are made. First, the fiber bundles of the weave microstructure are treated as individual constituents even though they are unidirectional composites themselves. Second, even though the undulation in the fiber bundles causes them to no longer be transversely isotropic in a continuum sense they are still assumed to be transversely isotropic for failure considerations. The reader is referenced to Key (2000) for a detailed description and numerical justification of these assumptions.

Using these assumptions, the two-constituent MCT failure criterion described previously can be extended to predict the failure of the fiber bundles inside the woven composite model.

To begin developing failure criteria for the woven composite, a set of isotropic stress invariants has to be developed for both the warp and fill fiber bundles. This is necessary since these fiber bundles lay perpendicular to one another and both are contained in the same global coordinate system. Therefore, their local stress invariants must be written in terms of the global stress values. Since the fill bundles are transversely isotropic about the x_1 axis, their stress invariants are the same as those given previously in equation (3.36). However, since the warp bundles are transversely isotropic about the x_2 axis, their stress invariants in the global coordinate system are:

$$\begin{aligned} I_1 &= \sigma_{22}, \\ I_2 &= \sigma_{11} + \sigma_{33}, \\ I_3 &= \sigma_{11}^2 + \sigma_{33}^2 + 2\sigma_{13}^2, \\ I_4 &= \sigma_{21}^2 + \sigma_{23}^2, \\ I_5 &= \sigma_{11}\sigma_{21}^2 + \sigma_{33}\sigma_{23}^2 + 2\sigma_{21}\sigma_{23}\sigma_{13}. \end{aligned} \tag{3.47}$$

During a failure analysis of a three-constituent woven fabric composite material, each fiber bundle within the weave structure is checked for both matrix failure and fiber failure. This is done by utilizing equations (3.39) and (3.42), with a minor changes to the invariant coefficients to account for the fact that we are know treating the fiber bundle as a single constituent rather than a composite composed of two constituents. Using these equations, the fiber bundle failure criteria for the three-constituent woven fabric model are as follows.

Three-Constituent Longitudinal Failure (Fiber Failure with a Fiber Bundle)

The criterion for longitudinal (fiber) failure in a fiber bundle of a three-constituent composite is of the same form as Eq. (3.39). The criteria has the following form, where the invariant quantities are for the fiber bundle constituents:

$$\pm A_1^l (I_1)^2 + A_4^l I_4 = 1 \ , \tag{3.48}$$

where the coefficients A_1^l and A_4^l are given by

$$^{\pm}A_1^l = \frac{1}{(^{\pm}S_{11}^{\ b})^2} \tag{3.49}$$

and

$$A_4^{\,l} = \frac{1}{(S_{12}^{\;lb})^2} \tag{3.50}$$

Three-Constituent Transverse Failure

The criteria for transverse (resin) failure in a fiber bundle of a three-constituent composite is of the same form as Eq. (3.42) and is given by:

$$\pm A_2^{\,t}(I_2)^2 + A_3^{\,t} I_3 + A_4^{\,t} I_4 = 1 \;, \tag{3.51}$$

where

$${}^{\pm}A_2^{\,t} = \frac{1}{({}^{+}S_{22}^{\;b})}\left(1 - \frac{({}^{\pm}S_{22}^{\;b})^2}{2(S_{23}^{\;b})^2}\right), \tag{3.52}$$

$$A_3^{\,t} = \frac{1}{2(S_{23}^{\;b})^2} \tag{3.53}$$

and

$$A_4^{\,t} = \frac{1}{(S_{12}^{\;tb})^2}. \tag{3.54}$$

A detailed discussion of the three-constituent failure criterion including all changes made to the strength coefficients are contained in Key (2000, 2003). Key also details all potential failure states that may be achieved in the three-constituent woven fabric microstructure and a matrix of which states are implemented in the MCT model.

Binary Degradation Approach

The stress based failure criterion utilizes an instantaneous material property degradation approach to capture the material damage nonlinearities. In other words, this approach reduces both the failed constituent and composite material properties in a single time step during an analysis rather than in a continuous manner of multiple time steps. Typical reductions for material properties are to reduce the Young's Modulus (E) and the shear modulus (G) of the resin constituent by 50 – 90% when matrix failure is determined to have occurred. Likewise, for fiber failure the typical reduction for these moduli is 75 – 99% for both the fiber and resin constituent. The amount of degradation is typically determined through experimental data. Degradation values are incorporated into the micromechanics models for construction of the MCT material data file described in Section 4.4. These micromechanics model provide the reduced composite (or bundle for woven fabrics) stiffness properties that are required in the MCT material data file.

3.5. Damage Based Failure with Continuous Degradation

A continuous damage approach that utilizes a damage evolution for the matrix constituent is also available in the MCT model. The damage evolution is motivated by a one-dimensional damage model that exhibits the characteristics of stress and time dependence based on the kinetic theory of fracture. This model allows for rate-dependent continuous degradation of resin and composite properties.

Kinetic theory is centered on bond rupture at the molecular level in a material. Bond rupture occurs at the molecular level and manifests itself in the form of sub-microcracks. As loading continues, these microcracks coalesce resulting in macroscopic failure. The evolution of microcracks under uniaxial stress is represented by the following differential equation (Hansen and Baker-Jarvis 1990):

$$\frac{dN(t)}{dt} = \left(N_T - N(t)\right)K_b \quad , \tag{3.55}$$

where N is the number of sub-microcracks, N_T is a constant representing local "hot spots" such as amorphous-crystalline interfaces, etc., and K_b is the reaction rate for material breakage given by

$$K_b = \frac{1}{\tau_0} e^{\left(\frac{-(U-\gamma\sigma)}{kTA}\right)} . \tag{3.56}$$

In Eq. (3.56), τ_0 is the period of characteristic oscillation of atoms in a solid, k is Boltzmann's constant, A is Avogadro's number, T is the temperature, and U and γ are material constants.

Dividing Eq. (3.55) by N_r, where N_r represents the number of sub-microcracks at rupture, the degree of damage can be represented on a scale of $0 \leq n \leq 1$, where $n = 0$ represents no damage and $n = 1$ represents macroscopic material failure. The resulting differential equation representing the degree of damage accumulation within a material is given by

$$\frac{dn(t)}{dt} = \left(n_0 - n(t)\right)K_b \quad . \tag{3.57}$$

The extension of the one-dimensional kinetic theory damage model to three-dimensional stress states is achieved by introducing a second order continuum damage tensor, n_{ij}, given by

$$n_{ij} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ & n_{22} & n_{23} \\ sym & & n_{33} \end{bmatrix} . \tag{3.58}$$

The damage tensor components are assumed to satisfy the evolution equations given by

$$\frac{dn_{ij}(t)}{dt} = \left(n_0 - n_{ij}(t)\right) K_{ijb} \quad , \tag{3.59}$$

where

$$K_{ijb} = \frac{1}{\tau_0} e^{\left(-\left(R-\beta\sigma_{ij}^{m}\right)\right)} \quad . \tag{3.60}$$

In Eq. (3.60) R and β represent material constants.

In the above, we are associating damage in the composite with the corresponding stress component seen by the *matrix* material. As a result, the damage tensor is symmetric due to the symmetry of the stress tensor. Within a finite element program, the degree of damage is calculated at every Gauss point. Once the degree of damage is known, the damage accumulation is used to control the degradation of elastic material properties.

Two-Constituent Damage (Unidirectional Composite)

For the transversely isotropic unidirectional composite, we assume that matrix damage accumulates in such a way that the composite and *in-situ* matrix both remain transversely isotropic. Therefore, the damage is expressed in terms of the transverse isotropic damage invariants given by:

$$\begin{aligned} I_1 &= n_{11} \ , \\ I_2 &= n_{22} + n_{33} \ , \\ I_3 &= n_{22}^2 + n_{33}^2 + 2n_{23}^2 \ , \\ I_4 &= n_{12}^2 + n_{13}^2 \ , \\ I_5 &= n_{22}n_{12}^2 + n_{33}n_{13}^2 + 2n_{12}n_{13}n_{23} \ . \end{aligned} \tag{3.60}$$

As damage in the matrix material accumulates, the matrix properties are degraded based on the damage invariants of Eq. (3.60) to reflect reduced stiffness. The development of the material property degradation models are described in detail by Schumacher (2002). For brevity, only the functional forms of the degradation models are presented here. The form of the material degradation for G_{12}^m and G_{13}^m are a function of I_4 and are given by:

$$G_{12}^{'m} = G_{12}^{m}\,(1-\sqrt{I_4})^{\alpha} \quad , \tag{3.61}$$

and

$$G'^{m}_{13} = G^{m}_{13} (1 - \sqrt{I_4})^{\alpha} \quad , \tag{3.62}$$

where the primed (') value denotes the degraded stiffness.

Motivation for the degradation models of Equations (3.61-62) is provided by recalling the fourth invariant from Eq. (3.60) which is a function of n_{12} and n_{13}. Hence, for the case of a longitudinal shear test where $\sigma_{13} \neq 0$, Eq. (3.62) becomes

$$G'^{m}_{13} = G^{m}_{13} (1 - n_{13})^{\alpha} . \tag{3.63}$$

Notice when $n_{13} = 0$, the material is undamaged whereas $n_{13} = 1$ would degrade the shear modulus completely.

Now consider the damage components n_{22} and n_{33} caused by transverse tensile stresses. The damage accumulation is assumed to affect the matrix elastic properties E^{m}_{22}, E^{m}_{33} and, in addition, G^{m}_{23} by transverse isotropy. Similarly, in the case of transverse shear, damage, n_{23}, is assumed to affect the matrix elastic properties G^{m}_{23}, E^{m}_{22}, and E^{m}_{33}.

The assumed form of material degradation of E^{m}_{22}, E^{m}_{33}, and G^{m}_{23} is shown below where the material degradation is only dependent upon I_3, i.e.,

$$E'^{m}_{22} = E^{m}_{22} \left(1 - \sqrt{\frac{I_3}{2}} \right)^{\alpha} \quad , \tag{3.64}$$

$$E'^{m}_{33} = E^{m}_{33} \left(1 - \sqrt{\frac{I_3}{2}} \right)^{\alpha} \quad , \tag{3.65}$$

and

$$G'^{m}_{23} = G^{m}_{23} \left(1 - \sqrt{\frac{I_3}{2}} \right)^{\alpha} \quad . \tag{3.66}$$

Finally it is noted that the composite stiffness properties must be degraded in a manner consistent with the constituents. A detailed discussion of the functional degradation of the composite properties is provided by Schumacher (2002). However it is worth noting that, under a three-dimensional state of stress, the transverse tension and transverse shear composite elastic material properties are degraded simultaneously, thereby preserving transverse isotropy while taking into account the directional damage dependence.

A final piece of the damage based modeling approach is the development of a macroscopic failure criterion. The damage interactive failure criterion developed is similar to the stress interactive failure criterion presented previously in the stress based binary degradation model. Specifically, the matrix failure criterion is given by:

$$^{\pm}A_2^m\left(I_2^m\right)^2 + A_3^m I_3^m + A_4^m I_4^m = 1 \quad , \tag{3.67}$$

where the invariants are *matrix damage invariants* for the matrix constituent.

In Eq. (3.67), the coefficients A_i^m are determined from unidirectional composite experimental stress-strain behavior in a manner identical to that presented by Mayes (1999). Therefore, a transverse tension loading case is used to determine the coefficient A_2^m, as

$$A_2^m = \frac{1}{\left(F_{22}^{22m} + F_{33}^{22m}\right)^2}\left(1 - \frac{\left(F_{22}^{22m}\right)^2 + \left(F_{33}^{22m}\right)^2}{2\left(F_{23}^m\right)^2}\right) \quad . \tag{3.68}$$

The coefficients F_{jj}^{iim} represent the critical values of damage as determined from stress-strain experimental data. The double subscript notation is necessary to identify the damage term and the loading direction. In particular, F_{33}^{22m} represents damage in the 33 direction as the result of a stress in the 22 direction. Such notation is necessary because the stress state in the matrix material is fully three-dimensional under uniaxial composite stress. For the case of out-of-plane shear (transverse shear), the critical coefficient A_3^m is given by

$$A_3^m = \frac{1}{2\left(F_{23}^m\right)^2} \quad . \tag{3.69}$$

In the case of in-plane shear (longitudinal shear), the calculation of the critical coefficient, A^m_4 is given by

$$A_4^m = \frac{1}{\left(F_{12}^m\right)^2} \quad . \tag{3.70}$$

Once a damage based failure has occurred, all matrix properties with a fiber bundle are set to near zero values at the failed Gauss points.

