Geometric Transitions and D-term SUSY Breaking*

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We propose a new way of using geometric transitions to study metastable vacua in string theory and certain confining gauge theories. The gauge theories in question are $\mathcal{N} = 2$ supersymmetric theories deformed to $\mathcal{N} = 1$ by superpotential terms. We first geometrically engineer supersymmetry-breaking vacua by wrapping D5 branes on rigid 2-cycles in noncompact Calabi-Yau geometries, such that the central charges of the branes are misaligned. In a limit of slightly misaligned charges, this has a gauge theory description, where supersymmetry is broken by Fayet-Iliopoulos D-terms. Geometric transitions relate these configurations to dual Calabi-Yaus with fluxes, where H_{RR}, H_{NS} and dJ are all non-vanishing. We argue that the dual geometry can be effectively used to study the resulting non-supersymmetric, confining vacua.

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1. Introduction

In the interest of finding controllable, realistic string vacua, it is important to find simple and tractable mechanisms of breaking supersymmetry in string theory. A powerful method which has been put forward in [1] consists of geometrically engineering metastable vacua with D-branes wrapping cycles in a Calabi-Yau manifold, and using geometric transitions and topological string techniques to analyze them. In [1], metastable vacua were engineered by wrapping D5 branes and anti-D5 branes on rigid 2-cycles in a Calabi-Yau. The non-supersymmetric vacuum obtained in this fashion was argued to have a simple closed-string dual description, in which branes and antibranes are replaced by fluxes. In [2], geometric transitions were used to study the physics of D-brane theories that break supersymmetry dynamically. Namely, the authors showed that the instanton generated superpotential that triggers supersymmetry breaking can be computed by classical means in a dual geometry, where some of the branes are replaced by fluxes (For an alternative approach see [3]).

In this paper, we propose another way to use geometric transitions to study supersymmetry breaking. As in [1], we consider D5 branes wrapping rigid cycles in a noncompact Calabi-Yau X. If $b_2(X) > 1$, supersymmetry can be broken by choosing the complexified Kähler moduli so as to misalign the central charges of the branes,

$$Z_i = \int_{S_i^2} J + iB_{NS}.$$

Since the 2-cycles wrapped by the D5 branes are rigid, any deformation of the branes costs energy, and the system is guaranteed to be metastable.[1] In the extreme case of anti-aligned central charges, we recover the brane/antibrane configurations of [1,5]. For slightly misaligned central charges, the system has a gauge theory description in terms of an $\mathcal{N} = 2$ quiver theory deformed to $\mathcal{N} = 1$ by superpotential terms [6,7], and with Fayet-Iliopoulos D-terms turned on [8,9,10].[2] The latter trigger spontaneous supersymmetry breaking in the gauge theory.

We argue that the dynamics of this system is effectively captured by a dual Calabi-Yau with all branes replaced by fluxes. Turning on generic Kähler moduli on the open-string

[1] Supersymmetry breaking by missaligning the central charges of D5 branes wrapped on rigid curves was also studied in [4], in the context of compact Calabi-Yau manifolds.

[2] Some geometric aspects of supersymmetry breaking by F-terms in this context were recently discussed in [11].

side has a simple interpretation in the dual low-energy effective theory as turning on a more generic set of FI parameters than hitherto considered in this context, but which are allowed by the $\mathcal{N} = 2$ supersymmetry of the background. On-shell, this breaks some or all of the $\mathcal{N} = 2$ supersymmetry. Geometrically, this corresponds to not only turning on H_{NS} and H_{RR} fluxes on the Calabi-Yau, but also allowing for $dJ \neq 0$ [12,13].[3] Moreover, we show that the Calabi-Yau geometries with these fluxes turned on have non-supersymmetric, metastable vacua, as expected by construction in the open-string theory.

The paper is organized as follows: In section two, we review the physics of D5 branes on a single conifold and the dual geometry after the transition, paying close attention to the effect of Fayet-Iliopoulos terms. In section three, we consider the case of an A_2 geometry where misaligned central charges lead to supersymmetry breaking. We provide evidence that the dual geometry correctly captures the physics of the non-supersymmetric brane system. We show that the results are consistent with expectations from the gauge theory, to the extent that these are available. We also comment on the relation of this work to [2], and point out some possible future directions. In an appendix, we lay out the general case for larger quiver theories. We show that in the limit of large separations between nodes, the theory has metastable, non-supersymmetric vacua in all cases where they are expected.

2. The Conifold

In this section, we consider N D5 branes on the resolved conifold. We will first review the open-string theory on the branes, and then discuss the dual closed-string description. Our discussion will be more general than the canonical treatment in that we will consider the case where the D5 branes possess an arbitrary central charge.

2.1. The D-brane theory

To begin with, let us recall the well-known physics of N D5 branes wrapping the S^2 tip of the resolved conifold. This geometry can be represented as a hypersurface in $\mathbb{C}^4[u, v, z, t]$,

$$uv = z(z - mt).$$

[3] For another example of supersymmetry breaking by turning on H_{NS}, H_{RR} and dJ fluxes, see [14].

The geometry has a singularity at the origin of $\mathbb{C}^4$ which can be repaired by blowing up a rigid $\mathbb{P}^1$. This gives the $\mathbb{P}^1$ a complexified Kähler class

$$Z = \int_{S^2} (J + iB_{NS}) = j + i\, b_{NS}. \tag{2.1}$$

The theory on the D5 branes at vanishing j reduces in the field theory limit to a $d = 4$, $\mathcal{N} = 1$, $U(N)$ gauge theory with an adjoint valued chiral superfield of mass m. The bare gauge coupling is given by

$$\frac{4\pi}{g_{YM}^2} = \frac{b_{NS}}{g_s}, \tag{2.2}$$

for positive b_{NS}. In string theory, the tension of the branes generates an energy density related to the four dimensional gauge coupling by

$$V_* = \frac{2N}{g_{YM}^2} = N\frac{b_{NS}}{2\pi g_s}. \tag{2.3}$$

Turning on a small, nonzero j can be viewed as a deformation of this theory by a Fayet-Iliopoulos parameter for the $U(1)$ center of the gauge group [8,9]. This deforms the Lagrangian by

$$\Delta\mathcal{L} = \sqrt{2}\xi\, \mathrm{Tr} D \tag{2.4}$$

where D is the auxiliary field in the $\mathcal{N} = 1$ vector multiplet and

$$\xi = \frac{j}{4\pi g_s}, \tag{2.5}$$

where the factor of g_s comes from the disk amplitude. This deformation (2.4) breaks the $\mathcal{N} = 1$ supersymmetry which was linearly realized at $j = 0$. In particular, turning on j, increases the energy of the vacuum: integrating out D from the theory by completing the square in the auxiliary field Lagrangian,

$$\mathcal{L}_D = \frac{1}{2g_{YM}^2}\mathrm{Tr} D^2 + \sqrt{2}\xi \mathrm{Tr} D,$$

raises the vacuum energy to

$$V_* = N\frac{b_{NS}}{2\pi g_s}\Big(1 + \frac{1}{2}\frac{j^2}{b_{NS}^2}\Big). \tag{2.6}$$

Supersymmetry is *not* broken however. At nonzero j, a different $\mathcal{N} = 1$ supersymmetry is preserved[4] – one that was realized nonlinearly at vanishing j [15,16]. Which subgroup

