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Dense Inclined Flows: Theory and Experiments

Quarterly Technical Progress Report (January 1, 1995 to March 31, 1995)

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Summary of accomplishments

Rapid, gravity-driven flows of granular materials down inclines pose a challenge to our understanding. Even in situations in which the flow is steady and two-dimensional, the details of how momentum and energy are balanced within the flow and at the bottom boundary are not well understood. Thus we have undertaken a research program integrating theory, computer simulation, and experiment that will focus on dense entry flows down inclines. The effort involves the development of theory informed by the results of simultaneous computer simulations and the construction, instrumentation, and use of an experimental facility in which the variables necessary to assess the success or failure of the theory can be measured.

In the present reporting period, we have continued a series of measurements in the chute facility with a flat, frictional boundary. At several inclinations between 15.5° and 20°, and at several gate openings for each angle, we have measured mass flow rate and mass holdup, as well as granular temperature and collision frequency at the bottom wall of the chute. By recording simultaneously the collisinal normal stress at the bottom wall and the mass holdup above it, the experiments reveal the fraction of the weight of particles that is supported by direct impact.

1. Chute Experiments

A summary of the global results obtained in the present reporting period is illustrated in Fig. 1. In this Fig., the error bars represent the amplitude of waves that propagate through the flow. The mass flow rate $\dot{m} = \int \rho_s \nu u W dz$ is made dimensionless using

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$$m^{\dagger} \equiv \frac{\dot{m}}{\rho_s \sqrt{g\sigma}\, W\, \sigma}, \qquad (1)$$

where ρ_s, σ, and ν are the material density of the spheres, their diameter, and the solid volume fraction, respectively. W is the width of the chute, z is the coordinate perpendicular to the base, u is the velocity parallel to it, and g is the acceleration of gravity. The mass hold-up is

$$\int \nu \, d(z/\sigma) \equiv H^{\dagger} . \qquad (2)$$

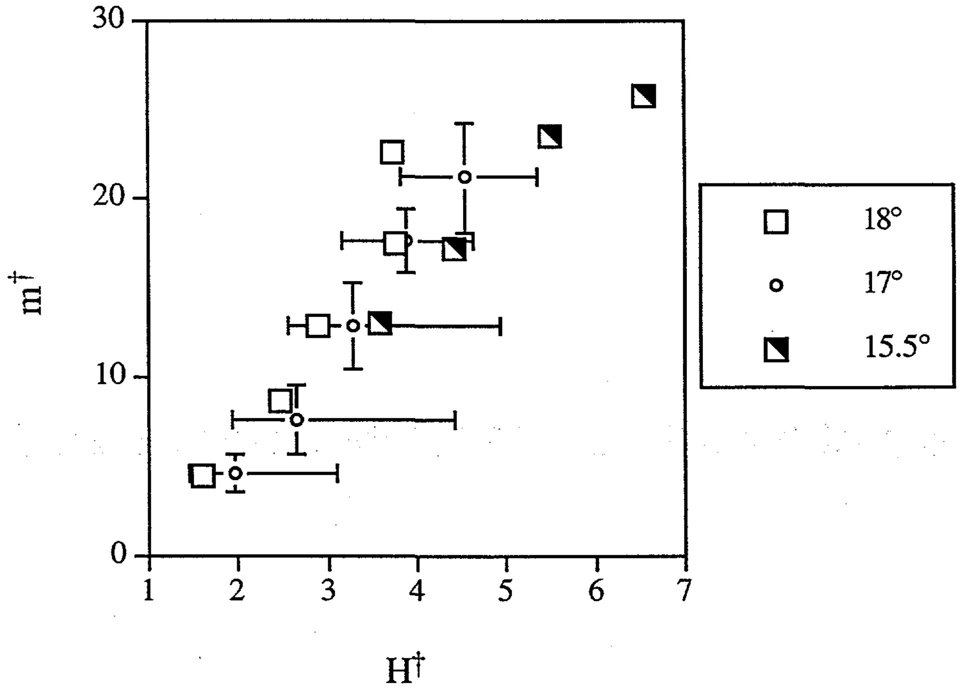

Fig. 1. Dimensionless mass flow rate against mass holdup. The error bars on the capacitance measurements are the possible excursions of the holdup that result from the presence of flood waves in the flow.

