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PROBABILISTIC TECHNIQUES USING MONTE CARLO SAMPLING FOR MULTI-COMPONENT SYSTEM DIAGNOSTICS*

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ABSTRACT

We outline the structure of a new approach at multi-component system fault diagnostics which utilizes detailed system simulation models, uncertain system observation data, statistical knowledge of system parameters, expert opinion, and component reliability data in an effort to identify incipient component performance degradations of arbitrary number and magnitude. The technique involves the use of multiple adaptive Kalman filters for fault estimation, the results of which are screened using standard hypothesis testing procedures to define a set of component events that could have transpired. Latin Hypercube sampling is then used to determine the likelihood of each of these feasible component events in terms of uncertain component reliability data and filter estimates. The capabilities of the procedure are demonstrated through the analysis of a simulated small magnitude binary component fault in a boiling water reactor balance of plant. The results show that the procedure has the potential to be a very effective tool for incipient component fault diagnosis.

I. INTRODUCTION

The ability to accurately and efficiently detect and diagnose failures and performance degradations in nuclear power systems is essential for their safe and efficient operation. These systems are typically composed of literally thousands of individual components that may interact in a very complex fashion. Although these components and the systems and subsystems they compose are in general very reliable, some level of random failure and/or degradation is

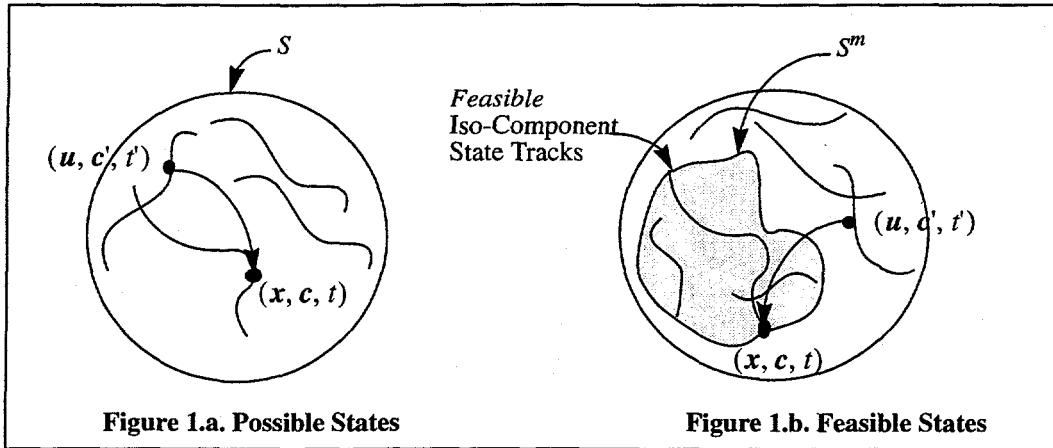
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to be expected.

Typically, catastrophic component or system faults leave very pronounced signatures on monitored process variables and are thus relatively simple to detect yet potentially difficult to diagnose. Small to moderate component failures and degradations are generally much more difficult to detect than catastrophic failures and potentially a significant challenge to diagnose. When component performance degrades slightly it may not always be readily apparent that a problem exists or the degradation may only manifest itself during certain plant maneuvers. Although such small component or system performance degradations may not pose an immediate challenge to the safety of the system as a whole, they can reduce the overall system efficiency, result in unexpected and costly maintenance outages, or eventually reduce the system safety margins. As nuclear power plants continue to age, the question of system safety, reliability, and efficiency takes on even greater importance due to age- and environment-related stress and wear that may result in a higher incidence of unexpected component degradations of varying magnitudes. These random component failures and performance degradations can have a significant deleterious effect on the overall performance of a nuclear power plant through the assumption of higher operating and maintenance costs, thus inducing a competitive handicap in today's already brutally competitive electric power industry. Also, because explicit credit is taken for extended capacity factors in marketing certain advanced reactor designs, advanced algorithms for component fault detection and diagnosis should also play a critical role in the next generation of nuclear power plants. It is therefore crucial that suitable techniques are available for the detection and diagnosis of not only large-scale system faults but also numerous small, simultaneous component degradations.