The final piece of the damage based failure criterion is that the fiber failure criterion for this approach is identical to that for the stress based approach presented previously. This is due to the fact that although the matrix constituent commonly accumulates damage in the form of microcracks in a composite material, the fibers in these systems are assumed to fail instantaneously.

Three-Constituent Damage (Woven Fabric Composites)

The previous damage model for unidirectional composites represents a critical component of the damage model for woven fabrics. Specifically, damage in the weave is attributed to matrix cracking occurring within the fiber bundles. Within the woven fabric composite, the fiber bundles are treated as unidirectional sub-composites. Accessing constituent information within a fiber bundle requires two additional MCT stress decomposition branches to be added to the weave analysis, as shown in Figure 3.12. Hence, the three-constituent MCT decomposition is executed first to generate stress/strain fields in the fiber bundles. The two-constituent MCT decomposition is then executed to determine the fiber and matrix stress and strain fields within a fiber bundle.

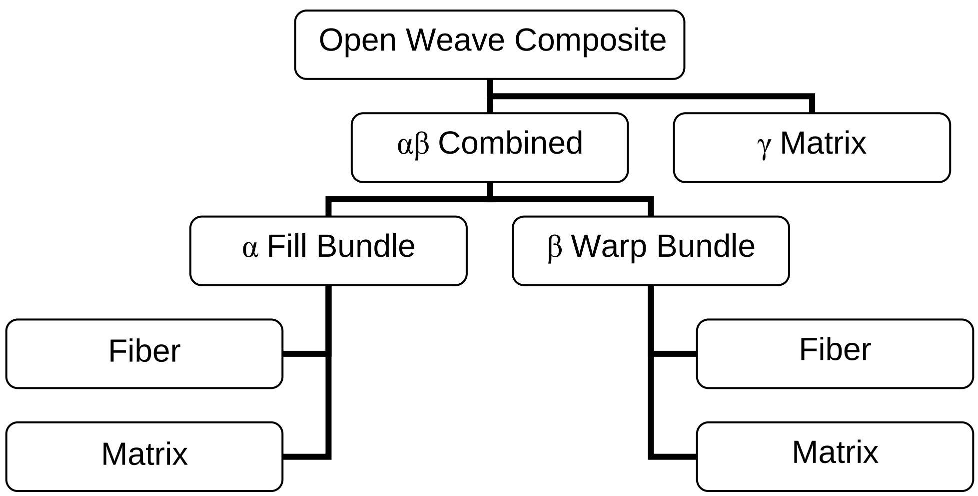


Figure 3.12 Three-constituent decomposition for damage modeling.

Because the fiber bundles within the woven fabric microstructure are considered to be individual unidirectional composites, the previously outlined failure criterion and degradation scheme are applicable. That is, Equations (3.60–3.70) are applied to the fiber bundles of the woven fabric composite material to predict the matrix failure within the bundles and matrix stiffness degradation according to the level of damage. Similar to the unidirectional composite, a functional relationship between the fiber bundle damage and the composite properties must be generated. The reader is referenced to Schumacher (2002) for a detail discussion of the functional degradation schemes for the woven fabric composite.

4. MODIFICATIONS FOR ANISOTROPIC MATERIALS

Due to the inherent anisotropy of composite materials, problems exist when modeling these materials in hydrocodes such as CTH where typical constitutive models consist of a spherical (pressure or change of volume) component determined with an equation of state and a deviatoric (shear, strength or change in shape) component. For an isotropic material the material pressure can be expressed as only a function of the spherical strain components. However, for an anisotropic material the pressure (spherical stress) is coupled with the deviatoric strain components. Therefore, modifications are needed in CTH for the EOS and deviatoric stress calculations to account for this coupling. The MCT model provides two different options to the user for incorporating this coupling. These include a method developed by Anderson *et al.* (1994) and a method by Lukyanov (2006, 2008). Both methods are available within the MCT model and are briefly outlined in the following sections.

4.1.Anderson Method

Anderson *et al.* (1994) developed anisotropic constitutive equations, which consider coupling between the pressure (EOS) and both spherical and deviatoric strains. He also developed relationships for the deviatoric stress that incorporate both the deviatoric and spherical strains. The development begins with the basic stress-strain relationship for orthotropic materials given by:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \begin{bmatrix} \frac{1-\nu_{yz}\nu_{zy}}{E_y E_z \Delta} & \frac{\nu_{yz}-\nu_{zx}\nu_{yz}}{E_y E_z \Delta} & \frac{\nu_{zx}-\nu_{yx}\nu_{xy}}{E_y E_z \Delta} & 0 & 0 & 0 \\ \frac{\nu_{yz}-\nu_{zx}\nu_{yz}}{E_y E_z \Delta} & \frac{1-\nu_{xz}\nu_{zx}}{E_x E_z \Delta} & \frac{\nu_{zy}-\nu_{xy}\nu_{zx}}{E_x E_z \Delta} & 0 & 0 & 0 \\ \frac{\nu_{zx}-\nu_{yx}\nu_{xy}}{E_y E_z \Delta} & \frac{\nu_{zy}-\nu_{xy}\nu_{zx}}{E_x E_z \Delta} & \frac{1-\nu_{xy}\nu_{yx}}{E_x E_y \Delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{xy} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{xz} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{yz} \end{bmatrix} * \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{yz} \end{bmatrix}, \tag{4.1}$$

where

$$\Delta = \frac{1-\nu_{xy}\nu_{yx}-\nu_{yz}\nu_{zy}-\nu_{zx}\nu_{xz}-2\nu_{yx}\nu_{zy}\nu_{xz}}{E_x E_y E_z}. \tag{4.2}$$

Deviatoric strains, ε_{ij}^{Dev}, can be expressed in terms of the total strain and the volumetric strain as:

$$\varepsilon_{ij}^{Dev} = \varepsilon_{ij} - \frac{D}{3}\delta_{ij} \tag{4.3}$$

where D is the trace of the total strain tensor.

Eq. (4.3) can now be solved for ε_{ij} and substituted into the right hand side of Eq. (4.1) to give a relationship between total stress and the spherical and deviatoric components of strain.

Recalling that the pressure in a material is defined as the mean stress we have the following relationship between pressure and stress:

$$P = -\frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right) \ . \tag{4.4}$$

Given the relationship for pressure of Eq. (4.4) and the σ_{xx} , σ_{yy} and σ_{zz} terms of Eq. (4.1) with Eq. (4.3) substituted in, one arrives at the following relationship for pressure as a function of both spherical and deviatoric strains.

$$\begin{aligned}
P = &-\frac{1}{9\Delta}\left\{\frac{1-\nu_{yz}\nu_{zy}}{E_y E_z} + \frac{1-\nu_{xz}\nu_{zx}}{E_x E_z} + \frac{1-\nu_{xy}\nu_{yx}}{E_x E_y} + 2\left(\frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_y E_z} + \frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_y E_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_x E_z}\right)\right\} D \\
&-\frac{1}{3\Delta}\left\{\frac{1-\nu_{yz}\nu_{zy}}{E_y E_z} + \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_y E_z} + \frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_y E_z}\right\}\varepsilon_{xx}^{Dev} \\
&-\frac{1}{3\Delta}\left\{\frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_y E_z} + \frac{1-\nu_{xz}\nu_{zx}}{E_x E_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_x E_z}\right\}\varepsilon_{yy}^{Dev} \\
&-\frac{1}{3\Delta}\left\{\frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_y E_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_x E_z} + \frac{1-\nu_{xy}\nu_{yx}}{E_x E_y}\right\}\varepsilon_{zz}^{Dev}
\end{aligned} \tag{4.5}$$

It is important to note here that if you have an isotropic material where all E and υ values are the same this relationship reduces to the traditional isotropic relationship relating pressure to volumetric strain through the bulk modulus.

Similar to Eq. (4.3), the total stress can be expressed in terms of a hydrostatic (pressure) component and a deviatoric (s) component as:

$$\sigma_{ij} = -P\delta_{ij} + s_{ij} \ . \tag{4.6}$$

By substituting Eq. (4.5), Eq. (4.1) and Eq. (4.3) into Eq. (4.6), the following relationships between deviatoric stress (s) and spherical and deviatoric strains are developed:

$$s_{xx} = \frac{1}{9\Delta}\left\{\frac{2(1-\nu_{yz}\nu_{zy})}{E_yE_z} - \frac{1-\nu_{xz}\nu_{zx}}{E_xE_z} - \frac{1-\nu_{xy}\nu_{yx}}{E_xE_y} + \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z} + \frac{\nu_{zx}+\nu_{yx}\nu_{xy}}{E_yE_z} - \frac{2(\nu_{zy}+\nu_{xy}\nu_{zx})}{E_xE_z}\right\}D$$
$$-\frac{1}{3\Delta}\left\{\frac{-2(1-\nu_{yz}\nu_{zy})}{E_yE_z} + \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z} + \frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_yE_z}\right\}\varepsilon_{xx}^{Dev}$$
$$-\frac{1}{3\Delta}\left\{\frac{-2(\nu_{yx}+\nu_{zx}\nu_{yz})}{E_yE_z} + \frac{1-\nu_{xz}\nu_{zx}}{E_xE_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z}\right\}\varepsilon_{yy}^{Dev} \quad (4.7a)$$
$$-\frac{1}{3\Delta}\left\{\frac{-2(\nu_{zx}+\nu_{yx}\nu_{zy})}{E_yE_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z} + \frac{1-\nu_{xy}\nu_{yx}}{E_xE_y}\right\}\varepsilon_{zz}^{Dev}$$

$$s_{yy} = \frac{1}{9\Delta}\left\{-\frac{1-\nu_{yz}\nu_{zy}}{E_yE_z} + \frac{2(1-\nu_{xz}\nu_{zx})}{E_xE_z} - \frac{1-\nu_{xy}\nu_{yx}}{E_xE_y} + \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z} + \frac{2(\nu_{zx}+\nu_{yx}\nu_{xy})}{E_yE_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z}\right\}D$$
$$-\frac{1}{3\Delta}\left\{\frac{1-\nu_{yz}\nu_{zy}}{E_yE_z} - \frac{2(\nu_{yx}+\nu_{zx}\nu_{yz})}{E_yE_z} + \frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_yE_z}\right\}\varepsilon_{xx}^{Dev} \quad (4.7b)$$
$$-\frac{1}{3\Delta}\left\{\frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z} - \frac{2(1-\nu_{xz}\nu_{zx})}{E_xE_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z}\right\}\varepsilon_{yy}^{Dev}$$
$$-\frac{1}{3\Delta}\left\{\frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_yE_z} - \frac{2(\nu_{zy}+\nu_{xy}\nu_{zx})}{E_xE_z} + \frac{1-\nu_{xy}\nu_{yx}}{E_xE_y}\right\}\varepsilon_{zz}^{Dev}$$

$$s_{yy} = \frac{1}{9\Delta}\left\{-\frac{1-\nu_{yz}\nu_{zy}}{E_yE_z} - \frac{1-\nu_{xz}\nu_{zx}}{E_xE_z} + \frac{2(1-\nu_{xy}\nu_{yx})}{E_xE_y} - \frac{2(\nu_{yx}+\nu_{zx}\nu_{yz})}{E_yE_z} + \frac{\nu_{zx}+\nu_{yx}\nu_{xy}}{E_yE_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z}\right\}D$$
$$-\frac{1}{3\Delta}\left\{\frac{1-\nu_{yz}\nu_{zy}}{E_yE_z} + \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z} - \frac{2(\nu_{zx}+\nu_{yx}\nu_{zy})}{E_yE_z}\right\}\varepsilon_{xx}^{Dev} \quad (4.7c)$$
$$-\frac{1}{3\Delta}\left\{\frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z} + \frac{1-\nu_{xz}\nu_{zx}}{E_xE_z} - \frac{2(\nu_{zy}+\nu_{xy}\nu_{zx})}{E_xE_z}\right\}\varepsilon_{yy}^{Dev}$$
$$-\frac{1}{3\Delta}\left\{\frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_yE_z} + \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z} - \frac{2(1-\nu_{xy}\nu_{yx})}{E_xE_y}\right\}\varepsilon_{zz}^{Dev}$$

$$s_{xy} = 2G_{xy}\varepsilon_{xy}^{Dev} = 2G_{xy}\varepsilon_{xy} \quad (4.7d)$$

$$s_{xz} = 2G_{xz}\varepsilon_{xz}^{Dev} = 2G_{xz}\varepsilon_{xz} \quad (4.7e)$$

$$s_{yz} = 2G_{yz}\varepsilon_{yz}^{Dev} = 2G_{yz}\varepsilon_{yz} \quad (4.7f)$$

Eq. (4.7) is implemented into the MCT constitutive model within CTH to provide the appropriate anisotropic deviatoric stresses for use in strength predictions.

