Application Of Chiral Two- And Three-Nucleon Interactions To The 4He Photo-Disintegration

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Application of chiral two- and three-nucleon interactions to the ^{4}He photo-disintegration

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Abstract. We report on an *ab initio* calculation of the ^{4}He total photo-absorption cross section using two- and three-nucleon interactions based upon chiral effective field theory. The microscopic treatment of the continuum problem is achieved using the Lorentz integral transform method, applied within the no-core shell model approach.

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INTRODUCTION

Chiral effective filed theory (χEFT) [1, 2] represents our best opportunity to reach a consistent picture of the interaction among nucleons, that is based on the underlying and fundamental theory of quantum chromo-dynamics (QCD). Indeed, χEFT may be thought as an effective theory for QCD in the low energy regime relevant for describing the properties of nuclei. In the framework of χEFT the nucleon-nucleon (NN) interaction is predicted at the leading order, together with a three-nucleon (NNN) interaction at the next-to-next-to-leading order or N^{2}LO [2, 3, 4], and even a four-nucleon (NNNN) interaction at the fourth order (N^{3}LO) [5]. The details of QCD are contained in parameters, the so-called low-energy constants (LECs), that are not fixed by the symmetry, but can be constrained by experiment. Therefore, it is of the utmost importance to apply χEFT to nuclei in an ab initio framework.

We performed an *ab initio* calculation [6] of the ^{4}He total photo absorption cross section in unretarded dipole approximation, using the high quality NN potential at the fourth order (N^{3}LO) in the χEFT expansion of Ref. [7], and the NNN interaction at the highest order presently available (N^{2}LO) [3, 5]. The two low-energy constants that parameterize the short-range NNN interaction were selected as discussed in Ref. [8]. The microscopic treatment of the continuum problem was achieved by means of the Lorentz integral transform (LIT) method [9], applied within the *ab initio* no-core shell model (NCSM) approach [10].

THE *AB INITIO* NO-CORE SHELL MODEL

The NCSM is a technique for the solution of the A-nucleon bound-state problem. Starting from an Hamiltonian containing realistic NN and NNN forces (both coordinate- and momentum-space interactions can be equally handled), the Schrrödinger equation is solved by expanding the wave functions in terms of a complete set of A-nucleon

harmonic oscillator (HO) basis states up to a maximum excitation $N_{max}\hbar\Omega$ above the minimum energy configuration, with Ω the HO frequency. Both Jacobi relative coordinates or Cartesian single-particle coordinates can be used. Indeed, in a complete $N_{max}\hbar\Omega$ space translational invariance is preserved even in the Slater-determinant basis. The convergence to the exact result with increasing N_{max} is accelerated by the use of an effective interaction tailored to the model-space truncation. The effective interaction is obtained using a unitary transformation in a $n-$body cluster approximation, where n is typically 2 or 3 [11].

TOTAL PHOTO-ABSORPTION CROSS SECTION VIA THE LORENTZ INTEGRAL TRANSFORM METHOD

In the long wave-length approximation, the total photo-absorption cross section

$$\sigma_\gamma(\omega) = 4\pi^2 \frac{e^2}{\hbar c} \omega R(\omega) , \tag{1}$$

is proportional to the inclusive dipole response function:

$$R(\omega) = \int d\Psi_f \left| \langle \Psi_f | \hat{D} | \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega) . \tag{2}$$

This is the sum of all the transitions from the ground state $|\Psi_0\rangle$ to the various allowed final states $|\Psi_f\rangle$ induced by the dipole operator:

$$\hat{D} = \sqrt{\frac{4\pi}{3}} \sum_{i=1}^{A} \frac{\tau_i^z}{2} r_i Y_{10}(\hat{r}_i) . \tag{3}$$

In the above equations ground- and final-state energies are denoted by E_0 and E_f, respectively, whereas τ_i^z and $\vec{r}_i = r_i\hat{r}_i$ represent the isospin third component and center of mass frame coordinate of the ith nucleon. The direct calculation of such response function is extremely difficult, especially for energies beyond the three-body breakup threshold. However, it is possible to obtain the response function by following a small detour, the LIT method [12].