We present a new approach at system diagnostics that makes an effective use of all available information for the purpose of off-line analyses of systems data for diagnosing single or multiple component faults of arbitrary magnitude. Given the detection of a possible system anomaly, a set of feasible component states, each characterized by a unique joint probability density function representative of a salient set of system/component attributes, can be hypothesized based on pertinent system observations and *a-priori* statistical knowledge. These density functions can then be utilized with fault magnitude- and system state-dependent distribution functions characterizing uncertain component failure rates to obtain the expected value of the likelihood of each hypothesized component state. Because of the complexity of the problem, a stratified Monte Carlo sampling procedure must be utilized to obtain the desired likelihoods. By approaching the fault diagnosis problem in this fashion we are able to make explicit use of all available information including system measurements, detailed engineering system models, *a priori* statistical information related to measurements and nominal component states, *a posteriori* statistical information, component reliability data, expert opinion, and the powerful computing resources now widely available. Our diagnostics framework thus allows us to transform sometimes disparate and conflicting pieces of system *information* into statistically meaningful diagnostic *knowledge*.

Due to the broad scope of the technique, only the general structure and demonstration of the capabilities of the fault diagnosis procedure will be presented here. A general conceptual interpretation of the component fault process as well as a presentation of the salient probabilistic expressions for component fault diagnosis is outlined in Section II. Section III then outlines the basic concepts involved in the solution of the probabilistic expressions of Section II while Section IV presents an illustrative example of the capabilities of the new procedure. A summary and



conclusions are then presented in Section V.

II. PROBABILISTIC EXPRESSIONS FOR FAULT DIAGNOSIS

A. General System Description

We consider a general physical system characterized by a set of directly or indirectly observable system state variables x , e.g., system power, pressure, etc. The system is comprised of N physical components, each of which is described via some continuous *component characteristic* c^v , e.g., valve flow areas, pump characteristics, etc., the value of which defines the state c of the component. The combination of the individual component states then defines the component state c of the system. The functional relationship between system and component state can be expressed as:

$$\dot{x} = f(x(t), c(t)) \quad (1)$$

which indicates that the system state is a functional of component state.

The components comprising the system are influenced by random faults, which we assume behave in a Markov fashion. Thus, although the system state is defined to behave deterministically for a *constant* component state, the functional relationship of Eq. (1) indicates that the joint system/component state will in general behave in a stochastic fashion due to the influence of random component faults. Thus, the trajectory of the system in phase space is characterized by state transitions $(u, c', t') \rightarrow (x, c, t)$. This type of system behavior can be visualized as in Figure 1.a where each "track" in the universe S of all possible tracks represents the system behavior corresponding to a given component state. Note that there are an infinite number distinct tracks, any one of which the system could realize at any given time. In general, the problem of system fault diagnosis is then to determine which of this infinitum of tracks the system is following at a given time. However, if one has either a direct or indirect (noisy) system observation of the form $y = h(x(c)) + v$, then in practice one need only diagnose the correct track from a subset S^m of the entire universe S . This is because only certain component states, the *feasible component states*, could result (to a sufficient level of certainty) in a system/component state that could account for the observed system behavior.

B. Salient Probabilistic Expressions

Successful diagnosis of component faults in a system of the type thus described can be accomplished by first defining and then solving a pertinent set of probabilistic expressions describing the temporal "flow" of probability. System diagnosis involves asking the following question: What is the probability that the system is in system state x and component state c given (uncertain) system observation(s) y ? The answer to this is contained in the following Bayesian expression:¹

$$p(c, t/y) = \int \frac{p(x, c, t) p(x, t/y)}{\sum_{c \in \{c\}^m} p(x, c, t)} dx \quad (2)$$

where $c \in \{c\}^m$ denotes a component state c in the feasible set $\{c\}^m$ contained within S^m . The next task is to determine a suitable set of requisite joint and conditional density functions. It can be shown^{1,2} that, starting from the Chapman-Kolmogoroff equation, one can derive an expression for the joint density function $p(x, c)$ that embodies the system characteristics described in the previous section as:

