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Abstract

This report presents a modification of a previous model for the statistical distribution of linear antenna impedance. With this modification a simple formula is determined which yields accurate results for all ratios of modal spectral width to spacing. It is shown that the reactance formula approaches the known unit Lorentzian in the lossless limit.

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1 INTRODUCTION

A previous paper [1] introduced a simplified model, based on power balance, for the statistical distribution of the input impedance of a linear antenna in an electrically large cavity. This model yielded simple expressions for the distributions of input resistance and reactance. These were compared to Monte Carlo simulations of the modal series [4], [5] using statistical distributions for the eigenvalues and eigenfunctions [7], [8], [9] of the cavity. The parameter (ratio of modal width to modal spacing)

$$\alpha = \frac{k^3 V}{2\pi Q} \tag{1}$$

determined the form of the results, ranging from near free space values for large α [2], [3] to large variations for small values of α.

The previously proposed approximate statistical distribution for the field reflected from the walls [1] is modified in this short paper to accommodate the known limits of the distribution. The resulting form is shown to give a uniformly valid accurate formula for the input impedance.

2 NEW DISTRIBUTIONS FOR REFLECTED FIELD AND INPUT IMPEDANCE

We repeat a brief sketch of the power balance method applied to a small dipole with center current $I(0)$ [1]. The power into the cavity from the antenna is

$$P_{in} = \frac{1}{2}\left(R_{rad} + R_{wall}\right)\left|I(0)\right|^2 = \frac{1}{2} R_{in} \left|I(0)\right|^2 \tag{2}$$

where R_{wall} is the real part of the impedance resulting from the wall, R_{rad} is the radiation resistance of the dipole in free space, and R_{in} is the input resistance. Using the definition of the Q (this is the Q of the cavity assuming the dipole absorbs no power, since it can be terminated in an ideal current source to measure the input impedance), we have

$$Q = \frac{\omega V U}{P_{in}} \tag{3}$$

where U is the time average mean energy density (isotropy is assumed) and is given by

$$U = \frac{3}{2}\epsilon_0 \left\langle \left|E_z\right|^2 \right\rangle_V \tag{4}$$

with the subscript V denoting volumetric mean. The voltage at the short dipole resulting from field due to the wall currents is (see Appendix 1)

$$V_{ref} \sim -h E_z^{ref} \tag{5}$$

where h is the physical half height of the short antenna. Thus the dipole impedance resulting from the reflected field is (time dependence $e^{-i\omega t}$ is suppressed)

$$Z_{wall} = R_{wall} - iX_{wall} = V_{ref}/I(0) \tag{6}$$

Then we can write, using (2) through (6),

$$Z_{wall} \approx \frac{-E_z^{ref}}{\sqrt{\left\langle \left|E_z\right|^2 \right\rangle_V}} h \sqrt{\frac{2U}{3\varepsilon_0 \left|I(0)\right|^2}} = \frac{-E_z^{ref}}{\sqrt{\left\langle \left|E_z\right|^2 \right\rangle_V}} h \sqrt{\frac{2QP_{in}}{3\varepsilon_0 \omega V \left|I(0)\right|^2}} = \frac{-E_z^{ref}}{\sqrt{\left\langle \left|E_z\right|^2 \right\rangle_V}} \sqrt{\frac{Qh^2 R_{in}\eta_0}{3kV}}$$

$$= \frac{-E_z^{ref}}{\sqrt{\left\langle \left|E_z\right|^2 \right\rangle_V}} \sqrt{\left(\frac{2\pi Q}{k^3 V}\right) R_{in} R_{rad}} = \frac{-E_z^{ref}}{\sqrt{\left\langle \left|E_z\right|^2 \right\rangle_V}} \sqrt{R_{in} R_{rad}/\alpha} \tag{7}$$

where we note that the radiation resistance in free space is $R_{rad} \sim \eta_0 (kh)^2/(6\pi)$, and η_0 is the impedance of free space. Taking real and imaginary parts of this equation and normalizing by R_{rad} (with $R_{wall}/R_{rad} = r_{wall}$, $(R_{in} - R)/R_{rad} = r_{in}$, and $X_{wall}/R_{rad} = x_{wall}$) gives [1]

$$r_{wall} = r_{in} - 1 = \tau \sqrt{r_{in}/\alpha} \tag{8}$$

$$x_{wall} = \zeta \sqrt{r_{in}/\alpha} \tag{9}$$

(note that the total input reactance is the sum of the wall part and the local or free space part, i. e., $X_{in} = X_{wall} + X$) where the normalized reflected fields are

$$\tau = \frac{-\operatorname{Re}\left(E_z^{ref}\right)}{\sqrt{\left\langle \left|E_z\right|^2 \right\rangle_V}} \tag{10}$$

$$\zeta = \frac{\operatorname{Im}\left(E_z^{ref}\right)}{\sqrt{\left\langle \left|E_z\right|^2 \right\rangle_V}} \tag{11}$$

The normalized input impedance of the antenna is then [1]

$$z_{in} = r_{in} - i x_{wall} = (Z_{in} - Z)/R_{rad} \tag{12}$$

where $Z = R - iX$ is the local impedance (R is the ohmic loss and X is the local reactance). Thus we have achieved, via the power balance method, a means to express the input impedance in terms of the reflected field from the cavity wall, whose real and imaginary parts are treated as independent random variables.

