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Kinetic Theory and Boundary Conditions for Flows of Highly Inelastic Spheres.

Quarterly Progress Report

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Introduction

In this quarter, we conducted a study to determine a range of parameters over which steady, fully developed, gravity driven granular flows of identical, smooth, inelastic spheres down bumpy inclines could be maintained. The appropriate boundary value problem has been described in detail in our Quarterly Progress Report from April 1, 1993 to June 30, 1993. The numerical solution procedure that we employed to solve the boundary value problem has been described in detail in our Quarterly Progress Report from July 1, 1993 to September 30, 1993. In what follows, we describe the parameters relevant to the inclined flows investigated, the range of these parameters that we included in our study, and the resulting ranges over which steady, fully developed flows could be maintained.

Parameters

We are concerned with steady, fully developed, gravity driven flows of identical, inelastic spheres down bumpy inclines. The diameter of each sphere is σ, the coefficient of restitution between them is e. and the angle between the incline and the horizontal is ϕ.

Here we focus on inclines that are flat surfaces to which identical, smooth, hemispherical particles of diameter d are randomly attached at an average distance s apart. Dimensionless measures of the boundary geometry are $r \equiv \sigma/d$ and $\Delta \equiv s/d$. In order to prevent flow particles from colliding with the flat part of the boundary, the maximum value of Δ is equal to $-1+(1+2r)^{1/2}$. When a flow particle collides with a boundary particle the energy dissipated is fixed by the coefficient of restitution e_w between them.

Additional parameters are the granular temperature at the "top" of the flow, and the "depth" of flow. However in our previous progress report, we showed that if the stresses and energy flux vanish at the top of the flows, then the kinetic theory predicts that the flows are infinitely deep, and that the stresses, energy flux, and solid fraction each approach zero asymtotically from the base. This presents a numerical difficulty because the integrations proceed *from* the top of the flow *to* the base, and requires somewhat artificial definitions of the "top" of the flow and the "depth" of the flow.

To address these issues, we define the "top" of the flow as the location at which the energy flux vanishes but where the solid fraction is equal to a small non-zero value (typically 10^{-6}). With the granular temperature τ_{top} fixed at a nonzero value there, this allows the integrations to be initiated from this location and is equivalent to relaxing very slightly the normal stress condition at the "top" of the flow. Because the ratio of the shear stress to normal stress is everywhere equal to $\tan\phi$, in this manner the shear stress condition is also relaxed very slightly at the "top" of the flow. The dimensionless parameter corresponding to τ_{top} is $T_{top} \equiv \tau_{top}/\sigma g$, where g is the vertical acceleration due to gravity.

However, it is not appropriate to define the "depth" of flow as the distance from the base to the location at which the integrations are initiated. This is because the integrations may include great distances over which the solid fraction is extremely small, and these distances may be quite sensitive to the arbitrary value that the solid fraction is assigned at the "top." For these reasons, proceeding from the "top" of the flow, we instead take the "depth" of flow L to be the height below which ninety-nine percent of the mass of the flow resides. Defined in this manner, the depth does not include the great distances over which the solid fraction is extremely small, and is not dependent on the small value of solid fraction that we choose to initiate the integrations. The dimensionless parameter measuring this "depth" is $\beta \equiv L/\sigma$.

We have experimented with values of solid fraction at the "top" ranging from 10^{-3} to 10^{-7} and have observed that while the distances required for the solid fraction to reach .01, for example, may be quite sensitive to these values and may be quite large, the variations in all other mean fields from their values at the "top" are quite small over these same distances. Consequently, the dimensionless value of the granular temperature at the upper portion of the flow may be accurately approximated by T_{top}. Most importantly, we have observed that the profiles of all the mean fields are quite insensitive to the value of the solid fraction chosen to initiate the integration from the "top."

In summary, the *input* parameters of interest here are the angle of inclination ϕ, the coefficient of restitution e between flow particles, the boundary bumpiness parameters r and Δ, the coefficient of restitution e_w between flow particles and boundary particles, and the granular temperature at the "top" of the flow T_{top}. The only *output* parameters that we are concerned with in this study are the depth β and the solid fraction ν.

Range of Parameters

In this study, we focus on boundaries for which r=1. These are boundaries with bumps that have the same diameters as the flow particles. Consequently, the range on Δ is between 0 (i.e. no spacing between bumps) to the maximum value .732 beyond which flow particles will collide with the falt part of the boundary. We consider all angles of inclination ϕ between 0° and 90°, but limit our attention to cases in which the coefficients of restitution e and e_w are equal and between .5 and 1.0. In addition, we accept a solution to the boundary value problem only if the flow depth predicted by the theory is greater than one particle diameter, and only if the theory predicts that the solid volume fraction ν is everywhere less than .65.