[4] This is true even in the field theory limit, despite the presence of the constant FI term. Namely, a second, nonlinearly realized supersymmetry is present in the gauge theory as long as there is only a constant energy density [15]. We thank A. Strominger for explaining this to us.

of the background $\mathcal{N}=2$ supersymmetry is preserved by the branes is determined by Z in (2.1), the BPS central charge in the extended supersymmetry algebra.[5] For any Z, the open-string theory on the branes has an alternative description which is manifestly $\mathcal{N}=1$ supersymmetric, with *vanishing* FI term and with a bare gauge coupling related to the magnitude of the central charge [7],

$$\frac{1}{\tilde{g}_{YM}^2} = \frac{\sqrt{b_{NS}^2 + j^2}}{4\pi g_s}. \tag{2.7}$$

Geometrically, this is just the quantum volume of the resolving $\mathbb{P}^1$. As such, the central charge also determines the exact tension of a single D5 brane at nonzero j, so

$$V_* = N\frac{\sqrt{b_{NS}^2 + j^2}}{2\pi g_s}. \tag{2.8}$$

For small Kähler parameter,

$$j \ll b_{NS},$$

this agrees with the vacuum energy in the field theory limit (2.6).

For any j, the theory is massive; it is expected to exhibit confinement and gaugino condensation at low energies, leaving an effective $U(1)$ gauge theory in terms of the center of the original $U(N)$ gauge group. We'll show next that the strongly coupled theory has a simple description for *any* value of Z in terms of a large N dual geometry with fluxes.

2.2. The geometric transition at general Z

We'll now discuss the large N dual geometry for general values of the central charge Z. Special cases (either vanishing j or vanishing b_{NS}) have been considered in the literature, but the present, expanded discussion is, to our knowledge, new.[6] We'll see that the dual geometry exactly reproduces the expected D5 brane physics. From the perspective of the low-energy effective action, the consideration of general central charge corresponds to turning on a more general set of $\mathcal{N}=2$ FI terms than previously considered in this context. Geometrically, this will lead us to consider generalized Calabi-Yau manifolds, for which dJ is nonvanishing in addition to having H_{NS} and H_{RR} fluxes turned on. This

[5] Strictly speaking, the central charge of N branes is NZ. In this paper, we will always take the number of branes N to be positive, so that we interpolate between the branes and antibranes by varying Z.

[6] See related discussion in [7].

will provide a local description of the physics for each set of branes in the more general supersymmetry-breaking cases of sections three and four.

To begin, let us recall the large N dual description of the D5 brane theory at vanishing j. This is given in terms of closed-string theory on the deformed conifold geometry,

$$uv = z(z - mt) + s. \tag{2.9}$$

This is related to the open-string geometry by a geometric transition which shrinks the $\mathbb{P}^1$ and replaces it with an S^3 of nonzero size,

$$S = \int_A \Omega,$$

where A is the 3-cycle corresponding to the new S^3, and the period of the holomorphic three-form over A is related to the parameters of the geometry by $S = s/m$. The D5 branes have disappeared and have been replaced by N units of Ramond-Ramond flux through the S^3,

$$\int_A H^{RR} = N. \tag{2.10}$$

There are Ramond-Ramond and Neveu-Schwarz fluxes through the dual, noncompact B-cycle as well,

$$\alpha = \int_B^{\Lambda_0} (H_{RR} + iH_{NS}/g_s) = b_{RR} + ib_{NS}/g_s, \tag{2.11}$$

which corresponds to the complexified gauge coupling of the open-string theory,

$$\alpha = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{YM}^2}.$$

The B-cycle is cut off at the scale, Λ_0, at which α is measured.[7] The dependence of α on the IR cutoff in the geometry corresponds to its renormalization group running in the open-string theory.

If it were not for the fluxes, the theory would have $\mathcal{N} = 2$ supersymmetry, with S being the lowest component of an $\mathcal{N} = 2$ $U(1)$ vector multiplet. That theory is completely described by specifying the prepotential, $\mathcal{F}_0(S)$, which can be determined by a classical geometry computation,

$$\int_B \Omega = \frac{\partial}{\partial S}\mathcal{F}_0.$$

[7] For simplicity, the IIB axion is set to zero in this paper.

The presence of nonzero fluxes introduces electric and magnetic Fayet-Iliopoulos terms in the low-energy theory for the $U(1)$ vector multiplet and its magnetic dual [17,18,19,20]. The effect of the fluxes (2.10),(2.11) can also be described in the language of $\mathcal{N}=1$ superspace as turning on a superpotential for the $\mathcal{N}=1$ chiral superfield with S as its scalar component,

$$\mathcal{W}(S) = \int_X \Omega \wedge (H^{RR} + iH^{NS}/g_s).$$

For the background in question, this takes the form

$$\mathcal{W}(S) = \alpha S - N\frac{\partial}{\partial S}\mathcal{F}_0. \tag{2.12}$$

In terms of the parameters of the D-brane theory, S is identified with the vev of the gaugino condensate. One way to see this is by comparing the superpotentials on the two sides of the duality. The αS superpotential on the closed string side corresponds to the classical superpotential term $\frac{\alpha}{4}TrW_\alpha W^\alpha$ on the gauge theory side.

What does the FI term deformation of the D-brane theory correspond to in the closed-string theory? To begin with, let us address this question from the perspective of the low-energy effective action. We know that the $U(1)$ gauge field after the transition coincides [6,21] with the $U(1)$ gauge field that is left over after the $SU(N)$ factor of the gauge group confines. This suggests that we should simply *identify* Fayet-Iliopoulos D-terms on the two sides. More precisely, the Lagrangian of the theory after the transition can be written in terms of $\mathcal{N}=1$ superfields,

$$\mathcal{S} = S + \sqrt{2}\theta\psi + \theta\theta F, \tag{2.13}$$

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D + \frac{i}{2}(\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu} \tag{2.14}$$

as an $\mathcal{N}=2$ action deformed to $\mathcal{N}=1$ by the superpotential (2.12),

$$\mathcal{L} = \frac{1}{4\pi}\mathrm{Im}\Big(\int d^2\theta d^2\bar\theta\, \bar{\mathcal{S}}\frac{\partial\mathcal{F}_0}{\partial\mathcal{S}} + \int d^2\theta\, \frac{1}{2}\frac{\partial^2\mathcal{F}_0}{\partial\mathcal{S}^2}\, W^\alpha W_\alpha + 2\int d^2\theta\, \mathcal{W}(\mathcal{S})\Big). \tag{2.15}$$

The Fayet-Iliopoulos deformation (2.4) should produce an additional term in this Lagrangian,

$$\Delta\mathcal{L} = \frac{j}{2\sqrt{2}\pi g_s}D, \tag{2.16}$$

corresponding to an FI term as in (2.5). Note that on the D-brane side, the center of mass $U(1)$ corresponds to $1/N$ times the identity matrix in $U(N)$, so that the normalization of (2.16) precisely matches (2.4).