Anderson also made adjustments to the EOS model within CTH when it is used with the anisotropic MCT model to account for the coupling of pressure and deviatoric strains. The adjustments are made to the Mie-Gruneisen form of the EOS, which is traditionally written as:

$$P = P_H\left(1+\frac{\Gamma}{2}\mu\right)+\frac{\Gamma}{V}E \quad . \tag{4.8}$$

Adjustments are needed in Eq. (4.8) because this form of the EOS does not provide the coupling between the pressure and the deviatoric strains. In order to adjust the pressure to account for the contribution of the deviatoric strains in an anisotropic material, Eq. (4.5) is utilized. Anderson simply adds the deviatoric strain contribution developed for Eq. (4.5) to the Mie-Gruneisen EOS, resulting in the following modified EOS, which can be utilized within the MCT model:

$$\begin{aligned} P = P_H\left(1+\frac{\Gamma}{2}\mu\right)+\frac{\Gamma}{V}E &- \frac{1}{3\Delta}\left\{\frac{1-\nu_{yz}\nu_{zy}}{E_yE_z}+\frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z}+\frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_yE_z}\right\}\varepsilon_{xx}^{Dev} \\ &- \frac{1}{3\Delta}\left\{\frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{E_yE_z}+\frac{1-\nu_{xz}\nu_{zx}}{E_xE_z}+\frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z}\right\}\varepsilon_{yy}^{Dev} \\ &- \frac{1}{3\Delta}\left\{\frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{E_yE_z}+\frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{E_xE_z}+\frac{1-\nu_{xy}\nu_{yx}}{E_xE_y}\right\}\varepsilon_{zz}^{Dev} \end{aligned} \tag{4.9}$$

The reader is referred to Anderson (1994) for further details and derivations of the method outlined herein.

4.2. Lukyanov Method

Lukyanov's (2006, 2008) work initially follows the work of Anderson but modifies the pressure term for an anisotropic material to be consistent with an isotropic material. In other words, Lukyanov's work focuses on the fact that a hydrostatic pressure applied to an anisotropic material results in an anisotropic state of strain, which is inconsistent with the definition of "generalized pressure". To be mathematically consistent with the "generalized pressure", Lukyanov modified the pressure term for an anisotropic material so that it only causes a change in volume with no corresponding change in shape.

In order for the pressure applied to an anisotropic material to cause only a change in volume and not a change in shape, the pressure must be described in terms of a tensor. Eq. (4.10) gives the total pressure tensor in terms of a constant (p*) and a tensor α_{ij}.

$$\widetilde{P} = -p^*\alpha_{ij} \tag{4.10}$$

In Eq. (4.10) α_{ij} is defined such that the diagonal terms define the vector for the direction of the pressure and the cross-terms are zero. In expanded matrix form α_{ij} looks like:

$$\alpha_{ij} = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} \tag{4.11}$$

where

$$\alpha_{ij} = C_{lk}\delta_{kk}\delta_{il}\delta_{lj} \cdot 3\overline{K}_c, \tag{4.12}$$

$$\overline{K}_c = \frac{1}{\sqrt{3 \cdot \left[\left(C_{11} + C_{12} + C_{13} \right)^2 + \left(C_{12} + C_{22} + C_{23} \right)^2 + \left(C_{13} + C_{23} + C_{33} \right)^2 \right]}} \tag{4.13}$$

and

$$K_c = \frac{1}{9\overline{K}_c} \quad . \tag{4.14}$$

Keeping the requirement of mathematically consistent pressure, the deviatoric stress must be independent of the pressure. Therefore, their contraction product must be zero.

$$p^* \alpha_{ij} s_{ij} = 0 \quad \text{or} \quad \alpha_{ij} s_{ij} = 0 \tag{4.15}$$

Using this relationship and Eq. (4.6), an expression for the generalized deviatoric part of the stress tensor can be written as follows:

$$s_{ij} = \sigma_{ij} - \alpha_{ij} \cdot \frac{1}{\|\alpha\|} \sigma_{kl} \alpha_{kl} \quad . \tag{4.16}$$

Eq. (4.16) is implemented into the MCT constitutive model within CTH to provide the appropriate anisotropic deviatoric stresses for use in damage and failure predictions.

Lukyanov next breaks the total pressure, p*, into two components. The first component, which is coupled directly with the spherical strain, is derived by first stating Hooke's Law with a decomposed strain tensor as:

$$\sigma_{ij} = C_{ijkl}\delta_{kl}\frac{1}{3}D + C_{ijkl}\varepsilon_{kl}^{Dev} = K_c D \cdot \alpha_{ij} + C_{ijkl}\varepsilon_{kl}^{Dev} \tag{4.17}$$

where

$$p = -K_c D \quad . \tag{4.18}$$

Multiplying Eq. (4.17) by the tensor β_{ij}, which has the same form as α_{ij}, but uses the compliance terms rather than the stiffness terms, and assuming $\beta_{ij} C_{ijkl} \varepsilon_{kl}^{Dev} = 0$, one arrives at the following expression for the component of the pressure that is coupled with the spherical strains:

$$p = -\frac{\beta_{ij}\sigma_{ij}}{\beta_{ij}\alpha_{ij}}. \tag{4.19}$$

Where p is a part of the "generalized" pressure and is directly linked to the spherical part of the strain tensor.

Decomposing the total stress tensor of Eq. (4.19) into a "generalized" pressure component and a deviatoric stress component and solving for the "generalized" pressure results in the following equation:

$$p^* = p + \frac{\beta_{ij} s_{ij}}{\alpha_{ij}\beta_{ij}}. \tag{4.20}$$

This new definition of "generalized" pressure reduces to the standard definition of pressure under the assumption of isotropy.

In order to provide an accurate description of the pressure for anisotropic materials at high pressures such as those under a shock loading, the appropriate EOS must be substituted. For the case of the MCT model within CTH, a Mie-Gruniessen form of the EOS is used, resulting in a material pressure that has the following form:

$$p^* = p^{EOS} + \frac{\beta_{ij} s_{ij}}{\alpha_{ij}\beta_{ij}} \tag{4.21}$$

where P^{EOS} is the standard Mie-Gruneisen relationship given in Eq. (4.8).

The reader is reference to Lukyanov (2006, 2008) for further details and derivation of the method outlined herein.

4.3. Using the Equation of State Coupling Methods

Setting the MCTES keyword performs the selection of the Anderson or Lukanov methods. When the MCTES keyword is set to 1, the Lukanov method is used and when set to 2, the Anderson method is used. By default the Lukanov method is used and an example is shown below in Figure 4.1.

epdata
matep=1 mct visc_2_dam_f mctes=2 *overrides the default to the Anderson method
ende

Figure 4.13 Equation of state coupling methods input example.

5. CTH MODIFICATIONS

This section outlines the modifications made to existing subroutines added to the CTH library of codes for the implementation of the MCT model, layering capability and interstitial layer.

Include m07p0a.h (modified)
Added layering capability and interstitial layer
Include m07p1.h (modified)
Added layering capability and interstitial layer
Include m07p3.h (modified)
Added layering capability and interstitial layer
Include m26p0a.h (new)
MCT model
Include m26p0b.h (new)
MCT model
Include m26p0c.h (new)
MCT model
Include m26p0d.h (new)
MCT model
Include m26p1.h (new)
MCT model
Include m26p2.h (new)
MCT model
Include m26p3.h (new)
MCT model
Subroutine COMPOSITE (new)
Layering routine for non cylindrical objects
Subroutine COMPOSITE_WRAP (new)
Layering routine for cylindrical objects
Subroutine COMPOSITE_MODULE (new)
Module for data storage of necessary info for the layering capability
Subroutine CUTCELLS_MODULE (new)
Routine for cutting CTH cell for layer volume fractions
Subroutine DBMWTF (modified)
Added calls for MCTRTR and MCTRTW for restart capability
Subroutine DIATOM_COMP (new)
This is the data interface from CTH to Diatom. (moving data from C to FORTRAN and vice versa "DIMCOMP")
Subroutine DIATOM_PARSE_INPUT (modified)
Added lookup for cmp_rotate and cmp_wrap
Subroutine DIATOM_VARIABLE (modified)
Added layering capability variables
Subroutine DIATOM_VOLUME_FRACTION (modified)
Added layering capability
Subroutine DIOM_DBASE (modified)

This routine contains the code that sets the extra variables for the layering capability through Diatom

Subroutine DIOM_GLOBALS (modified)
Added variables for laying capability

Subroutine ELEB (modified)
Added call to UINCMP

Subroutine ELSG (modified)
Added MCT model (m26p3.h)

Subroutine ELSGD (modified)
Added necessary room for MCT and TI model scratch storage

Subroutine EOS (modified)
Added NMAT to EOSCPT call

Subroutine EOSCCE (new)
EOS to strength coupling capability for orthotropic materials

Subroutine EOSCCII (new)
EOS to strength coupling capability for orthotropic materials

Subroutine EOSCCT (new)
EOS to strength coupling capability for orthotropic materials

Subroutine EOSCCVV (new)
EOS to strength coupling capability for orthotropic materials

Subroutine EOSCCX (new)
EOS to strength coupling capability for orthotropic materials

Subroutine EOSCPE (modified)
Added call to EOSCCE

Subroutine EOSCPI (modified)
Added call to EOSCCII

Subroutine EOSCPK (modified)
Added call to EOSCVV

Subroutine EOSCPT (modified)
Added call to EOSCCT

Subroutine EOSCPX (modified)
Added call to EOSCCX

Subroutine EOSMRE (modified)
Added NMAT to EOSCPE call

Subroutine EREB (modified)
Added fracturing capability for MCT. Currently commented out since fracture is handled through FRACSP.

Subroutine ERPHE (modified)
Added routine call for MCTNORM

Subroutine FRACSP (modified)
Added fracturing capability for MCT

Subroutine MATFRM (modified)
Updated scratch storage arrays

Subroutine MATFRM_LAYER (new)
This routine is similar to MATFRM, but for the layering capability. This routine has been simplified for a general rotation case rather than just the transverse isotropic case.

Subroutine MCT (new)
MCT model
Subroutine MCT_MODULE (new)
MCT model
Subroutine MCTCHK (new)
MCT model
Subroutine MCTDRV (new)
MCT model
Subroutine MCTEXV (new)
MCT model
Subroutine MCTNORM (new)
MCT model
Subroutine MCTRTR (new)
MCT model
Subroutine MCTRTW (new)
MCT model
Subroutine MCTSP (new)
MCT model
Subroutine PLANE_GEOMETRY_MODULE (new)
Module for general analytical geometry capabilities
Subroutine POLYHEDRA_MODULE (new)
Module for determining the cutting points for the cut_cell routine
Subroutine TICHK (modified)
Added layering capability and interstitial layer
Subroutine TIDRV (modified)
Added layering capability and interstitial layer
Subroutine TIEXV (modified)
Added layering capability and interstitial layer
Subroutine TRISO (modified)
Added layering capability and interstitial layer
Subroutine UINDIM (modified)
Added layering capability and interstitial layer
Subroutine UINEP (modified)
Added MCT model (m26p0a.h, m26p0b.h, m26p0c.h and m26p0d.h)
Subroutine UINCMP (new)
Routine for reading in the composite input based on keywords
Subroutine UINCHK (modified)
Added MCT model (m26p1.h)
Subroutine UINISV (modified)
Added MCT model (m26p2.h)

6. USE OF THE MODEL

6.1. MCT Extra Variables

MCT requires many extra variables including variables to store constituent stress and strain, failure states, damage levels, layer orientations, plastic strains, and yield surfaces. Table lists all extra variables for the MCT model that a user may wish to post-process. The number of extra variables allocated varies depending on the MCT analysis type. Additional extra variables are allocated as required for calculation purposes, but are not listed in Table as they provide no relevant information to the user.

