Bounds on Unparticles from the Higgs Sector

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ABSTRACT: We study supersymmetric QCD in the conformal window as a laboratory for unparticle physics, and analyze couplings between the unparticle sector and the Higgs sector. These couplings can lead to the unparticle sector being pushed away from its scale invariant fixed point. We show that this implies that low energy experiments will not be able to see unparticle physics, and the best hope of seeing unparticles is in high energy collider experiments such as the Tevatron and the LHC. We also demonstrate how the breaking of scale invariance could be observed at these experiments.

Contents

1. Introduction and Conclusions

Recently there has been a lot of interest [1–8] in unparticle theories [9, 10] in which the Standard Model (SM) is coupled to a conformal sector (called the unparticle sector). As shown in [9, 10], the conformal sector can have interesting and unexpected consequences. In this note, we shall investigate the effects on the conformal sector from the Higgs sector, and we will show that this leads to surprising new bounds on unparticle physics.

The main point of our analysis is that the coupling to the Higgs sector is the most important operator; in fact, if there is a scalar unparticle operator of dimension less than 2, the coupling to the Higgs sector is through a relevant operator. When the Higgs gets a vacuum expectation value, this operator breaks the conformal invariance of the hidden sector. For unparticle physics to be relevant, this breaking scale should be sufficiently low. This operator may also lead to strong effects on Higgs physics from unparticles, which would be interesting to investigate.

This requirement imposes strong constraints on the unparticle sector. If these constraints are satisfied, low energy experiments will not be able to probe any aspects of unparticle physics. The only place where unparticle physics will be relevant is in high energy experiments like the Tevatron and the LHC, which can indeed probe unparticle physics. In fact, the breaking of conformal invariance may also be measurable as deviations from the predictions of unparticle physics.

We will begin by discussing a model of unparticle physics, which is different from the previously suggested models. We take the conformal sector to be a supersymmetric gauge theory, which at low energies flows to a conformal theory. Using supersymmetry, we will be able to explicitly calculate the dimensions of chiral operators in this theory, and show that this is a good model for unparticle physics. We note that anomalous dimensions in these theories can be large, in contrast to Banks-Zaks fixed points which are weakly coupled.

We then couple this sector to the Standard Model, focusing on the Higgs sector. We show that conformal invariance is broken at low energies, and for reasonable choices of scales the inclusion of the Higgs-unparticle operator means that low energy experiments are unable to see the effects of unparticles. Finally, we propose a toy model for a theory with unparticles and a breaking of scale invariance and calculate experimental predictions of this effect.

2. Supersymmetric QCD as a model of unparticle physics

We would like to have an example of a conformal field theory in which it is possible to do semi-quantitative calculations. This is challenging since we also require the theory be strongly coupled in order that anomalous dimensions can be large. Remarkably such an example exists in the literature [11,12], in the form of supersymmetric QCD (SQCD)in a certain regime. We will therefore consider SQCD in the conformal window as a laboratory for unparticle physics.

We briefly review the results of SQCD (for a comprehensive review see [13]). Consider SQCD with gauge group $SU(N_C)$ and N_F vector-like quark superfields $(Q, \bar{Q})$ with $\frac{3}{2}N_C < N_F < 3N_C$ (we call this the electric theory). Such a theory flows to a strongly coupled conformal fixed point in the infrared (IR). At the fixed point the theory has a dual (magnetic) description, with gauge group $SU(N_F - N_C)$, N_F dual-quark superfields $(q, \bar{q})$, a gauge singlet meson superfield M (transforming in the bifundamental representation of the $SU(N_F) \times SU(N_F)$ flavor symmetry, and superpotential,

$$W_{mag} = \bar{q}Mq \,. \tag{2.1}$$

The meson of the magnetic description corresponds to the gauge invariant composite $(\bar{Q}Q)$ of the electric theory.