We now show that the deformation (2.16) leads to precisely the physics that we expect on the basis of large N duality. After turning on the FI term, the effective potential of the theory becomes

$$V = \frac{1}{4\pi} G^{S\bar{S}} \left(|\partial_S \mathcal{W}|^2 + |j/g_s|^2 \right) + const. \tag{2.17}$$

where

$$G_{S\bar{S}} = \frac{1}{2i}(\tau - \bar{\tau}), \qquad \tau = \frac{\partial^2}{\partial S^2}\mathcal{F}_0.$$

We have also shifted the potential by an (arbitrary) constant, which we choose to be the tension of the branes at vanishing j,

$$const. = N\frac{b_{NS}}{2\pi g_s},$$

for convenience. We can then rewrite (2.17) as

$$\begin{aligned} V &= \frac{i}{2\pi(\tau - \bar{\tau})}\Big(|\alpha - N\tau|^2 + |j/g_s|^2 \Big) + const. \\ &= \frac{i}{2\pi(\tau - \bar{\tau})} |\tilde{\alpha} - N\tau|^2 + const. \end{aligned} \tag{2.18}$$

where

$$\tilde{\alpha} = b_{RR} + \frac{i}{g_s}\sqrt{b_{NS}^2 + j^2}$$

and the constant has shifted. As expected from the D-brane picture, the effective potential of the theory with the FI term turned on and with gauge coupling (2.2) is the same as that of the theory without the FI term and with gauge coupling (2.7).

In this simple example, the prepotential is known to be given exactly by

$$2\pi i \mathcal{F}_0(S) = \frac{1}{2} S^2 \left(\log(\frac{S}{\Lambda_0^2 m}) - \frac{3}{2} \right).$$

The vacuum of the theory is determined by the minimum of (2.18), which occurs at

$$\tilde{\alpha} - N\tau = 0, \tag{2.19}$$

or, in terms of the expectation value of the gaugino bilinear, at

$$S_* = m\Lambda_0^2 \exp(2\pi i \tilde{\alpha}/N). \tag{2.20}$$

Finally, we note that the energy in the vacuum (2.20) is larger than that in the $j = 0$ vacuum by the constant that enters (2.18), so

$$V_* = N\frac{\sqrt{b_{NS}^2 + j^2}}{2\pi g_s}, \tag{2.21}$$

which is precisely the tension of the brane after turning on j. This is a strong indication that we have identified parameters correctly on the two sides of the duality.

It is easy to see that in the vacuum, neither the F-term

$$\partial_S \mathcal{W} \neq 0,$$

nor the D-term vanishes. Nevertheless, as will now show, this new vacuum preserves half of the $\mathcal{N} = 2$ supersymmetry of the theory we started with, though not the one manifest in the action as written. Defining the $SU(2)_R$ doublet of fermions

$$\Psi = \begin{pmatrix} \psi \\ \lambda \end{pmatrix},$$

the relevant part of the supersymmetry transformations of the $\mathcal{N} = 2$ theory are

$$\delta\Psi^i = X^{ij}\epsilon_j$$

where X is a matrix of F- and D-terms, shifted by an imaginary part due to the presence of a "magnetic" FI term (see for example [1] and references therein)

$$X = \frac{i}{\sqrt{2}}\begin{pmatrix} -Y_1 - iY_2 + N & Y_3 \\ Y_3 & Y_1 - iY_2 + N \end{pmatrix} \tag{2.22}$$

where the $\mathcal{N} = 2$ auxiliary fields are identified with the auxiliary F-term of $\mathcal{S}$ in (2.13) and the D-term of the gauge field in (2.14) according to

$$(Y_1 + iY_2) = 2iF \qquad Y_3 = \sqrt{2}D.$$

Note that the triplet $\vec{Y} = (Y_1, Y_2, Y_3)$ transform like a vector of the $SU(2)_R$ symmetry of the $\mathcal{N} = 2$ theory. In the vacuum (2.19)

$$X = \frac{iN}{\sqrt{2(b_{NS}^2 + j^2)}}\begin{pmatrix} b_{NS} - \sqrt{b_{NS}^2 + j^2} & j \\ j & -b_{NS} - \sqrt{b_{NS}^2 + j^2} \end{pmatrix}.$$

The supersymmetry manifest in (2.15) corresponds to ϵ_1, and it is clearly broken in the vacuum for nonvanishing j, since neither the F- nor the D-term vanish. However, the

determinant of X vanishes, and so there is a zero eigenvector corresponding to a preserved supersymmetry.

So far, we have identified turning on j with turning on an FI term in the low-energy effective action. It is natural to ask what this corresponds to geometrically in the Calabi-Yau manifold? In [17,18] (following [22,23]) it was shown that turning on a subset of the FI terms of the low-energy $\mathcal{N} = 2$ theory arising from IIB compactified on a Calabi-Yau manifold corresponds to turning on H_{NS} and H_{RR} fluxes in the geometry. This is what we used in (2.12). The question of what corresponds to introducing the full set of FI terms allowed by $\mathcal{N} = 2$ supersymmetry was studied, for example, in [12,13]. To make the $SU(2)_R$ symmetry of the theory manifest, we can write the triplet of the $\mathcal{N} = 2$ FI terms as

$$E = \frac{i}{\sqrt{2}} \begin{pmatrix} -E_1 - iE_2 & E_3 \\ E_3 & E_1 - iE_2 \end{pmatrix}$$

where (E_1, E_2, E_3) transform as a vector under $SU(2)_R$ and enter the action as

$$\frac{1}{4\pi} Re(\mathrm{Tr} X \bar{E})$$

These are given in terms of ten dimensional quantities by[8]

$$E_1 = \int_B H_{NS}/g_s, \qquad E_2 = \int_B H_{RR}, \qquad E_3 = \int_B dJ/g_s.$$

Note that this agrees precisely with what we have just derived using large N duality. Just as the bare gauge coupling $\int_{S^2} B_{NS}/g_s = b_{NS}/g_s$ gets mapped to $\int_B H_{NS}/g_s$ after the transition due to running of the coupling, $SU(2)_R$ covariance of the theory demands that turning on $\int_{S^2} J/g_s = j/g_s$ before the transition get mapped to turning on $\int_B dJ/g_s$ after the transition. Moreover, we saw in this section that the latter coupling gets identified as a Fayet-Iliopoulos D-term for the $U(1)$ gauge field on the gravity side. This exactly matches the result of [12,13], since E_3 is the Fayet-Iliopoulos D-term parameter. It is encouraging to note that [12,13] reach this conclusion via arguments completely orthogonal to ours. Finally, we observe that an $SO(2) \subset SU(2)_R$ rotation can be used to set the Fayet-Iliopoulos D-term $E_3 = j/g_s$ to zero, at the expense of replacing $E_1 = b_{NS}/g_s$ by $E_1 = \sqrt{b_{NS}^2 + j^2}/g_s$, and this directly reproduces (2.18).

[8] This follows from equation (3.53) of [13] up to an $SU(2)_R$ rotation and specializing to a local Calabi-Yau. More precisely, to derive this statement one needs to look at the the transformations of the $\mathcal{N} = 2$ gauginos, not the gravitino as in [13], but these are closely related. See, for example, [23].

3. An A_2 Fibration and the Geometric Engineering of a Metastable Vacuum

By wrapping D5 branes on rigid $\mathbb{P}^1$'s in more general geometries with $b_2(X) > 1$, we can engineer vacua which are guaranteed to be massive and break supersymmetry by choosing the central charges of the branes to be misaligned. Since the D-brane theories experience confinement and gaugino condensation at low energies, we expect to be able to study the dynamics of these vacua in the dual geometries where the branes are replaced by fluxes.