In order to determine a reasonable range of the final input parameter T_{top}, we imagine the flow as consisting of an observable collisional portion of depth β above which there exists a diffuse saltating region of height h. The saltating region occurs because particles with mean square fluctuation speeds $3\tau_{top}$ at the upper layer of the collisional portion of the flows rarely collide with other particles and are influenced primarily by gravity alone. Simple conservation of energy arguments dictate that $3\tau_{top} \leq 2gh$, or in dimensionless form,

$$T_{top} \leq \frac{2h}{3\sigma} \quad , \tag{1}$$

where the height h should be at least half of the depth of the collisional layer. A flow depth $\beta=100$ would dictate that $h=50$, and therefore that, for fixed values of r, Δ, e, e_w, and ϕ, we consider all values of $T_{top} \leq 33$. However, in the interest of saving computer time and because we have found that most solutions have depths no greater than 30 particle diameters, we instead take $h=15$ and consider the smaller range $0 \leq T_{top} \leq 10$. In this manner, we consider saltating layers that are at least half the height of the collisional portion of most flows, and more typically are of the same heights or of greater heights than the collisional portion of the flow.

In summary, the ranges of *input* parameters that we consider here are:

$$r = 1 \qquad \text{and} \qquad 0 \leq \Delta \leq .732 \quad ; \tag{2}$$

$$.5 \leq e{=}e_w \leq 1.0 \qquad \text{and} \qquad 0^\circ \leq \phi \leq 90^\circ \quad ; \tag{3}$$

and

$$0 \leq T_{top} \leq 10 \quad . \tag{4}$$

The ranges of *output* parameters are:

$$\beta \geq 1 \qquad \text{and} \qquad \nu \leq .65 \quad . \tag{5}$$

Solution Search Procedure

A recurring question that we answer in the current search is the following: for prescribed values of r(=1), Δ, $e=e_w$, and ϕ, can a steady, fully developed, gravity-driven flow within the ranges (4) and (5) be maintained?

In order to answer this question, we pick a value for T_{top} within range (4) and integrate downward until we either find a solution (i.e. satisfy the basal boundary conditions) within each of ranges (5), or until either ν exceeds .65, or until it appears that the basal boundary condition will not be satisfied regardless of how far the integrations proceed. If we find a solution, then we have demonstrated that at least one such steady, fully developed flow can be maintained for the prescribed values of $r(=1)$, Δ, $e=e_w$, and ϕ, and we need not proceed further. If we do not find a solution, then we increment the value of T_{top} and repeat the downward integration. We continue to increment T_{top} until we either find a solution for some value of T_{top} within range (4), or until we have varied T_{top} throughout the entire range (4) and have not found any such solutions.

In order to determine, for prescribed values of $r(=1)$, Δ, and $e=e_w$, the *range* of angles ϕ between which steady, fully developed, gravity-driven flows can be maintained, we repeat the process described above for all values of ϕ between 0° and 90° incremented by 1°.

Results and Discussion

In Figure 1, we show as a darkened area in the ϕ-e plane the values of ϕ and e for which steady, fully developed, flows are possible within the bounds described by inequalities (4) and (5) for $.5<e=e_w<1.0$, when $r=1$ and $\Delta=.414$. As expected, as $e=e_w$ increases from .5, the typical inclinations at which steady, fully developed flows are possible generally decrease. In these flows, the rate at which energy is dissipated must help to balance the rate at which it is supplied by gravity. Furthermore, the rate at which energy is supplied to the flow by gravity decreases as the angle of inclination decreases. Consequently, as the flows become less dissipative, the work done by gravity, and therefore the angles of inclination decrease. When $e=e_w=1.0$, no energy is dissipated within the flow or at the boundary, and the work done by gravity must vanish. In this limit, the theory predicts that steady, fully developed flows can be maintained only when $\phi=0°$.