The magnetic conformal theory can now be coupled to the Standard Model, and will then be a candidate for the unparticle sector. In general, we can write the UV coupling of an operator of dimension d_{UV} in the unparticle sector to a SM operator of dimension l:

$$\frac{1}{M_{\mathcal{U}}^{l+d_{UV}-4}} \mathcal{O}_{SM} \mathcal{O}_{UV} \tag{2.2}$$

and below the strong coupling scale, these couplings flow to

$$\mathcal{C} \frac{\Lambda_{\mathcal{U}}^{d_{UV}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{l+d_{UV}-4}} \mathcal{O}_{SM} \mathcal{O}_{IR} \tag{2.3}$$

(in the notation of Georgi $k = l + d_{UV} - 4$).

Supersymmetric QCD allows us to make this explicit. For example adding superpotential coupling in the UV regime of the magnetic description

$$W_c = \frac{1}{M_{\mathcal{U}}} HL\bar{e} \operatorname{Tr} M \tag{2.4}$$

leads, among others to the following terms in in the Lagrangian

$$\frac{1}{M_{\mathcal{U}}} H L \bar{e} \ \mathrm{Tr} \ M + \left(\frac{1}{M_{\mathcal{U}}} H \tilde{L} \tilde{\bar{e}} q^* \bar{q}^* + h.c. \right) , \tag{2.5}$$

which have the form (2.2) with $\mathcal{O}_{UV} = \mathrm{Tr} \ M$ of dimension $d_{UV} = 1$ and $\mathcal{O}_{UV} = q\bar{q}$ of dimension $d_{UV} = 2$. Below the strong coupling scale, once the theory reaches its conformal fixed point, the dimensions of these operators can be computed from R-charges to be $d_{\mathcal{U}} = 3\frac{N_C - NF}{N_F}$ and $d_{\mathcal{U}} = 3\frac{N_C}{N_F}$ respectively. Furthermore, in the conformal window the dimension of both operators lie between 1 and 2, making them perfect candidates for the operator $\mathcal{O}_{\mathcal{U}}$ of the unparticle conformal sector [9].

A couple of comments are in order:

1. For generic choices of N_C and N_F, in the conformal window, the dimensions of the operators qq, M significantly differ from integer values. This is unlike the Banks-Zaks theory [14] which has a weakly coupled fixed point where all operators have dimension close to their classical value. In particular, all gauge invariant operators in BZ theory have almost integer dimensions.

2. We can perturb the theory by adding a term to the action $\lambda \mathcal{O}_{IR} = \lambda \ \mathrm{Tr} \ M$. This corresponds to adding a mass term for the quarks of the electric description. The result of this mass term is that at low energies, the quarks can be integrated out, and the theory becomes a pure super-Yang Mills theory, which is no longer conformal.

3. Operator analysis and experimental constraints

The couplings of the unparticle sector and the SM sector can have interesting effects. Most interest has concentrated on operators involving SM fermions and gauge bosons (with the goal of determining low energy signatures of unparticles) and consequently on operators with $l \geq 3$. Because the operator of lowest dimension in the unparticle sector has dimension greater than 1 this means that the coupling operator is irrelevant $l + d_{UV} - 4 \geq 0$.

However there is another type of coupling between the SM and the unparticle sector, involving the SM Higgs boson. The coupling is of the form

$$\frac{1}{M_{\mathcal{U}}^{d_{UV}-2}} |H|^2 \mathcal{O}_{UV} \tag{3.1}$$

which flows in the IR to

$$C_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{UV}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{UV}-2}} |H|^2 \mathcal{O}_{IR} \tag{3.2}$$

The dimension of $\mathcal{O}_{IR}$ is usually assumed to lie between 1 and 2 [10], as is indeed the case for SQCD. For such operators, this coupling is *relevant* in the CFT and can significantly change the low energy physics of the unparticle sector. We note there is no symmetry that can forbid

this operator without simultaneously forbidding fermion and gauge boson operators coupling to the unparticle sector.

It is clear that once the Higgs acquires a vev, this operator introduces a scale into the CFT. This relevant operator will cause the unparticle sector to flow away from its conformal fixed point and the theory will become non-conformal at a scale $\Lambda_{\mathcal{U}}$, where

$$\Lambda_{\mathcal{U}}^{4-d_{\mathcal{U}}} = \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{d_{UV}-d_{\mathcal{U}}} M_{\mathcal{U}}^{2-d_{\mathcal{U}}} v^2 \,. \tag{3.3}$$

Below this scale the unparticle sector presumably becomes a traditional particle sector. For consistency $\Lambda_{\mathcal{U}} < \Lambda_{\mathcal{U}}$ and if there is to be any sense in which the theory is truly conformal the two scales should be well separated. This means that is does not make sense to talk of $\Lambda_{\mathcal{U}} < \left(v^2 M_{\mathcal{U}}^{2-d_{UV}}\right)^{\frac{1}{4-d_{UV}}}$.

