

Aligned vertical fractures, *HTI* reservoir symmetry, and Thomsen seismic anisotropy parameters

James G. Berryman
E-mail: JGBerryman@LBL.GOV
University of California,
Lawrence Berkeley National Laboratory,
1 Cyclotron Road, MS 90R1116,
Berkeley, CA 94720, USA

Abstract:

The Sayers and Kachanov (1991) crack-influence parameters are shown to be directly related to Thomsen (1986) weak-anisotropy seismic parameters for fractured reservoirs when the crack density is small enough. These results are then applied to seismic wave propagation in reservoirs having *HTI* symmetry due to aligned vertical fractures. The approach suggests a method of inverting for fracture density from wave speed data.

INTRODUCTION

Aligned vertical fractures provide a commonly recognized source of azimuthal (surface angle dependent) seismic anisotropy in oil and gas reservoirs (Lynn *et al.*, 1995). For analysis, *VTI* earth media are much easier to understand and analyze than *HTI* media. Nevertheless, when the source of the anisotropy is aligned vertical fractures, we can make very good use of the simpler case of horizontal fracture analysis by making a rather minor change of our point of view that easily gives all the needed results. We can also understand very directly the sources of the anisotropy due to fractures by considering a method introduced by Sayers and Kachanov (1991). Elastic constants, and therefore the Thomsen (1986) parameters, can be conveniently expressed in terms of the Sayers and Kachanov (1991) formalism. Furthermore, in the low crack density limit [which is also consistent with the weak anisotropy approach of Thomsen (1986)], we obtain direct links between the Thomsen parameters and the fracture properties. These links suggest a method of inverting for fracture density from wave speed data.

THOMSEN'S SEISMIC WEAK ANISOTROPY METHOD

Thomsen's weak anisotropy method (Thomsen, 1986), being an approximation designed specifically for use in velocity analysis for exploration geophysics, is clearly not exact. Approximations incorporated into the formulas become most apparent for greater angles θ from the vertical, especially for compressional and vertically polarized shear velocities $v_p(\theta)$ and $v_{sv}(\theta)$, respectively. Angle θ is measured from the z-vector pointing into the earth.

Exact velocity formulas for *P*, *SV*, and *SH* seismic waves at all angles in a *VTI* elastic medium are known and available in many places (Ruger, 2002; Musgrave, 2003), so will not be listed here. Expressions for phase velocities in Thomsen's weak anisotropy limit can also be found in many places, including Thomsen (1986) and Ruger (2002). The pertinent expressions for phase velocities in *VTI* media as a function of angle θ are:

$$v_p(\theta) \simeq v_p(0)(1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta), \quad (1)$$

$$v_{sv}(\theta) \simeq v_s(0) \left(1 + \left[\frac{v_p^2(0)}{v_s^2(0)} \right] (\epsilon - \delta) \sin^2 \theta \cos^2 \theta \right), \quad (2)$$

and

$$v_{sh}(\theta) \simeq v_s(0)(1 + \gamma \sin^2 \theta). \quad (3)$$

In our present context, $v_s(0) = \sqrt{\frac{c_{44}}{\rho_0}}$, and $v_p(0) = \sqrt{\frac{c_{33}}{\rho_0}}$, where c_{33} , c_{44} , and ρ_0 are two stiffnesses of the cracked medium and the mass density of the isotropic host elastic medium. We assume that the cracks have insufficient volume to affect the mass density significantly.

In each case, Thomsen's approximation has included a step that removes the square on the left-hand side of the exact equation, by then expanding a square root on the right hand side. This step introduces a factor of $\frac{1}{2}$ multiplying the $\sin^2\theta$ terms on the right hand side, and --- for example --- immediately explains how equation (3) is obtained from the exact result. The other two equations for $v_p(\theta)$ and $v_{sv}(\theta)$, *i.e.*, (1) and (2), involve additional approximations we will not attempt to explain here.

The three Thomsen (1986) seismic parameters for weak anisotropy with VTI symmetry are $\gamma = \frac{c_{66} - c_{44}}{2c_{44}}$, $\epsilon = \frac{c_{11} - c_{33}}{2c_{33}}$, and δ , which is determined by $2c_{33}(c_{33} - c_{44})\delta = ((c_{13} + c_{44})^2 - (c_{33} - c_{44})^2)$. All three of these parameters can play important roles in the velocities given by (1)-(3) when the crack densities are high enough. If crack densities are very low, then the SV shear wave will actually have no dependence on angle of wave propagation. Note that the so-called anellipticity parameter $A = \epsilon - \delta$, vanishes when $\epsilon \equiv \delta$, which we will soon see does happen for low crack densities.