In Figure 2, we show as a darkened area in the ϕ-Δ plane the values of ϕ and Δ for which steady, fully developed, flows are possible within the bounds described by inequalities (4) and (5) for $0<\Delta<.732$, when $r=1$ and $e=e_w=1.0$. (If Δ exceeds .732 when $r=1$, then the flow particles could collide with the flat part of the boundary.) As expected, as Δ increases from 0 to its maximum value, the typical inclinations at which steady, fully developed flows are possible generally increase. As Δ increases, the boundary becomes bumpier, the slip velocity decreases, and the energy supplied to the flow by the working of the traction at the boundary decreases. In order to balance the energy dissipated due to inelastic collisions in the flow and at the boundary, the work done by gravity, and therefore the angles of inclination, must increase.

Figure Captions

Figure 1: The area in the ϕ-e plane in which steady, fully developed, gravity driven flows are possible when $r=1$, $\Delta=.414$, and $0<e=e_w<1.0$.

Figure 2: The area in the ϕ-Δ plane in which steady, fully developed, gravity driven flows are possible when $r=1$, $e=e_w=.5$, and $0<\Delta<.732$.

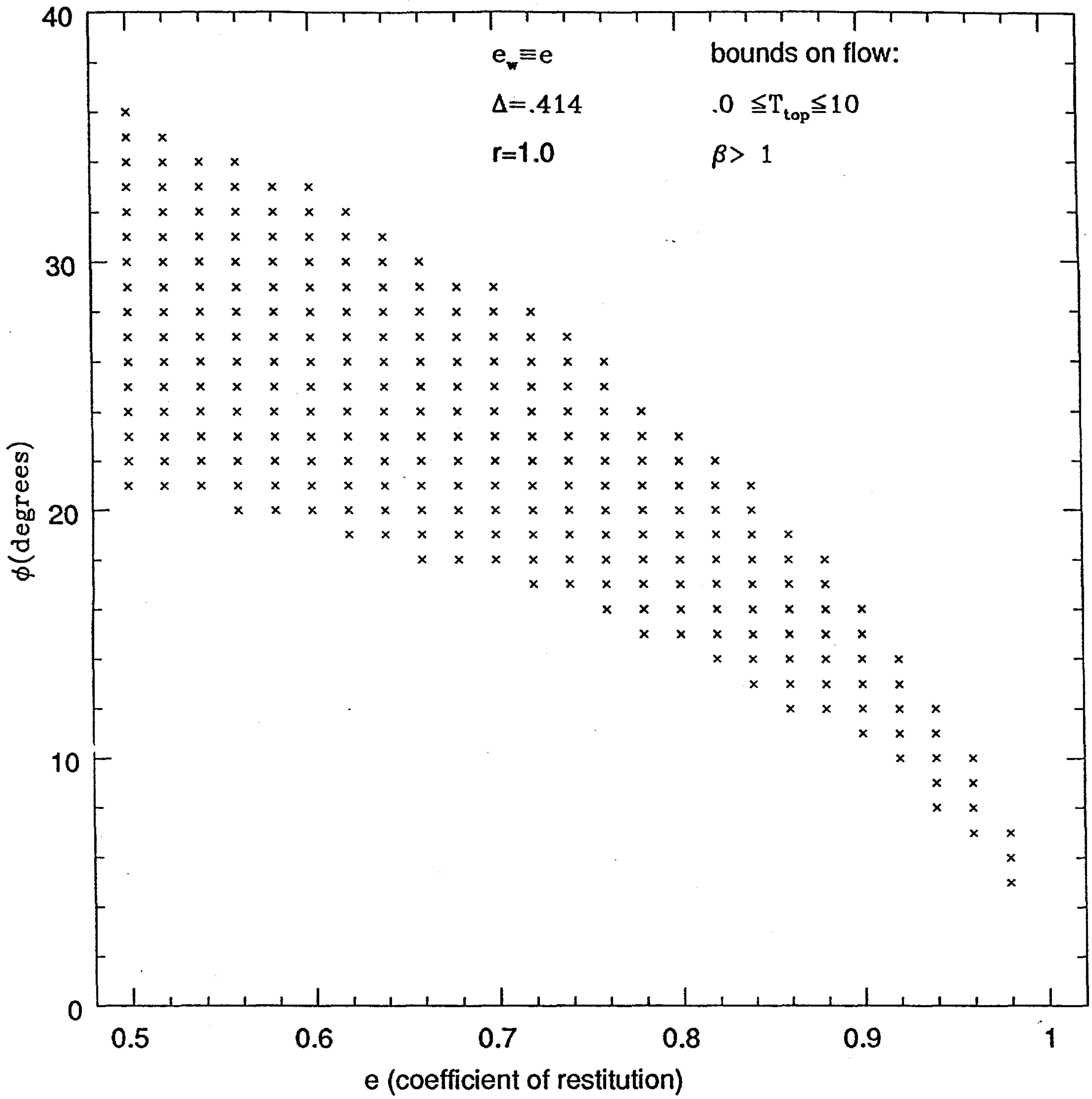

Figure 1: The area in the ϕ-e plane in which steady, fully developed, gravity driven flows are possible when r=1, Δ=.414, and $0<e=e_w<1.0$.

