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Effective Supergravity from the Weakly Coupled Heterotic String*†

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Abstract

The motivation for Calabi-Yau-like compactifications of the weakly coupled $E_8 \otimes E_8$ heterotic string theory, its particle spectrum and the issue of dilaton stabilization are briefly reviewed. Modular invariant models for hidden sector condensation and supersymmetry breaking are described at the quantum level of the effective field theory. Their phenomenological and cosmological implications, including a possible origin for R-parity, are discussed.

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Disclaimer

1 The weakly coupled $E_8 \otimes E_8$ Heterotic String

1.1 Bottom up: the case for supergravity

The primary phenomenological motivation for supersymmetry (SUSY) is the observed large hierarchy between the Z mass, characteristic of the scale of electroweak symmetry breaking, and the reduced Planck scale m_P:

$$m_Z \approx 90\text{GeV} \ll m_P = \sqrt{\frac{8\pi}{G_N}} \approx 2 \times 10^{18}\,\text{GeV}.$$

This hierarchy can be technically understood in the context of SUSY. The conjunction of SUSY and general relativity (GR) implies supergravity (SUGRA). The absence of observed SUSY partners (sparticles) requires broken SUSY in the vacuum, and the observed particle spectrum constrains the mechanism of SUSY-breaking in the observable sector: spontaneous SUSY-breaking is not viable, leaving soft SUSY-breaking as the only option that preserves the technical SUSY solution to the hierarchy problem. This means introducing SUSY-breaking operators of dimension three or less–such as gauge invariant masses–into the Lagrangian for the SUSY extension of the Standard Model (SM). The unattractiveness of these *ad hoc* soft terms suggests they arise from spontaneous SUSY breaking in a "hidden sector" of the underlying theory. Based on the above facts, a number of standard scenarios have emerged. These include: *i*) Gravity mediated SUSY-breaking, usually understood as "minimal SUGRA" (mSUGRA) [1]. This scenario is typically characterized by $m_{\text{scalars}} = m_0 \sim m_{\text{gravitino}} = m_{\frac{3}{2}} > m_{\text{gauginos}} = m_{\frac{1}{2}}$ at the weak scale. *ii*) Anomaly mediated SUSY-breaking [2, 3], in which $m_0 = m_{\frac{1}{2}} = 0$ classically; these models are characterized by $m_{\frac{3}{2}} >> m_0,\ m_{\frac{1}{2}}$, and typically $m_0 > m_{\frac{1}{2}}$. An exception is the Randall-Sundrum (RS) "separable potential", constructed [3] to mimic SUSY-breaking on a brane spatially separated from our own in a fifth dimension; in this scenario $m_0^2 < 0$ and m_0 arises first at two loops. In general, the scalar masses at one loop depend on the details of Planck-scale physics [4]. *iii*) Gauge mediated SUSY uses a hidden sector that has renormalizable gauge interactions with the SM particles, and is typically characterized by small $m_{\frac{1}{2}}$.

1.2 Top down: the case for superstring theory

Here the driving motivation is that superstring theory is at present the only known candidate for reconciling GR with quantum mechanics. These theories are consistent in ten dimensions; in recent years it was discovered that all the consistent [5] superstring theories are related to one another

by dualities, namely S-duality: $\alpha \to 1/\alpha$, and T-duality: $R \to 1/R$, where α is the fine structure constant of the gauge group(s) at the string scale, and R is a radius of compactification from dimension D to dimension D-1. These theories, as well as D $= 11$ SUGRA, are now understood as particular limits of M-theory. Recently, there has been considerable activity in type I and II theories, or more generally in theories with branes. Similarly, the Hořava-Witten (HW) scenario [6] and its inspirations have received considerable attention. Compactification of one of the 11 dimensions of M-theory gives the HW scenario with two 10-D branes, each having an E_8 gauge group. As the length of the 11th dimension is shrunk to zero, the two branes coincide, and the WCHS scenario [7] is recovered. This is the scenario addressed here.

1.3 Calabi-Yau (like) compactification

The zero-slope (infinite string tension) limit of heterotic string theory is ten dimensional supergravity coupled to a supersymmetric Yang-Mills theory with an $E_8 \otimes E_8$ gauge group. To make contact with the real world, six of these ten dimensions must be compact and here are assumed to be of order $m_P \sim 10^{-32}$cm. If the topology of the extra dimensions were a six-torus, which has a flat geometry, the 8-component spinorial parameter of $N = 1$ supergravity in ten dimensions would appear as the four two-component parameters of $N = 4$ supergravity in four dimensions. A Calabi-Yau (CY) manifold leaves only one of these spinors invariant under parallel transport; the group of transformations under parallel transport (holonomy group) is the $SU(3)$ subgroup of the maximal $SU(4) \cong SO(6)$ holonomy group of a six dimensional compact space. This breaks $N = 4$ supersymmetry to $N = 1$ in four dimensions. The only phenomenologically viable supersymmetric theory at low energies is $N = 1$, because it is the only one that admits complex representations of the gauge group that are needed to describe quarks and leptons. For this solution, the classical equations of motion impose the identification of the affine connection of general coordinate transformations on the compact space (described by three complex dimensions) with the gauge connection of an $SU(3)$ subgroup of one of the E_8's: $E_8 \ni E_6 \otimes SU(3)$, resulting in $E_6 \otimes E_8$ as the gauge group in four dimensions [8]. Since the early 1980's, E_6 has been considered the largest group that is a phenomenologically viable candidate for a Grand Unified Theory (GUT) of the SM. Hence E_6 is identified as the gauge group of the "observable sector", and the additional E_8 is attributed to a "hidden sector", that interacts with the former only with gravitational strength couplings. Orbifolds, which are flat spaces except for points of infinite curvature, are more easily studied than CY manifolds, and orbifold compactifications that closely mimic CY compactification, and that yield realistic spectra with just three generations of quarks and leptons, have been found [9, 10].