$$\begin{aligned} p(x, c, t) = & \int p(u, c, 0) \exp \left\{ - \int_0^t \Gamma(x, c, s) ds \right\} \delta[x - g(u, c, t)] du \\ & + \int_0^t d\tau \int \delta[x - g(v, c, t - \tau)] \exp \left\{ - \int_\tau^t \Gamma(x, c, s) ds \right\} \\ & * \int W(c/c', x, \tau) p(u, c, \tau) dc' du \quad (0 \leq \tau \leq t) \end{aligned} \quad (3)$$

where $g(u, c, t)$ is the solution to Eq. (1) at time t with initial condition $x(0)=u$ and component state c , $W(c/c', x, t)$ is the probability per unit time of the system undergoing a transition from $c' \rightarrow c$, sometimes referred to as the *component state transition rate*, and $\Gamma(x, c, t) = \int W(c'/c, x, t) dc'$ is the probability per unit time of a transition from state c . It is important to note that the component transition rates $W(c/c', x, t)$ are a function of the individual component characteristic transition rates $W(c^v/c^v')$, which are themselves uncertain quantities, and system state x . Also, $W(c^v/c^v')$ in general is obtained as a synthesis of both historical observation data and expert opinion.³ If one has an appropriate mathematical model of the system, Eq. (1), a set of representative component transition rates, and an expression for the probability density $p(x/y)$, then in principle it is possible to determine the probability associated with each feasible component state c , thus solving the diagnosis problem as previously defined.

III. SOLUTION TECHNIQUE

In most practical situations the direct solution of Eqs. (2) and (3) is far from tractable. However, we can extract the desired information from these equations in the following fashion: Assume that, given a system observation y at the time a fault is detected, one can obtain a *finite, representative* set of component states $c \in \{c\}^m$, each defined

via a *unique joint density function* in c^v and $g(u, c^v, t)$, where the $g(u, c^v, t)$ corresponding to each $c \in \{c\}^m$ are *equally probable*. This can be accomplished by utilizing specially designed adaptive extended Kalman filters (EKF),¹ each utilizing an appropriate phenomenological system model, to test (hypothesize) every possible combination of component fault/no-fault conditions. This procedure is accomplished not by hypothesizing a specific component fault magnitude, but rather by introducing an appropriate uncertainty into each individual model in such a way as to elicit the proper (unknown) adjustment in parameters corresponding to the observed manifestation of the component fault(s). The choice of uncertainty and computational procedures are outlined in Reference 1.

There are, for most problems, a large number L of possible fault/no-fault combinations, where L is defined as the number of m simultaneous component faults in an N component system:

$$L \equiv \sum_{m=0}^N C(N, m) = \sum_{m=0}^N \frac{N!}{m!(N-m)!} = 2^N \quad (4)$$

thus, the problem dimensionality may quickly become unwieldy. However, if one is willing to consider a reduced number m_{max} of *maximum simultaneous component faults*, then the problem dimensionality may be reduced by the corresponding *reduction factor RF*:

$$RF = \sum_{m=0}^N C(N, m) / \sum_{m=0}^{m_{max}} C(N, m) \quad (5)$$

Typically, m_{max} can be taken to be 3 or 4 without a significant loss of accuracy.

The multiple filtering procedure thus outlined yields a *set* of joint density functions for the component/system characteristics c^v and $x(c^v)$, each of which will produce an (approximately) equally-likely match between computed and observed system characteristics, i.e., $p(x(c_1^v)/y) \approx p(x(c_2^v)/y) \approx \dots \approx p(x(c_L^v)/y)$. One can then "screen" this set of joint density functions via suitable hypothesis testing procedures¹ to determine which set of density functions constitutes a component state *significantly* different (statistically) from the *a priori* component states (in terms of both *a priori* and *a posteriori* density functions). In this manner we obtain a set of feasible component states $c \in \{c\}^m$, each defined via an *uncertain* set of system/component state attributes based on the current system observation. A set of algorithms for the solution of the above described problem has been developed and is detailed in Reference 1.