We note that if there is no scalar operator of dimension less than 2 in the unparticle sector, then the operator (3.2) is irrelevant. Any operator with dimension less than 2 would then have to be a vector or higher tensor operator. Such scenarios are difficult to realize in SQCD, but may be realized in more exotic theories. (For example, one may use AdS/CFT and consider the CFT dual of an AdS theory which only contains vector fields.) We will not consider this possibility further.

Breaking of the conformal invariance due to the new operators has important implications for unparticle phenomenology. For any given experiment, unparticle physics will only be relevant if

$$\Lambda_{\mathcal{U}} < Q \tag{3.4}$$

where Q is the typical energy of the experiment. For lower energies, the unparticle sector can be treated as a particle sector. With this observation the constraint of (3.4) takes the form

$$Q^{4-d_{\mathcal{U}}} > \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{d_{UV}-d_{\mathcal{U}}} M_{\mathcal{U}}^{2-d_{\mathcal{U}}} v^2 \,. \tag{3.5}$$

This suggests that low energy experiments may not be sensitive to unparticle physics. To see this explicitly we note that any observable effect of the operator (2.3) will be proportional to

$$\epsilon = \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{2d_{UV}-2d_{\mathcal{U}}} \left(\frac{Q}{M_{\mathcal{U}}}\right)^{2(d_{\mathcal{U}}+l-4)} \,. \tag{3.6}$$

Then the effects of the unparticle sector on observables are bounded by

$$\epsilon < \left(\frac{Q}{M_{\mathcal{U}}}\right)^{2l} \left(\frac{M_{\mathcal{U}}}{v}\right)^4 \,. \tag{3.7}$$

Note that this constraint is completely independent[1] of both the UV and IR scaling dimension of the CFT operator and the potential effects of the unparticle sector are constrained by only

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[1]This is not entirely true, there is dependence on $d_{\mathcal{U}}$ due to the modification of phase space. This results in order 1 numbers but the dependence on energy scales remains the same.

3 parameters: the experimental energy, scale of the electroweak symmetry breaking and the energy scale at which the interactions between the SM and unparticle sector are generated.

Let us now concentrate on the measurement of (g-2) of the electron. In this case the SM operator is simply[2] $\mathcal{O}_{SM} = \bar{e}e$ and the relevant energy scale is m_e. Therefore eqn. (3.7) becomes

$$\epsilon < \frac{m_e^6}{M_\mathcal{U}^2 v^4} \,. \tag{3.8}$$

Since effective field theory works at the electroweak scale, we expect $M_\mathcal{U} \gtrsim 100 \,\text{GeV}$. We then find $\epsilon < 10^{-28}$. This should be contrasted with the existing experimental constraint given in [15], $\epsilon < 10^{-11}$. Therefore the effects of the unparticle sector are completely invisible in (g-2) experiments.

It is clear from (3.6) that signals of unparticle physics increase with energy and the LHC is the most likely place where it can be discovered. For such an experimental discovery, $M_\mathcal{U}$ should not be too high. Assuming that we can detect deviations from the Standard Model of order $\epsilon \sim 1\%$, we see that unparticle physics will be visible at the LHC as long as $M_\mathcal{U} \simeq 10^5 \,\text{GeV}$.