In this case the surviving gauge group is $E_6 \otimes \mathcal{G}_o \otimes E_8$, $\mathcal{G}_o \in SU(3)$. The low energy effective field theory is determined by the massless spectrum, *i.e.*, the spectrum of states with masses very small compared with the string tension and compactification scale. Massless bosons have zero triality under an $SU(3)$ which is the diagonal of the $SU(3)$ holonomy group and the (broken) $SU(3)$ subgroup of one E_8. The ten-vectors A_M, $M = 0, 1, \ldots 9$, appear in four dimensions as four-vectors A_μ, $\mu = M = 0, 1, \ldots 3$, and as scalars A_m, $m = M - 3 = 1, \cdots 6$. Under the decomposition $E_8 \ni E_6 \otimes SU(3)$, the E_8 adjoint contains the adjoints of E_6 and $SU(3)$, and the representation $(\mathbf{27}, \mathbf{3}) + (\overline{\mathbf{27}}, \overline{\mathbf{3}})$. Thus the massless spectrum includes gauge fields in the adjoint representation of $E_6 \otimes \mathcal{G}_o \otimes E_8$ with zero triality under both $SU(3)$'s, and scalar fields in $\mathbf{27} + \overline{\mathbf{27}}$ of E_6, with triality ± 1 under both $SU(3)$'s, together with their fermionic superpartners. The number of $\mathbf{27}$ and $\overline{\mathbf{27}}$ chiral supermultiplets that are massless depends on the topology of the compact manifold. The important point for phenomenology is the decomposition under $E_6 \to SO(10) \to SU(5)$:

$$(\mathbf{27})_{E_6} = (\mathbf{16} + \mathbf{10} + \mathbf{1})_{SO(10)} = (\{\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}\} + \{\mathbf{5} + \bar{\mathbf{5}}\} + \mathbf{1})_{SU(5)}\,. \tag{1.1}$$

A $\overline{\mathbf{5}} + \mathbf{10} + \mathbf{1}$ contains one generation of quarks and leptons of the SM, a right-handed neutrino and their scalar superpartners; a $\mathbf{5} + \overline{\mathbf{5}}$ contains the two Higgs doublets needed in the supersymmetric extension of the SM and their fermion superpartners, as well as color-triplet supermultiplets. While all the states of the SM and its minimal supersymmetric extension are present, there are no scalar particles in the adjoint representation of the gauge group. In conventional models for grand unification, these (or other large representations) are needed to break the GUT group to the SM. In string theory, this symmetry breaking can be achieved by the Hosotani or "Wilson line", mechanism in which gauge flux is trapped around "holes" or "tubes" in the compact manifold, in a manner reminiscent of the Arahonov-Bohm effect. The vacuum value of the trapped flux $< \int d\ell^m A_m >$ has the same effect as an adjoint Higgs, without the difficulties of constructing a potential for large Higgs representations that actually reproduces the observed vacuum. When this effect is included, the gauge group in four dimensions is

$$\begin{aligned} &\mathcal{G}_{obs} \otimes \mathcal{G}_{hid}, \quad \mathcal{G}_{obs} = \mathcal{G}_{SM} \otimes \mathcal{G}' \otimes \mathcal{G}_o, \quad \mathcal{G}_{SM} \otimes \mathcal{G}' \in E_6, \quad \mathcal{G}_o \in SU(3), \\ &\mathcal{G}_{hid} \in E_8, \quad \mathcal{G}_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_w. \end{aligned} \tag{1.2}$$

There are many other four dimensional string vacua in addition to those described above. The attractiveness of the above picture is that the requirement of $N = 1$ SUSY naturally results in a phenomenologically viable gauge group and particle spectrum, and the gauge symmetry can be broken to a product group embedding the SM without introducing large Higgs representations.

