OFFICE OF CIVILIAN RADIOACTIVE WASTE MANAGEMENT SCIENTIFIC ANALYSIS COVER SHEET

0.0-146- 41	T !11 _			Page: 1 Or. 25
2. Scientific Analyses Pretest Calculations	of Temperature for	Field Thermal Conductivity Tes	sts	
3. DI (including Revisi CAL-EBS-MD-000	on Number) 028 Rev 00			• , ,
4. Total Attachments	5. A I-11	ttachment Numbers - Number of page 1, II-9, III-2, IV-5, V-3, VI-3, VI	ges in each II-6, VIII-3	
		Print Name	Signature	Date
6. Originator		Nancy S. Brodsky	My Brody	6-25-02
7. Checker		John B. Case Norma E. Biggar	Noima E. Byca	6-28-02 1 6-28-02
8. QER	-	Kenneth O. Gilkerson	Balkern	4/28/02
9. Responsible Manag	er/Lead	Cliff Howard	My Saides For C. Howard	1 6-28-02
10. Responsible Mana	ger	Thomas Doering	\$1.5	6.20.02
	uthored by John B.	Case. A technical check of this	Attachment was performed by Houy	Miller and Rochell
Technical Contact/Dep	partment:			
		Revision Hist	ory	
12. Revision/ICN No.		13. Descrip	tion of Revision/Change	
00	Initial Issue			
	·			

TABLE OF CONTENTS

		Page
1.	PURPOSE	4
2.	QUALITY ASSURANCE	4
3.	USE OF SOFTWARE	4
4.	INPUTS	5
Δ	.1 DATA AND PARAMETERS	
	.2 CRITERIA	
4	.3 CODES AND STANDARDS	
5.	ASSUMPTIONS	5
5	.1 METHOD OF HEAT TRANSFER FROM HEATER TO ROCK	5
-	.2 MECHANISMS OF HEAT TRANSFER WITHIN ROCK	
_	.3 ISOTROPIC HOMOGENEOUS MEDIUM	
_	.4 THERMAL TRANSPORT PROPERTIES	
_	.5 TEST GEOMETRY	
_	.6 HEATER POWER	
	.7 VAPOR DIFFUSION	
6.	SCIENTIFIC ANALYSIS DISCUSSION	8
6	.1 DESCRIPTION OF TEST RELEVANT TO PRETEST PREDICTION	18
6	.2 PREDICTION TEMPERATURES AROUND A HEAT SOURCE	9
6	.3 CALCULATION OF TEMPERATURES AROUND A CYLINDRICA	
	HEATER AS IN TEST 1	
6	.4 CALCULATION OF THERMAL CONDUCTIVITY AND THERMA	
	DIFFUSIVITY OR THERMAL CONDUCTANCE FROM FIELD DA	
6	.5 RESULTS	
	6.5.1 TEMPERATURE PREDICTIONS FOR TWO-HOLE AND 6-HOTESTS	_
	6.5.2 ANALYSIS UNCERTAINTY	
_		
7.	CONCLUSIONS	
8.	INPUTS AND REFERENCES	22
8	.1 DOCUMENTS CITED	
	.2 CODES, STANDARDS, REGULATIONS, AND PROCEDURES	
8	.3 SOURCE DATA	24
I	IST OF ATTACHMENTS	25

LIST OF FIGURES

	Page
Figure 1. Plan view of first heater test.	
Figure 2. Test 1: Predicted temperature at midpoint of instrumentation hole at 5 of	days 9
Figure 3. Mathcad Step 1	12
Figure 4. Mathcad Step 2	13
Figure 5. Mathcad Step 3. Data are read into inputfile array. First row gives thermocouple number. Rows 2, 3, and 4 give x, y, and z coordinates, respectively used in calculation are listed in column 1 starting in row 5	
Figure 6. Mathcad Step 4	14
Figure 7. Mathcad Step 5	15
Figure 8. Mathcad Step 6.	15
Figure 9. Mathcad Step 7	16
Figure 10. Mathcad Step 8.	16
Figure 11. Mathcad Step 9	17
Figure 12. Mathcad Step 10	17
Figure 13. Predicted temperature profiles in instrumentation borehole up to 30 da initiating power to the heater.	
Figure 14. Predicted temperature profiles in three instrumentation boreholes for test (Test 2) after 30 days of heating at 500W.	
LIST OF TABLES	
Table 1. Property Values Used for Thermal Diffusivity Calculation	6
Table 2. Input Change versus Temperature Change	20
Table II-1. Summary of Lithophysal Porosity	II-3
Table II-2. Saturated Rock Mass Thermal Conductivity for the Tptpll Unit	II-4
Table II-3. Dry Rock Mass Thermal Conductivity for the Tptpll Unit	II-4
Table II-4. Data Sources for Core Properties	II-4
Table II-5. Saturated Values from Selected Boreholes	II-5

1. PURPOSE

A large volume fraction of the potential monitored geologic repository at Yucca Mountain may reside in the Tptpll (Tertiary, Paintbrush Group, Topopah Spring Tuff, crystal poor, lower lithophysal) lithostratigraphic unit. This unit is characterized by voids, or lithophysae, which range in size from centimeters to meters. A series of thermal conductivity field tests are planned in the Enhanced Characterization of the Repository Block (ECRB) Cross Drift. The objective of the pretest calculation described in this document is to predict changes in temperatures in the surrounding rock for these tests for a given heater power and a set of thermal transport properties. The calculation can be extended, as described in this document, to obtain thermal conductivity, thermal capacitance (density × heat capacity, J·m⁻³·K⁻¹), and thermal diffusivity from the field data. The work has been conducted under the *Technical Work Plan For: Testing and Monitoring* (BSC 2001).

One of the outcomes of this analysis is to determine the initial output of the heater. This heater output must be sufficiently high that it will provide results in a reasonably short period of time (within several weeks or a month) and be sufficiently high that the heat increase is detectable by the instruments employed in the test. The test will be conducted in stages and heater output will be step increased as the test progresses. If the initial temperature is set too high, the experiment will not have as many steps and thus fewer thermal conductivity data points will result.

2. QUALITY ASSURANCE

The OCRWM Quality Assurance program applies for this activity as determined in the Activity Evaluation performed per AP-2.21Q *Quality Determinations and Planning for Scientific, Engineering, and Regulatory Compliance Activities* (BSC 2001, Attachment I). The provisions of AP-SV.1Q, *Control of the Electronic Management of Information*, do not apply as no data are collected on electronic media.

This analysis is documented in accordance with AP-SIII.9Q, *Scientific Analyses*. The analysis reports on natural barriers that are included in the Q-List (YMP 2001) as items important to waste isolation. However, this analysis only contributes to the analysis and modeling of data for performance assessment and site characterization; it does not directly impact engineering, construction, or operational tasks associated with the Q-list items as discussed in AP-2.22Q *Classification Criteria and Maintenance of the Monitored Geologic Repository Q-List*.