After the set of feasible component states and associated attributes has been uncovered we must then determine a suitable $W(c/c', x, t)$ corresponding to each feasible $c \in \{c\}^m$ in order to solve for $p(c/y)$. If the individual transition rates are known exactly, then for *each* point in the $(x(c^v), c^v)$ continuum that defines *each* feasible component state, one could solve for $p(c/y)$ based on $W(c^v/c', x, t)$ using standard Boolean algebraic expressions.⁴ However, because the values of c^v and $x(c^v)$ are *uncertain* (described via the density functions obtained via the multiple adaptive filtering procedure) and because, for *each* $[x(c^v), c^v]$ combination in the continuum of possibilities, the individual transition rates $W(c^v/c', x, t)$ are *uncertain* (typically described by a lognormal distribution function⁴), one must obtain a suitable *estimator* of the likelihood $p(c/y)$ of each feasible component state $c \in \{c\}^m$ based on the *uncertain* information that is available. This can be accomplished by

completely sampling, for every $c \in \{c\}^m$, each initial distribution $[x(c^v), c^v]$, each final distribution $[x(c^v), c^v]$, and for each of these $[(x(c^v), c^v), (x(c^v), c^v)]$ pairs, completely sampling each lognormal distribution corresponding to the individual $W(c^v/c^v, x, t)$. It can be shown¹ that the application of traditional Monte Carlo techniques for problems involving even a small number of components would be intractable, requiring $>10^9$ histories. However, Latin Hypercube sampling (LHS),⁵ a stratified sampling technique, does allow for a suitable sampling procedure to be designed¹ to yield a tractable solution to the above described problem.

Once we complete the above procedure for determining the likelihood of each $c \in \{c\}^m$, it is then possible to obtain a set of *marginal density functions* that describe the probability density of each individual component characteristic c^v based on all available system information. This *marginalization* is accomplished by sampling a random number $\xi \in [0, 1]$ and choosing feasible component state J such that:

$$\sum_{j=1}^{J-1} p(c_j/y) \leq \xi < \sum_{j=1}^J p(c_j/y) . \quad (6)$$

For each feasible component state thus chosen, we then *completely* sample the joint distributions of component/system state characteristics (via LHS) and accumulate (bin) the individual values of component characteristics, thus effectively “weighting” each component characteristic density function by its associated likelihood. If this procedure is performed a suitable number of times, one can then construct a marginal density function for each individual component characteristic based on all available information including uncertain system observations, uncertain state-dependent component reliability data, and system models.

IV. ILLUSTRATIVE EXAMPLE

To illustrate the usefulness of the component diagnostic technique, we attempt to diagnose a simulated transient in the balance of plant (BOP) of a boiling water reactor. The system of interest consists of a high pressure turbine fed via a main steam admission valve and exhausting wet steam to a steam dryer. The saturated steam from the dryer passes through a reheat, which is fed bleed steam via a tap in the main steam line, and into a low pressure turbine. Steam is also bled from the low and high pressure turbines, combined with condensed steam from the reheat, and passed through a series of high and low pressure feedwater heaters. A simple non-linear time-lag representation consisting of 11 non-linear differential equations is used to represent the system.¹ The manifestation of a simulated transient resulting from a simultaneous increase of 10% in reheat steam valve flow area and a 5% decrease in low pressure turbine bleed on five system observations y (with 1% white noise superimposed) is represented in Figures 2 through 7. Nine component characteristics, presented in Table 1, are to be considered (component characteristic c_2^v and c_4^v faulted in the transient represented by Figures 2 through 7). Note that component characteristics need not correspond to an actual physical quantity related to a given component, e.g., valve flow area, rather they may simply be descriptive of the overall operability of the *combination* of many components, e.g., turbine efficiencies. If every possible fault/no-fault combination were to be considered, the problem would involve [see Eq. (4)] $L=512$ hypotheses (filter runs) to determine the set $\{c\}^m$. However, we consider the possibility that no more than $m_{max}=3$ simultaneous component faults are likely, resulting in $L=130$ fault/no-fault hypothesis to consider, corresponding to a reduction factor $RF=3.9$.