1.4 Gaugino Condensation and the Runaway Dilaton

The $E_8 \otimes E_8$ string theory provides a hidden sector needed for spontaneous SUSY-breaking. Specifically, if some subgroup $\mathcal{G}_c$ of $\mathcal{G}_{hid}$ is asymptotically free, with a β-function coefficient $b_c > b_{SU(3)}$, defined by the renormalization group equation (RGE)

$$\mu \frac{\partial g_c(\mu)}{\partial \mu} = -b_c g_c^3(\mu) + O(g_c^5), \tag{1.3}$$

confinement and fermion condensation will occur at a scale $\Lambda_c \gg \Lambda_{QCD}$, and hidden sector gaugino condensation $< \bar{\lambda}\lambda >_{\mathcal{G}_c} \neq 0$, may induce [11] supersymmetry breaking. To discuss supersymmetry breaking in more detail, we need the low energy spectrum resulting from the ten-dimensional gravity supermultiplet that consists of the 10-D metric g_{MN}, an antisymmetric tensor b_{MN}, the dilaton ϕ, the gravitino ψ_M and the dilatino χ. For the class of CY and orbifold compactifications described above, the massless bosons in four dimensions are the 4-D metric $g_{\mu\nu}$, the antisymmetric tensor $b_{\mu\nu}$, the dilaton ϕ, and certain components of the tensors g_{mn} and b_{mn} that form the real and imaginary parts, respectively, of complex scalars known as Kähler moduli. The number of moduli is related to the number of particle generations (# of **27**'s $-$ # of $\overline{\mathbf{27}}$'s). In three generation orbifold models there are at least three moduli t_I whose *vev*'s $< \mathrm{Re}t_I >$ determine the radii of the three tori of the compact space. They form chiral multiplets with fermions χ_I^t obtained from components of ψ_m. The 4-D dilatino χ forms a chiral multiplet with with a complex scalar field s whose *vev* $< s >= g^{-2} - i\theta/8\pi^2$ determines the gauge coupling constant and the θ parameter of the 4-D Yang-Mills theory. The "universal" axion $\mathrm{Im}s$ is obtained by a duality transformation from the antisymmetric tensor $b_{\mu\nu}$: $\partial_\mu \mathrm{Im}s \leftrightarrow \epsilon_{\mu\nu\rho\sigma}\partial^\nu b^{\rho\sigma}$. Because the dilaton couples to the (observable and hidden) Yang-Mills sector, gaugino condensation induces a superpotential for the dilaton superfield[1] S:

$$W(S) \propto e^{-S/b_c}. \tag{1.4}$$

The vacuum value $< W(S) > \propto \left\langle e^{-S/b_c} \right\rangle = e^{-g^{-2}/b_c} = \Lambda_c$ is governed by the condensation scale Λ_c as determined by the RGE (1.3). If it is nonzero, the gravitino acquires a mass $m_{\frac{3}{2}} \propto < W >$, and local supersymmetry is broken.

The superpotential (1.4) results in a potential for the dilaton of the form $V(s) \propto e^{-2\mathrm{Re}s/b_c}$, which has its minimum at vanishing vacuum energy and vanishing gauge coupling: $< \mathrm{Re}s > \to \infty$, $g^2 \to 0$. This is the notorious runaway dilaton problem. The effective potential for s is in fact determined

[1] Throughout I use capital Greek or Roman letters to denote a chiral superfield, and the corresponding lower case letter to denote its scalar component.

from anomaly matching: $\delta\mathcal{L}_{eff}(s,u) \longleftrightarrow \delta\mathcal{L}_{hid}(\text{gauge})$, where u, $\langle u\rangle = \langle\bar{\lambda}\lambda\rangle_{\mathcal{G}_c}$, is the lightest scalar bound state of the strongly interacting, confined gauge sector. Just as in QCD, the effective low energy theory of bound states must reflect both the symmetries and the anomalies, *i.e.* the quantum induced breaking of classical symmetries, of the underlying Yang-Mills theory. It turns out that the effective quantum field theory (QFT) is anomalous under T-duality. Since this is an exact symmetry of heterotic string perturbation theory, it means that the effective QFT is incomplete. This is cured by including model dependent string-loop threshold corrections [12] as well as a "Green-Schwarz" (GS) counter-term [13], analogous to the GS mechanism in 10-D SUGRA. This introduces dilaton-moduli mixing, and the gauge coupling constant is now identified as

$$g^2 = 2\,\langle\ell\rangle\,, \quad \ell^{-1} = 2\text{Re}s - b\sum_I \ln(2\text{Re}t^I), \tag{1.5}$$

where $b \leq b_{E_8} = 30/8\pi^2$ is the coefficient of the GS term, and and ℓ is the scalar component of a linear superfield L that includes the two-form $b_{\mu\nu}$ and is dual to the chiral superfield S in the supersymmetric version of the two-form/axion duality mentioned above. The GS term introduces a second runaway direction, this time at strong coupling: $V \to -\infty$ for $g^2 \to \infty$. The small coupling behavior is unaffected, but the potential becomes negative for $\alpha = \ell/2\pi > .57$. This is the strong coupling regime, and nonperturbative string effects cannot be neglected; they are expected [14] to modify the Kähler potential for the dilaton, and therefore the potential $V(\ell,u)$. It has been shown [15] that these contributions can indeed stabilize the dilaton. Retaining just one or two terms of the suggested parameterizations [14, 16] of the nonperturbative string corrections: $a_n\ell^{-n/2}e^{-c_n/\sqrt{\ell}}$ or $a_n\ell^{-n}e^{-c_n/\ell}$, the potential can be made positive-definite everywhere and the parameters a_n, c_n can be chosen to fit two data points: the coupling constant $g^2 \approx 1/2$ and the cosmological constant $\Lambda \simeq 0$. This is fine tuning, but it can be done with plausible (order 1) values for the parameters c_n, a_n. If there are several condensates with different β-functions, the potential is dominated by the condensate with the largest β-function coefficient b_+, and the result is essentially the same as in the single condensate case, except that a small mass is generated for the axion $a = \text{Im}s$. In these models the presence of β-function coefficients generate mass hierarchies that have interesting implications for cosmology and the spectrum of sparticles–the supersymmetric partners of the SM particles.