We observe the formation of waves along the length of the chute. Figures 2 and 3 show the dependence of the time history of these waves on gate height and chute inclination. The most remarkable waves travel upstream against the main flow. By recording their passage at two different locations (Fig. 4), we can establish their speed and wavelength (Fig. 5 and 6). We note that, although the waves can be very slow, their wavelength do not exceed the length of the chute.

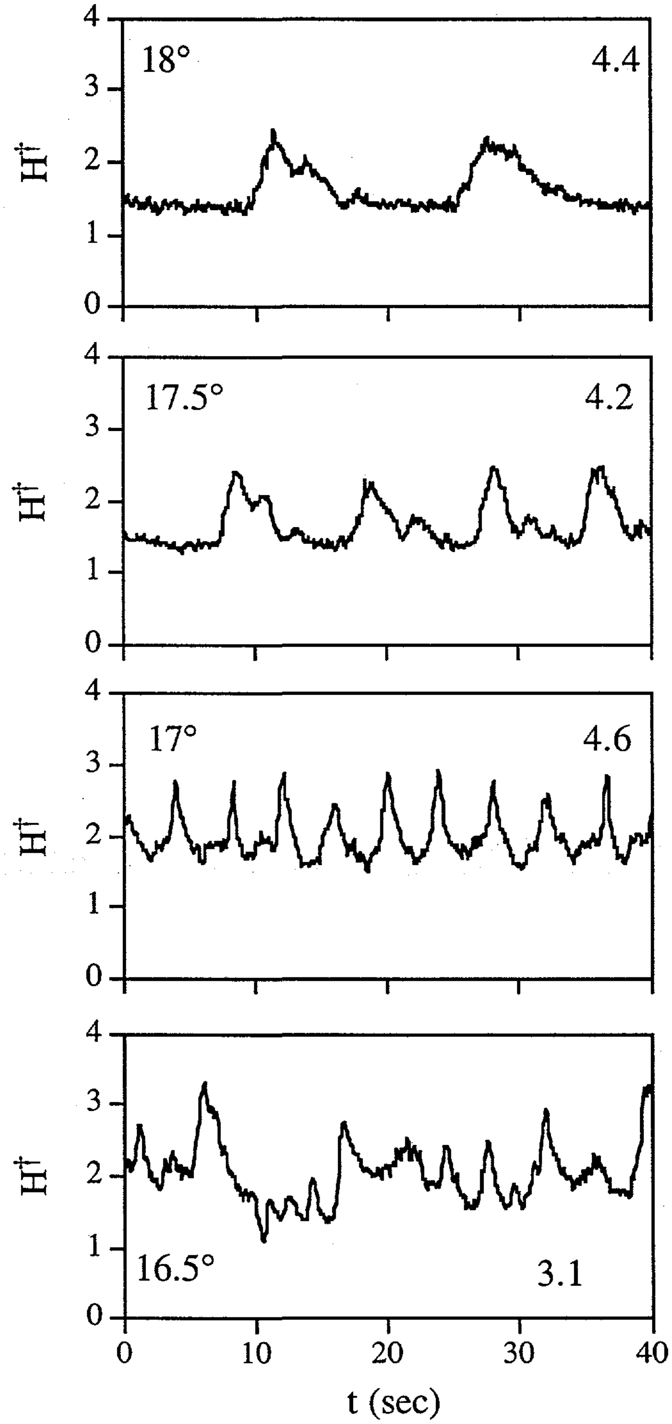

Fig. 2. Typical wave patterns at different inclinations but identical gate heights. The corresponding dimensionless mass flow rate is shown in each figure.