As an example, we may take $\Lambda_\mathcal{U} \approx 10^4 GeV$ and $d_\mathcal{U} = 1.5$, then $\Lambda_{\not{\mathcal{U}}} \approx 40 \,\text{GeV}$, showing that unparticle physics will indeed be a valid description at LHC scales for this choice of parameters. On the other hand, if $\Lambda_\mathcal{U} \approx M_\mathcal{U} = 10^5 \,\text{GeV}$ and $d_\mathcal{U} = 1.5$ we will find $\Lambda_{\not{\mathcal{U}}} \approx 400 \,\text{GeV}$ which is large and will imply deviations from unparticle physics. In Figure 1 we show how the breaking scale $\Lambda_{\not{\mathcal{U}}}$ varies with $d_\mathcal{U}$ and $\Lambda_\mathcal{U}$ for two different choices of d_{UV}; in both cases $M_\mathcal{U} = 10^5$ GeV, although for $d_{UV} = 2$ there is no dependence on $M_\mathcal{U}$. From these plots we can see that the range over which the unparticle sector is conformal can be made large only at the expense of increasing the UV dimension of the unparticle operator or the scale $M_\mathcal{U}$. The former then requires very large anomalous dimensions to allow $d_\mathcal{U}$ to lie between 1 and 2, and the latter may make the discovery of unparticles beyond the reach of any experiment. On the other hand, as we will see in the next section if $\Lambda_{\not{\mathcal{U}}}$ is not very small, then the deviations from both from particle and unparticle physics may be measurable.

4. New effects in non-unparticle physics

In a pure conformal theory, the correlator can be written as

$$\langle O_\mathcal{U}(x) O_\mathcal{U}(0) \rangle = \int \frac{d^4P}{(2\pi)^4} e^{-ipx} |\langle 0|O_\mathcal{U}|P\rangle|^2 \rho(P^2) \tag{4.1}$$

with

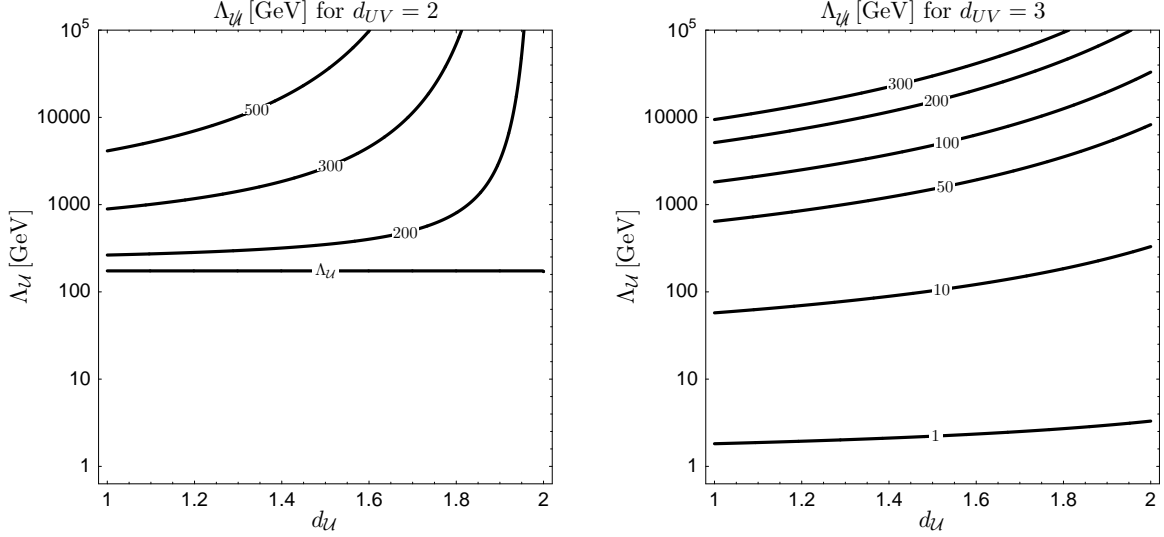$$|\langle 0|O_\mathcal{U}|P\rangle|^2 \rho(P^2) = A_{d_\mathcal{U}} \theta(P^0) \theta(P^2) (P^2)^{d_\mathcal{U}-2} \,. \tag{4.2}$$

We propose a simple toy model where conformal invariance is broken at a low energy μ by modifying the above equation to

$$|\langle 0|O_\mathcal{U}|P\rangle|^2 \rho(P^2) = A_{d_\mathcal{U}} \theta(P^0) \theta(P^2 - \mu^2) (P^2 - \mu^2)^{d_\mathcal{U}-2} \,. \tag{4.3}$$

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[2]There is also the possibility of a pseudoscalar operator $\bar{e}\gamma_5 e$ but this follows the same scaling arguments.