2 Modular invariant gaugino condensation

In this section I will summarize results [17, 18] from the study of modular (T-duality) invariant effective Lagrangians for gaugino condensation. These are characterized in particular by

- Dilaton dominated supersymmetry breaking. The auxiliary fields of the T-moduli (or Kähler moduli) have vanishing vacuum values (*vev*'s): $\left\langle F^T \right\rangle = 0$, thus avoiding a potentially dangerous source of flavor changing neutral currents (FCNC).
- The constraint of vanishing (or nearly so) vacuum energy leads to a variety of mass hierarchies that involve the β-function coefficient of the condensing gauge group.

One starts above the (reduced) Planck scale m_P with the heterotic string theory in 10 dimensions. Just below the string scale $\mu_s = g_s m_P$, where g_s is the gauge coupling at the string scale, physics is described by $N = 1$ modular invariant supergravity in four dimensions, where here modular invariance refers to T-duality under which the Kähler moduli T transform as

$$T \to \frac{aT - ibT}{icT + d}, \qquad ad - bc = 1, \qquad a, b, d, c \in \mathcal{Z}. \tag{2.6}$$

Modular invariance – and in many compactifications [19] a $U(1)$ gauge group factor called $U(1)_X$ – is broken by anomalies at the quantum level of the effective field theory, and the symmetry is restored by an appropriate combination of threshold effects [12] and four dimensional GS term(s) [13, 20]. The precise form of these loop effects in the Yang-Mills sector of the effective supergravity theory have been determined by matching the string and field theory amplitudes at the quantum level [21].

If an anomalous $U(1)$ is present, the corresponding GS term leads to a Fayet-Illiopoulos (FI) D-term in the effective Lagrangian [20] and some $U(1)$-charged scalars ϕ^A acquire *vev*'s at a scale μ_D one or two orders of magnitude below the Planck scale such that the overall D-terms vanish:

$$\left\langle \frac{1}{\ell(s,\bar{s})} \sum_A q_A^a \prod_I (t^I + \bar{t}^I)^{n_I^A} |\phi^A|^2 \right\rangle = \frac{1}{2} \delta_X \delta_{Xa}, \tag{2.7}$$

where $\delta_X \ell$ is the coefficient of the FI term, n^A is the modular weight of ϕ^A, q_A^a is its charge under the gauge group factor $U(1)_a$, and t, s are the scalar components of the Kähler moduli and dilaton chiral superfields T, S. The function $\ell(s, \bar{s}) = \ell(s + \bar{s})$ is the dilaton field in the dual, linear supermultiplet formulation; in the classical limit $\ell = (s + \bar{s})^{-1}$. The combination of fields that gets a vacuum value is modular invariant. Thus modular invariance, as well as local supersymmetry,

is unbroken at this scale, and the moduli fields s, t remain undetermined [22]. The ϕ^A vacuum is generically characterized by a high degree of further degeneracy [23] that may lead to problems for cosmology.

At a lower scale μ_c, a gauge group $\mathcal{G}_c$ in the hidden sector becomes strongly coupled, and gauginos as well $\mathcal{G}_c$-charged matter condense. The potential generated for the moduli is T-duality invariant and the Kähler moduli T are stabilized at self-dual points with $\left\langle F^T \right\rangle = 0$, while $\left\langle F^S \right\rangle \neq 0$, so that, in the absence of an anomalous $U(1)$, supersymmetry breaking is dilaton mediated [17]. In the presence of an anomalous $U(1)$, *vev*'s of D-terms are generically generated as well and tend to dominate supersymmetry breaking; these may be problematic for phenomenology. On the plus side, at least some of the degeneracy of the ϕ^A vacuum is lifted by ϕ^A couplings to the condensates [18].

To briefly summarize the phenomenology of these models, the condition of vanishing vacuum energy introduces the β-function coefficient of the condensing gauge group $\mathcal{G}_c$ into the supersymmetry breaking parameters in such a way as to generate a variety of mass hierarchies. Defining

$$b_c = \frac{1}{16\pi^2}\left(3C^c - C^c_M\right), \tag{2.8}$$

where $C^c(C^c_M)$ is the adjoint (matter) quadratic Casimir for $\mathcal{G}_c$, in the absence of an anomalous $U(1)$ one has at the condensation scale [17] (one can also have $m_0 \sim m_T \gg m_{\frac{3}{2}}$ if gauge-charged matter couples to the GS term)