3. USE OF SOFTWARE

Standard functions of commercial off-the-shelf (COTS) software are used in these calculations. The COTS software includes Mathcad 2001 Professional (this software does not have a version number) and Excel 2000 (9.0.3821 SR-1), both run on a Dell OptiPlex GX110 PC, under Windows 2000. These are exempt software products in accordance

with AP-SI.1Q *Software Management*, Section 2.1.6. The MathCad 2001 and EXCEL 2000 calculations use standard functions, and the results are not dependent on these software programs themselves. The results derived using standard functions can be reproduced and checked by hand using the information provided in this document.

The following information, required for usage of COTS software, is given in this document:

- The formula or algorithm used (see Section 6.2 and printout of the file mcad 2holefwd pretest.mcd, given in Attachment IV)
- A listing of the inputs (spatial coordinates and times) to the formula or algorithm (see Section 6.3 and printout of the file predict_into_mcad.xls, given in Attachment V)
- A listing of outputs from the formula or algorithm (see Section 6.3 and printout of the file predict outof mcad.xls, given in Attachment VI)
- Other information which would be required for a checker or other independent person to reproduce the computation (see Section 6).

4. INPUTS

4.1 DATA AND PARAMETERS

The input data used for this calculation comprise thermocouple locations, heater geometry and power, and values for thermal conductivity and thermal diffusivity. The assigned values are assumptions and are discussed in Sections 5.4, 5.5 and Attachment II.

4.2 CRITERIA

There are no applicable criteria pertaining to this document.

4.3 CODES AND STANDARDS

There are no applicable codes and standards pertaining to this document.

5. ASSUMPTIONS

5.1 METHOD OF HEAT TRANSFER FROM HEATER TO ROCK

Heat transfer from the heater(s) to the borehole wall(s) is assumed to be radially symmetric, based on the radial symmetry of the heaters themselves. Each heater will be comprised of one or two 4,000 W (maximum output) heaters contained in a central heating element, centered inside a copper tube. The heaters may be run at less than full power. The outer copper tubes will be as large in diameter as possible to minimize movement of air between the heater and rock wall, so convection within the borehole is not anticipated to affect the temperature distribution on the borehole wall. However, if possible, thermocouples will be placed above and below the outer copper tube to indicate temperature distribution and verify this assumption. Insulation will be placed in front and behind the heater(s) to minimize heat losses out the borehole. The field thermal conductivity tests will determine if this assumption is justified and the assumption requires no further confirmation. This assumption is used throughout.

5.2 MECHANISMS OF HEAT TRANSFER WITHIN ROCK

It is assumed that conduction is the only mechanism of heat transfer through the rock. In solids, conduction is by far the dominant heat transfer mechanism (Carslaw and Jaeger, 1959, p. 1). Radiation and convection are not included in this calculation; however, their potential effects will be assessed as part of the field data evaluation. This assumption is used throughout and does not require further confirmation for the purpose of this analysis.

5.3 ISOTROPIC HOMOGENEOUS MEDIUM

The rock is assumed to be an isotropic homogeneous medium. The field tests will assist in evaluating the anisotropy, if any, in heat transport properties. The Tptpll lithostratigraphic unit contains lithophysae and so it is inhomogeneous on a small scale. These tests, therefore, are conducted on a scale on the order of meters to minimize the effects of inhomogeneity. An isotropic homogeneous medium is assumed for ease of computation and because there is no basis to assume otherwise. Results shown in *Laboratory Measurements of Thermal Conductivity as a Function of Saturation State for Welded and Nonwelded Tuff Specimens* (SNL 1998, Figure 2) supports this assumption. Little difference is shown in the figure between thermal conductivity from cores taken horizontally or vertically and thus anisotropy is judged to be none. This assumption requires no further confirmation and is used throughout.

5.4 THERMAL TRANSPORT PROPERTIES

The technical bases for the assumed values of thermal properties, used as input to the pretest calculation, are given in Attachments II and III. Attachment II uses volume averaging to obtain a rock mass thermal conductivity value of approximately 1.7 W/mK for the lower lithophysal unit. The rock mass thermal diffusivity is calculated from thermal conductivity, density, and specific heat, using Equation II-1 and the values given in Table 1. This calculation is shown in Attachment III and summarized here.

Property	Value	Source
Grain Density (kg/m³)	2526	Finley et al. 1998, p. 148
Grain Specific Heat (J/kg-K)	928	Finley et al. 1998, p. 148
Matrix Porosity	0.11	Finely et al. 1998, p. 148
Lithophysal Porosity	0.125	See Attachment II
Saturation	0.76	See Attachment II
Water Specific Heat at 25 °C (J/kg-K)	4180	Weast (1972, p. D128)
Water Density at 25 °C (kg/m ³)	997	Weast (1972, p. F11)

Table 1. Property Values Used for Thermal Diffusivity Calculation

The Finley et al. (1998) reference in the table above was chosen for the matrix properties because the location tested would have matrix properties of grain density, grain specific heat, and matrix porosity that are similar to the location for the Field Thermal Conductivity Measurements Test. The Weast (1972) values in Table 1 are from a standard handbook.

The volumetric heat capacity for the rock mass, $\rho \cdot C_p$, is obtained as follows (CRWMS M&O 2000, p. 9, equation 3):

$$(\rho \cdot C_p)_{rm} = (1 - \phi_l) \cdot (1 - \phi_m) \cdot (\rho \cdot C_p)_{grain} + (1 - \phi_l) \cdot \phi_m \cdot S \cdot (\rho \cdot C_p)_{water}$$

where:

 $(\rho \cdot C_p)_{rm}$ = Volumetric heat capacity of rock mass (J/m³K) $(\rho \cdot C_p)_{grain}$ = Volumetric heat capacity of rock grains (J/m³K) $(\rho \cdot C_p)_{water}$ = Volumetric heat capacity of water (J/m³K)

 ϕ_l = volumetric heat capacity of w ϕ_l = Lithophysal porosity (m³/m³) ϕ_m = Matrix porosity (m³/m³) S = Saturation (m³/m³)

Attachment III shows substitution of Table 1 values into Equation 1 and then into Equation 8 to obtain a thermal diffusivity value of approximately 8×10^{-7} m²/s for the rock mass. The contribution of air to the rock mass thermal diffusivity is negligible. The field thermal conductivity tests will determine if this assumption is justified and the assumption requires no further conformation. These assumed properties are used throughout.

5.5 TEST GEOMETRY

Estimated values of borehole diameter (0.076 m), and heater length (5 m) are assumed for Tests 1 and 2. For Test 1, the heater and thermocouple string are assumed to be perpendicular, with the thermocouple string passing 0.012 m above the heater. For Test 2, 3 heaters spaced nominally 0.5 m apart and nearly parallel are assumed. The 3 thermocouple strings used for Test 2 are assumed to be nominally perpendicular to the heaters, parallel to one another, and spaced 1 m apart. Locations of thermocouples, relative to the center of the heater, are given in Attachment V. This assumption is necessary as the test geometry has not been entered into the Technical Data Management System. This assumption requires no further justification as the test has been constructed with the geometry as described. These assumed test geometries are used throughout.

5.6 HEATER POWER

As mentioned in Section 1, heater power must not be set too high or too low. This analysis assumes a value of 500 W as the initial heater output power. This heater setting requires no further justification as the test has been conducted with this power setting. This assumption is used throughout.