Table 6.1 Extra variables associated with the MCT model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Composite Failure State	FAILURE	X		Saved for MCT model
Cell Center X Location	MCTCEN_1	X		Saved for MCT model
Cell Center Y Location	MCTCEN_2	X		Saved for MCT model
Cell Center Z Location	MCTCEN_3	X		Saved for MCT model
Material Orientation Vector 1-Component	MC1%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Material Orientation Vector 2-Component	MC2%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Material Orientation Vector 3-Component	MC3%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Lamina Failure State	COMP_FAILURE%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 11 – Strain	STN01%%## (%% = layer number) (## = material number)		X	Saved for MCT model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Composite 22 – Strain	STN02%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 33 – Strain	STN03%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 12 – Strain	STN04%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 13 – Strain	STN05%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 23 – Strain	STN06%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Lamina Failure	COMPFAIL%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 11 – Strain	STN07%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 22 – Strain	STN08%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 33 – Strain	STN09%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 12 – Strain	STN10%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 13 – Strain	STN11%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 23 – Strain	STN12%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Matrix (2-cons) or Fill Bundle (3-cons) 11 – Strain	STN13%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 22 – Strain	STN14%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 33 – Strain	STN15%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 12 – Strain	STN16%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 13 – Strain	STN17%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 23 – Strain	STN18%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Pure Resin (3-cons) 11 – Strain	STN19%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 22 – Strain	STN20%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 33 – Strain	STN21%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 12 – Strain	STN22%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 13 – Strain	STN23%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 23 – Strain	STN24%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Warp Bundle Fiber 11 -Strain	STN25%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 22 –Strain	STN26%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 33 –Strain	STN27%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 12 –Strain	STN28%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 13 –Strain	STN29%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 23 -Strain	STN30%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 11 – Strain	STN31%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 22 – Strain	STN32%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 33 – Strain	STN33%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 12 – Strain	STN34%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 13 – Strain	STN35%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 23 – Strain	STN36%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 11 -Strain	STN37%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 22 –Strain	STN38%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Fill Bundle Fiber 33 –Strain	STN39%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 12 -Strain	STN40%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 13 –Strain	STN41%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 23 -Strain	STN42%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 11 – Strain	STN43%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 22 – Strain	STN44%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 33 – Strain	STN45%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 12 – Strain	STN46%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 13 – Strain	STN47%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 23 – Strain	STN48%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Composite 11 – Stress	STS01%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 22 – Stress	STS02%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 33 – Stress	STS03%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 12 – Stress	STS04%%## (%% = layer number) (## = material number)		X	Saved for MCT model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Composite 13 – Stress	STS05%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Composite 23 – Stress	STS06%%## (%% = layer number) (## = material number)		X	Saved for MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 11 – Stress	STS07%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 22 – Stress	STS08%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 33 – Stress	STS09%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 12 – Stress	STS10%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 13 – Stress	STS11%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Fiber (2-cons) or Warp Bundle (3-cons) 23 – Stress	STS12%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 11 – Stress	STS13%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 22 – Stress	STS14%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 33 – Stress	STS15%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Matrix (2-cons) or Fill Bundle (3-cons) 12 – Stress	STS16%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 13 – Stress	STS17%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Matrix (2-cons) or Fill Bundle (3-cons) 23 – Stress	STS18%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT model
Pure Resin (3-cons) 11 – Stress	STS19%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 22 – Stress	STS20%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 33 – Stress	STS21%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 12 – Stress	STS22%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 13 – Stress	STS23%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Pure Resin (3-cons) 23 – Stress	STS24%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT model
Warp Bundle Fiber 11 -Stress	STS25%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 22 –Stress	STS26%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 33 –Stress	STS27%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Warp Bundle Fiber 12 –Stress	STS28%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 13 –Stress	STS29%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Fiber 23 -Stress	STS30%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 11 – Stress	STS31%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 22 – Stress	STS32%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 33 – Stress	STS33%%## \(%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 12 – Stress	STS34%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 13 – Stress	STS35%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Warp Bundle Matrix 23 – Stress	STS36%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 11 -Stress	STS37%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 22 –Stress	STS38%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 33 –Stress	STS39%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 12 -Stress	STS40%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Fiber 13 –Stress	STS41%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Fill Bundle Fiber 23 -Stress	STS42%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 11 – Stress	STS43%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 22 – Stress	STS44%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 33 – Stress	STS45%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 12 – Stress	STS46%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 13 – Stress	STS47%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Fill Bundle Matrix 23 – Stress	STS48%%## (%% = layer number) (## = material number)		X	Saved only for 3 cons MCT Damage model
Damage Value in 11-direction in Matrix (2-cons) or Warp Bundle (3-cons)	N11%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT Damage model
Damage Value in 22-direction in Matrix (2-cons) or Warp Bundle (3-cons)	N22%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT Damage model
Damage Value in 33-direction in Matrix (2-cons) or Warp Bundle (3-cons)	N33%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT Damage model
Damage Value in 12-direction in Matrix (2-cons) or Warp Bundle (3-cons)	N12%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT Damage model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Damage Value in 13-direction in Matrix (2-cons) or Warp Bundle (3-cons)	N13%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT Damage model
Damage Value in 23-direction in Matrix (2-cons) or Warp Bundle (3-cons)	N23%%## (%% = layer number) (## = material number)		X	Saved for 2 and 3 cons MCT Damage model
Damage Value in 11-direction of the Fill Bundle	N11R%%## (%% = layer number) (## = material number)		X	Saved for 3 cons MCT Damage model
Damage Value in 22-direction of the Fill Bundle	N22R%%## (%% = layer number) (## = material number)		X	Saved for 3 cons MCT Damage model
Damage Value in 33-direction of the Fill Bundle	N33R%%## (%% = layer number) (## = material number)		X	Saved for 3 cons MCT Damage model
Damage Value in 12-direction of the Fill Bundle	N12R%%## (%% = layer number) (## = material number)		X	Saved for 3 cons MCT Damage model
Damage Value in 13-direction of the Fill Bundle	N13R%%## (%% = layer number) (## = material number)		X	Saved for 3 cons MCT Damage model
Damage Value in 23-direction of the Fill Bundle	N23R%%## (%% = layer number) (## = material number)		X	Saved for 3 cons MCT Damage model
Total Flow Strain in the 11 - direction	TSTNFLW_1%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain in the 22 - direction	TSTNFLW_2%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain in the 33 - direction	TSTNFLW_3%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model

Variable	SPYMASTER/SPYHIS Variable Name	Cell Variable	Layer/ Material Variable	Remarks
Total Flow Strain in the 12 - direction	TSTNFLW_4%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain in the 13 - direction	TSTNFLW_5%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain in the 23 - direction	TSTNFLW_6%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain Matrix 11	TSTNFLW_7%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain Matrix 22	TSTNFLW_8%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain Matrix 33	TSTNFLW_9%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain Matrix 12	TSTNFLW_10%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain Matrix 13	TSTNFLW_11%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Total Flow Strain Matrix 23	TSTNFLW_12%%## (%% = layer number) (## = material number)		X	Saved for MCT Visco Model
Yield Strength for the SEP interlaminar Ply	MCTSEPYLD%%## (%% = layer number) (## = material number)		X	Saved for MCT-SEP Model
Plastic Strain for the SEP interlaminar Ply	MCTPLSSTN%%## (%% = layer number) (## = material number)		X	Saved for MCT-SEP Model

6.2. MCT Failure States

The user can post-process the failure state of the MCT material either at the cell level (FAILURE) or at the lamina level (COMP_FAILURE). A description of the failure state that can occur for the two- and three-constituent MCT analyses are shown in Table 6.2, Table 6.3, and Table 6.4. The user can also post-process the damage level of the resin constituent if the damage option is selected for the MCT model. The resin damage level is stored in the N## (two-and three-constituents) and N##R (three-constituents only) extra variables and can be post-processed along with the discrete failure state.

Table 6.2 Failure states for a two-constituent (unidirectional) analysis.

Failure Description	***Failure State #***
Unfailed Composite	1
Failed Resin	2
Failed Composite (fiber and resin failure)	3

Table 6.3 Failure states for a three-constituent (weave) analysis with no damage.

Failure Description	***Failure State #***
Unfailed Composite	1
Resin Failure in Warp Bundle	2
Resin Failure in Fill Bundle	3
Warp Bundle Failure	4
Fill Bundle Failure	5
Resin Failure in Warp and Fill Bundle	6
Failed Composite	7

Table 6.4 Failure states for a three-constituent (weave) analysis with damage.

Failure Description	***Failure State #***
Unfailed Composite	1
Resin Failure in Warp Bundle	2
Resin Failure in Fill Bundle	3
Warp Bundle Failure	4
Fill Bundle Failure	5
Resin Failure in Warp and Fill Bundle	6
Resin Failure in Warp Bundle and Complete Failure in Fill Bundle	7
Resin Failure in Fill Bundle and Complete Failure in Warp Bundle	8
Failed Composite	9

6.3. CTH Input

For each material represented by the MCT model, the following line must appear after the associated MATEP identifier in the EPDATA input block for CTH:

mct cmat

where, *cmat* is a material identifier listed in Table 6.5. By specifying a MCT material (*cmat*) in the EPDATA input set, the code will automatically read the separate file, VP_data, containing the appropriate control parameters for the material.

Table 6.5 Material identifiers for the MCT model.

Material Name	***cmat***
2-Constituent (Unidirectional Composite) Elastic Analysis	ELAS_2
2-Constituent (Unidirectional Composite) Elastic Analysis with Failure	ELAS_2_F
2-Constituent (Unidirectional Composite) Elastic Analysis with Continuous Damage and Failure	ELAS_2_DAM_F
2-Constituent (Unidirectional Composite) Viscoelastic Analysis	VISC_2
2-Constituent (Unidirectional Composite) Viscoelastic Analysis with Failure	VISC_2_F
2-Constituent (Unidirectional Composite) Viscoelastic Analysis with Continuous Damage and Failure	VISC_2_DAM_F
3-Constituent (Woven Fabric Composite) Elastic Analysis	ELAS_3

3-Constituent (Woven Fabric Composite) Elastic Analysis with Failure	ELAS_3_F
3-Constituent (Woven Fabric Composite) Elastic Analysis with Continuous Damage and Failure	ELAS_3_DAM_F

If either the MCT damage or failure options are specified for the MCT model, then the PFRAC parameter is reset internally to CTH to an artificial value. Fracture is then controlled by the material failure parameters in the *zmctdata* file and the specified criterion. Once fracture is determined through the MCT model, the CTH routines insert void volume into the cell to account for fracture and void growth. Models not using failure or damage use the CTH PFRAC value as inputted by the user.

6.4. Material Data File (zmctdata#)

Use of the MCT model within CTH requires that the user provide a formatted material data file that contains the appropriate material parameters needed for the MCT analysis. The material data file should reside in the same directory as the model input deck. The material data file should be named according to the material number from the CTH input deck that corresponds with the MCT material. For example, if the MCT material in the CTH model is material = 3, then the material data file should be named *zmctdata3.mct*.

The material parameters contained in the MCT material data file are generated from micromechanics models and experimental test data. The user is cautioned that editing any properties in a *zmctdata* file without extensive knowledge of the file itself and how the properties were generated could result in inconsistent composite and constituent material properties and hence, incorrect results. The user should contact Shane Schumacher (scschum@sandia.gov, 505-284-0610) to obtain MCT material data files for composite material systems of interest.

Appendix A provides a detailed description of the MCT material data file for both two- and three-constituent materials.

7. EXAMPLE

The sample problem below is a 2D example problem with a thin metal plate impacting an oblique composite material. The composite material is modeled using the MCT model. The composite laminate consists of 4 lamina. Results are shown below.