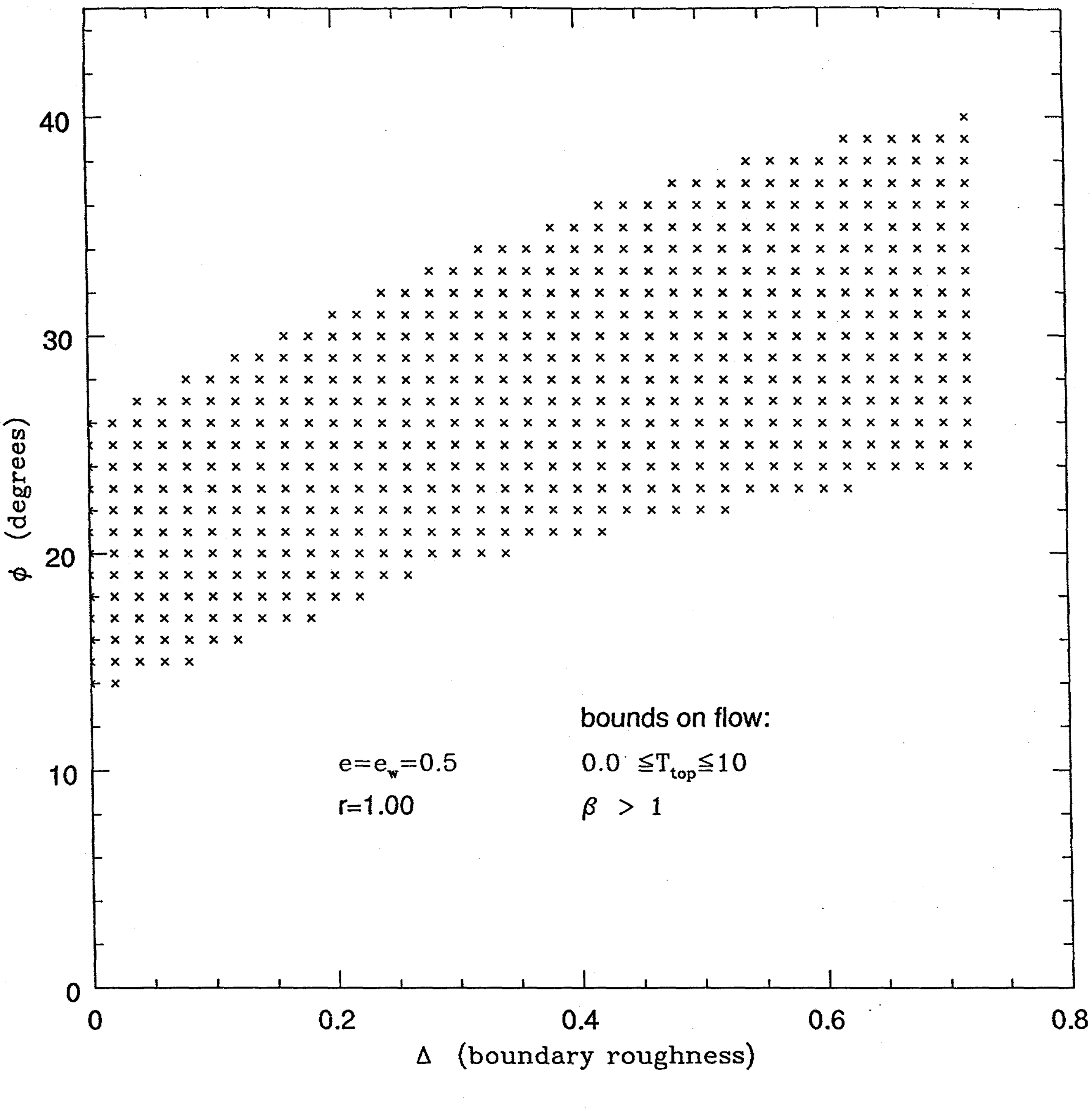

Figure 2: The area in the ϕ-Δ plane in which steady, fully developed, gravity driven flows are possible when r=1, $e=e_w=.5$, and $0<\Delta<.732$.