5.7 VAPOR DIFFUSION

Vapor diffusion could significantly affect the thermal conductivity because of the heat carrying capacity of water. It is assumed that vapor diffusion does not play an important role for the rock thermal conductivity to be measured as part of the field test program because measurements will be made either well below boiling, or above boiling after all vapor has been boiled out of the test region. *Laboratory Measurements of Thermal Conductivity as a Function of Saturation State for Welded and Nonwelded Tuff Specimens* (SNL 1998, Fig. 2) supports this assumption as the thermal conductivity shown in the Figure changes linearly with an increase in saturation. If vapor diffusion played an important part in the thermal conductivity of tuff rock, the linear relationship would be more curvilinear (as vapor at the higher saturation level would carry more heat and thus affect the thermal conductivity).

6. SCIENTIFIC ANALYSIS DISCUSSION

Multiple field tests are planned within the ECRB so that a database for in situ thermal properties can be established. The first test involves a single heater and single instrumentation borehole. The second test comprises an array of 3 heaters and 3 instrumentation boreholes to sample a rock volume several times the volume of the first test. The third test employs a single heater, and instrumentation holes both above and below the heater to measure any effects of air/ moisture convection within the borehole on temperature distribution. Additional test geometries will be established based on information gained from the initial three tests. Only the first test is described in detail in Sections 6.1 - 6.3 as an aid to understanding the pretest calculation. The following presents a discussion of the test geometry (Section 6.1), the methods used for assessing the insitu thermal conductivity (Section 6.2), and the method used in the pretest calculation (Section 6.3).

6.1 DESCRIPTION OF TEST RELEVANT TO PRETEST PREDICTION

The first planned in situ field test includes a single heater borehole and a single instrumentation borehole, as shown in Fig. 1. The heater is 5 m in length, 76 mm in diameter, and is inset at least 3 m from the ECRB drift wall. A second borehole, perpendicular to the first, of the same diameter, and approximately 12 cm above it at the intersection point contains a 5-m-long array of 30 thermocouples. The heater will be run at a constant power output, slowly heating the surrounding rock. The heater power, heater and thermocouple locations (see Sections 5.4 and 5.5), and thermocouple outputs will be used to determine bulk values of thermal conductivity and diffusivity for the rock mass. This test will be run initially below 100°C to minimize changes in moisture content.

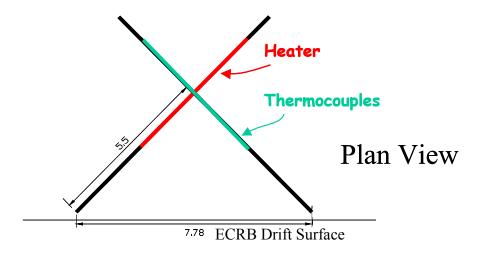


Figure 1. Plan view of first heater test. (Dimensions in meters)

6.2 PREDICTION OF TEMPERATURES AROUND A HEAT SOURCE

The temperature change at a point any distance from a heated sphere in an isotropic homogeneous medium (see Section 5.3) is calculated using the following equation (Carslaw and Jaeger 1959, p. 248, eqn.4), which assumes only heat conduction through the rock (see Section 5.2):

$$v = \frac{a^2 F_0}{Kr} \left\{ erfc \ \frac{r-a}{2(kt)^{1/2}} - \exp\left[\frac{r-a}{a} + \frac{kt}{a^2}\right] erfc \left[\frac{r-a}{2(kt)^{1/2}} + \frac{(kt)^{1/2}}{a}\right] \right\}$$
 Equation 2

where:

v = Temperature change (K)

a = Radius of sphere (m)

 F_0 = Heat flux at surface of sphere (W/m²). This is a constant flux at r = a.

K = Thermal conductivity (W/mK)

r = Distance from center of sphere to measurement point (m)

k = Thermal diffusivity (m²/s)

t = Time (s).

Figure 2 checks the accuracy of using 1312 spheres for calculation purposes as shown below. The temperatures around a finite length line source heater are predicted by approximating the heater as a series of small overlapping spheres, each with a diameter equal to the borehole diameter. The predicted temperature at each thermocouple location is calculated by summing the contributions from each sphere based upon the principle of superposition (Carslaw and Jaeger 1959, p. 262). A total of 65.6 small spheres would be required to model a 5-m-long heater if the spheres touched end-to-end¹. If 20 overlapping spheres are used to model the space taken up by each of the touching spheres, then a total of 1312 spheres² would be needed. Fig. 2 shows that using 1000 or more spheres results in an error of only a small fraction of a degree C, and so 1312 is a sufficient number of spheres. The method used to generate Figure 2 is given in a MathCAD file in Attachment I. The use of MathCAD is further explained in Section 6.3.

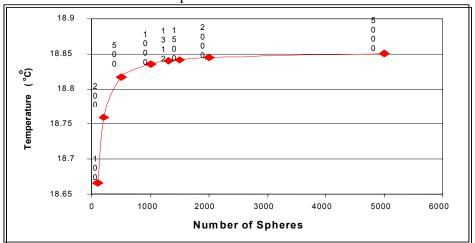


Figure 2. Test 1: Predicted temperature at midpoint of instrumentation hole at 5 days.

_

¹ 5 m (length of heater) divided by 0.0762 m (diameter of borehole) results in 65.6 touching spheres ²20 multiplied by 65.6 spheres results in 1312 spheres

In order to perform the calculation more efficiently, the form of the equation was arranged as described here. The first error function term,

$$\left\{ erfc \left[\frac{r-a}{2(kt)^{1/2}} \right] \right\}$$
 Equation 3

was evaluated using a standard function of commercial off-the-shelf (COTS) software, the Mathcad "ERFC" standard function. The exponential term,

$$\left\{ \exp \left[\frac{r-a}{a} + \frac{kt}{a^2} \right] \right\},$$
 Equation 4

reaches values of over 10^{300} at moderate values of t (10 days) and is discussed further below.

The second erfc term,

$$\left\{ erfc \left[\frac{r-a}{2(kt)^{1/2}} + \frac{(kt)^{1/2}}{a} \right] \right\}$$
 Equation 5

was evaluated as zero using standard functions of both Excel and Mathcad for large values of *r* and *t*. From Abramowitz and Stegun (1972, p. 298, eqn. 7.1.23), an asymptotic expansion can be used to evaluate an erfc function, as follows:

$$erfc(z) \cong (\sqrt{\pi}z \exp(z^2))^{-1} \left[1 + \sum_{m=1}^{\infty} (-1)^m \frac{1 \cdot 3 ... (2m-1)}{(2z^2)^m} \right].$$
 Equation 6

Using 2 terms of the expansion was sufficient to provide the same results as 8 terms, to within 6 decimal places. The expansion terms were therefore used through m=3. Substituting Equation 6 into Equation 2, and rearranging, the following is obtained:

$$v = \frac{a^2 F_0}{Kr} \left\{ erfc \frac{r-a}{2(kt)^{1/2}} - \frac{1}{\sqrt{\pi}z} \exp\left[\frac{r-a}{a} + \frac{kt}{a^2} - z^2\right] \cdot \left[1 + \sum_{m=1}^{\infty} (-1)^m \frac{1 \cdot 3 \dots (2m-1)}{(2z^2)^m}\right] \right\}$$

Equation 7

where

$$z = \frac{r - a}{2(kt)^{1/2}} + \frac{(kt)^{1/2}}{a} .$$

By combining all exponents before applying the exponential function, extreme values such as 10^{300} for the exponential term in Equation 2, and 10^{-300} for the second erfc term in Equation 2, are avoided.