HORIZONTAL FRACTURES, CRACK-INFLUENCE PARAMETERS, & VTI SYMMETRY

To illustrate the Sayers and Kachanov (1991) crack-influence parameter method, consider the situation in which all the cracks in the system have the same vertical (or z -axis) of symmetry. (We use 1,2,3 and x, y, z notation interchangeably for the axes.) Then, the cracked/fractured system is not isotropic, and we have the first-order compliance correction matrix for horizontal fractures, which is:

$$\Delta S_{ij}^{(1)} = \rho_c \begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}, \quad (4)$$

where $i,j = 1,2,3$. The two lowest order crack-influence parameters from the Sayers and Kachanov (1991) approach are η_1 and η_2 . The scalar crack density parameter is defined -- for penny-shaped cracks having number density $n = \frac{N}{V}$ and radius in the plane of the crack equal to a -- to be $\rho_c = na^3$. The aspect ratio of the cracks is $\frac{b}{a}$. Considering orientational averages of (4) provides a direct connection to the isotropic case, which is of great practical importance, because it permits us to estimate the parameters η_1 and η_2 by studying isotropic cracked/fractured systems, using well-understood effective medium theories (Zimmerman, 1991; Berryman and Grechka, 2006).

Now consider horizontal fractures, as just illustrated by the correction matrix ΔS_{ij}^1 . The axis of fracture symmetry is uniformly vertical, and so such a reservoir would exhibit *VTI* symmetry. The resulting expressions for the Thomsen parameters in terms of the Sayers and Kachanov (1991) parameters η_1 and η_2 are given by

$$\gamma_h = \frac{c_{66} - c_{44}}{2c_{44}} = \rho_c \eta_2 G_0, \quad (5)$$

and

$$\epsilon_h = \frac{c_{11} - c_{33}}{2c_{33}} = \rho_c [(1 + \nu_0)\eta_1 + \eta_2] \left(\frac{E_0}{1 - \nu_0^2} \right) \simeq \left(\frac{2\rho_c \eta_2 G_0}{1 - \nu_0} \right). \quad (6)$$

Background shear modulus is G_0 , with corresponding Poisson ratio is ν_0 . Young's modulus is $E_0 = 2(1 + \nu_0)G_0$. Also $\delta = \epsilon$ to the lowest order in crack density parameters. We chose to neglect the term η_1 in the final expression of (6), as this is on the order of a 1% correction to the term retained. Values of η_1 and η_2 can be determined from simulations and/or effective medium theories (Zimmerman, 1991; Berryman and Grechka, 2006). They depend on the elastic constants of the background medium, and on the shape of the cracks (assumed to be penny-shaped in these examples).

HTI RESERVOIR SYMMETRY FROM ALIGNED VERTICAL FRACTURES

The trick to get from horizontal fractures and *VTI* to aligned vertical fractures and *HTI* symmetry is relatively simple. We need not relabel the c_{ij} 's. Instead we change the meaning of the labels. As long as we stay mentally oriented in the reference frame of the fractures themselves, we can view the z -direction as the symmetry axis and the xy -plane, as the plane of the fractures. The only change we need to make arises because we shoot our seismic survey on the earth surface at 90° from the fracture plane. This observation implies that, wherever the angle θ (measured in radians) appeared in our previous formulas, now we replace it by $\frac{\pi}{2} - \theta$ radians. Thus, $\sin^2 \theta \rightarrow \cos^2 \theta$, and vice versa in the formulas. This algorithm is exactly right only for those planes that are vertical and also perpendicular to the fracture plane, *i.e.*, at azimuthal angles $\phi = \pm\pi/2$. For all angles, we actually need to replace $\sin^2 \theta$ by $\cos^2 \theta \sin^2 \phi$. Then, when $\phi = 0$ or π , there is no angular dependence, as our point of view is within the plane of the fracture itself. Backing up one step in the Thomsen derivation and restoring squares, thereby “unexpanding” the square root is also helpful. Certain approximations are then undone, and final formulas are more accurate.