Upon execution of the procedures outlined in Section III, 10 feasible, unique component states were discovered, the fault sequences for which are presented in Table 2. The LHS sampling procedure was carried out¹ using 10 batches of 4.4×10^5 histories (total) per batch, resulting in an estimate for the likelihood of each feasible component state, also presented in Table 2. We note that the correct fault sequence is identified (via the multiple filter/hypothesis testing procedure) twice (c_3 and c_6), the component characteristic estimates for which are presented in Table 3. Each estimate for the 9 component characteristics corresponds to the mean of a normal distribution with associated standard deviation. It is important to note that whether or not a fault is declared for a specific component will depend on both the *a priori* uncertainty and *a posteriori* uncertainty (uncertainty in estimate). A fault will not necessarily be declared simply because *some* deviation from nominal appears in the estimate - the deviation must be statistically significant. We note from Table 3 that the specially designed filters do an excellent job of identifying the correct magnitude of the small component faults (c_6). Table 4 presents a comparison of the feasible component state probabilities computed using representative point (mean) values for all pertinent variables instead of completely

Table 1. Component State Variables

Component Characteristics		
Variable	Component	Description
c_1^v	H.P. Bleed Taps & Assoc. Piping	High Pressure Steam Bleed (%)
c_2^v	L.P. Bleed Taps & Assoc. Piping	Low Pressure Steam Bleed (%)
c_3^v	Main Steam Valve	Main Steam Admission Valve Flow Area (m^2)
c_4^v	Reheat Steam Valve	Reheat Steam Valve Flow Area (m^2)
c_5^v	Reheater	Heat Transfer Coefficient - Reheater (J/kg-K)
c_6^v	H.P. Feedwater Heater	Heat Transfer Coefficient - H.P. FWH (J/kg)
c_7^v	L.P. Feedwater Heater	Heat Transfer Coefficient - L.P. FWH (J/kg)
c_8^v	H.P. Turbine	High Pressure Turbine Efficiency (%)
c_9^v	L.P. Turbine	Low Pressure Turbine Efficiency (%)

Table 2. Feasible Component States and Associated Likelihoods

Feasible State	$p(c/y)$	Component Characteristics ^a								
		c_1^v	c_2^v	c_3^v	c_4^v	c_5^v	c_6^v	c_7^v	c_8^v	c_9^v
c_1	$0.825\% \pm 4.7 \times 10^{-2}\%$	NF	NF	NF	NF	F	NF	NF	F	NF
c_2	$0.834\% \pm 4.2 \times 10^{-2}\%$	NF	NF	NF	NF	F	NF	F	F	NF
c_3	$65.364\% \pm 1.5 \times 10^{-1}\%$	NF	F	NF	F	NF	NF	NF	NF	NF
c_4	$1.621\% \pm 4.9 \times 10^{-2}\%$	NF	NF	NF	F	F	NF	NF	NF	NF
c_5	$0.037\% \pm 5.2 \times 10^{-3}\%$	NF	NF	NF	NF	F	F	NF	NF	NF
c_6	$9.651\% \pm 2.8 \times 10^{-2}\%$	NF	F	NF	F	NF	NF	NF	NF	NF
c_7	$<10^{-5}\%$	NF	F	NF	F	F	NF	NF	NF	NF
c_8	$2.591\% \pm 2.7 \times 10^{-2}\%$	F	NF	NF	F	NF	NF	NF	NF	NF
c_9	$0.637\% \pm 3.3 \times 10^{-2}\%$	F	NF	NF	NF	F	NF	NF	NF	NF
c_{10}	$18.44\% \pm 1.7 \times 10^{-1}\%$	F	F	NF						

a. NF=No Fault Detected; F=Fault Detected

sampling the distributions. This comparison shows that Monte Carlo sampling is indeed very important due to the uncertainty in the available information. Three of the 9 marginal density functions, obtained via the procedure outlined in Section III, using 1000 random Monte Carlo samples of Eq. (6) and 20 LHS of each joint density function per random sample, are presented in Figures 7 through 9. Note that Figures 7 and 8 seem to indicate a significant probability that c_2^v and c_4^v have deviated from assumed nominal values, while Figure 9 indicates little likelihood of such a deviation in c_7^v . Figures of this type could prove to be quite useful for the continuous monitoring of component performance.