Thermal diffusivity is defined as (Carslaw and Jaeger, 1959, p. 9, eqn. 5):

$$k = \frac{K}{\rho \cdot C_p}$$
, Equation 8

where:

 ρ = Density (kg/m³) C_n = Heat capacity (J/(kg·K)

Equation 7 can be substituted into Equation 6 as follows so that temperature can be obtained as a function of thermal conductivity and thermal capacitance ($\rho \cdot C_p$).

$$v = \frac{a^{2} F_{0}}{Kr} \left\{ erfc \frac{r - a}{2 \left(\frac{Kt}{\rho C_{p}}\right)^{1/2}} - \frac{1}{\sqrt{\pi} z} exp \left[\frac{r - a}{a} + \frac{\left(\frac{Kt}{\rho C_{p}}\right)}{a^{2}} - z^{2} \right] \cdot \left[1 + \sum_{m=1}^{\infty} (-1)^{m} \frac{1 \cdot 3 \dots (2m-1)}{(2z^{2})^{m}} \right] \right\}$$

Equation 9

where

$$z = \frac{r - a}{2\left(\frac{Kt}{\rho C_p}\right)^{1/2}} + \frac{\left(\frac{Kt}{\rho C_p}\right)^{1/2}}{a}.$$

Predicted temperatures can be calculated equivalently using either Equations 7 or 9 and estimated values of thermal conductivity and thermal diffusivity or thermal capacitance, respectively.

6.3 CALCULATION OF TEMPERATURES AROUND A CYLINDRICAL **HEATER AS IN TEST 1**

Standard functions of Mathcad 2001 were used to predict temperatures as a function of time for Tests 1 and 2. The calculation for Test 1 will be shown here in detail.

Equation 7 is used to calculate the temperature at each thermocouple location using estimated values for thermal conductivity and thermal diffusivity (Section 5.4). Equation 9 can be used equivalently, using estimated values of thermal conductivity and thermal capacitance. The Mathcad routine used to calculate predicted temperatures, mcad 2holefwd pretest.mcd, is given in Attachment IV and presented step by step in this section.

The Mathcad routine begins with a statement about the function of the routine and then initial or estimated values (Sections 5.4, 5.5, and 5.6) are defined, as shown in Step 1

(Fig. 3). Values are defined in Mathcad using the ":=" symbol; values calculated by Mathcad are shown by using the "=" symbol. Lengths are given in meters (m), power in watts (W), thermal conductivity in watts per meter degree K (W/mK), and thermal diffusivity in meters squared per second (m²/s).

As discussed in Section 6.2, the heater is approximated as 1312 heated spheres and the contributions of each heated sphere to the temperature change at each thermocouple location are summed. As noted in Fig. 3, the origin of the coordinate system is the center of the heater, and the y-axis is coincident with the heater axis. The z-axis is vertical and the x-axis is perpendicular to the y and z-axes. In step 2, shown in Fig. 4, the y-coordinate for the center of each sphere is calculated. Values of y-coordinates (ysphere) are printed out and checked against hand calculations.

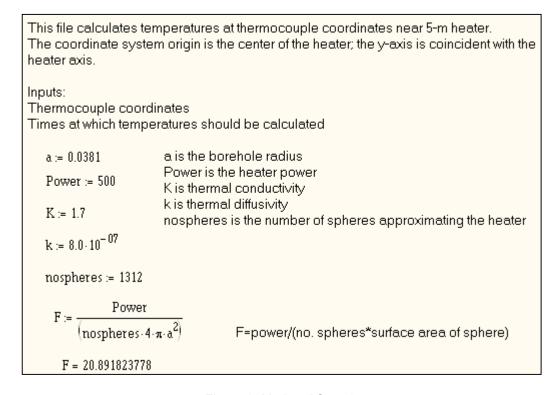


Figure 3. Mathcad Step 1

```
Calculate y coordinates of each sphere ysphere_{1} := -2.5 + a
i := 2... 1312
ysphere_{i} := ysphere_{1} + \frac{(5 - 2 \cdot a) \cdot (i - 1)}{nospheres - 1}
ysphere_{1} = -2.4619
ysphere_{2} = -2.458144241
ysphere_{3} = -2.454388482
ysphere_{4} = -2.450632723
ysphere_{5} = -2.446876964
```

Figure 4. Mathcad Step 2

A set of thermocouple coordinates are read in from an Excel spreadsheet, predict_into_mcad.xls (Attachment V), as are the times at which temperatures will be calculated. These data are read into the Mathcad routine as shown in Step 3 (Fig 5). Only the first 8 of 30 thermocouples are shown here, but the full set of coordinates is given in Attachment V. The x, y, and z coordinates are in rows just below the thermocouple numbers, and for this test case, the coordinates of thermocouples used for the 2-hole test shown in Fig. 1 are used. Only times of 0.1, 0.25, 0.5, 1, 2, and 3 days are shown in Fig. 5 (listed in column 1 starting in row 5); however, temperatures are also calculated for times of 4, 5, 6, 7, 8, 9, 10, 15, 20, 25, and 30 days, as can be verified by viewing predict_into_mcad.xls (Attachment V). These data are read into an array entitled "Inputfile" as shown in Fig. 5.

		-	_		-	-	-	
TCs	1	2	3	4	5	ь	7	
×	-3	-2.666	-2.333	-2	-1.584	-1.146	-0.882	-0.69
Y	0	0	0	0	0	0	0	(
Z	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.123
0.1	0	0	0	0	0	0	0	-
0.25	0	0	0	0	0	0	0	-
0.5	0	0	0	0	0	0	0	(
1	0	0	0	0	0	0	0	-
2	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	

Note: Data are read into inputfile array. First row gives thermocouple number. Rows 2, 3, and 4 give x, y, and z coordinates in meters, respectively (see Section 5.5). Times (in days) used in calculation are listed in column 1, starting in row 5.

Figure 5. Mathcad Step 3.