The quadratic equation (8) can be solved as

$$r_{in} = \left(\sqrt{\tau^2 + 4\alpha} + \tau\right)^2 / (4\alpha) \tag{13}$$

The previous paper [1] estimated these normalized reflected fields with the real part (with respect to a current that is taken to be real) (10) approximated as an asymmetrical Gaussian [1]

$$f(\tau) \approx \frac{p(\alpha)}{\sqrt{2\pi}} e^{-\tau^2/2}\ ,\ 0 < \tau < \infty$$

$$\approx \frac{2-p(\alpha)}{\sqrt{2\pi}} e^{-\tau^2/2} \ , \quad -\infty < \tau < 0 \tag{14}$$

where $p(\alpha)$ was adjusted to go to zero as $\alpha \to 0$, to approach unity as $\alpha \to \infty$, and to make the mean of the input resistance equal to unity. Although (14) fits the experimental and simulation results well, it has a discontinuity at $\tau = 0$ thus producing a "kink" for the cumulative distribution [1]. In what follows, a smooth density function is derived that provides even better agreement with simulation results and is very easily manipulated to determine other distributions of interest.

Near $\tau = 0$, and for small α, a density function can be derived from the derivative of equation (68) of [1], valid for uniformly spaced eigenvalues

$$f(\tau) \sim \frac{2}{\pi} \frac{1}{\sqrt{2\pi}} \left[1 - \frac{\tau}{\sqrt{\tau^2 + 4\alpha}} \right] \ , \ \alpha << 1 \ , \ \tau << 1 \tag{15}$$

Taking a clue from this asymptotic form we modify the asymmetrical (and discontinuous) Gaussian density (14) to the expression

$$f(\tau) = \frac{1}{\sqrt{2\pi\sigma_r^2}} \left(1 - \frac{\tau}{\sqrt{\tau^2 + 4\alpha}} \right) e^{-\tau^2/\left(2\sigma_r^2\right)} \ , -\infty < \tau < \infty \, , \, 0 < \alpha < \infty \tag{16}$$

where we had to introduce a multiplicative factor $\pi/2$ from the normalization condition

$$\int_{-\infty}^{\infty} f(\tau)\, d\tau = 1 \tag{17}$$

and we have introduced the parameter σ_r^2. Note that if $\alpha \to 0$ we obtain the negative sided Gaussian discussed previously [1]. If $\alpha \to \infty$ we obtain the unit Gaussian density. The corresponding distribution function is

$$F(\tau) = \int_{-\infty}^{\tau} f(\tau)\, d\tau = 1 - \frac{1}{2} \left[\mathrm{erfc}\left(\tau/\sqrt{2\sigma_r^2} \right) - e^{2\alpha/\sigma_r^2} \mathrm{erfc}\left(\sqrt{\frac{\tau^2 + 4\alpha}{2\sigma_r^2}} \right) \right] \tag{18}$$

where erfc(x) is the complementary error function [11]. The mean value of the normalized input resistance, using the new density function (16), is

$$\langle r_{in} \rangle = \int_{-\infty}^{\infty} r_{in} f(\tau)\, d\tau = 1 \tag{19}$$

as it should. Introducing the change of variables [12] we have

$$f(r_{in}) = \left| \frac{d\tau}{dr_{in}} \right| f(\tau) = \frac{1}{r_{in}\sqrt{2\pi r_{in}\sigma_r^2/\alpha}} e^{-\alpha(r_{in}-1)^2/\left(2r_{in}\sigma_r^2\right)} \tag{20}$$

and

$$F(r_{in}) = 1 - \frac{1}{2} \left[\mathrm{erfc}\left(\frac{r_{in} - 1}{\sqrt{2\sigma_r^2 r_{in}/\alpha}} \right) - e^{2\alpha/\sigma_r^2} \mathrm{erfc}\left(\frac{r_{in} + 1}{\sqrt{2\sigma_r^2 r_{in}/\alpha}} \right) \right] \tag{21}$$

Suppose $\alpha >> 1$ and $r_{in} = 1 + \rho_r$ then we obtain a Gaussian distribution

$$f\left(\rho_r\right) \sim \frac{1}{\sqrt{2\pi\sigma_r^2/\alpha}} e^{-\alpha\rho_r^2/\left(2\sigma_r^2\right)} \ , \ \alpha \to \infty \tag{22}$$

and as $\alpha \to 0$

$$f\left(r_{in}\right) \sim \frac{1}{r_{in}\sqrt{2\pi r_{in}\sigma_r^2/\alpha}} \ , \ \alpha \to 0 \tag{23}$$

The imaginary part (11) is taken to be a Gaussian density [1]

$$f_\zeta\left(\zeta\right) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\zeta^2/\left(2\sigma_i^2\right)} \ , \ -\infty < \zeta < \infty \tag{24}$$

and distribution

$$F\left(\zeta\right) = \int_{-\infty}^{\zeta} f\left(\zeta\right) d\zeta = 1 - \frac{1}{2}\mathrm{erfc}\left(\zeta/\sqrt{2\sigma_i^2}\right) \tag{25}$$

where we have introduced the variance σ_i^2. From (9) we can write

$$x_{wall} = \zeta v \tag{26}$$

where

$$f_v\left(v\right) = \frac{\sqrt{2/\left(\pi\sigma_r^2\right)}}{v^2} e^{-\left(\alpha v^2-1\right)^2/\left(2v^2\sigma_r^2\right)} \tag{27}$$