Table 3. Estimated Component Characteristics For Correct Fault Sequences

Component Characteristic	True Value (After Fault)	Feasible Component State c_3	Feasible Component State c_6
		Estimated Characteristics	Estimated Characteristics
c_1^v	8.8000×10^0	$8.8793 \times 10^1 \pm 4.4 \times 10^{-2}$	$8.8018 \times 10^0 \pm 4.4 \times 10^{-2}$
c_2^v	2.2150×10^1	$2.2951 \times 10^1 \pm 9.9 \times 10^{-2}$	$2.2022 \times 10^1 \pm 3.8 \times 10^{-1}$
c_3^v	5.2471×10^{-2}	$5.2492 \times 10^{-2} \pm 5.8 \times 10^{-5}$	$5.2438 \times 10^{-2} \pm 6.2 \times 10^{-1}$
c_4^v	7.338×10^{-4}	$6.9249 \times 10^{-4} \pm 1.1 \times 10^{-5}$	$7.4289 \times 10^{-4} \pm 2.3 \times 10^{-5}$
c_5^v	7.9564×10^4	$7.9586 \times 10^4 \pm 3.1 \times 10^2$	$7.9562 \times 10^4 \pm 3.9 \times 10^4$
c_6^v	7.5900×10^5	$7.5986 \times 10^5 \pm 3.8 \times 10^3$	$7.5933 \times 10^5 \pm 3.7 \times 10^3$
c_7^v	8.0300×10^5	$8.0333 \times 10^5 \pm 3.7 \times 10^4$	$8.0275 \times 10^5 \pm 3.7 \times 10^3$
c_8^v	8.6000×10^1	$8.6061 \times 10^1 \pm 3.7 \times 10^0$	$8.6213 \times 10^1 \pm 3.7 \times 10^{-1}$
c_9^v	8.3000×10^1	$8.3458 \times 10^1 \pm 1.1 \times 10^{-1}$	$8.3007 \times 10^1 \pm 2.1 \times 10^{-1}$

Table 4. Feasible Component State Likelihoods - Point Estimate vs. Sampled

Feasible Component States Probabilities (%)										
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
Point Estimate	0.	0.	81.529	0.	0.	0.0122	$<10^{-5}$	0.0051	0.	18.344
Sampled Estimate	0.825	0.834	65.364	1.621	0.037	9.651	$<10^{-5}$	2.591	0.637	18.440

V. SUMMARY AND CONCLUSIONS

We have outlined the basic structure of a unique scheme for component fault diagnosis based on multiple adaptive Kalman filtering and Latin Hypercube sampling. Upon detection of a given fault, a series of specially designed adaptive Kalman filters is employed to effectively test every possible component fault/no-fault combination (or a subset of). The filters utilize special phenomenological system models, each possessing a unique uncertainty structure, to obtain best estimates for the various hypothesized faults. The joint density functions provided by the multiple adaptive EKFs are then screened via hypotheses testing techniques to determine which joint density functions characterize a unique set of component attributes, or feasible component states, significantly different from

nominal. The defining attributes of these states are then used with the nominal state attributes (joint density functions) and uncertain component reliability data to determine the probability, or likelihood, of each feasible component state by performing a LHS of the distributions.