The "Inputfile" array is checked to be sure that the correct number of thermocouples and number of times are read in as shown in Step 4 (Fig. 6). The coordinates of the thermocouples are checked, as shown in Step 5 (Fig. 7), which provides the transpose of the "Inputfile" array.

```
Inputfile := Inputfile^{T}
numTCs := rows(Inputfile) - 1
                                           tempc := cols(Inputfile)
           numtimes := tempc - 4
   ir := 1... numTCs
                                                                                Check values
   ic := 1.. tempc - 4
                                                                                       numtimes = 17
                                                                                        numTCs = 30
   Meastemp<sub>in, ic</sub> := Inputfile<sub>in+1, ic+4</sub>
                                                                                        tempc = 21
maxtimes := numtimes
 j = 1.. numtimes
                                                                                        maxtimes = 17
 t_i := 86400 \cdot Inputfile_{1, i+4}
                                                                                        t_4 = 86400
```

Figure 6. Mathcad Step 4

Spot check input values											
		1	2	3	4	5	6	7	8	9	10
	1	"TCs"	"X"	"Y"	"Z"	0.1	0.25	0.5	1	2	3
	2	1	-3	0	0.122	0	0	0	0	0	0
	3	2	-2.666	0	0.122	0	0	0	0	0	0
	4	3	-2.333	0	0.122	0	0	0	0	0	0
	5	4	-2	0	0.122	0	0	0	0	0	0
Inputfile =	6	5	-1.584	0	0.122	0	0	0	0	0	0
Tip wills	7	6	-1.146	0	0.122	0	0	0	0	0	0
	8	7	-0.882	0	0.122	0	0	0	0	0	0
	9	8	-0.696	0	0.122	0	0	0	0	0	0
	10	9	-0.556	0	0.122	0	0	0	0	0	0
-	11	10	-0.446	0	0.122	0	0	0	0	0	0
	12	11	-0.358	0	0.122	0	0	0	0	0	0
	13	12	-0.285	0	0.122	0	0	0	0	0	0

Figure 7. Mathcad Step 5

The x, y, and z-coordinates for each thermocouple (see Section 5.5) are then assigned to arrays as shown in Step 6 (Fig. 8). The distance, R, from each thermocouple to each sphere is calculated using the distance formula (Beyer 1987, p. 206) as shown in Step 7 (Fig. 9).

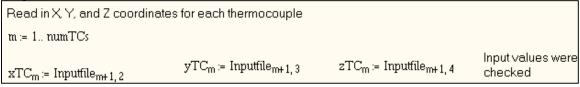


Figure 8. Mathcad Step 6.

```
 \begin{array}{c} \text{Calculate R, distance from each sphere to each thermocouple} \\ \text{zTC} = \text{z coordinate of thermocouple} \\ \text{xTC=x coord of thermocouple} \\ \text{yTC=y coord of thermocouple} \\ \text{R(m,s)= distance from thermocouple m (measurement point) to sphere s.} \\ \\ \text{s:= 1... nospheres} \\ \\ \text{R_{m,s}:=} \sqrt{(\text{xTC}_{m}-0)^{2} + (\text{yTC}_{m}-\text{ysphere}_{s})^{2} + (\text{zTC}_{m}-0)^{2}}} \\ \\ \text{spot check of values:} \\ \\ \text{R_{17,1} = 2.464921015} \\ \text{R_{29,656} = 1.588692395} \\ \\ \text{R_{17,2} = 2.461169866} \\ \text{R_{28,1} = 2.718299397} \\ \\ \text{R_{17,3} = 2.457418731} \\ \text{R_{27,2} = 2.614437054} \\ \\ \text{R_{17,4} = 2.453667611} \\ \\ \text{R_{30,656} = 2.003718425} \\ \end{array}
```

Figure 9. Mathcad Step 7

Equation 7, which gives temperature as a function of thermal diffusivity, thermal conductivity, distance from heat source, and time, is represented in the Mathcad routine in Step 8 (Fig. 10).

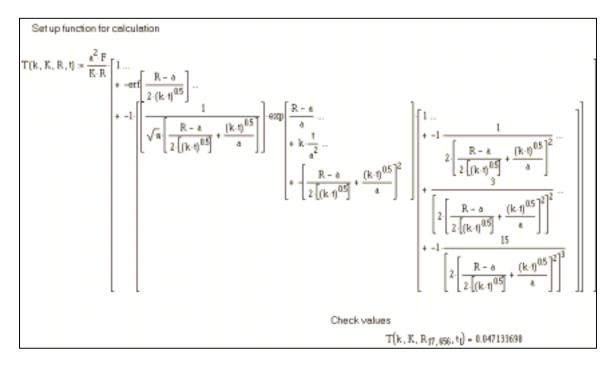


Figure 10. Mathcad Step 8.

The last step in the pretest temperature prediction is to calculate and store the predicted temperatures in an output array, mcadout. The calculation is shown in Step 9 (Fig. 11). The temperature at any thermocouple, m, at any time, j, is found by summing the contributions for each sphere used to approximate the heater. The first few rows and columns of mcadout are shown in Step 10 (Fig. 12). The full output array is copied into sheet 1 of predict_outof_mcad.xls (Attachment VI). The x coordinates of the thermocouples are also pasted into that spreadsheet so that the data can be plotted. The plotted data are given in Fig. 13. If a lower value of thermal conductivity had been used, then the predicted temperatures would be higher.

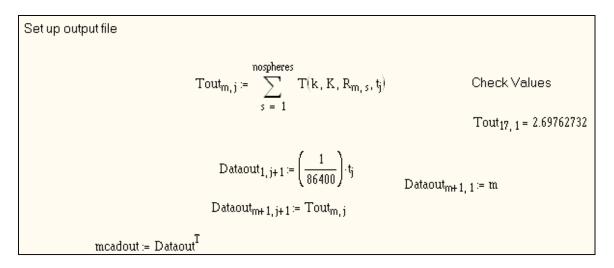


Figure 11. Mathcad Step 9.

0	1	2	3	4	5	6	7
0.1	-1E-140	-3E-111	-2.3E-85	-5.8E-63	-7.4E-40	-2.4E-21	1.93E-13
0.25	-1.7E-57	-1.1E-45	-2.9E-35	-3E-26	2.67E-17	1.7E-09	5.49E-06
0.5	-1.1E-29	-1E-23	-1.7E-18	5.58E-14	3.77E-09	3.54E-05	0.002576
1	1.22E-15	1.39E-12	7.15E-10	1.71E-07	5.57E-05	0.007216	0.075891
2	2.46E-08	9.28E-07	2.38E-05	0.000424	0.009352	0.136798	0.530145
3	7.94E-06	9.6E-05	0.000903	0.006698	0.059415	0.412309	1.128854
4	0.000154	0.001052	0.005966	0.028481	0.159257	0.754663	1.725244
5	0.000955	0.004609	0.019268	0.070446	0.297575	1.116173	2.281742
6	0.003305	0.01266	0.043134	0.131809	0.460804	1.474579	2.791713
mcadout							

Figure 12. Mathcad Step 10.

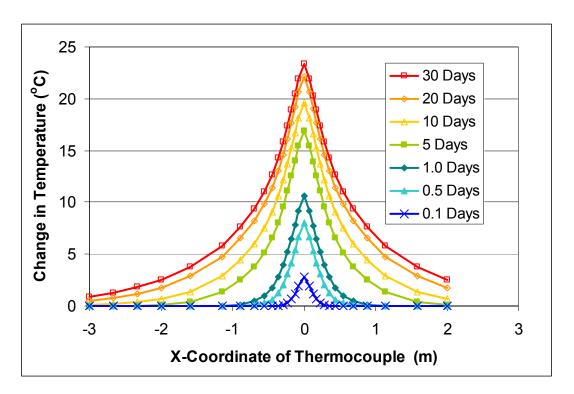


Figure 13. Predicted temperature change profiles in instrumentation borehole up to 30 days after initiating power to the heater.