$$\begin{aligned} m_0 &= m_{\frac{3}{2}}, \qquad m^a_{\frac{1}{2}} = \frac{4b_c^2}{9} g_a(\mu_c) m_{\frac{3}{2}}, \\ m_T &\approx \frac{b}{b_c} m_{\frac{3}{2}}, \qquad m_S \sim b_c^{-2} m_{\frac{3}{2}}, \qquad m_a = 0. \end{aligned} \tag{2.9}$$

where $m_{0,\frac{1}{2},\frac{3}{2}}$ refer to observable sector scalars and gauginos, and the gravitino, respectively; $m_{T,S,a}$ are the Kähler moduli, dilaton and universal axion masses. The expression for m_T assumes $b \gg b_c$, where b is the β-function coefficient appearing in the modular invariance restoring GS term [13]. For example in the absence of Wilson lines, $b = b_{E_8} \approx .57$, and viable scenarios for electroweak symmetry breaking [24] and for neutralinos as dark matter [25] require $b_c \approx .05 - .06$. These numbers give desirably large moduli and dilaton masses, while the scalar/gaugino mass ratio is perhaps uncomfortably large, but no worse than in many other models.

When Wilson lines are present the condition $b \gg b_c$ may not hold; for example $b_c = b$ in a Z_3 compactification [10] with an $SO(10)$ hidden sector gauge group; this would give vanishing T-moduli masses in the above class of models. However when an anomalous $U(1)$ is present, the T-moduli couplings to the condensates are modified, giving additional contributions to their masses,

and a hierarchy with respect to the gravitino mass can still be maintained [18]. In this scenario the gaugino, dilaton and axion masses are determined only by the dilaton potential, as before. A stable vacuum with a positive metric for the dilaton is most easily achieved in a "minimal" class of models in which the number of Standard Model (SM) gauge singlets that get *vev*'s at the scale μ_D is equal to the number m of broken $U(1)$'s (in which case there are no massless "D-moduli" [23] associated with the degeneracy of the $U(1)$-charged ϕ^A vacuum), or N replicas of these with identical $U(1)$ charges [yielding $(N-1)m$ D-moduli]. In this case the gaugino, dilaton and axion[2] masses are unchanged from (2.9). The most significant change from the above scenario is a D-term contribution to scalar squared masses m_0^2 that is proportional to their $U(1)$ charges. At weak coupling, and neglecting nonperturbative effects, this term dominates the one in (2.9) by a factor $b_c^{-2} \gg 1$, and is not positive semi-definite. Thus unless SM particles are uncharged under the broken $U(1)$'s (or have charges that, in a well-defined sense [18], are orthogonal to those of the ϕ^A with large *vev*'s), these models are seriously challenged by the SM data: a very high scalar/gaugino mass ratio for positive m_0^2, and the danger of color and electromagnetic charge breaking if $m_0^2 < 0$.

3 QFT quantum corrections

The above results were obtained at tree level in the effective supergravity theory for gaugino condensation, which includes QFT and string quantum corrections to the strongly coupled gauge sector whose elementary degrees of freedom have been integrated out, as well as the four dimensional Green-Schwarz (GS) terms needed at the quantum level to cancel field theory anomalies. In addition, the logarithmically divergent and finite ("anomaly mediated" [2, 3, 27]) one-loop corrections to soft supersymmetry-breaking parameters have been extensively studied [28, 4]. These analyses did not include quadratically divergent loop corrections which are proportional to terms in the tree Lagrangian, and are suppressed by the loop expansion parameter

$$\epsilon = 1/16\pi^2. \tag{3.10}$$

However, since some of these terms have coefficients proportional to the number of fields in the effective supergravity theory, it has been argued that they may not be negligible. In particular, their contributions to the cosmological constant [29] and to flavor changing neutral currents [30] have been emphasized. Both are important for the phenomenology of the above condensation models; thus we need to revisit [31] their effects.

[2]The possibility that an axion mass may be generated by higher dimension operators [16] is under study [26].

When local supersymmetry is broken, there is a quadratically divergent one-loop contribution to the vacuum energy [32]

$$\langle V_{1-\mathrm{loop}}\rangle \ni \frac{\Lambda^2}{32\pi^2}\left\langle \mathrm{STr}\mathcal{M}^2\right\rangle, \tag{3.11}$$

where $\mathcal{M}$ is the field-dependent mass matrix, and the gravitino contribution is gauge dependent. For example in minimal supergravity [1] with N_χ chiral and N_G Yang-Mills superfields, one obtains, using the gravitino gauge fixing procedure of Ref [33].

$$\langle \delta V_{1-\mathrm{loop}}\rangle \ni \frac{\Lambda^2}{16\pi^2}\left(N_\chi m_0^2 - N_G m_{\frac{1}{2}}^2 + 2m_{\frac{3}{2}}^2\right). \tag{3.12}$$

In the MSSM we have $N_\chi = 49$ and $N_G = 12$. The much larger field content of a typical Z_3 orbifold compactification [34, 19] of the $E_8\otimes E_8$ heterotic string has $N_\chi \gtrsim 300$ and $N_G \lesssim 65$, suggesting [29] that this contribution to the vacuum energy is always positive.