If ζ and τ are taken to be independent then the product has density [12], [13]

$$f\left(x_{wall}\right) = \int_{-\infty}^{\infty} f_v\left(v\right) f_\zeta\left(x_{wall}/v\right) dv/\left|v\right|$$

$$= \frac{1}{\pi\sigma_r\sigma_i}\int_0^\infty e^{-\left\{\left(\alpha v^2-1\right)^2+x_{wall}^2\sigma_r^2/\sigma_i^2\right\}/\left(2v^2\sigma_r^2\right)} dv/v^3 = \frac{\alpha/\left(\pi\sigma_r\sigma_i\right)}{\sqrt{1+x_{wall}^2\sigma_r^2/\sigma_i^2}} e^{\alpha/\sigma_r^2} K_1\left(\frac{\alpha}{\sigma_r^2}\sqrt{1+x_{wall}^2\sigma_r^2/\sigma_i^2}\right) \tag{28}$$

where $K_1\left(x\right)$ is the modified Bessel function [11].

We will take the distribution parameters to be equal

$$\sigma_i^2 = \sigma_r^2 \tag{29}$$

so that

$$f\left(x_{wall}\right) = \frac{\alpha/\left(\pi\sigma_r^2\right)}{\sqrt{1+x_{wall}^2}} e^{\alpha/\sigma_r^2} K_1\left(\frac{\alpha}{\sigma_r^2}\sqrt{1+x_{wall}^2}\right) \tag{30}$$

from which the positive distribution is

$$F\left(x_{wall}\right) = 2\int_0^{x_{wall}} f\left(x_{wall}\right) dx_{wall} = 2\frac{\alpha}{\pi\sigma_r^2} e^{\alpha/\sigma_r^2} \int_1^{\sqrt{1+x_{wall}^2}} K_1\left(u\alpha/\sigma_r^2\right) \frac{du}{\sqrt{u^2-1}} \tag{31}$$

Note that as $\alpha \to 0$ we obtain from (30) (this is also true of the approximate distribution given previously

[1]) a unit Lorentzian distribution

$$f\left(x_{wall}\right) \sim \frac{1/\pi}{1+x_{wall}^2}\ ,\ \alpha \to 0 \tag{32}$$

in agreement with [14]. Note that as $\alpha \to \infty$

$$f\left(x_{wall}\right) \sim \frac{1}{\sqrt{2\pi\sigma_r^2/\alpha}} e^{-\alpha x_{wall}^2/\left(2\sigma_r^2\right)}\ ,\ \alpha \to \infty \tag{33}$$

2.1 Variance With Rayleigh Spacing

The previous paper [1] examined various asymptotic limits of the modal series for the input impedance with a uniform eigenvalue spacing and concluded that the normalized reflected field should be taken to have unit variance. However when Rayleigh (or Wigner) spacing is introduced, the variance in the undermoded limit $\alpha \to 0$ appears to be increased to $\pi/2$. The previous analysis [1] is unaffected in the overmoded limit $\alpha \to \infty$. Note that $\langle\tau\rangle = -1$ corresponds to $\sigma_r^2 = \pi/2$ when $\alpha \to 0$.

A fit which encompasses these two limits for Rayleigh spacing is

$$\sigma_r^2 = \arctan\left(\frac{1}{\sqrt{4\alpha}}\right) + \frac{1}{1+1/\sqrt{4\alpha}} \tag{34}$$

2.2 Numerical Comparisons

Figure 1 shows comparisons of the normalized reflected field in the highly undermoded limit $\alpha \to 0$ from Monte Carlo simulations of the random mode series (see Appendix 2) with both Rayleigh and uniform spacing of eigenvalues versus the power balance fit with variance formula (34) and unit variance. Figures 2 through 7 show comparisons of Monte Carlo simulations and the preceding power balance formulas for both the normalized reflected field and input impedance for the two values of α for which experimental data from a mode stirred chamber was discussed previously [1]. The preceding formulas eliminate the “kink” discrepancy in the real part of the reflected field and input impedance [1]. Furthermore the variance (34) seems to improve the correspondence between the Monte Carlo simulations with Rayleigh spacing and the power balance results.

Figures 8 and 9 show comparisons of the input impedance from Monte Carlo simulations versus the approximate formulas for a broad range of values of α. Figures 10 and 11 show the approximate formulas for the density functions.

3 CONCLUSIONS

The power balance method introduced in a previous paper is improved with a modification of the reflected field distribution to yield more accurate formulas for the input impedance of a linear antenna in a cavity. It is noted that the asymptotic form of the present result in the undermoded limit matches the known Lorentzian limit of reactance.