While in conventional approaches one usually starts from Eq. (2), in the LIT method one obtains $R(\omega)$ after the inversion of its integral transform with a Lorentzian kernel

$$L(\sigma_R, \sigma_I) = \int d\omega \frac{R(\omega)}{(\omega - \sigma_R)^2 + \sigma_I^2} = \langle \widetilde{\Psi} | \widetilde{\Psi} \rangle . \tag{4}$$

Indeed, in order to calculate such a transform it is sufficient to solve the in-homogeneous "Schrödinger-like" equation

$$(H - E_0 - \sigma_R + i\sigma_I)|\widetilde{\Psi}\rangle = \hat{D}|\Psi_0\rangle. \tag{5}$$

Because of the presence of an imaginary part σ_I in Eq. (5) and the fact that the right-hand side of this same equation is localized ($\langle \Psi_0 | \hat{D}^\dagger \hat{D} | \Psi_0 \rangle < \infty$), one has an asymptotic
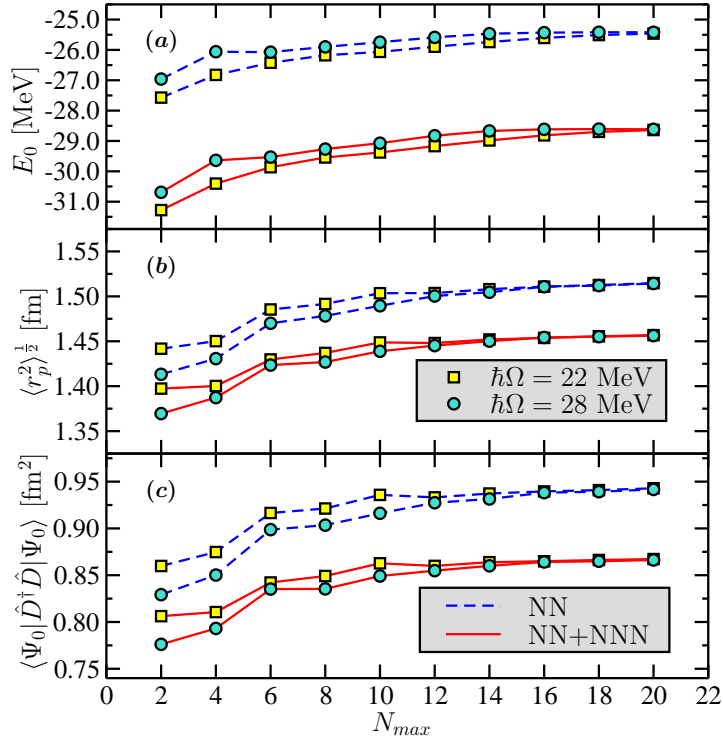
FIGURE 1. The ^{4}He ground-state (*a*) energy, (*b*) point-proton root-mean-square radius $\langle r_p^2 \rangle^{1/2}$, and (*c*) total dipole strength $\langle \Psi_0 | \hat{D}^\dagger \hat{D} | \Psi_0 \rangle$ as functions of the model-space truncation N_{max}. Present results with χEFT interactions for $\hbar\Omega = 22$ and 28 MeV.

RESULTS

We start by discussing our results for the ground state of the α particle, which enters the driving term of the LIT equation (5). In Fig. 1 we show the convergence patterns for three different ground-state observables, calculated with and without inclusion of the NNN terms of the interaction: binding energy, point-proton radius and total dipole strength. For all of them we obtain very similar and smooth convergence patterns, using effective interactions at the three-body cluster level. In particular, already for $N_{max} = 18$ we find independence from both model space and harmonic oscillator parameter. At the ground-state level, the inclusion of the NNN force affects mostly the energy, providing 3.21 MeV additional binding, while the point-proton radius undergoes only a weak reduction. The total dipole strength follows the same pattern as the radius. Indeed, these two observables
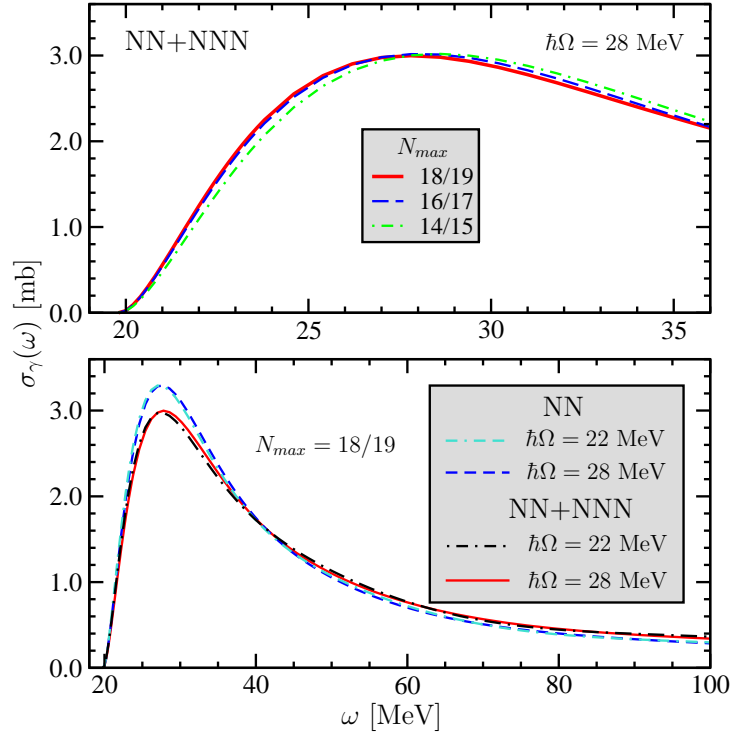
FIGURE 2. The ^{4}He total photo-absorption cross section as a function of the excitation energy ω. Present results with χEFT interactions, and in particular: (upper panel) convergence pattern of the NN+NNN calculation with respect to the model-space truncation N_{max} for $\hbar\Omega = 28$ Me; (lower panel) frequency dependence of the best ($N_{max} = 18/19$) results with and without inclusion of the NNN force.