Note: The material used to run this problem is fictitious.

```
************************************************************************
*eor*cthin
*
 2D MCT Oblique Impact Problem
*
************************  control block  ****************************
control
  tstop = 50.0e-6
  mmp0
  ntbad 99999999
endcontrol
*
*  start of geometry block 1
*
mesh
  block 1  geom=2dr    type=e

      x0  -5.0
      x1  n=200  dxf=.05              w=10.0
    endx

      y0   0.00
      y1  n=200  dyf=.05              w=10.0
    endy

      xact =  -0.5, 0.5
      yact =  4.1, 4.2

  endb
*
endmesh                  *   end geometry blocks
*
*--------------------------------------------------------------------
*
spy
```

```
Save("VOLM,P,M,PM,T,TM,EM,MC112,MC212,MC122,MC222,MC132,MC232,MC142,MC
242,VY,COMPFAIL12");
 SaveTime(0, 1e-6);
 PlotTime(0, 1e-6);

 define main()
 {
   pprintf(" PLOT: Cycle=%d, Time=%e\n",CYCLE,TIME);
   MatColors(LIGHT_STEEL_BLUE,NAVAJO_WHITE);
   MatNames("Steel","Composite");

   XLimits(-5,5);
   YLimits(0,10);

   ColorMapClipping(ON,OFF);

   Image("zfiberlam_mat_vector1",WHITE,BLACK);
    Window(0,0,0.95,1);
    FontSize(0.04);
    Label("MCT Material Vector Plot");
    Plot2DMats;
    VectorPlot2D("MC112","MC212",0.25,3,0.7);
    Draw2DMatContour;
    Draw2DTracers(3);
    FontAlignment(RIGHT,BOTTOM);
   EndImage;

   Image("zfiberlam_mat_vector2",WHITE,BLACK);
    Window(0,0,0.95,1);
    FontSize(0.04);
    Label("MCT Material Vector Plot");
    Plot2DMats;
    VectorPlot2D("MC122","MC222",0.25,3,0.7);
    Draw2DMatContour;
    Draw2DTracers(3);
    FontAlignment(RIGHT,BOTTOM);
   EndImage;

   Image("zfiberlam_mat_vector3",WHITE,BLACK);
    Window(0,0,0.95,1);
    FontSize(0.04);
    Label("MCT Material Vector Plot");
    Plot2DMats;
    VectorPlot2D("MC132","MC232",0.25,3,0.7);
    Draw2DMatContour;
```

```
   Draw2DTracers(3);
   FontAlignment(RIGHT,BOTTOM);
  EndImage;

  Image("zfiberlam_mat_vector4",WHITE,BLACK);
   Window(0,0,0.95,1);
   FontSize(0.04);
   Label("MCT Material Vector Plot");
   Plot2DMats;
   VectorPlot2D("MC142","MC242",0.25,3,0.7);
   Draw2DMatContour;
   Draw2DTracers(3);
   FontAlignment(RIGHT,BOTTOM);
  EndImage;

 }
 SaveHis("VMAG");
 SaveTracer(ALL);
 HisCycle(0,1);

 define spyhis_main()
 {
  HisLoad(1,"hscth");
  HisImageName("impactor_velocity");
  Label("Velocity Magnitude at Tracer 1");
  TPlot("VMAG.1",1,AUTOSCALE);
 }
endspy
*
*--------------------------------------------------------------------
*
diatom
    package 'Fiber'
      material 2
      numsub 10
      insert uds
        p1 = -5.0, 0.0
        p2 = 5.0, 10.0
        p3 = 5.0, 11.414
        p4 = -5.0, 1.414
      endinsert
      cmp_rotate
    endpackage
    package 'Steel Disk'
      material 1
      numsub 10
```

```
      yvel 1.5e5
      insert box
        p1 = -.5, 3.8
        p2 = 0.5, 4.3
      endinsert
    endpackage
 enddiatom
*
 composite
  compmat = 2
  layers = 4
  origin = -5.0, 0.0, 0.0
  pt3 = -5.707106, 0.707106, 0.0
  pt13 = 1.0, 0.1, 0.0
  lamina = 1
  axisp = 3
  anglep = 0
  axisdp = 2
  angledp = 0
  thickness = 0.25
  lamina = 2
  axisp = 3
  anglep = 0
  axisdp = 2
  angledp = 90
  thickness = 0.25
  lamina = 3
  axisp = 3
  anglep = 0
  axisdp = 2
  angledp = 0
  thickness = 0.25
  lamina = 4
  axisp = 3
  anglep = 0
  axisdp = 2
  angledp = 90
  thickness = 0.25
 endcomposite
*
***************
eos
  mat2 cceos plaminate
  mat1 sesame RHA
endeos
***************
```

```
epdata
  mix=3
  matep=2 mct visc_2_dam_f
  matep=1  yield=2.758e9  poisson=0.27
ende
*
tracer
 add 0.0 4.05
endtracer
*
convct
   convection = 1
   interface = smyra
endc
*
edit
  shortt
    time = 0.0   , dt = 3.0e-6
  ends

  longt
    time = 0.0   , dt = 1.0
  endl
ende
*  spall parameters
 fracts
   pfrac2 -20e19
   pfrac1 -10.e9
   pfmix  -20e19
   pfvoid -20e19
 endf
*
boundary
   bhydro
     bl 1
       bxbot = 1 , bxtop = 1
       bybot = 2 , bytop = 2
     endb
   endh
endb
*
mindt
   time = 0.  dt = 1.e-10
endm
```

The results shown below summarize the deformation of the composite material. In Figures 7.11-14, the deformation is shown where the separate lamina. In each particular case the material direction is plotted for each individual lamina. This provides two main forms of information: the first is to ensure the location of each lamina is correct and the second is tracking the lamina deformations. The result shows the fragmentation of the composite and the location of each lamina fragment.

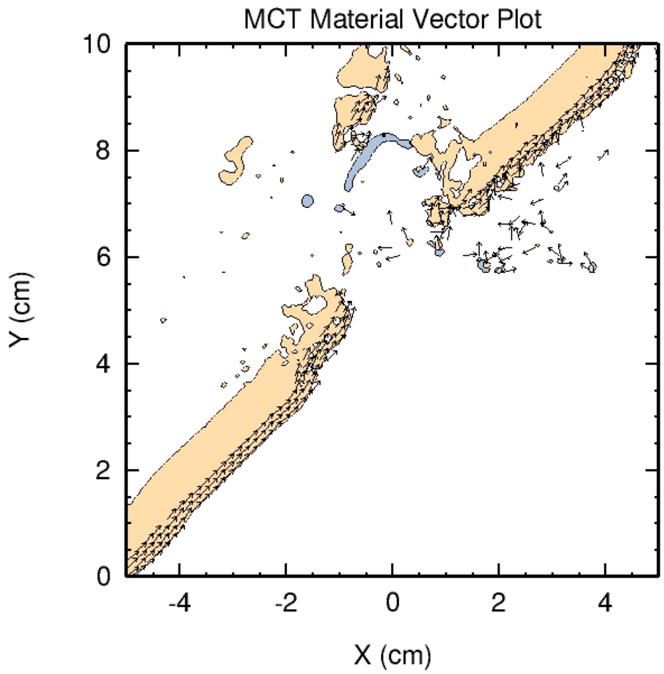


Figure 7.14 Layer 1 deformation.

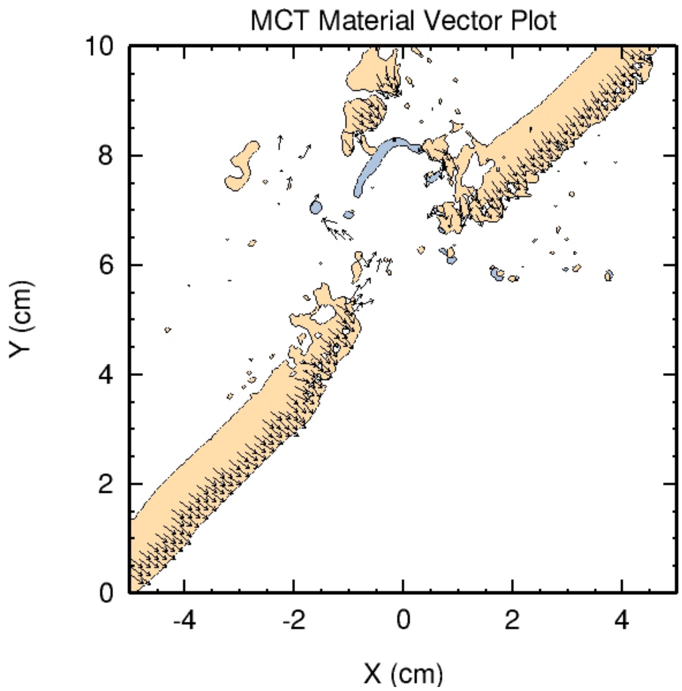


Figure 7.15 Layer 2 deformation.

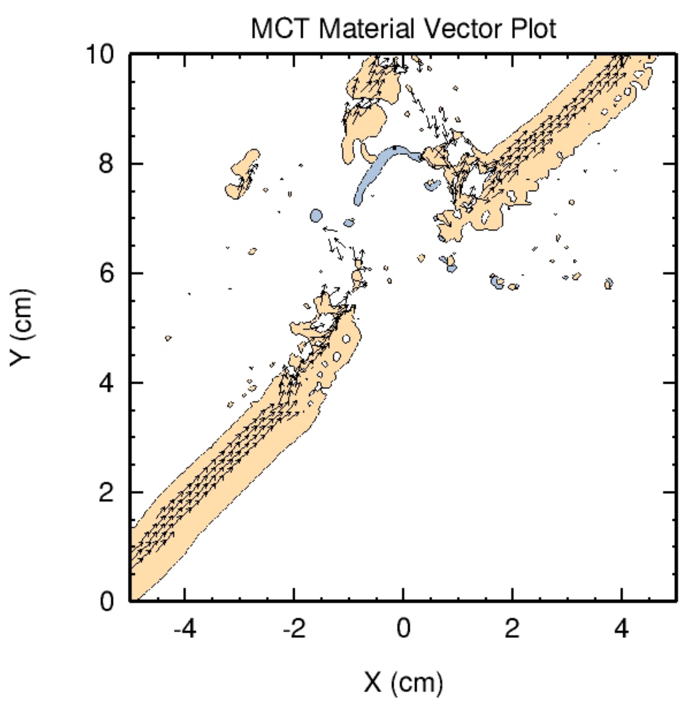


Figure 7.16 Layer 3 deformation.

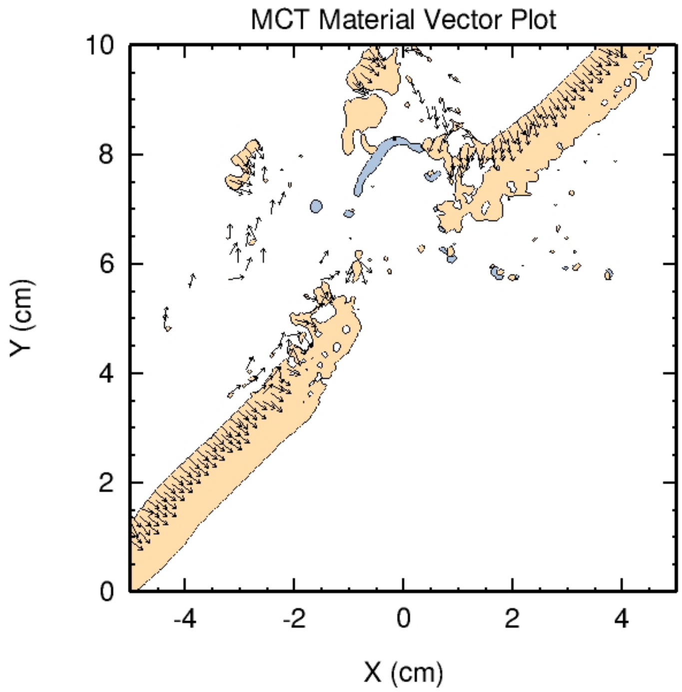


Figure 7.17 Layer 4 deformation.

Figure 7.15 shows the magnitude of the velocity of the penetrator. The velocity is tracked using a tracer point at the center of the penetrator.

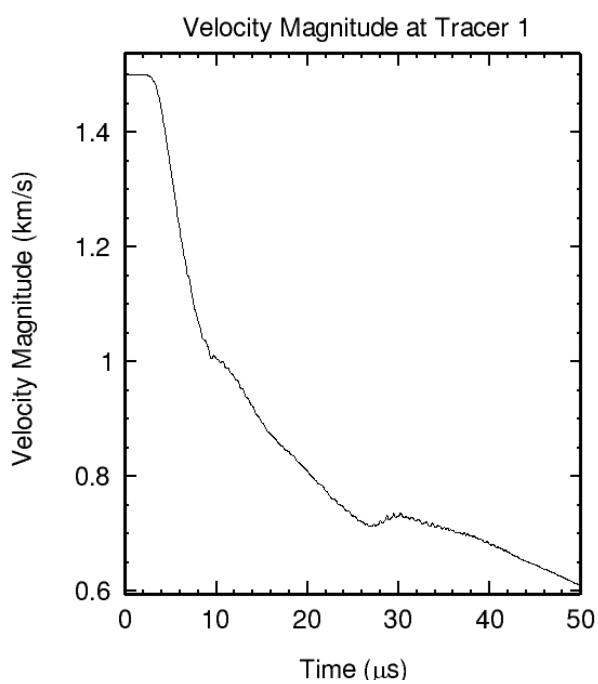


Figure 7.18 Penetrator velocity at tracer point.