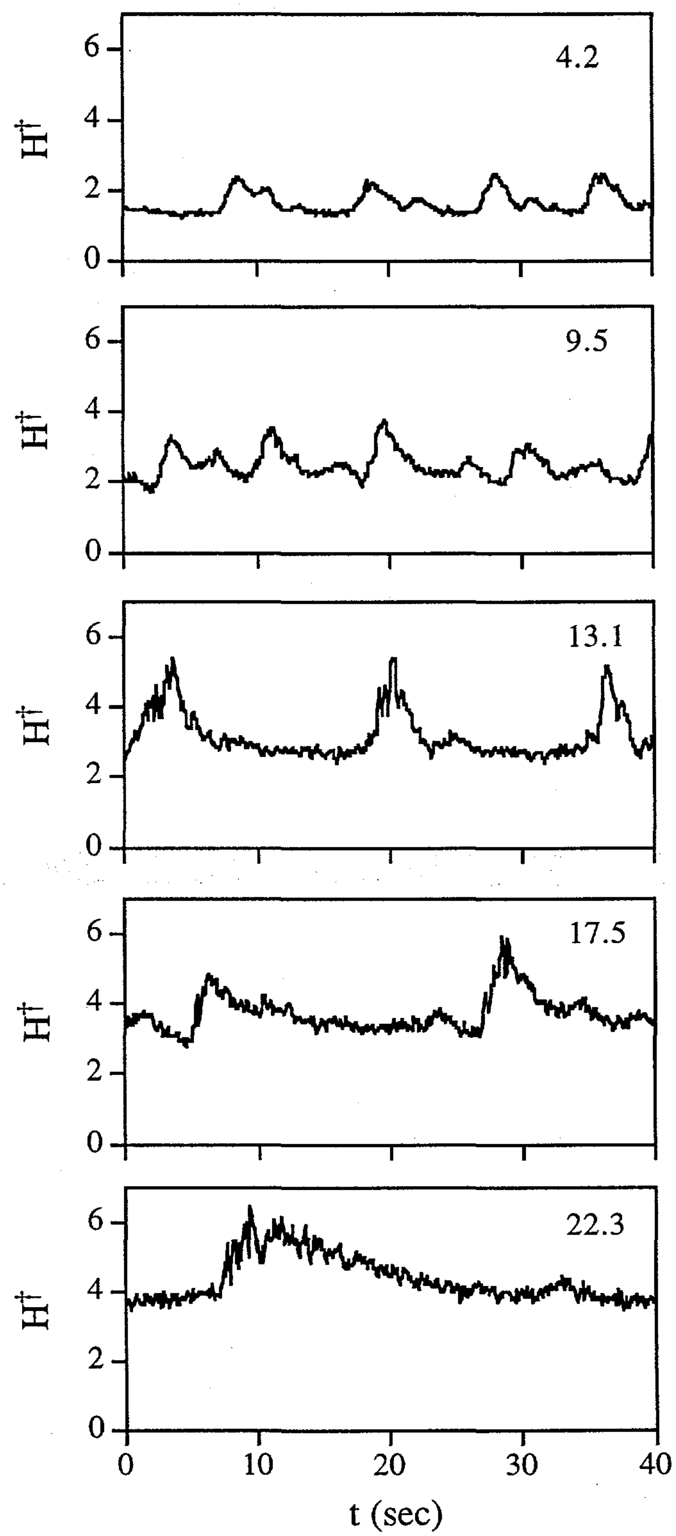

Fig. 3. Typical wave patterns at different gate heights for an inclination of 17.5°. The corresponding dimensionless mass flow rate is shown in each figure.