In this section, we'll consider the simple example of an A_2 singularity fibered over the complex plane $\mathbb{C}[t]$. This is described as a hypersurface in $\mathbb{C}^4$,

$$uv = z(z - mt)(z - m(t - a)). \tag{3.1}$$

This geometry has two singular points at $u, v, z = 0$ and $t = 0, a$. The singularities are isolated, and blowing them up replaces each with a rigid $\mathbb{P}^1$. The two $\mathbb{P}^1$'s are independent in homology, and the local geometry near each of them is the same as that studied in the previous section.

Consider now wrapping N_1 D5 branes on the $\mathbb{P}^1$ at $t = 0$ and N_2 branes on the $\mathbb{P}^1$ at $t = a$. If the central charges of the branes,

$$Z_i = \int_{S_i^2} J + iB_{NS} = j_i + ib_{NS,i} \tag{3.2}$$

are aligned (e.g., if the Kähler parameters j_i both vanish), the theory on the branes has $\mathcal{N} = 1$ supersymmetry. At sufficiently low energies, it reduces to a $U(N_1) \times U(N_2)$ gauge theory with a bifundamental hypermultiplet $Q, \tilde{Q}$, a pair of adjoint-valued chiral fields $\Phi_{1,2}$ and a superpotential given by

$$W = \frac{m}{2} Tr\Phi_1^2 - \frac{m}{2} Tr\Phi_2^2 - aTrQ\tilde{Q} + Tr(Q\Phi_1\tilde{Q} - Q\tilde{Q}\Phi_2). \tag{3.3}$$

For a small relative phase of the central charges, e.g., by deforming the theory at vanishing Kähler parameters j_i by

$$j_i/b_{NS,i} \ll 1, \tag{3.4}$$

we expect this to have a pure gauge theory description at low energies in terms of the supersymmetric theory with Fayet-Iliopoulos terms for the two $U(1)$'s.

Misaligning the central charges such that

$$Z_1 \neq c_{12} Z_2, \tag{3.5}$$

for any positive, real constant c_{12}, should break all the supersymmetries of the background. Nevertheless, for large enough m and a, the vacuum should be stable. Since the theory is massive, we expect it to exhibit confinement at very low energies, with broken supersymmetry. Nevertheless, as we'll now argue, the dynamics of the theory can be studied effectively for any j_i in the dual geometry, where the branes have been replaced by fluxes.

3.1. Large N dual geometry

The Calabi-Yau (3.1) has a geometric transition which replaces the two $\mathbb{P}^1$'s by two S^3's,

$$S_i^2 \qquad \to \qquad S_i^3 \qquad\qquad\qquad i = 1, 2.$$

The complex structure of the geometry after the transition is encoded in its description as a hypersurface,

$$uv = z(z - mt)(z - m(t - a)) + ct + d, \tag{3.6}$$

where c, d are related to the periods, $S_{1,2}$, of the 3-cycles, $A_{1,2}$, corresponding to the two S^3's,

$$S_i = \int_{A_i} \Omega, \qquad \frac{\partial}{\partial S_i}\mathcal{F}_0 = \int_{B_i} \Omega.$$

As before, B_i are the noncompact 3-cycles dual to A_i, and $\mathcal{F}_0$ is the prepotential of the $\mathcal{N} = 2$ theory. The prepotential in this geometry is again given by an exact formula,

$$2\pi i\mathcal{F}_0 = \frac{1}{2}\, S_1^2\, (\log(\frac{S_1}{\Lambda_0^2 m}) - \frac{3}{2}) + \frac{1}{2}\, S_2^2\, (\log(\frac{S_2}{\Lambda_0^2 m}) - \frac{3}{2}) - S_1 S_2\, \log(\frac{a}{\Lambda_0}).$$

The theory with N_i D5 branes on the $\mathbb{P}_i^1$ before the transition is dual to a theory with N_i units of RR flux through S_i^3 after the transition:

$$\int_{A_i} H_{RR} = N_i.$$

There are additional fluxes turned on through the noncompact, dual B-cycles,

$$\alpha_i = \int_{B_i} (H_{RR} + iH_{NS}/g_s) = b_{RR,i} + ib_{NS,i}/g_s,$$

corresponding to running gauge couplings, and

$$\int_{B_i} dJ/g_s = j_i/g_s$$

corresponding to Fayet-Iliopoulos terms. The fluxes generate a superpotential,

$$\mathcal{W} = \int_X \Omega \wedge (H_{RR} + iH_{NS}/g_s)$$

or

$$\mathcal{W} = \sum_i \alpha_i S_i - N_i \frac{\partial}{\partial S_i}\mathcal{F}_0,$$

and Fayet-Iliopoulos D-terms,

$$\Delta\mathcal{L} = \sum_i \frac{j_i}{2\sqrt{2}\pi g_s} D_i,$$

where D_i are auxiliary fields in the two $\mathcal{N} = 1$ vector multiplets.

Large N duality predicts that for misaligned central charges (3.5), the fluxes should break all supersymmetries, and moreover, that the non-supersymmetric vacuum should be metastable. We'll now show that this indeed the case. The tree-level effective potential of the theory is

$$V = \frac{1}{4\pi} G^{i\bar{k}} \left(\partial_i \mathcal{W} \overline{\partial_k \mathcal{W}} + j_i j_k / g_s^2 \right) + const,$$

where

$$G_{i\bar{k}} = \frac{1}{2i}(\tau - \bar{\tau})_{ik} \qquad \tau_{ik} = \frac{\partial^2}{\partial S_i \partial S_k}\mathcal{F}_0,$$

and the Kähler metric is determined by the off-shell $\mathcal{N} = 2$ supersymmetry of the background. We have shifted the zero of the potential energy by the tension of the branes at vanishing j_i,

$$const. = \sum_{i=1,2} N_i \frac{b_{NS,i}}{2\pi g_s}.$$

In the case at hand,

$$\tau_{11} = \frac{1}{2\pi i}\log(\frac{S_1}{\Lambda_0^2 m}), \qquad \tau_{22} = \frac{1}{2\pi i}\log(\frac{S_2}{\Lambda_0^2(-m)}),$$

whereas τ_{12} is a constant[9] independent of the S_i,

$$\tau_{12} = -\frac{1}{2\pi i}\log(a/\Lambda_0).$$

It is straightforward to see that the critical points of the potential correspond to solutions of

$$Re(\alpha_i) + Re(\tau_{ik})N^k = 0$$
$$G^{ji}G^{jk}\left(Im(\alpha_i)Im(\alpha_k) + j_i j_k/g_s^2\right) = (N^j)^2.$$

[9] For convenience, we will take τ_{12} to be purely imaginary.