6.4 CALCULATION OF THERMAL CONDUCTIVITY AND THERMAL DIFFUSIVITY OR THERMAL CONDUCTANCE FROM FIELD DATA

The calculation of thermal conductivity and thermal diffusivity or thermal conductance from field data is not a pretest prediction and is therefore outside the scope of this calculation. Documentation of these calculations will be provided in the final field test report. However, for completeness, a brief description of that calculation is given here. Predicted temperature changes are calculated using initial guesses for thermal conductivity and diffusivity as shown in Sections 6.2 and 6.3. The measured and predicted temperature changes are compared, and the error is taken as the sum of the squares of the differences between measured and predicted values. Minerr, a standard function of COTS Mathcad software, adjusts the predicted values of thermal conductivity and thermal diffusivity so as to minimize this error.

6.5 RESULTS

6.5.1 Temperature Change Predictions for Two-Hole and 6-Hole Tests

Temperature changes predicted at different times after initiating power to the heater for Test 1 are shown in Fig. 13. These were calculated using a thermal conductivity of 1.7 W/mK and a thermal diffusivity of 8.0×10^{-7} m²/s (Section 5.4). A calculation was performed to predict temperature rise around the heaters for Test 2, the 6-hole test, using the same method as given here. Instead of a single heater, three heaters were used and each was approximated as 1312 spherical heat sources. Using thermal conductivity and thermal diffusivity values of 1.7 W/mK and 8×10^{-7} m²/s, respectively, temperature rise

predictions for each of the three instrumentation boreholes in Test 2 were calculated and are shown in Fig. 14. The three instrumentation boreholes are designated as TMK006, TMK007, and TMK008. All are assumed to be nominally perpendicular to the heaters, parallel to one another, and in plan view they are spaced approximately 1 m apart. TMK007 passes approximately 0.3 m above the heater centerlines. TMK006, angled downwards, passes about 0.6 m below the heaters and TMK008, angled slightly upwards, passes about 0.6 m above the heaters. TMK006 and 008 are each offset horizontally approximately 1 m from the heater centerlines, in opposite directions. The temperature peaks are slightly off of the heater centerline because one of the outer heater boreholes is slightly angled and closer to the center heater than the other outer heater borehole. The Mathcad procedure used for this calculation, mcad_6holefwd_pretest.mcd, is given in Attachment VII. For this test, the "Inputfile" contains an additional 30 columns for each additional thermocouple string.

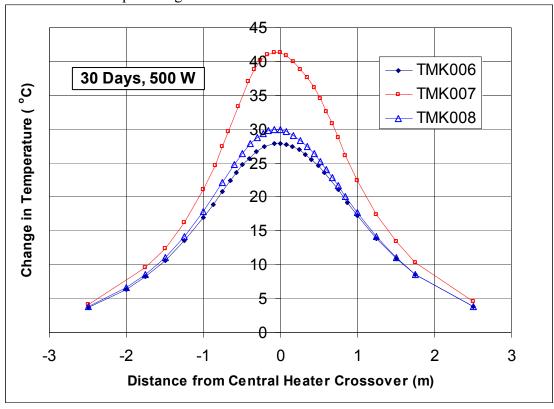


Figure 14. Predicted temperature profiles in three instrumentation boreholes for 6-hole test (Test 2) after 30 days of heating at 500W per heater. The designations TMK006, TMK007, and TMK008 refer to the instrumentation boreholes.

6.5.2 Analysis Uncertainty

Possible sources of uncertainty in this analysis are: errors in the equations used to describe the increase in temperature, errors in the inputs, and errors in the assumptions. Each of these errors will be discussed below.

The equations used to describe the increase in temperature are from standard heat transfer reference books (Carslaw and Jaeger 1959) and (Holman 1997). The possibility that

these equations are wrong is remote. The possibility that the equations are misused or not used correctly in this analysis is also remote as the report was written and checked by knowledgeable scientists and engineers. As mentioned in Section 6.2 the method used to calculate the thermal conductivity uses 1312 spheres. This method introduces a small error into the analysis as shown in Figure 2. This error is less than 0.02°C.

Errors in the inputs are possible. The inputs are determined from averaging data from different tests or from a reference handbook (Weast 1972). The variability of input data is the major source of uncertainty in this analysis. For example, Attachment II shows lithophysal porosity varies from 0.05 to 0.30 over the Tptpll rock unit, with an average of 0.125. This average value is used to characterize the volume of rock. Table 2 is presented to show how changes in the inputs could affect the thermal properties calculated in Attachments II and III, and thereby affect the calculated peak temperature.

Input	Input Value Range ^(a)		Range of (m ² /s	Diffusivity x10 ⁻⁷)	Peak Temperature @ 30 Days (°C) ^(b)		
	Upper	Lower	Upper	Lower	Upper	Lower	
Grain Density	2530	2526	7.97	7.98	26.6	26.6	
Grain Specific Heat	948	928	7.84	7.98	26.6	26.6	
Matrix Porosity	0.11	0.089	7.98	8.04	26.6	26.7	
Saturation ^(c)	0.995	0.08	8.38	7.11	24.4	33.7	
Lithophysal Porosity (d)	0.3 ^(e)	0.05 ^(†)	8.13 ^(e)	8.09 ^(†)	32.8 ^(e)	24.3 ^(†)	

Table 2. Input Change versus Temperature Change

- (a) Values are from the reference cited in Attachment III or the data provide in Attachment II.
- (b) Note that the nominal peak temperature at 30 days is 26.6 (from Attachment VI).
- (c) Upper and lower limits of thermal conductivity (1.86 and 1.32 W/mK) are calculated using the nominal value of lithophysal porosity (0.125) and upper and lower limits of saturation (0.995 and 0.08 from Table II-5), using the method given in Attachment II
- (d) Upper and lower limits of thermal conductivity (1.86 and 1.32 W/mK) are calculated using the nominal value of saturation (0.76) and upper and lower limits of lithophysal porosity (0.3 and 0.05 from Table II-1) using the method given in Attachment II.
- (e) Lithophysal porosity of 0.3 is used to obtain a thermal conductivity of 1.385 using the method of Attachment II. These lithophysal porosity and thermal conductivity values are then used to obtain a thermal diffusivity of 8.13 using the method of Attachment III. The thermal conductivity and thermal diffusivity values are then used to calculate peak temperature.
- (f) Lithophysal porosity of 0.05 is used to obtain a thermal conductivity of 1.87 using the method of Attachment II. The lithophysal porosity and thermal conductivity values are then used to obtain a thermal diffusivity of 8.09 using method of Attachment III. These thermal conductivity and thermal diffusivity values are then used to calculate peak temperature.

Variations in saturation and lithophysal porosity change peak temperature by more than 1°C from nominal (26.6°C) by modifying the calculated values of thermal conductivity and thermal diffusivity used in the analysis. It is worth noting that because thermal conductivity at a given temperature is a function of saturation, lithophysal porosity, matrix porosity and rock grain thermal conductivity, it is variability in these physical quantities and in the functional form used to calculate thermal conductivity, that produce variability in thermal conductivity and therefore in predicted peak temperature.

Alternatively, an error analysis can be used to approximate these required accuracies as shown in Attachment VIII. This method is useful for variables that are not independent; however, it provides only a very rough estimate because in order to make this calculation tractable, the heater is approximated as a single heated sphere. That analysis shows peak temperature deviating by less than 3°C, or 17.5 percent. Although this percentage is large, the temperatures are substantially below boiling and this potential uncertainty will not alter the conclusions of this pretest calculation.