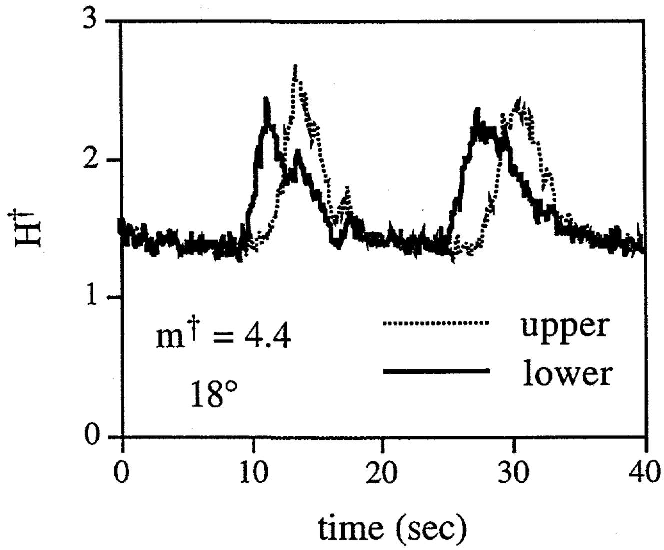

Fig. 4. Passage of a typical wave patterns at two different locations separated by approximately 82cm.