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APPENDIX A. MCT MATERIAL DATA FILE

As described in Section 4.4, a unique material data file (*zmctdata#*) is required for each MCT material model. The MCT material data file contains both composite and constituent material stiffness parameters, composite and constituent strengths (if required), viscoelastic parameters (if required), damage parameters (if required), and interstitial SEP properties (if required). This appendix provides an overview of the MCT material data file format.

The MCT material data file is a sequential keyword file. In what follows, a list of the material data file keywords along with the format of the data lines required for each keyword is provided. It is noted here that any information in *italics* is for informative purposes only and is not used by the MCT model.

Two example MCT material data files are included at the end of this appendix.

***UNIDIRECTIONAL** -or- ***WEAVE** –

Either of these two keywords must begin the first line of a MCT material data file. The keyword *UNIDIRECTIONAL is required for 2 constituent models, while the keyword *WEAVE is required for 3 constituent models. If the appropriate keyword corresponding to the chosen MCT material model is not found in the material data file CTH will terminate with a message.

***MATLDEFINITION** –

This keyword must follow either the *UNI or *WEAVE keyword. This keyword defines the material number associated with each failure state of the composite microstructure and also the material numbers for each constituent that make up the individual failure states.

The input format for the *MATLDEFINITION keyword is as follows:

- <u>If the keyword follows *UNI</u> -

2 *MATLDEFINITION
3 Composite I.D. (Failure State 1), Fiber I.D. , Resin I.D.
4 Composite I.D. (Failure State 2), Fiber I.D. , Resin I.D.
.
.
.
n+2 Composite I.D. (Failure State n), Fiber I.D. , Resin I.D.

\- or –

- <u>If the keyword follows *WEAVE</u> -

2 *MATLDEFINITION
3 Composite I.D. (Failure State 1), Warp Bundle I.D. , Fill Bundle I.D. , Resin I.D. , (Warp Fiber I.D., Warp Resin I.D. , Fill Fiber I.D. , Fill Resin I.D.)*
4 Composite I.D. (Failure State 2), Warp Bundle I.D. , Fill Bundle I.D. , Resin I.D. , (Warp Fiber I.D., Warp Resin I.D. , Fill Fiber I.D. , Fill Resin I.D.)*

.
.
.

n+2 Composite I.D. (Failure State n), Warp Bundle I.D. , Fill Bundle I.D. , Resin I.D. , (Warp Fiber I.D., Warp Resin I.D. , Fill Fiber I.D. , Fill Resin I.D.)*

*Note: The values in parentheses ( ) are only required if a 3 constitute damage model is requested.

***VOLUMEFRACTION** –

This keyword must follow the input for the *MATLDEFINITION keyword. For a two-constituent analysis this keyword provides the fiber volume fraction for the unidirectional composite. For a three-constituent analysis this keyword provides the volume fraction of the bundles relative to the entire composite and for a damage analysis. It also provides the fiber volume fraction of the individual fiber bundles.

The input format for the *VOLUMEFRACTION keyword is as follows:

2 constituents (Uni)

1 *VOLUMEFRACTION
2 Fiber Volume Fraction

- or -

3 constituents (Weave)

1 *VOLUMEFRACTION
2 Warp Bundle Volume Fraction
3 Fill Bundle Volume Fraction
4 (Warp Bundle <u>Fiber</u> Volume Fraction , Fill Bundle <u>Fiber</u> Volume Fraction)*

*Note: The values in parentheses ( ) are only required if a 3 constitute damage model is requested

---

***MATERIAL** –

This keyword provides all of the stiffness (and thermal) information for the composite and constituents input in the *MATLDEFINITION keyword. The material numbering is automatically assigned according to the order in which the materials are input. e.g. the first *MATERIAL card input should correspond to material 1 from the *MATLDEFINITION card and continue on sequentially.

The input format for the *MATERIAL keyword is as follows:

1 *MATERIAL, material information/comments
2 Exx , Eyy , Ezz
3 υ_{xy} , υ_{xz} , υ_{yz}
4 Gxy , Gxz , Gyz
5 α_x , α_y , α_z
6 density(ρ)[1] , dummy variable, dummy variable

[1] The input line for the density is required in the MCT material data file. However, the value is not currently used by the MCT model so a dummy value may be input.

***DAMAGE** - (if damage is requested)

The *DAMAGE keyword contains the 2nd order curve fit coefficients for the damage degradation of the composite or the warp and fill bundles. The degradation is given by the following relationship:

$$y(f) = Af^2 + Bf + C$$

The reader is reference to Schumacher (2002) for a complete discussion of the development of these parameters.

The input format for the *DAMAGE keyword is as follows:

2 constituents (Uni)

1 *DAMAGE, *material information/comments*
2 A for Eyy/Ezz , B for Eyy/Ezz , C for Eyy/Ezz
3 A for $\upsilon_{xy/}$ υ_{xz} , B for $\upsilon_{xy/}$ υ_{xz} , C for $\upsilon_{xy/}$ υ_{xz}
4 A for Gxy/Gxz , B for Gxy/Gxz , C for Gxy/Gxz
5 A for Gyz , B for Gyz , C for Gyz

- or -

3 constituents (Weave)

1 *DAMAGE, *material information/comments*
2 A for Exx/Eyy , B for Exx/Eyy , C for Exx/Eyy
3 A for Ezz , B for Ezz , C for Ezz
4 A for υ_{xy} , B for υ_{xy} , C for υ_{xy}
5 A for $\upsilon_{xz/}$ υ_{yz} , B for $\upsilon_{xz/}$ υ_{yz}, , C for $\upsilon_{xz/}$ υ_{yz}
6 A for Gxy , B for Gxy , C for Gxy
7 A for Gxz/Gyz , B for Gxz/Gyz , C for Gxz/Gyz

***CONDAM** – (if damage is requested)

The *CONDAM keyword sets the key to identify which of the MCT constituents are to be flagged for a damage analysis. If the user wants the constituent to be analyzed using the damage routine the corresponding flag should be set to 30. If the user does not want the constituent to be analyzed with the damage routine the flag should be set to 0.

The input format for the *CONDAM keyword is as follows:

2 constituents (Uni)

1 *CONDAM
2 Fiber damage flag (on = 30, off = 0)
3 Resin damage flag (on = 30, off = 0)

- or -

3 constituents (Weave)

1 *CONDAM
2 Warp bundle damage flag (on = 30, off = 0)
3 Fill bundle damage flag (on = 30, off = 0)
4 Resin damage flag (on = 30, off = 0)

Note: Damage is currently only available for the resin constituent of the two-constituent model and the resin constituent within the fiber bundles of the three-constituent model.

***ALPHA** – (if damage is requested)

The *ALPHA keyword provides the input for the α parameter used in Eq. (51-52) and (54-56). This parameter is commonly set to 1.0. The reader is reference to Schumacher (2002) for a complete discussion of the development of these parameters.

The input format for the *ALPHA keyword is as follows:

```
*ALPHA
 α
```

***DPARAM** – (if damage is requested)

The *DPARAM keyword provides the material parameters U, γ, Temperature, and Damage Limit associated with the damage model. These parameters are those associated with Eq. (45). The reader is referenced to Schumacher (2002) for a detailed description and discussion of these material damage parameters.

The input format for the *DPARAM keyword is as follows:

2 constituents (Uni)

```
*DPARAM
 U for normal loading , U for shear loading
 γ for normal loading ,  γ for shear loading
 Temperature (T) for normal loading , Temperature (T) for shear loading
 Damage Limit for normal loading , Damage limit for Shear loading
```

- or -

3 constituents (Weave)

```
*DPARAM  (Warp Bundle)
 U for normal loading , U for shear loading
 γ for normal loading ,  γ for shear loading
 Temperature (T) for normal loading , Temperature (T) for shear loading
 Damage Limit for normal loading , Damage limit for Shear loading
*DPARAM  (Fill Bundle)
 U for normal loading , U for shear loading
 γ for normal loading ,  γ for shear loading
 Temperature (T) for normal loading , Temperature (T) for shear loading
 Damage Limit for normal loading , Damage limit for Shear loading
```

***FAILURE** –

The *FAILURE keyword sets the failure strengths for the composite and its corresponding constituents. Tensile and compressive strengths are required for the composite and its constituents. However, none of the composite strength values are currently used by the MCT model. Hence, dummy values may be input for these strengths. The reader is referenced to Mayes (1999, 2001) and Key (2000, 2003) for a detailed description of the failure strengths and their analytical and experimental derivation.

The input format for the *FAILURE keyword is as follows:

2 constituents (Uni)

1 *FAILURE, *material information/comments*

2 **Material Sets* < --- Required comment line

3 # of material failure sets

4 *Comment line for labeling Composite failure data* < ------ Required comment line

5 Composite Sxx tensile, Composite Syy tensile, Composite Szz tensile,
Composite Sxy, Composite Sxz, Composite Syz

6 Composite Sxx compressive, Composite Syy compressive,
Composite Szz compressive, Composite Sxy, Composite Sxz, Composite Syz

7 *Comment line for labeling Fiber failure data* < ------ Required comment line

8 Fiber Sxx tensile, Fiber Syy tensile, Fiber Szz tensile,
Fiber Sxy, Fiber Sxz, Fiber Syz

9 Fiber Sxx compressive, Fiber Syy compressive,
Fiber Szz compressive, Fiber Sxy, Fiber Sxz, Fiber Syz

10 *Comment line for labeling Resin failure data* < ------ Required comment line

11 Resin Sxx tensile, Resin Syy tensile, Resin Szz tensile,
Resin Sxy, Resin Sxz, Resin Syz

12 Resin Sxx compressive, Resin Syy compressive,
Resin Szz compressive, Resin Sxy, Resin Sxz, Resin Syz

- or -

3 constituents (Weave with damage)

1 *FAILURE, *material information/comments*

2 **Material Sets* < --- Required comment line

3 # of material failure sets

4 *Comment line for labeling Warp Bundle failure data* < ------ Required comment line

5 Warp Bundle Sxx tensile, Warp Bundle Syy tensile Warp Bundle Szz tensile,
Warp Bundle Sxy, Warp Bundle Sxz, Warp Bundle Syz

6 Warp Bundle Sxx compressive, Warp Bundle Syy compressive,
Warp Bundle Szz compressive, Warp Bundle Sxy, Warp Bundle Sxz, Warp Bundle Syz

7 *Comment line for labeling Fill Bundle failure data* < ------ Required comment line

8 Fill Bundle Sxx tensile, Fill Bundle Syy tensile, Fill Bundle Szz tensile,
Fill Bundle Sxy, Fill Bundle Sxz, Fill Bundle Syz

9 Fill Bundle Sxx compressive, Fill Bundle Syy compressive,
Fill Bundle Szz compressive, Fill Bundle Sxy, Fill Bundle Sxz, Fill Bundle Syz

- or -

3 constituents (Weave)

1 *FAILURE, *material information/comments*

2 **Material Sets* < --- Required comment line

3 # of material failure sets

4 *Comment line for labeling Composite failure data* < ------ Required comment line

5 Composite Sxx tensile, Composite Syy tensile, Composite Szz tensile,
Composite Sxy, Composite Sxz, Composite Syz
6 Composite Sxx compressive, Composite Syy compressive,
Composite Szz compressive, Composite Sxy, Composite Sxz, Composite Syz
7 *Comment line for labeling Fill Bundle failure data* < ------ Required comment line
8 Fill Bundle Sxx tensile, Fill Bundle Syy tensile, Fill Bundle Szz tensile,
Fill Bundle Sxy, Fill Bundle Sxz, Fill Bundle Syz
9 Fill Bundle Sxx compressive, Fill Bundle Syy compressive,
Fill Bundle Szz compressive, Fill Bundle Sxy, Fill Bundle Sxz, Fill Bundle Syz
10 *Comment line for labeling Warp Bundle failure data* < ------ Required comment line
11 Warp Bundle Sxx tensile, Warp Bundle Syy tensile Warp Bundle Szz tensile,
Warp Bundle Sxy, Warp Bundle Sxz, Warp Bundle Syz
12 Warp Bundle Sxx compressive, Warp Bundle Syy compressive,
Warp Bundle Szz compressive, Warp Bundle Sxy, Warp Bundle Sxz, Warp Bundle Syz
13 *Comment line for labeling Resin failure data* < ------ Required comment line
14 Resin Sxx tensile, Resin Syy tensile, Resin Szz tensile,
Resin Sxy, Resin Sxz, Resin Syz
15 Resin Sxx compressive, Resin Syy compressive,
Resin Szz compressive, Resin Sxy, Resin Sxz, Resin Syz

Note: The last format option for *FAILURE can only be used when no *DAMAGE data is specified in the material data file. The resin strengths (lines 14 & 15) for this format are currently not used in the MCT model so dummy values may be input here.