4 APPENDIX 1: Power Balance for Electrically Longer Antenna

Note that if the antenna is not an electrically short dipole the power balance arguments still apply. The open circuit voltage at the dipole, resulting from the field due to the wall currents, can be found by splitting the field into direct (radiation in free space) and reflected parts and integrating the product of the field with the free space current distribution. The result is [15]

$$V_{ref} = -\frac{1}{I_t(0)} \int_{-h}^{h} E_z^{ref}(z)\, I_t(z)\, dz = Z_{wall} I(0) \tag{35}$$

where h is the physical half height of the antenna and $I_t(z) = I_t(0) \sin k(h - |z|) / \sin(kh)$ is the transmitting current distribution in free space. The mean square magnitude of the reflected voltage (35) is

$$\left\langle |V_{ref}|^2 \right\rangle_V = \left\langle |Z_{wall}|^2 \right\rangle_V |I(0)|^2 = \int_{-h}^{h}\int_{-h}^{h} \left\langle E_z^{ref}(z)\, E_z^{ref*}(z') \right\rangle_V \frac{I_t(z)}{I_t(0)} \frac{I_t^*(z')}{I_t^*(0)} dz dz'$$

$$\approx \left\langle |E_z|^2 \right\rangle_V \int_{-h}^{h}\int_{-h}^{h} \frac{\left\langle E_z(z)\, E_z^*(z') \right\rangle_V}{\left\langle |E_z|^2 \right\rangle_V} \frac{I_t(z)}{I_t(0)} \frac{I_t^*(z')}{I_t^*(0)} dz dz'$$

$$= \left\langle |E_z|^2 \right\rangle_V \int_{-h}^{h}\int_{-h}^{h} \rho_z(z, z') \frac{I_t(z)}{I_t(0)} \frac{I_t^*(z')}{I_t^*(0)} dz dz' = \left\langle |E_z|^2 \right\rangle_V \left(\frac{3\lambda^2}{2\pi\eta_0} \right) R_{rad}$$

$$= \frac{3\lambda^2}{2\pi\eta_0} R_{rad} \frac{2U}{3\varepsilon_0} = 2R_{rad} \frac{P_{in}}{\alpha} = R_{rad} R_{in} \frac{1}{\alpha} |I(0)|^2 \tag{36}$$

where we have taken the correlation of the reflected field to be approximately represented by the source free cavity field correlation function [3], [16]

$$\rho_z(z, z') = \frac{3}{2} \left(1 + \frac{1}{k^2} \frac{\partial^2}{\partial z^2} \right) \frac{\sin k |z - z'|}{k |z - z'|} \tag{37}$$

and R_{rad} is now the free space radiation resistance of the longer dipole [1]. Thus we find

$$\sqrt{\left\langle |Z_{wall}|^2 \right\rangle_V} = \sqrt{R_{rad} R_{in} / \alpha} \tag{38}$$

Now from (35), (36), (10), and (11) we take

$$Z_{wall} = \frac{-E_z^{ref}}{\sqrt{\left\langle |E_z|^2 \right\rangle_V}} \sqrt{R_{rad} R_{in}/\alpha} = (\tau - i\zeta) \sqrt{R_{rad} R_{in}/\alpha} \tag{39}$$

Normalizing by R_{rad} we find

$$z_{wall} = r_{wall} - i x_{wall} = r_{in} - 1 - i x_{wall} = (\tau - i\zeta) \sqrt{r_{in}/\alpha} \tag{40}$$

the same equation we used for the normalized electrically short impedance (the only difference being that the radiation resistance in free space is no longer the short value).

5 APPENDIX 2: Subtraction of Free Space and Local Impedance

The simulations were carried out here by use of Monte Carlo methods on the representation

$$z_{wall} = (Z_{in} - Z - R_{rad})/R_{rad} = \frac{\omega^2}{\alpha Q}\left[i\sum_n \frac{3A_{nz}^2(\underline{r})}{\omega^2(1+i/Q)-\omega_n^2} - i\int \frac{1}{\omega^2(1+i/Q)-\omega_n^2}\frac{d\omega_n}{\langle\omega_n\rangle}\right] \tag{41}$$

where $\langle\omega_n\rangle \sim \pi^2 c^3/\left(V\omega_n^2\right)$ is the mean modal spacing (Rayleigh statistics are imposed in some cases), ω_n the modal frequencies, A_{nz} the vector potential eigenfunctions of the cavity (taken to be Gaussian and normalized so that $\left\langle 3A_{nz}^2(\underline{r})\right\rangle_V = 1$), the integral, which is subtracted out from the sum, generates the free space solenoidal field as the short dipole is approached. Note that the form given in the previous paper, equation (1) of [1], had the quasistatic part subtracted out

$$z_{in} = z_{wall} + 1 = (Z_{in} - Z)/R_{rad} = \frac{\omega^2}{\alpha Q}\left[i\sum_n \frac{3A_{nz}^2(\underline{r})}{\omega^2(1+i/Q)-\omega_n^2} + i\sum_n \frac{3A_{nz}^2(\underline{r})}{\omega_n^2}\right]$$

$$\approx \frac{\omega^2}{\alpha Q} i\sum_n \frac{3A_{nz}^2(\underline{r})\,\omega^2/\omega_n^2}{\omega^2(1+i/Q)-\omega_n^2} \tag{42}$$

If the tails of the summation are summed by an integral approximation we find that (42) minus unity is approximately the same as (41) for high frequencies and large quality factor.