are correlated [14]:

$$\langle\Psi_0|\hat{D}^\dagger\hat{D}|\Psi_0\rangle \simeq \frac{ZN}{3(A-1)}\langle r_p^2\rangle . \tag{6}$$

This expression, which is exact for deuteron and triton, and for spatially symmetric systems, is violated of about 9% for the ^{4}He calculated both with and without NNN interactions terms.

By applying the LIT method we have obtained the ^{4}He total photo-absorption cross section shown in Fig. 2. Also for this observable we find a stable and accurate convergence thanks to the use of three-body effective interactions. From the bottom panel of the figure we also see that for the biggest model space used, $N_{max} = 18/19$, the dependence on the HO frequency is weak. The inclusion of the NNN interaction terms induces a suppression of the peak and an enhancement of the tail of the cross section. In particular, the reduction of the low-energy cross section is related, through the inverse energy-weighted sum rule to the the reduction found for the dipole strength

$$\int_{E_{th}}^\infty \frac{\sigma_\gamma(\omega)}{\omega}d\omega = 4\pi^2 \frac{e^2}{\hbar c}\langle\Psi_0|\hat{D}^\dagger\hat{D}|\Psi_0\rangle . \tag{7}$$

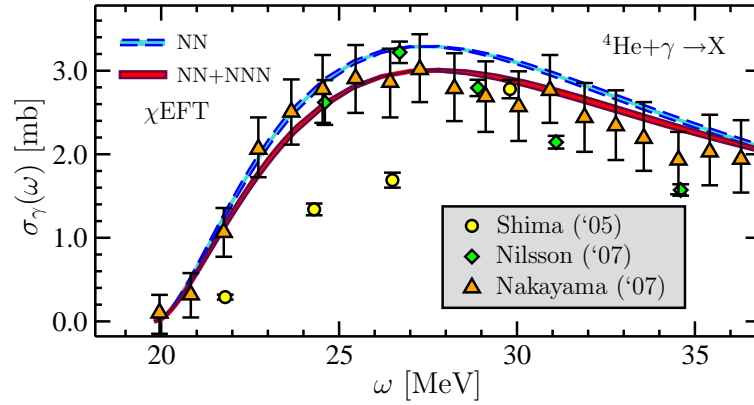**FIGURE 3.** The ^{4}He photo-absorption cross section as a function of the excitation energy ω. Present results with χEFT interactions compared to the most recent experiments [15, 16, 17].

Our final results, presented in Fig. 3 together with the most recent experiments, show a peak around the excitation energy of $\omega = 27.8$ MeV, with a pick height mildly sensitive to the NNN force. The experimental situation in the near-threshold region is controversial: two direct measurements performed using quasi-mono-energetic photons [15, 16] show discrepancies up to a factor of two on the absolute height of the cross-section peak. We find an overall good agreement with the photo-disintegration data from bremsstrahlung photons of Nilsson *et al*. [16], while we reach only the last of the experimental points of Ref. [15]. In particular, the confused experimental situation drawn by these two data sets does not allow to asses the role of the NNN force effect. Recently Nakayama *et al*. performed an indirect measurements of the α-particle total photo-absorption cross section [17] by observing its analog via the ^{4}He(^{7}Li,^{7}Be) reaction at an incident energy of 455 MeV and at forward scattering angles. Although the uncertainty on this extracted absolute cross section is 20% or more, the inclusion of the NNN terms of the interaction appear to improve the agreement of the calculated cross section with the latter indirect measurement.

ACKNOWLEDGMENTS

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