Errors in the handbook are possible, but the information in the handbook is peer reviewed and has been subjected to a wide usage for many years. The 53rd edition of the *CRC Handbook of Chemistry and Physics* (Weast 1972) was used. The information cited (Assumption 5.4) is for 25°C and temperatures in this analysis are from ambient to below boiling. The physical properties of water change slightly over this temperature range, but not enough to affect the predicted temperatures.

Errors in the assumptions can also lead to errors in the analysis. The assumptions are:

- Radial heat transfer from heater to rock,
- Heat transfer mechanism is conduction,
- Isotropic homogeneous medium,
- Thermal transport properties (discussed above and not reiterated here), and
- Test geometry.

Radial heat transfer from the heater to the rock means that there is no temperature difference between the top and bottom of the hole. The small size of the borehole used to insert the heater, the close contact between the heater and the rock, and the use of insulation all contribute to ensuring that heat flux is radial Therefore, errors associated with this assumption are minimal.

The errors associated with a conduction-only heat transfer mechanism (versus convection, radiation, or diffusion) should also be small. The small size of the borehole used to insert the heater, the close contact between the heater and the rock, below boiling temperatures, and insulation all contribute to a conduction-only heat transfer mechanism. The below boiling temperature ensures that effects from water diffusion are small, as there is no mechanism driving water to transfer heat and thus affect the thermal conductivity. Therefore, errors associated with this assumption should be minor.

An isotropic homogeneous medium is assumed and errors associated with the rock medium being anisotropic or variable are difficult to quantify. The use of long heaters in boreholes ensures that the heat is spread over several meters, thus making the anisotropy of the medium less important because the heat is diffused. Errors associated with this assumption should, therefore, be minor.

Test geometry is an assumption in this analysis, but the test area has already been drilled so the geometry is known. The actual test geometry is very similar to the assumption in this analysis and thus the error associated with test geometry is minimal.

7. CONCLUSIONS

A pretest calculation was performed to predict temperature increases occurring during a planned in-situ thermal conductivity field test in the lower lithophysal lithostratigraphic unit. The heater was approximated as a series of spherical heat sources, and temperature was predicted for a set of assumed thermocouple locations as shown in Figures 13 and 14. Based on this prediction, the temperature changes and heating rates will be reasonable (sufficient data can be collected) if the heater is powered at approximately 500W. No actual test data are reported here, and there are no restrictions for subsequent use of the information given in this report. Uncertainty was discussed in Section 6.5.2 and the series of tests supported by this calculation will help to reduce the uncertainty surrounding thermal conductivity.

8. INPUTS AND REFERENCES

8.1 DOCUMENTS CITED

Abramowitz, M. and Stegun, I.A., eds. 1972. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York, New York: Dover Publications. TIC: 240610.

Beyer, W.H., ed. 1987. *CRC Standard Mathematical Tables*. 28th Edition. 3rd Printing 1988. Boca Raton, Florida: CRC Press. TIC: 240507.

BSC (Bechtel SAIC Company) 2001. *Technical Work Plan For: Testing and Monitoring*. TWP-MGR-MD-000018 Rev 00. Las Vegas, Nevada: Bechtel SAIC Company. ACC: MOL.20011015.0022.

Carslaw H.S. and Jaeger, J.C. 1959. *Conduction of Heat in Solids*. 2nd Edition. Oxford, Great Britain: Oxford University Press. TIC: 206085.

Crane, R.A.; Vachon, R.I.; and Khader, M.S. 1977. "Thermal Conductivity of Granular Materials – A Review." *Proceedings of the Seventh Symposium on Thermophysical Properties, Held at National Bureau of Standards, Gaithersburg, Maryland, May 10-12, 1977.* Cezairliyan, A., ed. 109-123. New York, New York: The American Society of Mechanical Engineers. TIC: 246057.

CRWMS M&O (Civilian Radioactive Waste Management System Management and Operating Contractor) 2000. *Input Parameters for Thermal Modeling Using Conduction-Only Models*. CAL-NBS-TH-000001 REV 00. Las Vegas, Nevada: CRWM M&O. ACC: MOL.20001107.0365.

Finley, R.E.; Ballard, S.; Brodsky, N.S.; Francis, N.D.; George, J.T.; and Sobolik, S.R. 1998. *Single Heater Test Final Report*. Milestones SP1430MR and SPY148M4. Draft,

Version 00B. Albuquerque, New Mexico: Sandia National Laboratories. ACC: MOL.19981030.0113.

Flint, L.E. 1998. *Characterization of Hydrogeologic Units Using Matrix Properties, Yucca Mountain, Nevada*. Water-Resources Investigations Report 97-4243. Denver, Colorado: U.S. Geological Survey. ACC: MOL.19980429.0512.

Hadley, G.R. 1986. "Thermal Conductivity of Packed Metal Powders." *International Journal of Heat and Mass Transfer*, 29, (6), 909-920. [New York, New York]: Pergamon Journals. TIC: 249320.

Holman, J.P. 1997. *Heat Transfer*. 8th Edition. New York, New York: McGraw-Hill. TIC: 239954.

Mongano, G.S.; Singleton, W.L.; Moyer, T.C; Beason, S.C.; Eatman, G.L.W.; Albin, A.L.; and Lung, R.C., 1999. *Geology of the ECRB Cross Drift – Exploratory Studies Facility, Yucca Mountain Project, Yucca Mountain, Nevada.* [Deliverable SPG42GM3]. Denver, Colorado: U.S. Geological Survey. ACC: MOL.20000324.0614.

SNL (Sandia National Laboratories) 1998. *Laboratory Measurements of Thermal Conductivity as a Function of Saturation State for Welded and Nonwelded Tuff Specimens*. Albuquerque, New Mexico: Sandia National Laboratories. ACC: MOL.19980901.0177.

Weast, R. C., ed. 1972. *CRC Handbook of Chemistry and Physics*. 53rd Edition. Cleveland, Ohio: Chemical Rubber Company. TIC: 219220.

YMP (Yucca Mountain Site Characterization Project) 2001. *Q-List.* YMP/90-55Q, Rev. 7. Las Vegas, Nevada: Yucca Mountain Site Characterization Office. ACC: MOL.20010409.0366.

8.2 CODES, STANDARDS, AND PROCEDURES

AP-2.21Q, Rev 1, BSCN 1. *Quality Determinations and Planning for Scientific, Engineering, and Regulatory Compliance Activities*. Washington, D.C.: U.S. Department of Energy, Office of Civilian Radioactive Waste Management. ACC: MOL.20010212.0018.

AP-2.22Q, Rev 0. Classification Criteria and Maintenance of the Monitored Geologic Repository Q-List. Washington, D.C.: U.S. Department of Energy, Office of Civilian Radioactive Waste Management. ACC: MOL.20020314.0046.

AP-SI.1Q, Rev 3, ICN 4. *Software Management*. Washington, D.C.: U.S. Department of Energy, Office of Civilian Radioactive Waste Management. ACC: MOL.20020520.0283.

AP-SIII.9Q, Rev 0, ICN 1. *Scientific Analyses*. Washington, D.C.: U.S. Department of Energy, Office of Civilian Radioactive Waste Management. ACC: MOL.20020404.0083.