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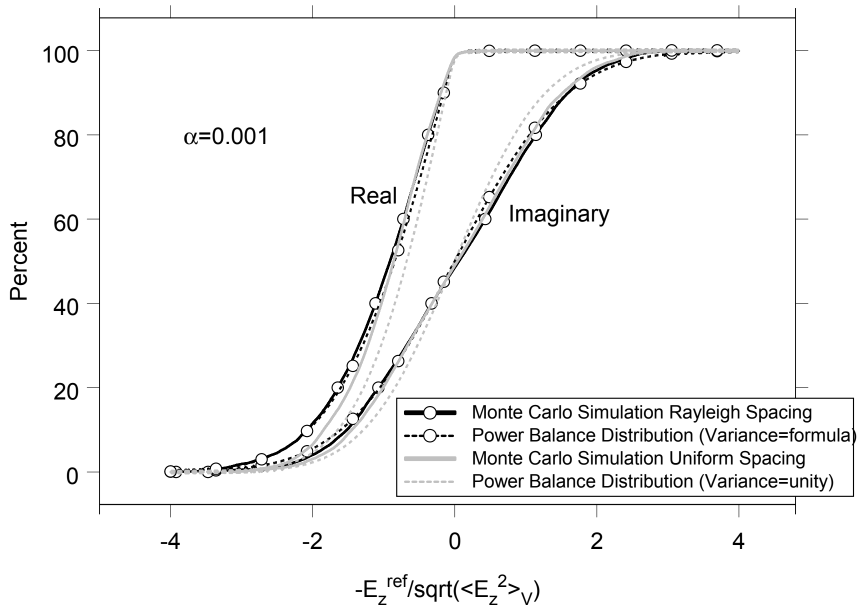

Figure 1. Comparison of normalized reflected field from Monte Carlo simulations at very high frequency (5 GHz, 100 kHz band, with the same volume as in the next two figures) in the highly undermoded limit. Both Rayleigh spacing of eigenvalues and unform spacing of eigenvalues are used (and compared with the approximate formula with near $\pi/2$ variance and with unit variance, respectively).

[1]

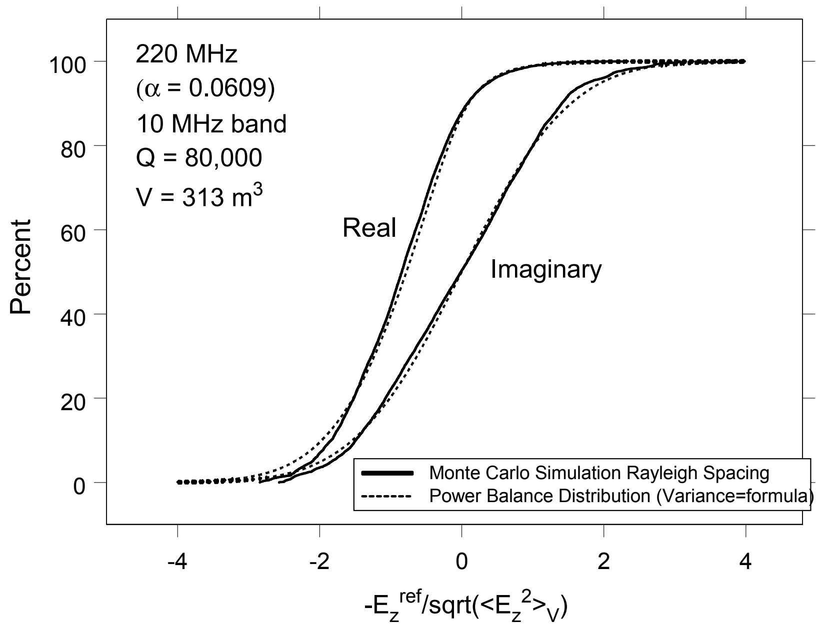

Figure 2. Comparison of normalized reflected field from Monte Carlo simulation and from the approximate formula in the undermoded case. The actual frequency was used as shown.

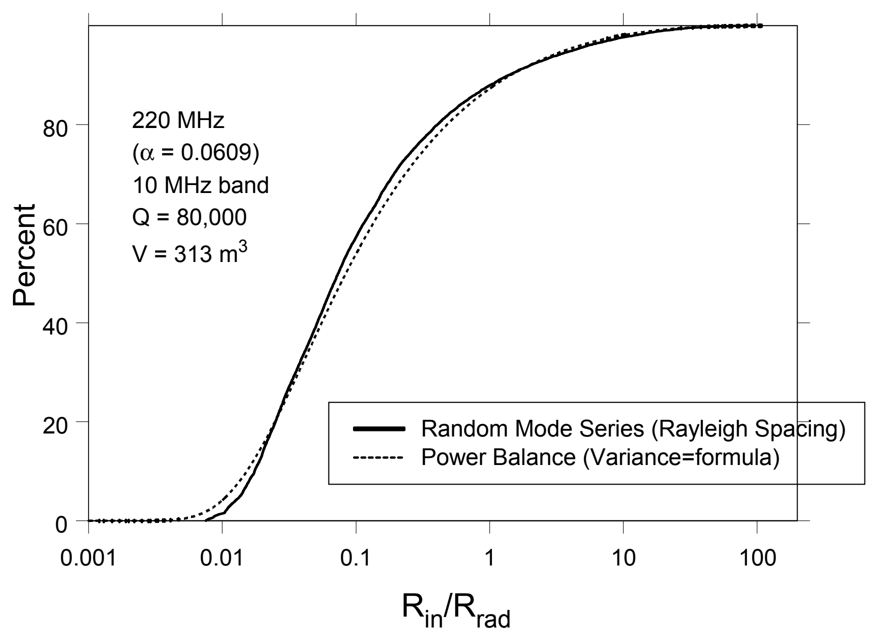

Figure 3. Corresponding real part of the normalized input impedance.