If ϵ , δ , and γ are the Thomsen parameters for the *VTI* symmetry (horizontal fracture), then, for example,

$$v_{sh}^2 \left(\frac{\pi}{2} - \theta \right) = v_s^2(0) \left[1 + 2\gamma \sin^2 \left(\frac{\pi}{2} - \theta \right) \right] = v_s^2(0) (1 + 2\gamma) \left[1 - \left(\frac{2\gamma}{(1+2\gamma)} \right) \sin^2 \theta \right]. \quad (7)$$

From this result, we deduce that, for *HTI*, $\gamma \rightarrow -\frac{\gamma}{1+2\gamma}$. This statement is rigorous for the form of equation (7) considered. Then, in the weak anisotropy limit we have $\gamma \rightarrow -\gamma$. But this final step is neither necessary nor recommended for some of the higher crack densities considered here.

Similar calculations for v_p^2 and v_{sv}^2 lead to the results $\epsilon \rightarrow -\frac{\epsilon}{1+2\epsilon} \simeq -\epsilon$, and $\delta \rightarrow \frac{\delta-2\epsilon}{1+2\epsilon} \simeq \delta - 2\epsilon$.

As a consistency check, note that $\epsilon - \delta \rightarrow \frac{\epsilon-\delta}{1+2\epsilon} \simeq \epsilon - \delta$. Similarly, the pertinent wave speeds are:

$v_p(0) \rightarrow \sqrt{\left\{ \frac{c_{33}(1+2\epsilon)}{\rho} \right\}} = \sqrt{\left\{ \frac{c_{11}}{\rho} \right\}}$ & $v_s(0) \rightarrow \sqrt{\{c_{44}(1+2\gamma)/\rho\}} = \sqrt{\left\{ \frac{c_{66}}{\rho} \right\}}$ in (7), but the remaining velocity [$v_{sv}(\theta)$] does not change at all since Thomsen's approximation (2) for $v_{sv}(\theta)$ is completely symmetric in θ and, therefore, must remain so, also having the same end points, after the switch from θ to $\frac{\pi}{2} - \theta$. These results were all previously known and can be found in Ruger (2002), p. 75.

Examples of these results for small ($\rho_c = 0.05$) and higher ($\rho_c = 0.1, 0.2$) crack densities are presented in Figure 1. See Berryman and Grechka (2006) for details of the methods used to obtain higher order Sayers and Kachanov (1991) parameters from simulation data, and Berryman (2007) for a full discussion of the reservoir applications.

CONCLUSIONS

The Sayers and Kachanov (1991) crack-influence parameters are ideally suited to analyzing mechanics in reservoirs having aligned fractures and exhibiting *HTI* symmetry. Detailed discussion of results obtained for the higher crack density examples presented in Figure 1 will be provided in the oral presentation, but the main ideas are all contained in Berryman and Grechka (2006) and Berryman (2007). Modeling presented here shows that Thomsen's weak anisotropy method is valid for crack densities up to about $\rho_c \simeq 0.05$, but should be replaced by more exact calculations if the crack density is $\rho_c \simeq 0.1$ or higher.

ACKNOWLEDGMENTS

The author thanks V. Grechka, M. Kachanov, S. R. Pride, and M. Schoenberg for helpful collaborations and conversations. Work performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Berkeley National Laboratory under Contract No. DE-AC03-76SF00098 and supported specifically by the Geosciences Research Program of the DOE, Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences and Biosciences.