The diagnosis procedure was applied to a simulated transient involving binary faults of small magnitude in the BOP of a boiling water reactor. The results demonstrate both the ability of the specially designed adaptive EKFs to uncover the correct fault magnitudes from relatively noisy system signals and the usefulness of the LHS procedure in obtaining the likelihood of each significant, feasible fault hypothesis. We also demonstrate how all of the data may be combined using a *marginalization* procedure to obtain representative marginal probability density functions for each of the component characteristics of interest.

The new scheme has the potential to provide a robust method for small, multiple component fault diagnosis. Although not suitable for real time analysis, this scheme could be very useful as an off-line analysis tool for diagnosing incipient component faults and thus aid in the efficient scheduling of preventative maintenance procedures.

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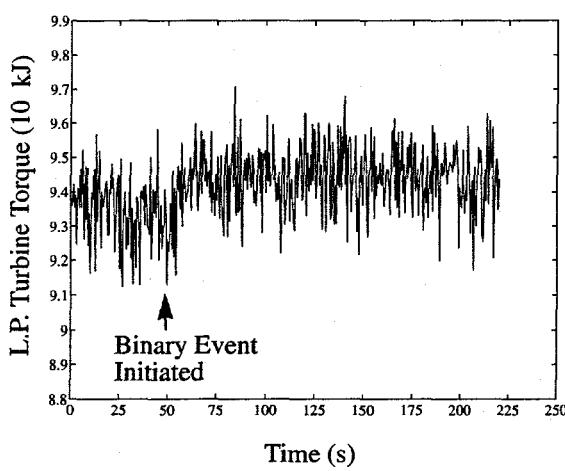