However, in order to maintain manifest supersymmetry, a supersymmetric regularization of ultraviolet divergences must be used. Pauli-Villars (PV) regularization [35] meets this criterion. The regulation of quadratic divergences requires *a priori* two subtractions; in the context of PV regularization, the number S of subtractions is the number of PV fields for each light field. Once the divergences are regulated (*i.e.* eliminated), we are left with the replacement

$$\Lambda^2\mathrm{STr}\mathcal{M}^2 \to \mathrm{STr}\mu^2\mathcal{M}^2\ln(\mu^2)\eta_S, \qquad \eta_S = \sum_{q=1}^{S}\eta_q\lambda_q\ln\lambda_q, \tag{3.13}$$

where μ represents the scale of new physics, and the parameter η_S reflects the uncertainty in the threshold for the onset of this new physics. The squared PV mass of the chiral supermultiplet Φ^q is $\lambda_q\mu^2$ (so $\lambda_q > 0$), and $\eta_q = \pm 1$ is the corresponding PV signature. The sign of the effective cut-off is determined by the sign of η_S, which is positive definite only[3] if $S \leq 3$. Cancellation of all the ultraviolet divergences of a general supergravity theory requires [37] at least 5 PV chiral multiplets for every light chiral multiplet and even more PV supermultiplets to regulate gauge loops. Therefore one cannot assume that the effective cut-offs are all positive.

More importantly, the Lagrangian constructed using a simple cut-off does not respect supersymmetry. With a supersymmetric PV regularization, PV masses arise from quadratic couplings in the superpotential

$$W_{PV} \ni \mu_{IJ}(Z^k)Z^I_{PV}Z^J_{PV}, \qquad Z^k\Big| = z^k. \tag{3.14}$$

[3]See appendix C of [36]. and the discussion in [31].

Then the squared cut-off in (3.12) is replaced by suitably weighted linear combinations of PV squared masses

$$\Lambda^2 \to (M^2)^I_J = e^K K^{I\bar{K}}(z) K^{\bar{L}M}(z) \bar{\mu}_{\bar{K}\bar{L}}(\bar{z}) \mu_{MJ}(z) \tag{3.15}$$

that are generally field-dependent. Moreover, the couplings (3.14) induce additional terms proportional to M^2 that cannot be obtained by a straight cut-off procedure. The resulting effective Lagrangian takes the form [35]

$$\mathcal{L}^1_{eff} = \mathcal{L}_{\rm tree}(g, K) + \mathcal{L}_{1-\rm loop} = \mathcal{L}_{\rm tree}(g_R, K_R) + O(\epsilon \ln \Lambda^2_{eff}) + O(\epsilon^2), \tag{3.16}$$

where

$$K_R = K + \Delta K \tag{3.17}$$

is the renormalized superpotential. The action obtained in this way is only perturbatively supersymmetric:

$$\delta S^1_{eff} = \int d^4x \delta \mathcal{L}^1_{eff} = O(\epsilon^2). \tag{3.18}$$

Writing

$$\Delta K = \frac{\epsilon}{2}\left[N\Lambda^2_\chi - 4N_G\Lambda^2_G + O(1)\Lambda^2_{\rm grav}\right] + O(\epsilon \ln \Lambda^2_{eff}) + O(\epsilon^2), \tag{3.19}$$

where $\Lambda_{\chi,G,\rm grav}$ are the effective cut-offs for chiral, gauge and gravity loops, and Λ_{eff} is a generic effective cutoff, if $N_\chi, N_G \sim \epsilon^{-1}$, we must retain the full effective Lagrangian as derived from K_R. This amounts to resuming the leading terms in $\epsilon N \Lambda^2_{eff}$, with the result, as dictated by supersymmetry, just a correction to the Kähler potential. I will discuss the consequences of this correction in the remainder of this section.

3.1 The vacuum energy

Consider first the possibility that we can choose the Z^k-dependence of the PV Kähler potential and superpotential such that the effective cutoffs are constant. For example, one needs PV superfields Z^I_{PV} with the same Kähler metric as the light superfields Z^i: $K^Z_{I\bar{M}} = K_{i\bar{m}}$. If we introduce superfields Y_I with Kähler metric: $K^{I\bar{M}}_Y = e^{-K} K^{-1}_{i\bar{m}} = e^{-K} K^{i\bar{m}}$, the superpotential coupling

$$W_{PV} = \mu Z^I Y_I \tag{3.20}$$

yields a constant squared mass $M^2 = \mu^2$ if μ is constant, and the quantum corrected potential just reads

$$V_{eff} = \mathcal{D} + e^{\Delta K}\left(F^i K_{i\bar{m}} F^{\bar{m}} - 3m^2_{\frac{3}{2}}\right)_{\rm tree} + O(\epsilon \ln \Lambda^2_{eff}). \tag{3.21}$$

If supersymmetry breaking is F-term induced: $\langle \mathcal{D} \rangle = 0$, the tree level condition $\left\langle F^i K_{i\bar{m}} F^{\bar{m}} = 3m_{\frac{3}{2}}^2 \right\rangle$ for vanishing vacuum energy is unmodified by these quantum corrections.