***VISCO** – (if a viscoelastic analysis is requested)

The *VISCO keyword defines all of the constants needed to perform a viscoelastic MCT analysis. The user is reminded that MCT currently on supports viscoelastic analyses of two-constituent (unidirectional composite) materials. The reader is referenced to Garnich (1996, 2000) and Schumacher (2002) for a detailed description of the MCT viscoelastic model and the corresponding constants.

The input format for the *VISCO keyword is as follows:

1 *VISCO, *material information/comments*
2 **Material #1* < -- Required comment line
3 # of terms in the Creep Compliance exponential curve fit, # of terms in the thermal expansion Visco curve fit.
4 *Mechanical Properties* < ------------------------------------ Required comment line
5 Term Descriptor , A for D11 term , B for D11 term , C for D11 term
6 Term Descriptor , A for D12 term , B for D12 term , C for D12 term
7 Term Descriptor , A for D13 term , B for D13 term , C for D13 term
9 Term Descriptor , A for D22 term , B for D22 term , C for D22 term
10 Term Descriptor , A for D23 term , B for D23 term , C for D23 term
11 Term Descriptor , A for D33 term , B for D33 term , C for D33 term
12 Term Descriptor , A for D44 term , B for D44 term , C for D44 term
13 Term Descriptor , A for D55 term , B for D55 term , C for D55 term

14 Term Descriptor , A for D66 term , B for D66 term , C for D66 term
15 Term Descriptor , A for π11 term , B for π11 term , C for π11 term
16 Term Descriptor , A for π12 term , B for π12 term , C for π12 term
17 Term Descriptor , A for π13 term , B for π13 term , C for π13 term
18 Term Descriptor , A for π22 term , B for π22 term , C for π22 term
19 Term Descriptor , A for π23 term , B for π23 term , C for π23 term
20 Term Descriptor , A for π33 term , B for π33 term , C for π33 term
21 Term Descriptor , A for π44 term , B for π44 term , C for π44 term
22 Term Descriptor , A for π55 term , B for π55 term , C for π55 term
23 Term Descriptor , A for π66 term , B for π66 term , C for π66 term
24 **Material #2*[1] < -- Required comment line
25 # of terms in the Creep Compliance exponential curve fit, # of terms in the thermal expansion Visco curve fit.
26 *Mechanical Properties* < ------------------------------------ Required comment line

.

.

.

(repeat for each material declared by the *MATERIAL card)

.

.

.

n **Material #n* < -- Required comment line
n+1 # of terms in the Creep Compliance exponential curve fit, # of terms in the thermal expansion Visco curve fit.
n+2 *Mechanical Properties*[2] < ------------------------------------ Required comment line
n+3 Term Descriptor , A for Jm Term , B for Jm Term , C for Jm Term
n+4 Term Descriptor , A for πm Term , B for πm Term, C for πm Term
n+5 Term Descriptor , Bulk Modulus

[1] If the # of terms in the Creep Compliance exponential curve fit and the # of terms in the thermal expansion viscoelastic curve fit are both set to zero (0), then the proceeding input parameter lines (Mechanical Properties) for that material are excluded.
[2] If the # of terms in the Creep Compliance exponential curve fit is set equal to a negative (-) value then this indicates an isotropic viscoelastic material and the input format is that show for material n in the above.

---

*ELPLASTIC – (if interstitial SEP model is requested)

The *ELPLASTIC keyword defines the elastic-plastic material constants for the isotropic interstitial (interface) plies if this option is requested within the MCT model. The keyword is applicable to all MCT material models.

The input format for the *ELPLASTIC keyword is as follows:

1 *ELPLASTIC
2 *Descriptor line* < --- Required comment line
3 $E_{elastic}$, $\upsilon_{elastic}$, Yield Strength , $E_{hardening}$, Beta

Note: Beta should be set equal to 1 for isotropic hardening.

```
* Unidirectional Eglass/8084, 51% FVF, Order important for failure analysis
*MatlDefinition
    1    4    6
    2    4    7
    3    5    7
*VolumeFraction
      0.51D+0
*Material,1,Undamaged composite
   38.4600D+9  11.6500D+9  11.6500D+9
      0.27309     0.27309      0.3431
    4.7390D+9   4.7390D+9   4.3370D+9
    0.3997D-3   0.3340D-2   0.3340D-2
     1.852D+3    0.000D+0    0.000D+0
*Material,2,Matrix Failed composite
     36.41D+9   1.3337D+9   1.3337D+9
      0.27291     0.27291     0.36428
    5.4644D+8   5.4644D+8   4.8880D+8
    0.9234D-5   0.3385D-2   0.3385D-2
     1.852D+3    0.000D+0    0.000D+0
*Material,3,Undamaged composite
   38.4600D+8  11.6500D+8  11.6500D+8
      0.27309     0.27309      0.3427
    4.7390D+8   4.7390D+8   4.3370D+8
    0.3997D-3   0.3340D-2   0.3340D-2
     1.852D+3    0.000D+0    0.000D+0
*Material,4,Undamaged fiber
     71.00D+9    71.00D+9    71.00D+9
      0.26000     0.26000     0.26000
     28.18D+9    28.18D+9    28.18D+9
    0.5040D-5   0.5040D-5   0.5040D-5
     2.500D+3    0.000D-0    0.000D-0
*Material,5,Failed fiber
     71.00D+8    71.00D+8    71.00D+8
      0.26000     0.26000     0.26000
     28.18D+8    28.18D+8    28.18D+8
    0.5040D-5   0.5040D-5   0.5040D-5
     2.500D+3    0.000D-0    0.000D-0
*Material,6,Undamaged matrix
    4.6560D+9   4.6560D+9   4.6560D+9
      0.29200     0.29200     0.29200
   1.80200D+9  1.80200D+9  1.80200D+9
 0.6480D-2   0.6480D-2   0.6480D-2
     1.122D+3    0.000D-0    0.000D-0
*Material,7,Failed matrix
    4.6560D+8   4.6560D+8   4.6560D+8
      0.29200     0.29200     0.29200
   1.80200D+8  1.80200D+8  1.80200D+8
    0.6480D-2   0.6480D-2   0.6480D-2
     1.122D+3    0.000D-0    0.000D-0
*Damage Eglass/D8084, 51% FVF where the fiber direction=11
*Composite damage curve fit coeff,E22,E33,NU12,G12,G23.
   0.0         0.0         0.0
   5.23390D-2  7.24564D-1  2.23097D-1
   1.88571D+0 -3.78464D+0  2.89893D+0
  -1.24693D-1  1.12005D+0  4.64300D-3
  -2.94225D-2  9.29512D-1  9.99105D-2
*Condamage n corresponding to fail state above, or fiber or matrix damage material
     0
    30
*ALPHA
       1.0D+0
*DParam FIBER/MATRIX
     1.337D+5   1.0875D+5
     1.500D-3      6.0D-4
     300.0D+0    300.0D+0
     2.303D-1    9.000D-1
```

(continued)

```
*Failure data Eglass/8084, 51% FVF - Verified R6
*Material Sets
    3
*Material,Composite,S11T,S22T,S33T,S12T,S13T,S23T,(repeat for compress)
     817.5D6     45.26D6      00.0D6     60.80D6      00.0D6     48.52D6
    -759.7D6    -144.3D6      00.0D6     60.80D6      00.0D6     48.52D6
*Material,Reinforcement,S11T,S22T,S33T,S12T,S13T,S23T,(repeat for compress)
    1507.0D6      00.0D6      00.0D6    120.00D6      00.0D6      00.0D6
   -1399.0D6      00.0D6      00.0D6    120.00D6      00.0D6      00.0D6
*Material,Matrix,S11T,S22T,S33T,S12T,S13T,S23T,(repeat for compress)
      00.0D6     37.10D6      2.20D6     85.00D6      00.0D6     50.42D6
      00.0D6    -118.1D6    -7.040D6     85.00D6      00.0D6     50.42D6
*Visco-elastic transversely isotropic 51% glass fiber - fiber in 1 direction
*Material #1 in Model:  Do, D1,....,Dn where n is NEXPC - Format 6(D14.6)
    2    0
m  -->Mechanical Properties
  D11 Terms:  2.706730D-11 -1.069360D-12        0.D+00 (1/Pa)
  D12 Terms: -9.174200D-12  2.084290D-12        0.D+00 (1/Pa)
  D13 Terms: -9.174200D-12  2.084290D-12        0.D+00 (1/Pa)
  D22 Terms:  1.857540D-10 -9.906140D-11        0.D+00 (1/Pa)
  D23 Terms: -1.180570D-10  8.879820D-11        0.D+00 (1/Pa)
  D33 Terms:  1.857540D-10 -9.906140D-11        0.D+00 (1/Pa)
  D44 Terms:  6.363250D-10 -4.253490D-10        0.D+00 (1/Pa)
  D55 Terms:  6.363250D-10 -4.253490D-10        0.D+00 (1/Pa)
  D66 Terms:  6.072000D-10 -3.751630D-10        0.D+00 (1/Pa)
 PI11 Terms:        0.D+00  2.655870D+01        0.D+00(Sec)
 PI12 Terms:        0.D+00  2.908860D+01        0.D+00(Sec)
 PI13 Terms:        0.D+00  2.908860D+01        0.D+00(Sec)
 PI22 Terms:        0.D+00  6.696360D+01        0.D+00(Sec)
 PI23 Terms:        0.D+00  7.661160D+01        0.D+00(Sec)
 PI33 Terms:        0.D+00  6.696360D+01        0.D+00(Sec)
 PI44 Terms:        0.D+00  7.827510D+01        0.D+00(Sec)
 PI55 Terms:        0.D+00  7.827510D+01        0.D+00(Sec)
 PI66 Terms:        0.D+00  7.126140D+01        0.D+00(Sec)
*-->Material #2
    0    0
*-->Material #3
    0    0
*-->Material #4
    0    0
*-->Material #5
    0    0
*-->Material #6
   -1    0
i  -->Isotropic Mechanical Properties
   Jm Terms:  6.547326D-10 -4.399569D-10  (1/Pa)
  PIm Terms:        0.D+00  78.42120D+00  (Sec)
   Bulk Mod:   3.73077D+09               (Pa)
*-->Material #7
    0    0
```

Figure A.1 Sample MCT material data file for a 2 constituent (unidirectional) model.

```
*WEAVE, Closed Weave Model, 71.3% VF, Order important for failure analysis
*MaterialDefinition
   1  10  13  16  18  27  29  31  33
   2  11  13  16  19  27  30  31  33
   3  10  14  16  20  27  29  31  34
   4  12  13  16  21  28  30  31  33
   5  10  15  16  22  27  29  32  34
   6  11  14  16  23  27  30  31  34
   7  11  15  16  24  27  30  32  34
   8  12  14  16  25  28  30  31  34
   9  12  15  17  26  28  30  32  34
*VOLUMEFRACTION
  0.35651D+0
  0.35651D+0
   0.749D+0    0.749D+0
*MATERIAL,1, composite
   26.50D+9    26.50D+9    15.30D+9
    0.15386     0.29163     0.29163
   5.520D+9    7.740D+9    7.740D+9
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
*MATERIAL,2, regular matrix failure
   25.36D+9    20.46D+9    4.222D+9
    0.10815     0.31902     0.33288
   3.127D+9    2.639D+9   4.617D+9
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
*MATERIAL,3, rotated matrix failure
   20.46D+9    25.36D+9    4.222D+9
    0.08725     0.33288     0.31902
   3.127D+9    4.617D+9    2.639D+9
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
*MATERIAL,4, composite failure in regular material
   8.812D+9    19.97D+9    3.946D+9
    0.10024     0.31345     0.28678
   3.061D+9    3.318D+9    2.973D+9
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
*MATERIAL,5, composite failure in rotated material
   19.97D+9    8.812D+9    3.946D+9
    0.22717     0.28678     0.31345
   3.061D+9    2.973D+9    3.318D+9
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
*MATERIAL,6, failure in regular and rotated matrix material
   19.66D+9    19.66D+9    2.143D+9
    0.03294     0.34561     0.34561
   9.122D+8    7.432D+8    7.432D+8
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
*MATERIAL,7, composite failure in rotated material, and matrix failure in regular material
   19.50D+9    3.720D+9    2.047D+9
    0.01630     0.30293     0.32558
   8.752D+8    6.523D+8    6.195D+8
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
*MATERIAL,8, composite failure in regular material, and matrix failure in rotated material
   3.720D+9    19.50D+9    2.047D+9
    0.00311     0.32558     0.30293
   8.759D+8    6.195D+8    6.523D+8
  0.0000D-6   0.0000D-6   0.0000D-6
   0.000D+3    0.000D-0    0.000D-0
```