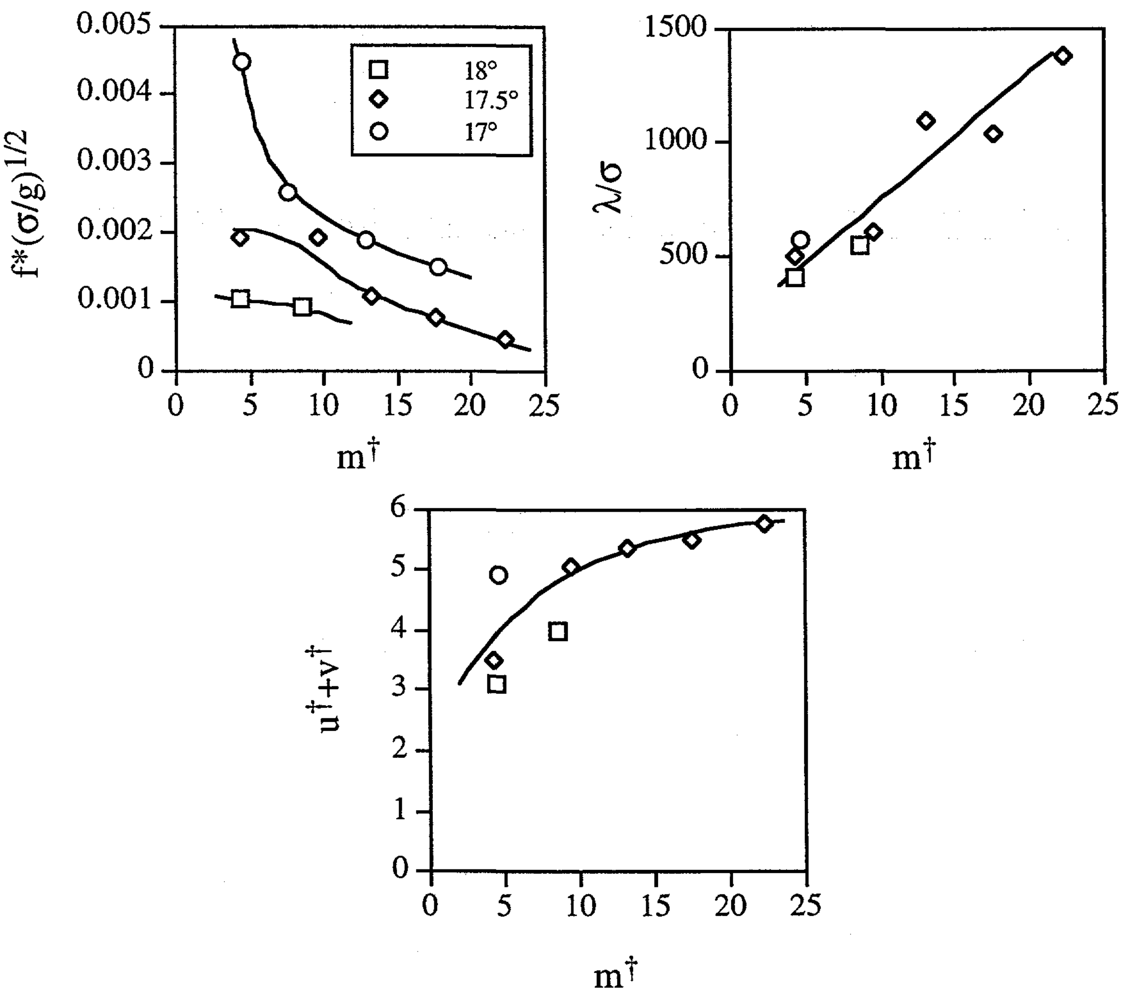

Fig. 5. Dimensionless wave frequency, wavelength and wave speed relative to the flow against mass flow rate. The length of the chute is $L/\sigma = 1220$.

Because the theory generally assumes that the granular flow is fully-developed, it is essential to establish whether the flow accelerates in a substantial way. A reasonable estimate of the acceleration can be inferred from changes of the mass holdup along the chute. We define a dimensionless convective acceleration $a^\dagger$ using

$$a^\dagger \equiv u^\dagger \frac{du^\dagger}{dx^\dagger} = \frac{m^{\dagger 2}}{2} \frac{d}{dx^\dagger} \left(\frac{1}{H^{\dagger 2}}\right) , \tag{3}$$

where the mean velocity $u^\dagger$ is $m^\dagger/H^\dagger$ and the dimensionless coordinate along the chute is $x^\dagger \equiv x/\sigma$. As Fig. 6 indicates, the convective acceleration is generally very small for most flows of interest. Note that it may alternate between positive and negative values for the same angle of inclination.

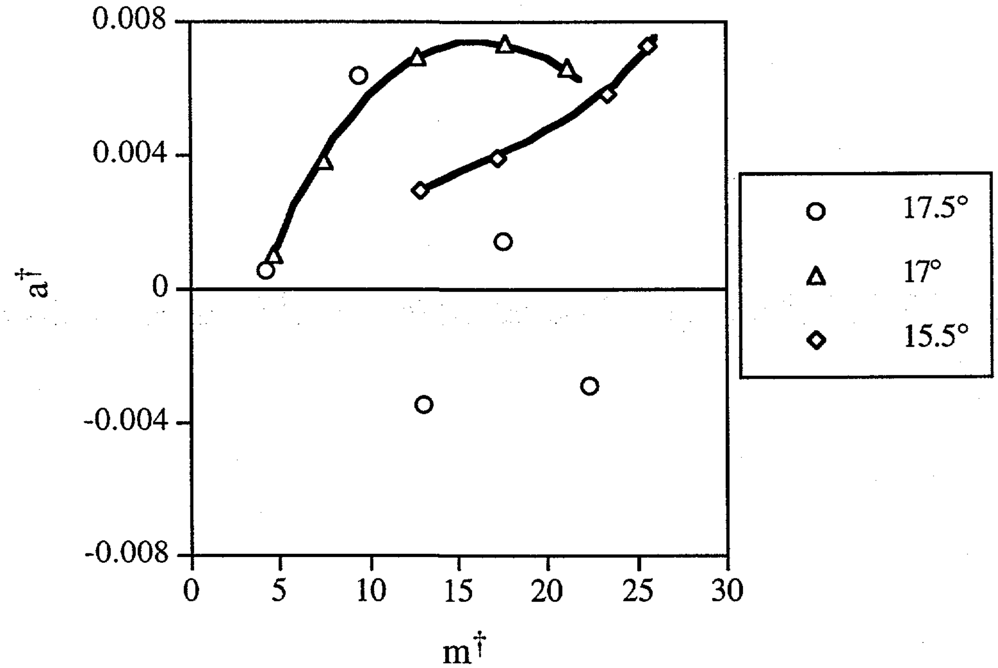

Fig. 6. Convective acceleration against mass flow rate for different inclinations.