However not all PV masses can be chosen to be constant because of the anomaly associated with Kähler transformations $K(Z,\bar{Z}) \to K(Z,\bar{Z}) + F(Z) + \bar{F}(\bar{Z})$ that leave the classical Lagrangian invariant. In the presence, for example, of an anomalous $U(1)_X$, with generator T_X, there is a quadratically divergent term proportional to $\mathrm{Tr} T_X \Lambda^2$ that cannot be canceled by $U(1)_X$-invariant PV mass terms, since the contribution to $\mathrm{Tr} T_X$ from each pair in the invariant superpotential cancels. As a consequence, there must be some PV masses $\propto e^{aV_X}$, where V_X is the $U(1)_X$ vector superfield. Similarly, in the presence of a Kähler anomaly there is a term

$$\mathcal{L}_{1-\mathrm{loop}} \ni c\epsilon K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \Lambda^2, \tag{3.22}$$

that cannot be canceled unless some PV superfields have masses $M_{PV}^2 \propto e^{\alpha K}$. In addition, PV regulation of the gauge + dilaton sector requires some PV masses proportional to the field-dependent string-scale gauge coupling constant: $M_{PV}^2 \propto g_s^2(s,\bar{s}) = 2(s+\bar{s})^{-1}$.

What might be the effects of this field-dependence on the condensation models described above? The modular invariance of these models assures that the moduli T are stabilized at self-dual points with vanishing auxiliary fields: $\left\langle F^T \right\rangle = 0$. Supersymmetry breaking is dilaton-dominated and the condition for vanishing vacuum energy at tree level in the effective theory relates $\left\langle F^S \right\rangle$ to the gravitino mass which in turn constrains the dilaton Käher metric:

$$\frac{2}{\sqrt{3}} b_c \approx .05 \leq \left\langle K_{S\bar{S}}^{-\frac{1}{2}} \right\rangle \ll \left\langle K_{S\bar{S}}^{-\frac{1}{2}} \right\rangle \Big|_{\mathrm{classical}} = 2 g_s^{-2} \approx 4, \tag{3.23}$$

with the approximate value of g_s inferred from low energy data. It is clear that (3.23) cannot be satisfied without a modification of the Kähler potential for the dilaton; the approach [15] used here is to invoke nonperturbative string [14] and/or QFT [16] corrections to the dilaton Kähler potential. Avoiding dangerously large D-term contributions to scalar masses in the presence of an anomalous $U(1)$ may further require [18]

$$-\langle K_S \rangle \approx \frac{3}{2} b_c^{-1} \approx 30 \gg -\langle K_S \rangle|_{\mathrm{classical}} = g_s^2/2 \approx 1/4, \tag{3.24}$$

suggesting that weak coupling may not be viable [6, 38, 39]. On the other hand, if $\Lambda \sim C e^{\alpha K}$, with $C\alpha^n$ large and positive, it might be possible to reinterpret part of the needed modification of the dilaton Kähler potential in terms of perturbative quantum corrections [31].

3.2 Flavor Changing Neutral Currents

The tree potential of an effective supergravity theory includes a term

$$V_{\rm tree} \ni e^K K_i K_{\bar{\jmath}} K^{i\bar{\jmath}} |W|^2, \tag{3.25}$$

and the quadratically divergent one-loop corrections generate a term

$$V_{1-{\rm loop}} \ni e^K K_i K_{\bar{\jmath}} R^{i\bar{\jmath}} |W|^2, \qquad R^{i\bar{\jmath}} = K^{i\bar{k}} R_{\bar{k}l} K^{k\bar{\jmath}}. \tag{3.26}$$

where $R_{i\bar{\jmath}}$ is the Kähler Ricci tensor. The contribution (3.26) simply reflects the fact that the leading divergent contribution in a nonlinear sigma model is a correction to the Kähler metric proportional to the Ricci tensor (whence, *e.g.*, the requisite Ricci flatness of two dimensional conformal field theories). Since the Ricci tensor involves a sum of Kähler Riemann tensor elements over all chiral degrees of freedom, a large, order N_χ, coefficient may be generated [30]. However, the supersymmetric completion of the potential in any given order in perturbation theory yields (in the absence of D-term contributions) the scalar squared mass matrix

$$(m^2)^i_j = \delta^i_j m^2_{\frac{3}{2}} - \left\langle \tilde{R} \right\rangle^i_{jk\bar{m}} \tilde{F}^k \bar{\tilde{F}}^{\bar{m}}, \tag{3.27}$$

where $\tilde{R}^i_{jk\bar{m}}$ is an element of the Riemann tensor derived from the fully renormalized Kähler metric, and $\tilde{F}^i$ is the auxiliary field for the chiral superfield Φ^i, evaluated by its equation of motion using the quantum corrected Lagrangian. Since the latter is perturbatively modular invariant, the Kähler moduli t^I are still stabilized at self-dual points with $\left\langle \tilde{F}^T \right\rangle = 0$. Classically we have $R^A_{BS\bar{S}} = 0$ where the indices A, B refer to gauge-charged fields in the observable sector. This need not be true at the quantum level. For example, if, as suggested above, the quantum correction to the Kähler potential includes a term

$$\Delta K = \frac{1}{32\pi^2} {\rm STr} \Lambda^2_{eff} \ni \frac{c N_\chi}{32\pi^2} e^{\alpha K}, \tag{3.28}$$

we get

$$\left\langle \tilde{R}^A_{BS\bar{S}} \right\rangle = \delta^A_B \frac{c N_\chi}{32\pi^2} \alpha^2 e^{\alpha K} \left(K_{S\bar{S}} + \alpha K_S K_{\bar{S}} \right), \tag{3.29}$$

which is flavor diagonal, and therefore FCNC safe.