Figure 2. System Observation y_1

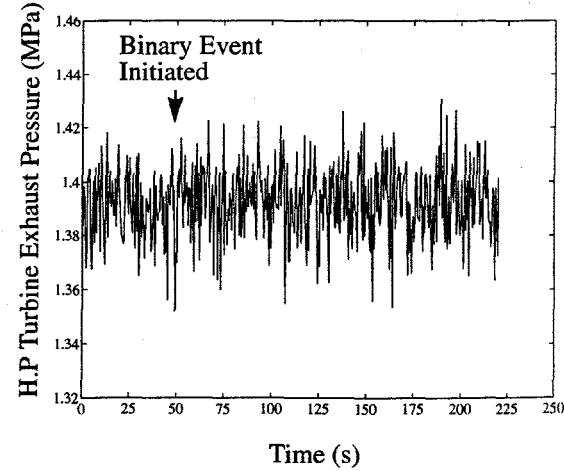


Figure 3. System Observation y_2

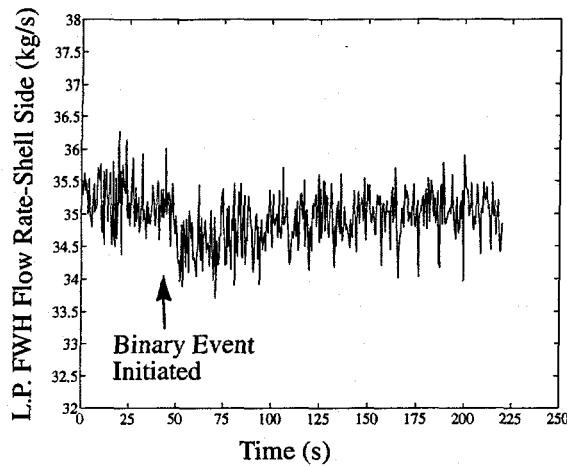


Figure 4. System Observation y_3

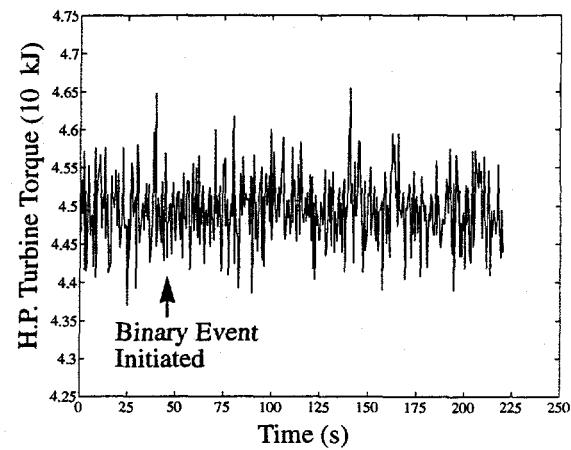


Figure 5. System Observation y_4

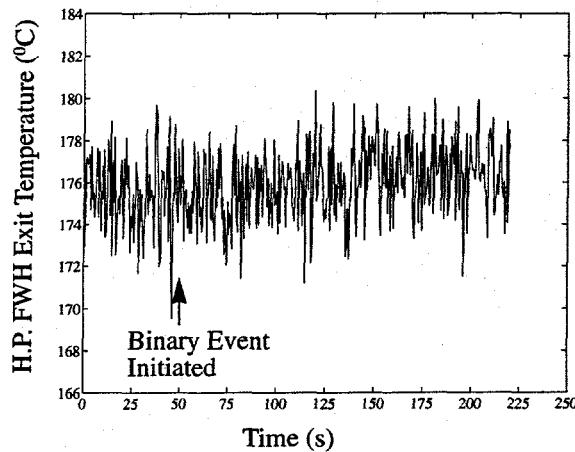


Figure 6. System Observation y_5

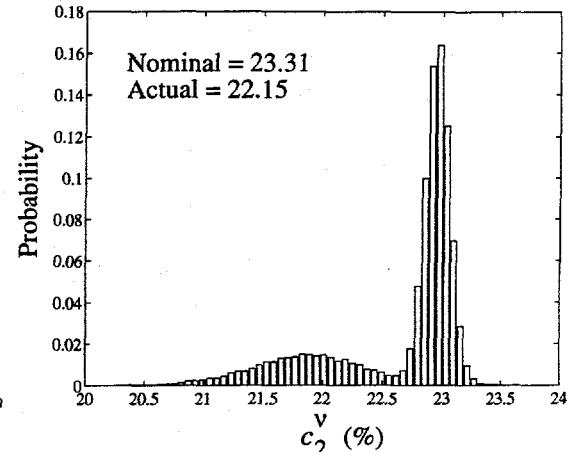


Figure 7. Marginalized Probability Density: c_2^v

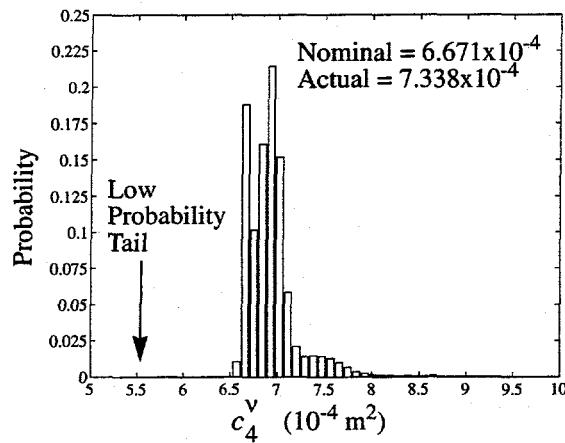


Figure 8. Marginalized Probability Density: c_4^v

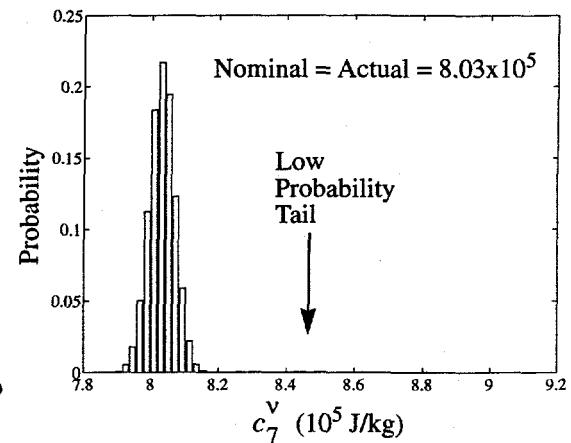


Figure 9. Marginalized Probability Density: c_7^v