Figure 1: Contours of fixed CFT breaking scale, $\Lambda_{\slashed{U}}$, as a function of the IR dimension of the unparticle operator, $d_{\mathcal{U}}$, and the scale at which it becomes conformal, $\Lambda_{\mathcal{U}}$. Two particular choices for the UV dimension of the unparticle operator are made, $d_{UV} = 2, 3$.

This modification corresponds to shifting the spectrum to remove modes with energy less than μ. This model maintains the unparticle nature of the hidden sector whilst including the effects of the breaking of scale invariance. There are other possibilities for instance, once scale invariance is broken there may be particle-like modes that would appear as isolated poles in the spectral function, but we ignore these effects in this simple model.

This modification can produce observable effects. To illustrate this, we will reconsider the effects of unparticle physics on the decay of the top through processes like $t \to u\, \mathcal{O}_{\mathcal{U}}$. The decay rate for this process can be computed following [9] to be

$$m_t \frac{d\log\Gamma}{dE_u} = 4d_{\mathcal{U}}(d_{\mathcal{U}}^2 - 1)\left(\frac{m_t}{M}\right)^6 \left(\frac{E_u}{m_t}\right)^2 \left(1 - 2\left(\frac{m_t}{M}\right)^2 \frac{E_u}{m_t}\right)^{d_{\mathcal{U}}-2} \tag{4.4}$$

with $M^2 = m_t^2 - \mu^2$. In Figure 2 we show this modification for various choices of μ and $d_{\mathcal{U}}$. Notice that the end point of the distribution is no longer $m_t/2$ but is now $\frac{m_t^2 - \mu^2}{2m_t}$ and that the normalisation of the distribution changes.

This is one simple modification from non-unparticle physics but it may be possible to look for effects of the breaking of scale invariance in other collider signatures, for instance the interference between unparticle and SM propogators in simple processes such as $e^+e^- \to \mu^+\mu^-$ [10]. It would also be interesting to see how the inclusion of the coupling between the SM Higgs and the unparticle sector affects Higgs physics.

Acknowledgements
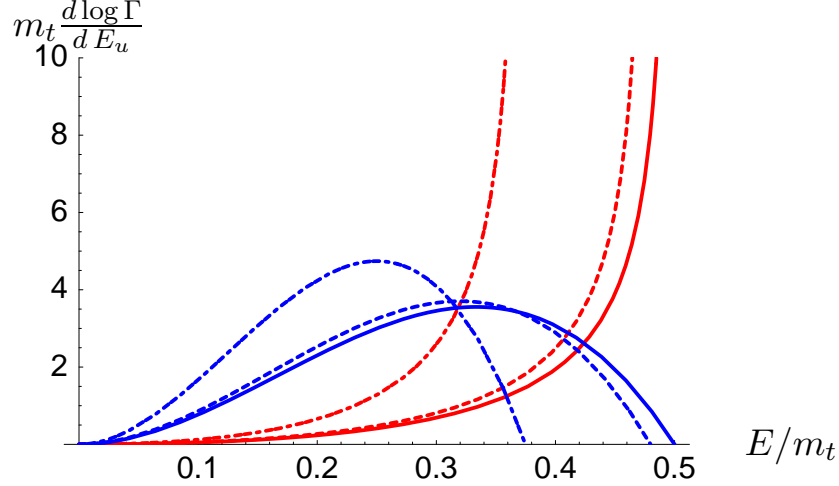
Figure 2: The differential decay rate, $m_t \frac{d \log \Gamma}{dE_u}$, for the decay $t \to u\, \mathcal{O}_{\mathcal{U}}$ as a function of final state quark energy. The red curves (concave)assume the IR dimension of the unparticle operator is $d_{\mathcal{U}} = 4/3$ and the blue (convex) assume $d_{\mathcal{U}} = 3$. In both cases the solid, dashed and dot-dashed curves label the scale, μ, below which there are no unparticle modes and correspond to $\mu/m_t = 0, 0.2, 0.5$ respectively.

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