The first equation fixes the phase of S_i's, and the second their magnitude. Consider the case where the two nodes are widely separated, namely, where the sizes S_i of the two S^3's are much smaller than the separation a between them. In this limit, the equations of motion can be easily solved to obtain

$$\begin{aligned} S_{1,*}^{N_1} =&(\Lambda_0^2 m)^{N_1}(\frac{a}{\Lambda_0})^{N_2\cos\theta_{12}}\exp(2\pi i\tilde{\alpha}_1)+\dots \\ S_{2,*}^{N_2} =&(-\Lambda_0^2 m)^{N_2}(\frac{a}{\Lambda_0})^{N_1\cos\theta_{12}}\exp(2\pi i\tilde{\alpha}_2)+\dots \end{aligned} \tag{3.7}$$

where θ_{ij} is the relative phase between the central charges Z_i and Z_j. We can see that in the limit where the Z_i are aligned, this reduces to the simple case without FI terms where the effective gauge coupling has been replaced with the parameter $\tilde{\alpha}_i$. The case of anti-aligned central charges was studied in [1,5]. The weak coupling limit of two widely separated nodes, in which our approximation is justified, corresponds to

$$S_{i,*} \ll a < \Lambda_0. \tag{3.8}$$

$S_{i,*}$'s should be identified with the vev's of gaugino condensates on the branes, and are the order parameters of the theory. This is the case even in the presence of FI terms, as explained in the previous section. For small FI terms, this relies only on the off-shell $\mathcal{N}=1$ supersymmetry of the theories on both sides of the duality and a comparison of superpotentials. In [1] it was conjectured that this also holds in the brane/antibrane case, where the central charges are anti-aligned and supersymmetry is maximally broken. It is natural, then, that the above limit should correspond to the the theory being weakly coupled at the scale of the superpotential (3.3).

In the same limit, the vacuum energy is given by

$$V_* = N_1\frac{\sqrt{b_{NS,1}^2+j_1^2}}{2\pi g_s}+N_2\frac{\sqrt{b_{NS,2}^2+j_2^2}}{2\pi g_s}+\frac{1}{4\pi^2}N_1N_2\log(\frac{a}{\Lambda_0})(1-\cos\theta_{12})+\dots \tag{3.9}$$

Note that in the limit of aligned central charges, the potential energy is simply the brane tension. This is in fact true *exactly*, and is related to the fact (which will demonstrate later on) that in this case supersymmetry is preserved. For any other value of the angle, there is an additional attraction. In the extreme case, when we increase θ_{12} from zero to π, we end up with a brane/antibrane system on the flopped geometry. We can view this as varying one of the Z_i's until the B_{NS} field through that cycle goes to *minus* itself.

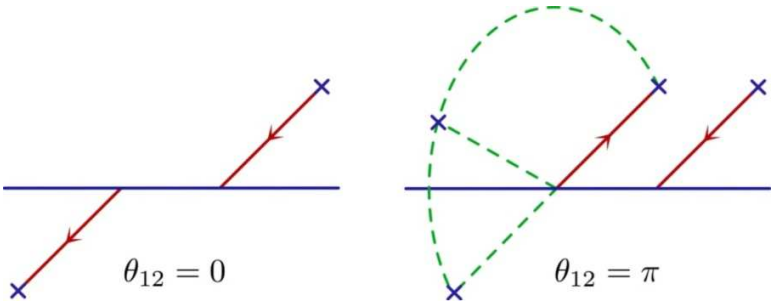

Fig. 1. The A_2 geometry from in the text, drawn in the T-dual NS5 brane picture. The D5 branes map to D4 branes and appear as red lines. The NS5 branes are drawn as blue lines/points. At $\theta_{12} = 0$ the system is supersymmetric. For any other value of θ_{12}, supersymmetry is broken. Varying θ_{12} continuously from zero to π produces a geometry which is related to the original A_2 geometry by a flop.

This is a flop, and by comparing to [5], it follows that the solution we found above for $\theta_{12} = \pi$ precisely corresponds to a brane/antibrane system in the flopped geometry.

To see that supersymmetry is broken by the vacuum at nonvanishing θ_{12}, we write the action (2.15) in an $\mathcal{N} = 2$ invariant way in terms of $\mathcal{N} = 2$ chiral multiplets $\mathcal{A}_i$ consisting of $\mathcal{N} = 1$ chiral multiplets $\mathcal{S}_i$, and $\mathcal{W}_i^\alpha$,

$$\mathcal{A}_i = (\mathcal{S}_i, W_i^\alpha)$$

or

$$\mathcal{A}_i = S_i + \theta^a \Psi_{a,i} + \theta^a \theta^b X_{ab,i} + \frac{1}{2}\epsilon_{ab}(\theta^a \sigma^{\mu\nu} \theta^b) F_{\mu\nu} + \dots .$$

The appropriate $\mathcal{N} = 2$ Lagrangian is given by

$$\mathcal{L} = \frac{1}{4\pi}\mathrm{Im}\Big(\int d^4\theta d^4x \; \mathcal{F}_0(\mathcal{A}_i)\Big) + \frac{1}{4\pi} Re(X_i^{ab} \bar{E}_{ab}^i)$$

where X_i^{ab} is defined as in (2.22). Then, the relevant supersymmetry variations of the fermions are given by

$$\delta_\epsilon \Psi_i^a = X_i^{ab} \epsilon_b, \tag{3.10}$$

and at the extrema of the effective potential,

$$X_i = \frac{i}{\sqrt{2}} \begin{pmatrix} N^i + G^{ik}\mathrm{Im}(\alpha_k) & -G^{ik} j_k / g_s \\ -G^{ik} j_k / g_s & N^i - G^{ik}\mathrm{Im}(\alpha_k) \end{pmatrix}. \tag{3.11}$$

The equations of motion imply that the determinants of both $X^{1,2}$ vanish. For each node, then, X^i has one zero eigenvalue. It can be shown (see the general discussion of appendix

A), that (3.10) and (3.11) imply that a global supersymmetry is preserved if and only if the central charges (3.2) are aligned, i.e., if a positive real constant c_{12} exists such that

$$Z_1 = c_{12} Z_2.$$

This is exactly as expected from the open-string picture, and provides a nice test of the large N duality conjecture for general central charges.

Now we'll show that the vacuum is indeed metastable. Consider the masses of bosonic fluctuations about the non-supersymmetric vacuum. As was found to be the case in [5], the Hessian of the scalar potential can be block diagonalized. After changing variables to bring the kinetic terms into their canonical form, the eigenvalues become

$$\begin{aligned} M^2_{\phi_{1,2}} &= \frac{(a^2+b^2+2abv) \pm \sqrt{(a+b)^2(a-b)^2 + 4abv(a+b)(b+a)}}{2(1-v)^2} \\ M^2_{\phi_{3,4}} &= \frac{(a^2+b^2+2abv\cos\theta) \pm \sqrt{(a+b)^2(a-b)^2 + 4abv(a+b\cos\theta)(b+a\cos\theta)}}{2(1-v)^2} \end{aligned} \tag{3.12}$$

where we've adopted the notation of [1] in defining

$$a = \frac{N_1}{2\pi G_{11}|S_1|}, \qquad\qquad b = \frac{N_2}{2\pi G_{22}|S_2|},$$

and

$$v = \frac{G_{12}^2}{G_{11}G_{22}},$$

with all quantities evaluated in the vacuum. In addition, we've introduced a new angle, which is defined by the equation

$$\vec{Y}^1 \cdot \vec{Y}^2 = N_1 N_2 \cos\theta,$$

where $\vec{Y}^i$ are related to X^i as in (2.22) for each i. In the limit (3.8),

$$\theta = \theta_{12} + \mathcal{O}(v),$$

so we can treat this as being the same as the phase which appears in (3.7),(3.9). Note that in the limit

$$\cos\theta \to 1, \tag{3.13}$$

we recover the results for a supersymmetric system, with the masses of bosons becoming pairwise degenerate. The other extreme of anti-aligned charges can be shown to correspond to

$$\cos\theta \to -1,$$

where the results of [5] should be recovered. Indeed by plugging in and rearranging terms we recover the mass formulas from page 25 of [5]. These then provide exact values for the tree-level masses of the component fields in the supersymmetry-breaking vacuum. For any θ, the masses of the bosons are all positive as long as

$$v < 1.$$

This, in turn, is ensured as long as the metric on moduli space is positive definite in the vacuum. So indeed, the system is metastable, as expected.