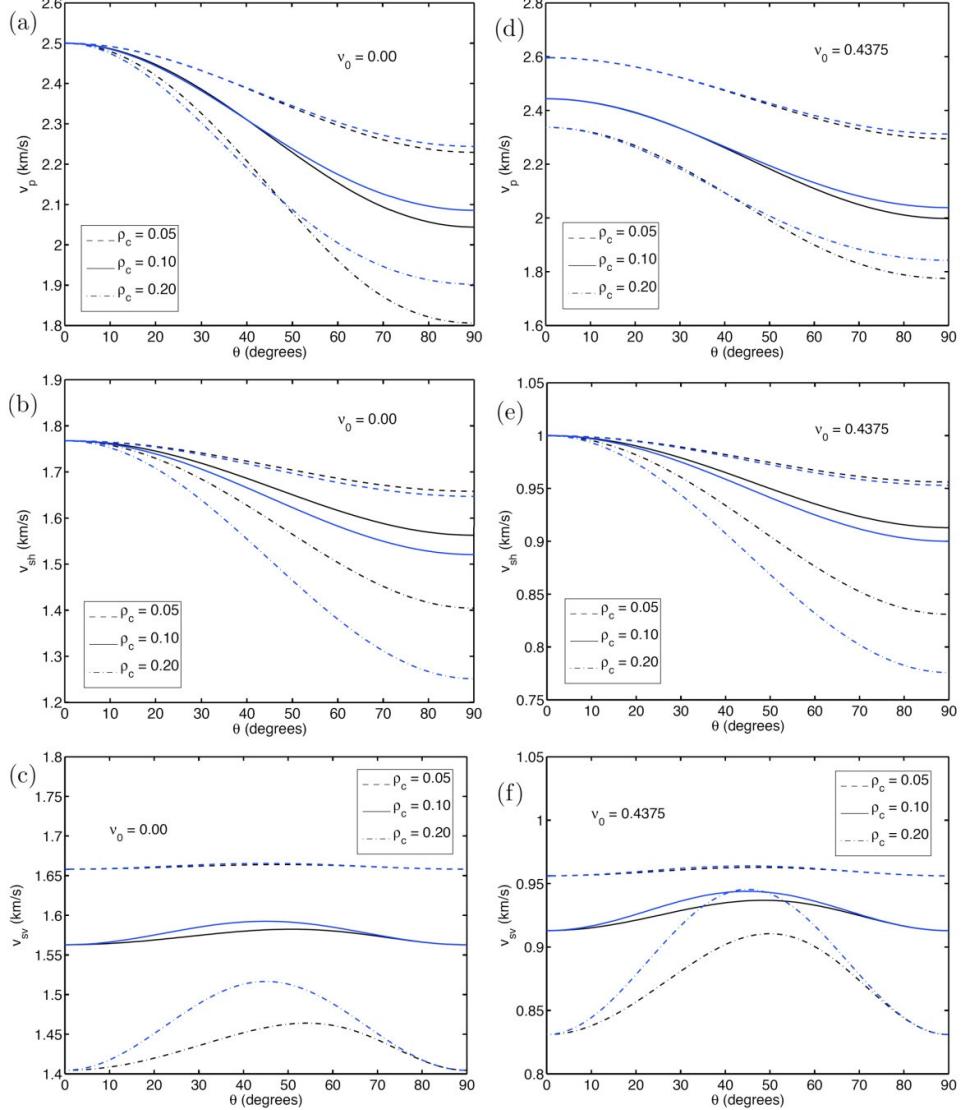


FIG. 1: For aligned vertical cracks, examples of anisotropic compressional wave speed, SH shear wave speed, and SV shear wave speed for two values of Poisson's ratio v_0 of the host medium: (a)-(c) $v_0 = 0.00$, (d)-(f) $v_0 = 0.4375$. Velocities in black are exact for the fracture model discussed. Thomsen weak anisotropy velocity curves for the same model (*i.e.*, the same c_{ij} 's) were then overlain in blue.

REFERENCES

Berryman, J.G., 2007, Seismic waves in rocks with fluids and fractures, submitted for publication.

Berryman, J. G., and Grechka, V., 2006, Random polycrystals of grains containing cracks: Model of quasistatic elastic behavior for fractured systems, *J. Appl. Phys.*, **100**, 113527.

Lynn, H. B., Simon, K. M., Layman, M. and Schneider, R., 1995, Use of anisotropy in *P*-wave and *S*-wave data for fracture characterization in a naturally fractured gas reservoir, *The Leading Edge*, **14**, 887—893.

Musgrave, M. J. P., 2003, *Crystal Acoustics*, Acoustical Society of America, Leetsdale, Pennsylvania, Chapter 8.

Ruger, A., 2002, *Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media*, Geophysical Monographs Series, Number 10, SEG, Tulsa, OK.

Sayers, C. M., and Kachanov, M., 1991, A simple technique for finding effective elastic constants of cracked solids for arbitrary crack orientation statistics, *Int. J. Solids Struct.*, **27**, 671—680.

Thomsen, L., 1986, Weak elastic anisotropy, *Geophysics*, **51**, 1954—1966.

Zimmerman, R. W., 1991, *Compressibility of Sandstones*, Elsevier, Amsterdam.