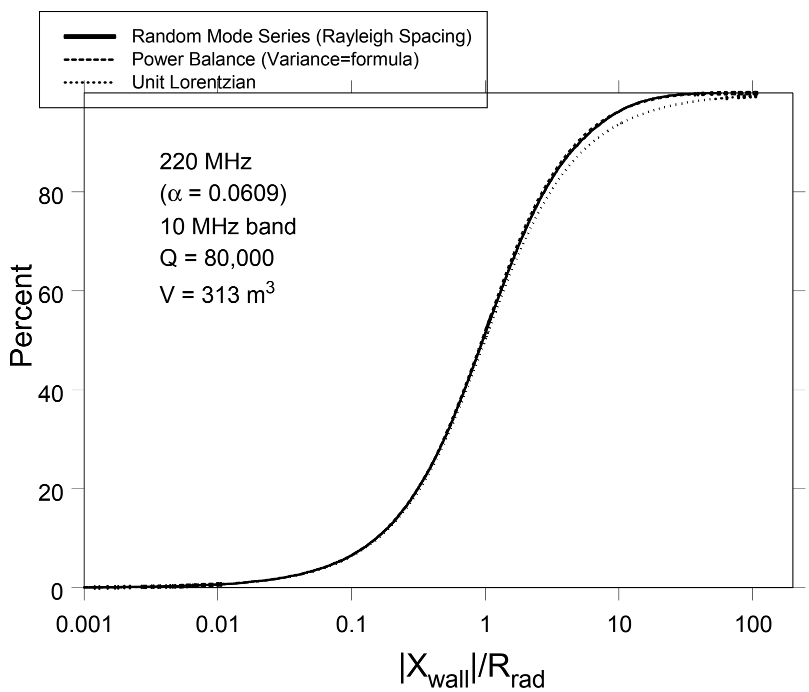

Figure 4. Corresponding imaginary part of the normalized input impedance due to the wall. The unit Lorentzian limit is also shown.

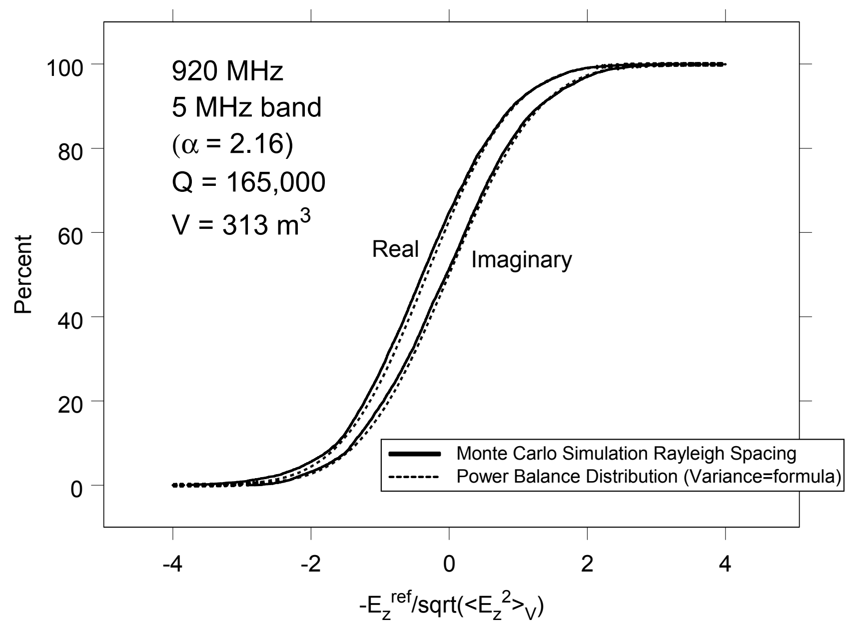

Figure 5.