Another difficulty in this chute of finite width is the influence of the side walls upon the flow. To establish the importance of this effect, we deposit large black spheres on the surface of the flow and we observe their descent along the chute with a video camera. As Fig. 6 indicates, the lateral profile of surface velocity is not flat.

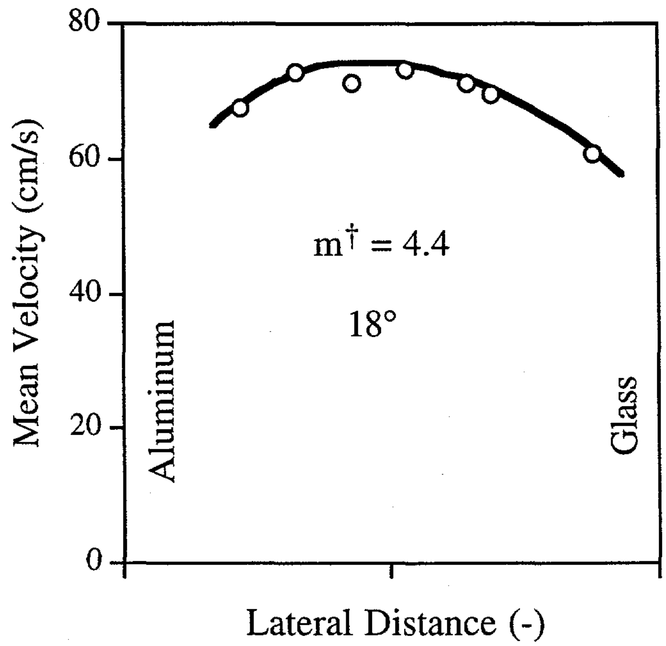

Fig. 6. Lateral profile of mean surface flow velocity.

Finally, our instrumentation permits simultaneous records of impulses exerted by the spheres on the bottom wall and the mass holdup across the flow at the same location. From the individual impulses $J = - m(1+e)\, c \cdot n$, we can calculate the normal stress associated with collisions,

$$N = -m(1+e)\, \mathbb{F}[c \cdot n]\,, \tag{4}$$

where the operator $\mathbb{F}[\psi]$ indicates the amount of quantity ψ striking the wall per unit time and unit surface,

$$\mathbb{F}[\psi] \equiv -g_{12}(0) \iint \psi\, f(c,\omega)\, c \cdot n \; dc\, d\omega\,. \tag{5}$$

In these expressions, m is the mass of each sphere and e is the coefficient of normal restitution between a sphere and the wall; c is the velocity vector of the center of mass of the sphere at contact; n is the unit normal to the wall; ω is the spin, and $f(c,\omega)$ is the velocity distribution function; $g_{12}(0)$ is the spatial distribution function at contact.

Assuming negligible mean flow velocity in the direction normal to the chute, the weight of the column of particles is balanced by the normal stress force applied by the wall on the grain assembly,

$$N = \rho_s\, g \cos\alpha \int \nu\, dz = \rho_s\, g \cos\alpha\, \sigma\, H^{\dagger}, \tag{6}$$

where α is the angle of inclination of the chute. Thus, a plot of the ratio

$$N^{\dagger}/H^{\dagger} \equiv N/\rho_s\, g \cos\alpha\, \sigma H^{\dagger} \qquad (7)$$

is a measure of the fraction of the weight of the granular assembly that is supported by collisional impulses at the base. As Fig. 7 illustrates, the fraction is often close to one, and it generally grows with mass holdup. This indicates that dense flows over a flat, frictional boundary are primarily supported by collisions with the base.