To get a measure of supersymmetry breaking, let's now compare the masses of the bosons and the fermions in this vacuum. The fermion masses arise from the superspace interaction which appears as

$$\frac{1}{4\pi} Im\left(\int d^4\theta d^4x\ \frac{1}{2}\mathcal{F}_{ijk}(\Psi_a^i\theta^a)(\Psi_b^j\theta^b)(X_{cd}^k\theta^c\theta^d)\right),$$

where $\mathcal{F}_{ijk} = \partial_i\partial_j\partial_k\mathcal{F}_0$. For the geometry in question, the prepotential is exact at one-loop order, and the third derivatives vanish except when all derivatives are with respect to the same field. We can then write the fermion mass matrices for a given node (and non-canonical kinetic terms) as

$$M_{ab}^i = \frac{1}{16\pi^2 S_i}\epsilon_{ac}X_{cd}^i\epsilon_{db}$$

Performing a change of basis to give the fermion kinetic terms a canonical form, we can diagonalize the resulting mass matrix and obtain the mass eigenvalues. There are two zero modes,

$$M_{\lambda_{1,2}} = 0,$$

corresponding to two broken supersymmetries. In addition there are two massive fermions, which we label by ψ_i,

$$M_{\psi_{1,2}} = \frac{(a+b) \pm \sqrt{(a-b)^2 + 2abv(1+\cos\theta)}}{2(1-v)}. \tag{3.14}$$

Note that in the supersymmetric limit (3.13), the masses of ψ_1 and ψ_2 match those of $\phi_{1,3}$ and $\phi_{2,4}$, which have become pairwise degenerate. For small misalignment, and large separation of the two nodes, the mass splittings of bosons and fermions are easily seen to go like

$$\frac{M_\phi^2 - M_\psi^2}{M_\phi^2 + M_\psi^2} \sim v\theta_{12}^2,$$

where v goes to zero in the limit of large separation, and θ_{12} measures the misalignment of the central charges.

3.2. Gauge theory limit

In the gauge theory limit (3.4), the vacuum energy (3.9) reduces to

$$V_* = \sum_i N_i \frac{b_{NS,i}}{2\pi g_s}(1 + \frac{1}{2}\frac{j_i^2}{b_{NS,i}^2}) + \frac{1}{8\pi^2} N_1 N_2 \log(\frac{a}{\Lambda_0})\Big(\frac{j_1}{b_{NS,1}} - \frac{j_2}{b_{NS,2}}\Big)^2 + \dots . \qquad (3.15)$$

The first terms are classical contributions, as we saw in section two. The last term comes from a one-loop diagram in string theory, with strings stretched between the two stacks of branes running around the loop.

To begin with, consider the Abelian case,[10]

$$U(1) \times U(1),$$

when the gauge theory has no strong dynamics at low energies. We should be able to reproduce (3.15) directly in the field theory by computing the one-loop vacuum amplitude in a theory with FI terms turned on. We can write the classical F- and D-term potential of the gauge theory as

$$V_{tree} = |F_{\Phi_1}|^2 + |F_{\Phi_2}|^2 + |F_Q|^2 + |F_{\tilde{Q}}|^2 + \frac{1}{2} g_{YM,1}^2 (|q|^2 - |\tilde{q}|^2 - \sqrt{2}\xi_1)^2 + \frac{1}{2} g_{YM,2}^2 (|\tilde{q}|^2 - |q|^2 - \sqrt{2}\xi_2)^2,$$

where

$$F_{\Phi_1} = m\phi_1 - q\tilde{q} \qquad F_{\Phi_2} = m\phi_2 - q\tilde{q} \qquad F_Q = \tilde{q}(a + \phi_2 - \phi_1) \qquad F_{\tilde{Q}} = q(a + \phi_2 - \phi_1)$$

[10] Although the rank of the gauge group is not large in this case, the geometric transition is still expected to provide a smooth interpolation between the open- and closed-string geometries. For a recent review, see [2], and references therein. It is natural to expect that for small deformations by FI terms that break supersymmetry, the two sides still provide dual descriptions of the same physics.

and ϕ_i, $q, \tilde{q}$ are the lowest components of the corresponding chiral superfields. The gauge theory quantities are related to those of the string theory construction by

$$\frac{1}{g^2_{YM,i}} = \frac{b_{NS,i}}{4\pi g_s}, \qquad \xi_i = \frac{j_i}{4\pi g_s}.$$

The identification between the field theory FI term and the string theory parameter is expected to hold only for small $j_i/b_{NS,i}$. For nonzero $\xi_{1,2}$, supersymmetry appears to be broken since the two D-term contributions cannot be simultaneously set to zero with the F-terms. In fact, we know that if the central charges are aligned, this is just a relic of writing the theory in the wrong superspace.

For large m, a, this potential has a critical point at the origin of field space. At this point, all the F-terms vanish, and there is pure D-term supersymmetry breaking. The spectrum of scalar adjoint and gauge boson masses is still supersymmetric at tree-level, since the only contribution to the masses in the Lagrangian is the FI-dependent piece for the bifundamentals. This means that the only relevant contribution to the one-loop corrected potential is from the bifundamental fields. The scalar components develop a tree-level mass which is simply given by[11]

$$m_q^2 = a^2 + r, \qquad m_{\tilde{q}}^2 = a^2 - r \tag{3.16}$$

while the fermion masses retain their supersymmetric value,

$$m_{\psi_q}^2 = m_{\psi_{\tilde{q}}}^2 = a^2.$$

We have defined the constant

$$r = \sqrt{2}(\xi_2 g^2_{YM,2} - \xi_1 g^2_{YM,1}) = \sqrt{2}(\frac{j_2}{b_{NS,2}} - \frac{j_1}{b_{NS,1}}). \tag{3.17}$$

The one-loop correction to the vacuum energy density is given by

$$V^{(1-loop)} = \frac{1}{64\pi^2}\left(\sum_b m_b^4 \log\frac{m_b^2}{\Lambda_0^2} - \sum_f m_f^4 \log\frac{m_f^2}{\Lambda_0^2}\right),$$

[11] One can easily check that for small r, the masses agree with what we expect from string theory. The bifundamental matter is the same as for the $0-4$ system, with small B-fields turned on along the D4 branes. See, e.g., [24,25].

where $m_{b,f}$ are the boson and the fermion masses, and Λ_0 is the UV cutoff of the theory. The limit in which we expect a good large N dual is when the charged fields are very massive, $r \ll a^2$, and at low energies the theory is a pure gauge theory. Expanding to the leading order in r/a^2, the one-loop potential is then given by

$$V = V_{tree} + \frac{1}{16\pi^2} r^2 \log \frac{a}{\Lambda_0}.$$

We have omitted the Λ_0 independent terms which correspond to the finite renormalization of the couplings in the Lagrangian and are ambiguous. We see that this exactly agrees with effective potential (3.9),(3.15) as computed in the dual geometry, after the transition.