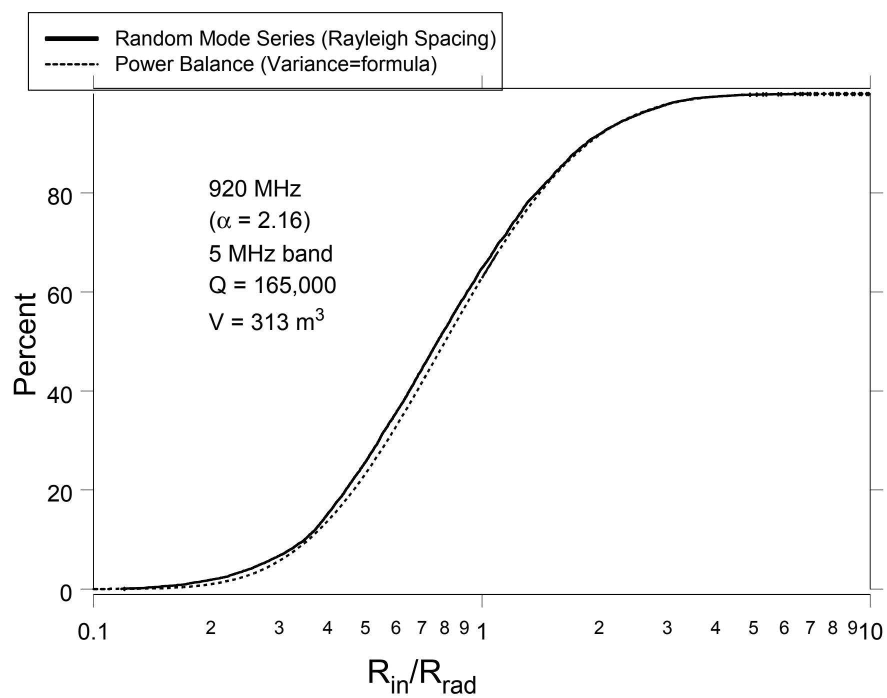

Figure 6. Corresponding real part of the normalized input impedance.

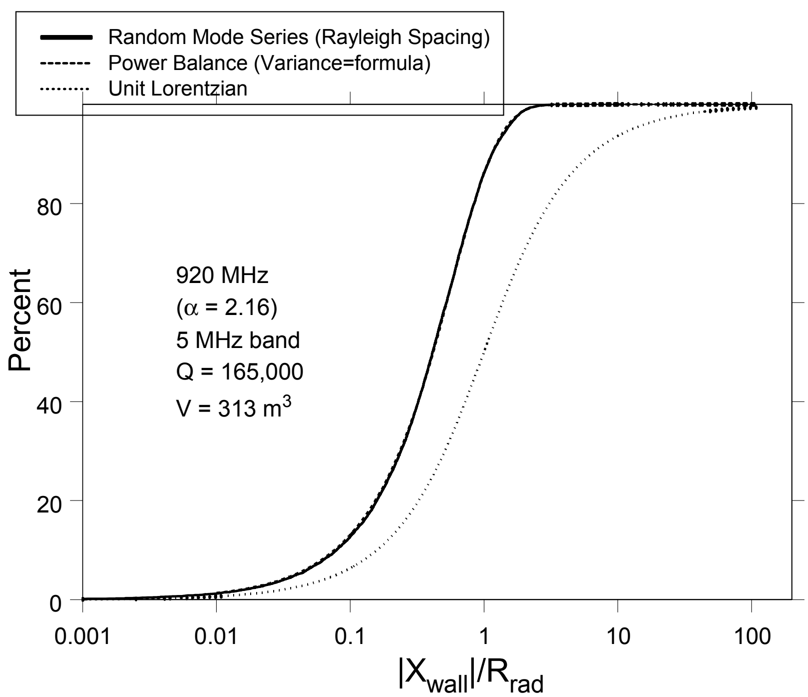

Figure 7. Corresponding imaginary part of the normalized input impedance due to the wall. The unit Lorentzian limit is also shown.

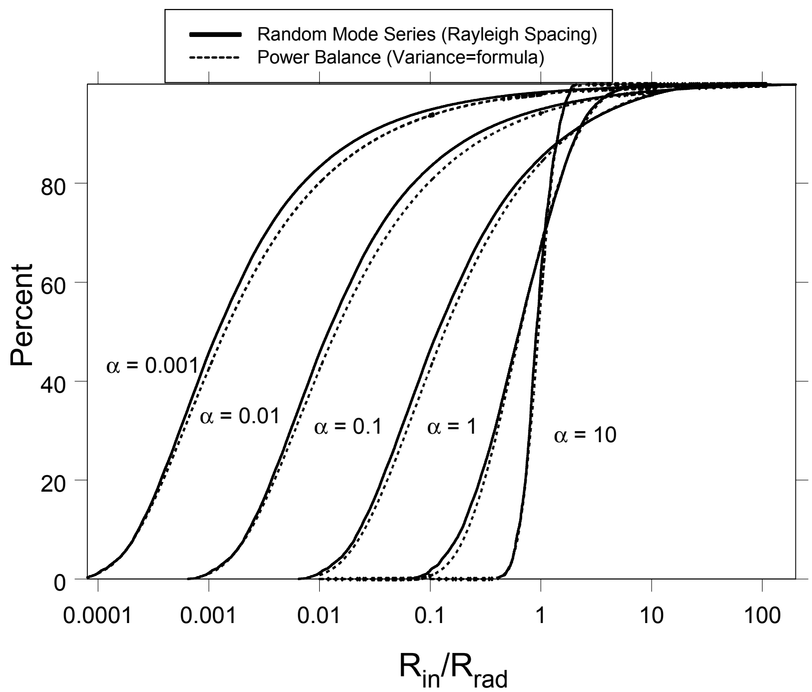

Figure 8. Comparison of real part of normalized input impedance from Monte Carlo simulations at very high frequencies (5 GHz) and from approximate formula for a broad range of parameter values. Notice limited amount of variation as the parameter becomes large.