4 R-parity

The self-dual vacua

$$\left\langle t^I \right\rangle = T_{sd} = 1 \quad \text{or} \quad e^{i\pi/6} \tag{4.30}$$

are invariant under (2.6) with

$$b^I = -c^I = \pm 1, \qquad a^I = d^I = 0 \quad \text{or} \quad \begin{cases} a^I = b^I,\ d^I = 0 \\ d^I = c^I,\ a^I = 0 \end{cases}, \qquad F^I = ni\frac{\pi}{2} \quad \text{or} \quad ni\frac{\pi}{3}. \tag{4.31}$$

The hidden sector condensates that get *vev*'s break this further to a subgroup G_R with

$$i\mathrm{Im}F = F = \sum_I F^I = 2ni\pi, \tag{4.32}$$

under which $\lambda_L \to e^{-\frac{i}{2}\mathrm{Im}F}\lambda_L = \pm\lambda_L$. Observable sector gauge-charged matter chiral supermultiplets transform as

$$\Phi^A \to e^{i\delta^A + \sum_I n_I^A F^I}\Phi^A = R(F^I, n_I^A)\Phi^A. \tag{4.33}$$

For example a Z_3 orbifold has untwisted sector fields U^{AI}, and twisted sector fields T^A and Y^{AI} with modular weights

$$\left(n_I^{AJ}\right)_U = \delta_I^J, \qquad n_I^A = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right), \qquad \left(n_I^{AJ}\right)_Y = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right) + \delta_I^J, \tag{4.34}$$

and moduli independent phases

$$\delta^U = 0, \qquad \delta^T = -\frac{2}{3}\delta, \qquad \delta^{Y^J} = -\frac{2}{3}\delta - 4\delta_I, \qquad \delta = \sum \delta^I, \tag{4.35}$$

with $\delta = 2\pi n$ for the subgroup defined by (4.32). If some modular covariant fields ϕ^A acquire large *vev*'s that break some $U(1)$ gauge factors near the string scale, the transformation property (4.33) can be modified to include a discrete $U(1)_a$ transformations such that the vacuum remains invariant. Similarly, below the electroweak scale where the Higgs fields acquire *vev*'s the residual symmetry involves a discrete transformation under the $U(1)_w$ of the SM such that $R(H_u) = R(H_d) = 1$ (the presence of a μ-term requires $R(H_u)R(H_d) = 1$). Then we obtain an effective R-parity that forbids baryon and lepton number violation while allowing other MSSM couplings provided the remaining MSSM chiral supermultiplets have R-charges $R(\Phi^A) = R(F^I, n_I^A, q_A^a)$ that satisfy [40]

$$R(Q) \;=\; e^{2i\pi\beta}, \qquad R(Q^c) = e^{-2i\pi\beta}, \qquad R(L) = e^{2i\pi\alpha}, \qquad R(L^c) = e^{-2i\pi\alpha}, \tag{4.36}$$

with $\beta \neq \frac{n}{3}$, $0 < \alpha, \beta < 1$. Since these phases need not be ± 1, dimension-five operators that violate baryon and lepton number will also be forbidden provided $3\beta + \alpha \neq n$, which is an advantage over the conventional definition of R-parity.

5 Issues and open questions

Other issues relevant to the viability of the WCHS are under active investigation. They include

- Can the universal axion be identified with the QCD/Peccei-Quinn axion [26]?
- Will the LHC be able to distinguish the WCHS from other scenarios [41]?
- Is there a specific vacuum of the WCHS such that
  - The β-function of the hidden sector condensing gauge group yields viable electroweak symmetry breaking and dark matter scenarios [*e.g.* $b_c \approx .05 - .06$ in the absence of an anomalous $U(1)$]?
  - D-term contributions to squark, slepton and Higgs masses are absent or highly suppressed?
  - The desired R-parity emerges [40]?
  - A see-saw mechanism for neutrino masses is present [42]?
  - A μ-parameter of about a TeV is natural [43]?
  - The correct Yukawa textures arise [44]?

In the present context suppression of Yukawa couplings could be due to string selection rules that allow some superpotential couplings only in terms of very high dimension: $W \ni Q_i Q_j^c H \prod_{A=1}^{n_{ij}} \Phi^A$, with *vev*'s $\left\langle \phi^A \right\rangle \sim (.1 - .01) m_{\rm Planck}$ arising at the $U(1)_X$-breaking scale Λ_D.

There is a complete classification of the observable [34] and hidden [19] sectors of Z_3 orbifolds, but only one of these [10] has been studied in detail, and it fails the above tests. A more complete survey of heterotic string vacua could help to determine if any scenario of the class considered here might be able to describe nature. A more general list of interesting questions about the relevance of string theory to the real world can be found in [45].

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