In the general, $U(N_1) \times U(N_2)$ case, we have a strongly coupled gauge theory at low energies. Nevertheless, since in the $(N_i, \bar{N}_i)$ sector supersymmetry is preserved, the one-loop contribution of that sector to the vacuum energy density should vanish beyond the classical contribution. Thus, we expect that only the bifundamental fields contribute to the vacuum energy at this level. The one-loop computation then goes through as in the Abelian case, up to the $N_1 N_2$ factor from multiplicity, once again reproducing the answer (3.15) from large N dual geometry.

3.3. Relation to the work of [2]

We close with a comment on the relation to the work of [2], to put the present work in context. The A_2 model at hand is the same as the geometry used to engineer the Fayet model in [2]. More precisely, the authors there engineered a "retrofitted" Fayet model. The parameter a that sets the mass of the bifundamentals was generated by stringy or fractional gauge theory instantons, and thus was much smaller than the scale set by the FI terms, which were taken to be generic. That resulted in F-term supersymmetry breaking which was dynamical.

In the present context, we still have a Fayet-type model, but we find ourselves in a different regime of parameters of the field theory, where $r/a^2 < 1$, with r defined in terms of the FI parameters as in (3.17). Outside of this regime, the vacuum at the origin of field space, with Q and $\tilde{Q}$ vanishing, becomes tachyonic even in the field theory, as can be seen from (3.16). Once this becomes the case, the large N dual presented here is unlikely to be a good description of the physics. For example, for $N_1 = N_2 = N$ and $r/a^2 > 1$, it was found in [2] that the theory has a non-supersymmetric vacuum where all the charged bifundamental fields are massive and the gauge symmetry is broken to $U(N)$. This may

still have a description in terms of some dual geometry with fluxes, but not the one at hand. This may be worth investigating.

Thus, unlike the models of [2], those considered here break supersymmetry spontaneously but not dynamically. It would be nice to find a way to retrofit the current models and to generate low scale supersymmetry breaking in this context. This would require finding a natural way of obtaining small FI terms. The mechanism of [2] does not apply here, since the terms in question are D-terms and not F-terms. This may be possible in the context of warped compactifications,[12] and compact Calabi-Yau manifolds, perhaps along the lines of [29].

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Appendix A. Fayet-Iliopoulos Terms for ADE Singularities

The large N duality we studied in the previous sections should generalize to other ADE fibered geometries. In this appendix we'll demonstrate that the large N dual geometries for these more general spaces have some of the same qualitative features. Consider the ADE type ALE spaces

$$\begin{aligned} A_k &: \quad x^2 + y^2 + z^{k+1} = 0 \\ D_r &: \quad x^2 + y^2 z + z^{r-1} = 0 \\ E_6 &: \quad x^2 + y^3 + z^4 = 0 \\ E_7 &: \quad x^2 + y^3 + y z^3 = 0 \\ E_8 &: \quad x^2 + y^3 + z^5 = 0 \end{aligned}$$

which are fibered over the complex t plane, allowing the coefficients parameterizing the deformations to be t dependent. The requisite deformations of the singularities are canonical

[12] The effects of warping in the context of [1] have been studied in [26]. In [27], the authors constructed supergravity solutions that were subsequently interpreted in [28], to correspond to D5 branes on the conifold with Fayet-Iliopolous terms turned on, i.e. the theory we studied in section two. We thank Y. Nakayama for pointing out to us the latter work.

(see [7] and references therein). In fibering this over the t plane, the z_i become polynomials $z_i(t)$. At a generic point in the t plane, the ALE space is smooth, with singularities resolved by blowing up r independent 2-cycle classes

$$S_i^2, \qquad i = 1, \ldots r$$

where r is the rank of the corresponding Lie algebra. This corresponds to turning on Kähler moduli

$$Z_i = \int_{S_i^2} (J + iB_{NS}) = j_i + ib_{NS,i}.$$

The 2-cycles S_i^2 intersect according to the ADE Dynkin diagram of the singularity. Consider now wrapping N_i D5 branes on the i'th 2-cycle class. The theory on the branes is an $\mathcal{N} = 2$ quiver theory with gauge group

$$\prod_i U(N_i),$$

with a bifundamental hypermultiplet Q_{ij}, Q_{ji} for each pair of nodes connected by a link in the Dynkin diagram. The fibration breaks the supersymmetry to $\mathcal{N} = 1$ by turning on superpotentials $W_i(\Phi_i)$ for the adjoint chiral multiplets Φ_i,

$$W_i'(t) = \int_{S_{i,t}^2} \omega^{2,0},$$

which compute the holomorphic volumes of the 2-cycles at fixed t. The superpotentials $W_i(t)$ can be thought of as parameterizing the choice of complex structure of the ALE space at each point in the t plane. The full tree-level superpotential of the theory is given by

$$W = \sum_i Tr W_i(\Phi_i) + \sum_{i<j} Tr(Q_{ij}Q_{ji}\Phi_i - Q_{ij}\Phi_j Q_{ji})$$

where the latter sum runs over nodes that are linked.

For vanishing j_i, the structure of the vacua of the theory was computed in [7]. For each positive root e_I of the lie algebra,

$$e_I = \sum_I n_I^i e_i$$

for positive integers n_I^i, one gets a rigid $\mathbb{P}^1$ at points in the t-plane

$$t = a_{I,p},$$

where

$$W_I'(a_{I,p}) = \sum_i n_I^i \, W_i'(a_{I,p}) = 0. \tag{A.1}$$

Here I labels the positive root and p runs over all the solutions to (A.1) for that root. The choice of vacuum breaks the gauge group down to

$$\prod_{I,p} U(M_{I,p})$$

where

$$N_i = \sum_I M_{I,p} n_I^i.$$

Turning on generic Fayet-Iliopoulos terms for the $U(1)$ centers of the gauge group factors,

$$\Delta\mathcal{L} = \sum_i \frac{j_i}{2\sqrt{2}\pi g_s} Tr D_i,$$

breaks supersymmetry while retaining (meta)stability of the vacuum as long as j_i is much smaller than the mass of all the bifundamentals in the vacuum.