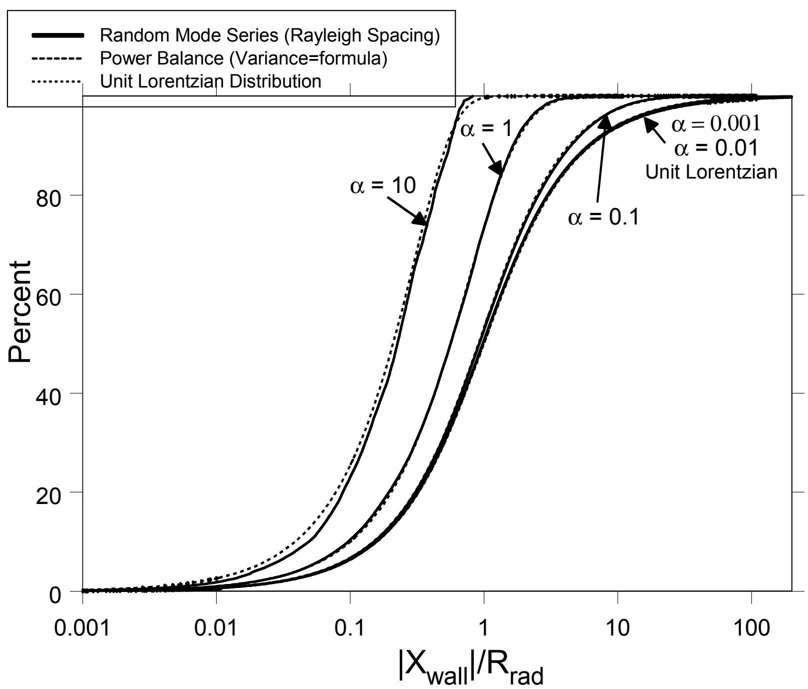

Figure 9. Comparison of imaginary part of normalized input impedance due to the wall from Monte Carlo simulations at very high frequencies (5 GHz) and from approximate formula for a broad range of parameter values. Notice approach to unit Lorentzian distribution as the parameter becomes small.

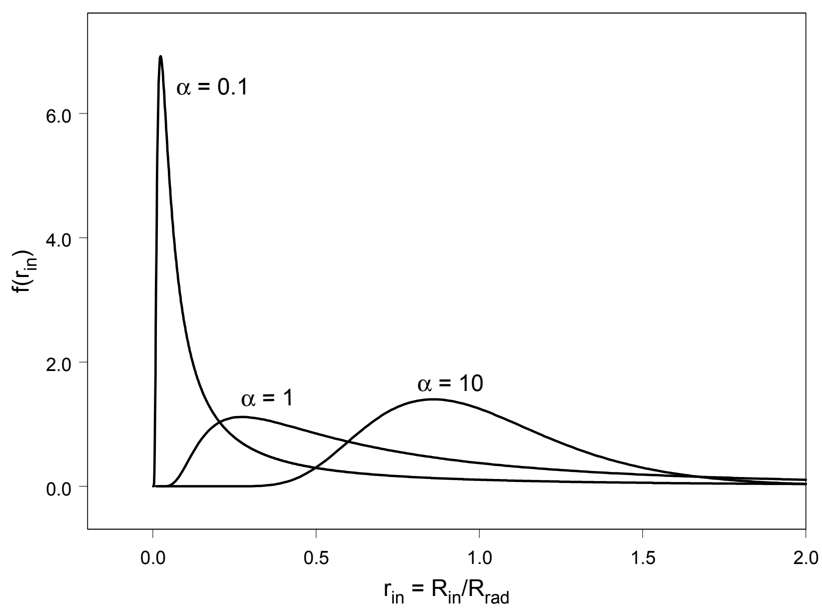

Figure 10. Approximate density function for real part of normalized input impedance.

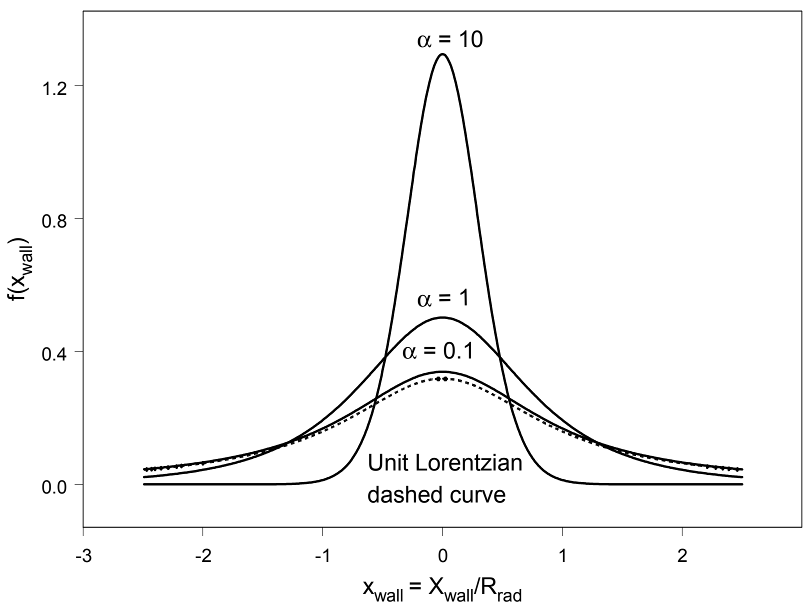

Figure 11. Approximate formula for imaginary part of normalized input impedance from the wall contribution.

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