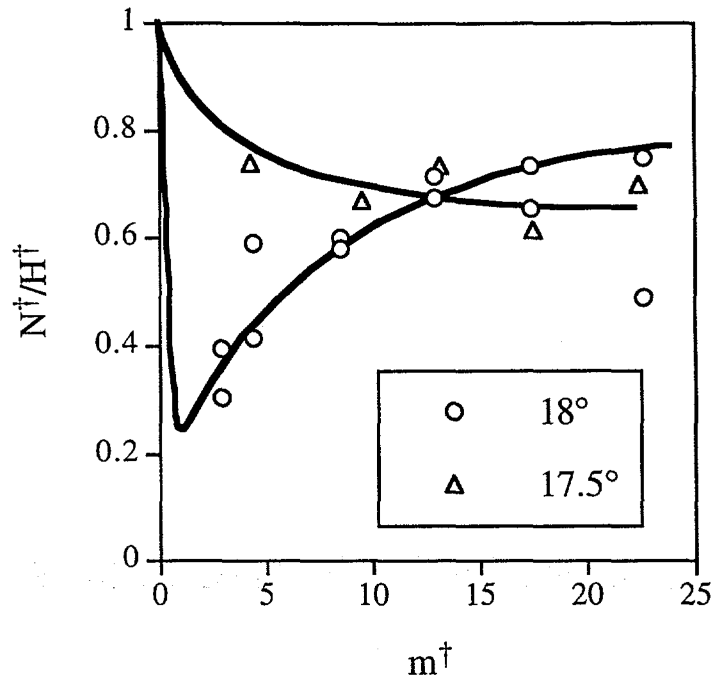

Fig. 7. Fraction of the weight of the granular assembly supported by collisional impulses at the base of the flow.

Another indication of the role of collisions is provided by our measurements of the granular temperature at the base. A robust way to determine the component of the temperature Θ_n in the normal direction is to build statistics of the collision frequency and the normal velocity at contact,

$$\Theta_n = \frac{\mathbb{F}[c.n^2]}{\mathbb{F}[1]} . \qquad (8)$$

Assuming negligible mean velocity in the normal direction, the balance of forces in (6) can be written in terms of the equation of state of the grain assembly at the wall,

$$N = \rho_s\, \nu\, g_{12}(0)\, \Theta_n\, (1+e) = \rho_s\, g\, \sigma\, H^{\dagger} \cos\alpha . \qquad (9)$$

This equation therefore predicts that the granular temperature should increase linearly with mass holdup. Figure 8 clearly confirms this prediction.

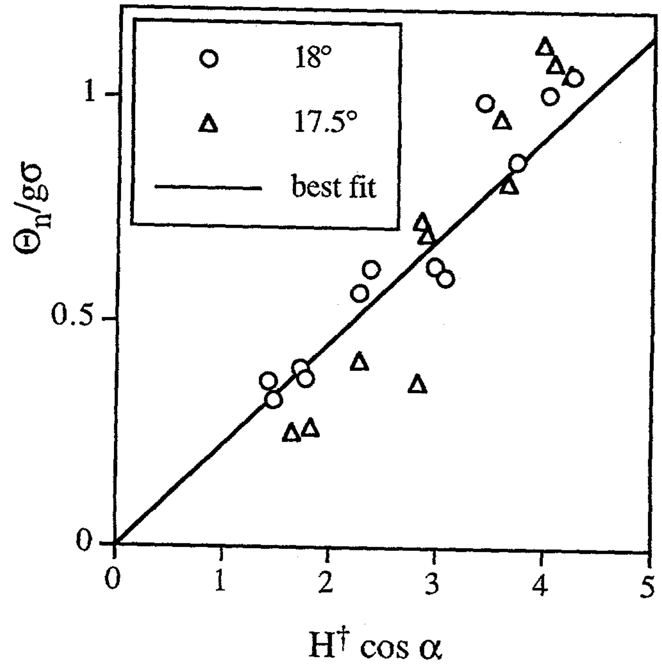

Fig. 8. Normal component of the granular temperature against mass holdup for relatively dense flows.

3) Next research

In the next reporting period, we anticipate to repeat the measurements shown above with a bumpy boundary consisting of glass spheres of 1mm randomly glued to the base of the chute.

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