The ALE fibrations have geometric transitions in which each $\mathbb{P}^1$ is replaced by a minimal S^3. The leading order prepotential $\mathcal{F}_0$ for all these singularities was computed in [30], and is given by

$$2\pi i \mathcal{F}_0(S) = \frac{1}{2} \sum_b \, S_b^2 \left(\log\Big(\frac{S_b}{W_I''(a_b)\, \Lambda_0^2} \Big) - \frac{3}{2} \right) + \frac{1}{2} \sum_{b \neq c} e_{I(b)} \cdot e_{J(c)} \, S_b \, S_c \, \log\Big(\frac{a_{bc}}{\Lambda_0} \Big) + \ldots, \tag{A.2}$$

where the sum is over all critical points

$$b = (I, p),$$

and $I(b) = I$ denotes the root I to which the critical point labeled by b corresponds. We are neglecting cubic and higher order terms in the $S_{I,p}$, which are related to higher loop corrections in the open string theory. Above, $e_I \cdot e_J$ is the inner product of two positive, though not necessarily simple, roots. Geometrically, the inner product is the same as minus the intersection number of the corresponding 2-cycles classes in the ALE space. In addition, there are fluxes turned on in the dual geometry which are determined by holography:

$$\int_{A_a} H_{RR} = M_a$$

$$\int_{B_a} (H_{RR} + \frac{i}{g_s} H_{NS}) = b_{RR,I(a)} + \frac{i}{g_s} b_{NS,I(a)} \quad , \qquad \int_{B_a} dJ = j_{I(a)}.$$

The theory on this geometry without fluxes is an $\mathcal{N}=2$, $U(1)^k$ gauge theory, where k is the number of S^3's. The effect of the fluxes on the closed-string theory in this background was determined in [12,13]. The result is a set of electric and magnetic $\mathcal{N}=2$ Fayet-Iliopoulos terms, which enter the $\mathcal{N}=2$ superspace Lagrangian,

$$\mathcal{L} = \frac{1}{4\pi}\mathrm{Im}\left(\int d^4\theta\ \mathcal{F}_0(\mathcal{A}^a)\right) + \frac{1}{4\pi}Re(\vec{Y}^a\cdot\vec{E}_a).$$

with

$$\vec{E}_a = \left(\ \frac{b_a^{NS}}{g_s}\ ,\ b_{RR,a}\ ,\ \frac{j_a}{g_s}\ \right),$$

and where the auxiliary fields $\vec{Y}^a$ are shifted by the magnetic FI term,

$$\vec{M}^a = (\ 0\ ,\ M^a\ ,\ 0\ ).$$

The auxiliary field Lagrangian then has the form

$$\mathcal{L}_{aux} = \frac{1}{8\pi}G_{ab}Re(\vec{Y})^a\cdot Re(\vec{Y})^b + \frac{1}{4\pi}Re(\tau_{ab})Re(\vec{Y})^a\cdot\vec{M}^b + \frac{1}{4\pi}Re(\vec{Y})^a\cdot\vec{E}_a$$

and integrating out the auxiliary fields sets them equal to their expectation values,

$$\vec{Y}^a = -G^{ab}\left(\vec{E}_b + Re(\tau_{bc})\vec{M}^c\right) + i\vec{M}^a$$

or, to be more precise,

$$-G_{ab}\vec{Y}^b = \left(b_a^{NS}/g_s\ ,\ b_a^{RR} + \bar{\tau}_{ab}M^b\ ,\ j_a/g_s\right).$$

We can make contact with the more familiar form of this action and its scalar potential by reducing to $\mathcal{N}=1$ superspace. There, the auxiliary fields $\vec{Y}^a$ get identified with auxiliary fields of the vector and chiral multiplets corresponding to the a'th S^3, and the fluxes give rise to the usual flux superpotential

$$\mathcal{W} = \sum_a \alpha_a S_a - M_a\partial_{S_a}\mathcal{F}_0(S).$$

In addition, there are Fayet-Iliopoulos terms for the $U(1)$'s, and the total scalar potential is given by

$$V = \frac{1}{4\pi}G^{ab}\left(\partial_a\mathcal{W}\overline{\partial_b\mathcal{W}} + j_a j_b/g_s^2\right)$$

where

$$\alpha_a = b_{RR,a} + i b_{NS,a}/g_s.$$

There are vacua at the field values which satisfy

$$\partial_a V \sim \mathcal{F}_{abc} G^{ce} G^{bd} \left((\alpha_e - \bar{\tau}_{em} M^m)(\bar{\alpha}_d - \bar{\tau}_{dn} M^n) + j_e j_d / g_s^2 \right) = 0.$$

At one-loop order, the prepotential has nonvanishing third derivatives only when all derivatives are with respect to the same field. The vacuum condition can be simplified to this order, and upon considering the equation as two real equations for the real and imaginary part, the conditions become

$$\begin{aligned} \left(b_{RR,a} - Re(\tau_{ab})\right) M^b &= 0 \\ G^{ac} G^{ad} \left(b_{NS,c} b_{NS,d} + j_c j_d\right) &= (M_a g_s)^2. \end{aligned}$$

The first of these can be solved easily for the phases of the S^a. Moreover, we see that it is equivalent to the condition that the real part of the auxiliary fields Y_2^a vanish for all a in the $\mathcal{N} = 2$ superspace Lagrangian,

$$G_{ab} Re(Y_2^b) = 0.$$

In light of that result, the second condition can be written as

$$\vec{Y}_a \cdot \vec{Y}_a = 0. \tag{A.3}$$

Since the supersymmetry transformations are

$$\delta_\epsilon \Psi_a = X_a \epsilon + \ldots$$

where

$$X_a = \frac{i}{\sqrt{2}} \begin{pmatrix} -Y_1^a - iRe(Y_2)^a + M^a & Y_3^a \\ Y_3^a & Y_1^a - iRe(Y_2)^a + M^a \end{pmatrix},$$

(A.3) is precisely the condition that there exists some supersymmetry transformation on each node which is locally preserved by the vacuum. Of course, for supersymmetry to be conserved globally, these supersymmetry transformations must match for all nodes. The condition for this to be the case is

$$\frac{Y_1^a}{M^a} = \frac{Y_1^b}{M^b} \qquad \frac{Y_3^a}{M^a} = \frac{Y_3^b}{M^b},$$

which, along with the requirement that the metric on moduli space be positive definite in the vacuum, requires that

$$Z_a = c_{ab} Z_b$$

for a positive, real constant c_{ab}. This conforms to our intuition from the open-string picture that preserving supersymmetry should require that the complex combination of the FI terms and gauge couplings should have the same phase on each node.

We can also see that the vacuum we just found is metastable, as we expect based on large N duality. Consider the Hessian of the potential,

$$4\pi \partial_a \partial_c V = \frac{1}{8\pi^2 S_a S_c g_s^2} \Big(G^{ia} G^{ac} G^{cj} (b^i_{NS} b^j_{NS} + j_i j_j) - G^{ac} M^a M^c g_s^2 \Big),$$

$$4\pi \partial_a \partial_{\bar{c}} V = \frac{1}{8\pi^2 S_a \bar{S}_c g_s^2} \Big(G^{ia} G^{ac} G^{cj} (b^i_{NS} b^j_{NS} + j_i j_j) + G^{ac} M^a M^c g_s^2 \Big),$$

and similarly for complex conjugates. The eigenvalues of the Hessian are manifestly positive in the limit where G_{ab} vanishes for $a \neq b$, which corresponds to widely separated nodes, and where the matrix $\partial\partial V$ is diagonal. Moreover, the determinant of the Hessian is strictly positive for any G_{ab}, so the one-loop Hessian remains positive definite for any G_{ab}.

Finally, we can compute the value of the vacuum energy in the limit where the branes are far separated. The relevant limit in this more general case is

$$S_{a,*} \ll a_{bc} < \Lambda_0.$$

The vacuum energy is then given by

$$V_* = \sum_b M^b \frac{\sqrt{b^2_{NS,b} + j_b^2}}{2\pi g_s} - \frac{1}{8\pi^2} \sum_{b \neq c} e_{I(b)} \cdot e_{J(c)} M^b M^c \log \frac{a_{bc}}{\Lambda_0} (1 - \cos\theta_{bc})$$

which reduces to the one-loop value in the gauge-theory limit, as in the A_2 case.

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