(continued)

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*MATERIAL,9, composite failure
    3.5607D+9   3.5607D+9   1.8434D+9
     0.15430     0.28327     0.28327
    8.4882D+8   6.1355D+8   6.1355D+8
    0.0000D-6   0.0000D-6   0.0000D-6
     0.000D+3    0.000D-0    0.000D-0
*MATERIAL,10, alpha (REGULAR Yarn)
     54.38D+9    22.39D+9    22.39D+9
     0.26607     0.26607     0.29526
     9.016D+9    9.016D+9    8.647D+9
    0.0000D-6   0.0000D-6   0.0000D-6
     0.000D+3    0.000D-0    0.000D-0
*MATERIAL,11, alpha (REGULAR Yarn) matrix failure
    53.267D+9  1.3054D+09  1.3054D+09
     0.26587     0.26587     0.29353
    5.1986D+8   5.1986D+8   5.0458D+8
    0.3997D-3   0.3340D-2   0.3340D-2
     1.852D+3    0.000D+0    0.000D+0
*MATERIAL,12, alpha (REGULAR Yarn) failure
    53.687D+8   1.1114D+9   1.1114D+9
     0.26595     0.26595     0.29541
    4.4554D+8   4.4554D+8   4.2899D+8
    0.3997D-3   0.3340D-2   0.3340D-2
     1.852D+3    0.000D+0    0.000D+0
*MATERIAL,13, beta (ROTATED Yarn)
     22.39D+9    54.38D+9    22.39D+9
     0.10954     0.29526     0.26607
     9.016D+9    8.647D+9    9.016D+9
    0.0000D-6   0.0000D-6   0.0000D-6
     0.000D+3    0.000D-0    0.000D-0
*MATERIAL,14, beta (ROTATED Yarn) matrix failure
   1.3054D+09   53.267D+9  1.3054D+09
     0.00652     0.29353     0.26587
    5.1986D+8   5.0458D+8   5.1986D+8
    0.3997D-3   0.3340D-2   0.3340D-2
     1.852D+3    0.000D+0    0.000D+0
*MATERIAL,15, beta (ROTATED Yarn) failure
    1.1114D+9   53.687D+8   1.1114D+9
     0.00551     0.29541     0.26595
    4.4554D+8   4.2899D+8   4.4554D+8
    0.3997D-3   0.3340D-2   0.3340D-2
     1.852D+3    0.000D+0    0.000D+0
*MATERIAL,16, gamma (Matrix)
     4.656D+9    4.656D+9    4.656D+9
     0.29190     0.29190     0.29190
     1.802D+9    1.802D+9    1.802D+9
    0.0000D-6   0.0000D-6   0.0000D-6
     0.000D+3    0.000D-0    0.000D-0
*MATERIAL,17, gamma (Matrix) Failure
     4.656D+9    4.656D+9    4.656D+9
     0.29190     0.29190     0.29190
     1.802D+9    1.802D+9    1.802D+9
    0.0000D-6   0.0000D-6   0.0000D-6
     0.000D+3    0.000D-0    0.000D-0
*MATERIAL,18, alpha-beta
     39.80D+9    39.80D+9    22.80D+9
     0.15792     0.29293     0.29293
     9.020D+9    8.860D+9    8.860D+9
    0.0000D-6   0.0000D-6   0.0000D-6
     0.000D+3    0.000D-0    0.000D-0
*MATERIAL,19, alpha-beta matrix failure in regular props
     38.59D+9    27.41D+9    3.596D+9
     0.12177     0.29424     0.31668
     4.048D+9    2.725D+9    6.110D+9
    0.0000D-6   0.0000D-6   0.0000D-6
     0.000D+3    0.000D-0    0.000D-0
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*MATERIAL,20, alpha-beta matrix failure rotated props
    27.41D+9   38.59D+9   3.596D+9
     0.08649    0.31668    0.29424
    4.048D+9   6.110D+9   2.725D+9
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,21, alpha-beta failure in regular material
    11.78D+9   26.81D+9   3.320D+9
     0.07566    0.25158    0.28317
    4.042D+9   3.541D+9   3.158D+9
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,22, alpha-beta failure in rotated material
    26.81D+9   11.78D+9   3.320D+9
     0.17219    0.28317    0.25158
    4.042D+9   3.158D+9   3.541D+9
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,23, alpha-beta failure in regular and rotated matrix material
    26.16D+9   26.16D+9   1.283D+9
     0.00212    0.36223    0.36223
    5.198D+8   6.221D+8   6.221D+8
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,24, alpha-beta failure in rotated material, and matrix failure in regular material
    25.97D+9   3.221D+9   1.283D+9
     0.08060    0.35069    0.32287
    4.836D+8   4.871D+8   4.860D+8
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,25, alpha-beta failure in regular material, and matrix failure in rotated material
    3.221D+9   25.97D+9   1.283D+9
     0.00999    0.32287    0.35069
    4.836D+8   4.860D+8   4.871D+8
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,26, alpha-beta failure
    3.100D+9   3.100D+9   1.163D+9
     0.08131    0.30584    0.30584
    4.454D+8   4.422D+8   4.422D+8
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,27, Eglass Fiber REGULAR
    71.00D+9   71.00D+9   71.00D+9
     0.26000    0.26000    0.26000
    28.18D+9   28.18D+9   28.18D+9
   0.5040D-5  0.5040D-5  0.5040D-5
    2.500D+3   0.000D-0   0.000D-0
*MATERIAL,28, Eglass Fiber REGULAR Faulure
    71.00D+8   71.00D+8   71.00D+8
     0.26000    0.26000    0.26000
    28.18D+8   28.18D+8   28.18D+8
   0.5040D-5  0.5040D-5  0.5040D-5
    2.500D+3   0.000D-0   0.000D-0
*MATERIAL,29, Matrix REGULAR
    4.656D+9   4.656D+9   4.656D+9
     0.29190    0.29190    0.29190
    1.802D+9   1.802D+9   1.802D+9
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
*MATERIAL,30, Matrix REGULAR Failure
    1.862D+8   1.862D+8   1.862D+8
     0.29190    0.29190    0.29190
    7.208D+7   7.208D+7   7.208D+7
   0.0000D-6  0.0000D-6  0.0000D-6
    0.000D+3   0.000D-0   0.000D-0
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*MATERIAL,31, Eglass Fiber ROATATED
    71.00D+9    71.00D+9    71.00D+9
     0.26000     0.26000     0.26000
    28.18D+9    28.18D+9    28.18D+9
   0.5040D-5   0.5040D-5   0.5040D-5
    2.500D+3    0.000D-0    0.000D-0
*MATERIAL,32, Eglass Fiber ROTATED Failure
    71.00D+8    71.00D+8    71.00D+8
     0.26000     0.26000     0.26000
    28.18D+8    28.18D+8    28.18D+8
   0.5040D-5   0.5040D-5   0.5040D-5
    2.500D+3    0.000D-0    0.000D-0
*MATERIAL,33, Matrix ROTATED
    4.656D+9    4.656D+9    4.656D+9
     0.29190     0.29190     0.29190
    1.802D+9    1.802D+9    1.802D+9
   0.0000D-6   0.0000D-6   0.0000D-6
    0.000D+3    0.000D-0    0.000D-0
*MATERIAL,34, Matrix ROTATED Failure
    1.862D+8    1.862D+8    1.862D+8
     0.29190     0.29190     0.29190
    7.208D+7    7.208D+7    7.208D+7
   0.0000D-6   0.0000D-6   0.0000D-6
    0.000D+3    0.000D-0    0.000D-0
*DAMAGE Eglass/D8084, 51% FVF where the fiber direction=11
*Composite damage curve fit coeff,E11,E22,E33,NU12,NU13,NU23,G12,G13,G23.
 -7.92000D-3  1.51080D-1  8.56840D-1
 -7.92000D-3  1.51080D-1  8.56840D-1
  1.15720D-1 -3.34630D-1  9.62410D-1
  2.30730D-1 -2.31350D-1  1.00062D+0
  1.55180D-1 -6.73200D-1  1.72568D+0
  1.55180D-1 -6.73200D-1  1.72568D+0
 -9.05200D-2  5.79250D-1  2.03580D-1
 -3.31700D-2  4.54200D-1  2.24280D-1
 -3.31700D-2  4.54200D-1  2.24280D-1
*Composite damage single curve fit coeff,E11,E22,E33,NU12,NU13&NU23,G12,G13,G23.
 -6.20600D-2  3.73310D-1  6.88750D-1
 -7.19300D-2  2.75490D-1  7.96440D-1
  2.95780D-1 -3.45310D-1  1.04953D+0
  5.27000D-1 -1.01340D+0  1.48640D+0
  2.62730D-1 -5.31580D-1  1.26885D+0
  1.94290D-1 -4.22200D-1  1.22791D+0
 -3.22650D-1  9.46400D-1  3.76250D-1
 -2.07810D-1  8.63370D-1  3.44440D-1
 -4.74500D-2  6.01570D-1  4.45770D-1
*alpha-beta damage curve fit coeff,E11,E22,E33,NU12,NU13,NU23,G12,G13,G23.
 -1.80000D-4  1.93280D-1  8.06900D-1
 -1.80000D-4  1.93280D-1  8.06900D-1
  1.66900D-2  2.51990D-1  4.29260D-1
  2.82010D-1 -2.94720D-1  1.01271D+0
  2.31370D-1 -9.33630D-1  1.94178D+0
  2.31370D-1 -9.33630D-1  1.94178D+0
 -7.78200D-2  6.38850D-1  3.35800D-2
  2.03400D-2  3.03590D-1  3.11460D-1
  2.03400D-2  3.03590D-1  3.11460D-1
*alpha-beta damage single curve fit coeff,E11,E22,E33,NU12,NU13&NU23,G12,G13,G23.
 -3.88900D-2  4.66650D-1  5.72240D-1
 -9.49100D-2  3.86630D-1  7.08280D-1
  4.73700D-1 -5.59910D-1  1.08621D+0
  7.53820D-1 -1.42980D+0  1.67598D+0
  4.38120D-1 -8.93300D-1  1.45518D+0
  4.16050D-1 -9.04030D-1  1.48798D+0
 -3.49330D-1  1.17870D+0  1.70630D-1
 -2.19130D-1  9.24140D-1  2.94990D-1
 -6.29800D-2  6.65990D-1  3.96990D-1
*fill and warp damage curve fit coeff,E22E33,NU12,G12,G23.
  5.29380D-2  5.55343D-1  3.91719D-1
  9.38710D-1 -1.87509D+0  1.93638D+0
 -3.10926D-1  1.27801D+0  3.29160D-2
 -4.58200D-2  7.59120D-1  2.86700D-1
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(continued)

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*CONDAM corresponding to fail state above, or fiber or matrix damage material
   30
   30
    0
*ALPHA
      1.0D+0
*DPARAM
    1.170D+5   1.0080D+5
    1.000D-3    7.500D-4
    300.0D+0    300.0D+0
    2.160D-1    8.600D-1
*DPARAM
    1.170D+5   1.0080D+5
    1.000D-3    7.500D-4
    300.0D+0    300.0D+0
    2.160D-1    8.600D-1
*FAILURE eglass/8084, 71.3% FVF, Weave Model
*Material Sets
    2
*Material,Fiber Regular, S11T,S22T,S33T,S12T,S13T,S23T,(repeat for compress)
    1507.0D6      00.0D6      00.0D6    120.00D6      00.0D6      00.0D6
   -1399.0D6      00.0D6      00.0D6    120.00D6      00.0D6      00.0D6
*Material,Fiber Rotated ,S11Tl,S22Tt,S33Tt,S12Tl,S12Tt,S23Tt,(repeat for compress)
      00.0D6    1507.0D6      00.0D6    120.00D6      00.0D6      00.0D6
      00.0D6   -1399.0D6      00.0D6    120.00D6      00.0D6      00.0D6
* ELPLASTIC Interstitial Properties                 |
*  SELAT, SPSNT, SYDT, SHRDT, SBTA                                                          0    !
 3.8500E+12  0.2640E+00  1.5000E+10  2.8200E+11  1.0000E+00
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Figure A.2 Sample MCT material data file for a 3 constituent (woven) material.

Distribution List

10	MS0836	Shane Schumacher	1516
1	MS0899	Technical Library	9536 (electronic copy)

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