AP-SV.1Q, Rev 0, ICN 2. *Control of the Electronic Management of Information*. Washington, D.C.: U.S. Department of Energy, Office of Civilian Radioactive Waste Management. ACC: MOL.20000831.0065.

ASME (American Society of Mechanical Engineers) PTC 19.1-1998. *Test Uncertainty Instruments and Apparatus*. New York, New York: American Society of Mechanical Engineers. TIC: 249327.

8.3 SOURCE DATA

GS000508312231.006. Physical Properties and Water Content from Borehole USW NRG-6, 3/19/94 to 3/27/95. Submittal date: 05/23/00.

GS950408312231.004. Physical Properties and Water Potentials of Core from Borehole USW SD-9. Submittal data: 03/01/1995.

GS950408212231.005. Physical Properties and Water Potentials of Core from Borehole USW UZ-14. Submittal date: 03/09/1995.

GS951108312231.009. Physical Properties, Water Content, and Water Potential for Borehole USW SD-7. Submittal date: 09/26/1995.

GS951108312231.010. Physical Properties and Water Content for Borehole USW# NRG-7/7A. Submittal date: 09/26/1995.

GS951108312231.011. Physical Properties, Water Content, and Water Potential for Borehole USW UZ-7A. Submittal date: 09/26/1995.

SNL01A05059301.005. Laboratory Thermal Conductivity Data for Boreholes UE25 NRG-4, NRG-5; USW NRG-6 and NRG-7/7A. Submittal date: 02/07/1996.

LIST OF ATTACHMENTS

Attachment I	TEMPERATURE AT CENTRAL THERMOCOUPLE AS A	
	FUNCTION OF NUMBER OF SPHERES USED TO	
	APPROXIMATE THE HEATER	I-1
Attachment II	ESTIMATION OF ROCK MASS THERMAL CONDUCTIVIT	YII-1
Attachment III	ESTIMATION OF ROCK MASS THERMAL DIFFUSIVITY	III-1
Attachment IV	PRINTOUT OF MATHCAD FILE	
	"MCAD_2HOLEFWD_PRETEST.MCD"	IV-1
Attachment V	PRINTOUT OF "PREDICT_INTO_MCAD.XLS"	V-1
Attachment VI	PRINTOUT OF "PREDICT_OUTOF_MCAD.XLS"	VI-1
Attachment VII	PRINTOUT OF "MCAD_6HOLEFWD_PRETEST.MCD"	VII-1
Attachment VIII	ERROR ANALYIS FOR ROUGH APPROXIMATION OF	
	UNCERTAINTY IN PREDICTED TEMPERATURES	VIII-1

ATTACHMENT I

TEMPERATURE AT CENTRAL THERMOCOUPLE AS A FUNCTION OF NUMBER OF SPHERES USED TO APPROXIMATE THE HEATER

This file calculates temperatures at thermocouple coordinates near 5-m heater.

The coordinate system origin is the center of the heater, the y-axis is coincident with the heater axis.

Inputs:

Thermocouple coordinates

Times at which temperatures should be calculated

a = 0.0381 a is the borehole radius

Power = 500 Power is the heater power

K is thermal conductivity

K = 1.7 k is thermal diffusivity

nospheres is the number of spheres approximating the heater

 $k := 8.0 \cdot 10^{-07}$

Read in TC coordinates and times at which temperatures are to be predicted

Inputfile >

TCs		17
Х		0
Υ		0
Z		0.122
	5	0

numtimes := tempc - 4

Inputfile := Inputfile^T

numTCs := rows[Inputfile] - 1 tempc := cols(Inputfile)

ir > 1.. numTCs

ic := 1.. tempc - 4 numtimes = 1

Check values

 $Meastemp_{iv,ic} := Imputfile_{iv+1,ic+4}$ mcmTCs = 1

maximes := numtimes tempc = 5

j = 1.. numtines maximes = 1

t_i = 86400-Inputfile_{1, j+4} t₁ = 432000

Spot check input values

Inputfile =
$$\begin{pmatrix} "TC_5" & "X" & "Y" & "Z" & 5 \end{pmatrix}$$

Read in X Y, and Z coordinates for each thermocouple

m := 1... numTCs

 $xTC_m := Inputfile_{m+1, 2}$ $yTC_m := Inputfile_{m+1, 3}$ $zTC_m := Inputfile_{m+1, 4}$ Input values were checked

Begin calculation for given number of spheres: Case 1= 100 spheres nospheres := 100 F=power/(no. spheres*surface area of sphere) F = 274.180727971 Calculate y coordinates of each sphere $ysphere_1 = -2.5 + a$ i > 2.. nospheres check calc, of ysphere $ysphere_i = ysphere_1 + \frac{[5-2 \cdot a] \cdot (i-1)}{nosphere_5 - 1}$ ysphere₁ = -2.4619 ysphere3 = -2.362429293 Calculate R, distance from each sphere to each thermocouple zTC = z coordinate of thermocouple xTC=x coord of thermocouple yTC- y coord of thermocouple R(m,s)= distance from thermocouple m (measurement point) to sphere s. s > 1.. nospheres $R_{m,j} = \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere_j)^2 + (zTC_m - 0)^2}$ R_{1, 1} = 2.464921015 R_{1.2} = 2.415247872 Set up function for calculation Check values T(k, K, R_{1,50}, t₁) - 1.656295035 Set up output file $Tout_{m,j} = \sum_{s=1}^{nopheres} T(k, K, R_{m,s}, t_j)$ Tout_{1, 1} = 18.666005161 $Dataout_{1, j+1} = \left(\frac{1}{86400}\right) \cdot t_j$ Dataout_{1,j+1}:= nospheres Dataoutmel, j+1 := Toutm, j mcadout := Dataout

mcadout

Begin calculation for given number of spheres: Case 2= 200 spheres nospheres := 200 F=power/(no. spheres*surface area of sphere) F = 137.050363985 Calculate y coordinates of each sphere $ysphere_1 = -2.5 + a$ i > 2.. nospheres check calc, of ysphere $ysphere_i = ysphere_1 + \frac{[5-2 \cdot a] \cdot (i-1)}{nosphere_5 - 1}$ ysphere₁ = -2.4619 ysphere3 = -2.412414573 Calculate R, distance from each sphere to each thermocouple zTC = z coordinate of thermocouple xTC=x coord of thermocouple yTC- y coord of thermocouple R(m,s)= distance from thermocouple m (measurement point) to sphere s. s > 1.. nospheres $R_{m,j} = \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere_j)^2 + (zTC_m - 0)^2}$ R_{1, 1} = 2.464921015 R_{1.2} = 2.440208933 Set up function for calculation Check values T(k, K, R_{1,50}, t₁) = 0.012250141 Set up output file $Tout_{m,j} = \sum_{s=1}^{nopheres} T(k, K, R_{m,s}, t_j)$ Check Values Tout_{1, 1} = 18.760140358 $Dataout_{1, j+1} = \left(\frac{1}{86400}\right) \cdot t_j$ Dataout_{1,j+1}:= nospheres Dataoutmel, j+1 := Toutm, j mcadout := Dataout

mcadout

Begin calculation for given number of spheres: Case 3= 500 spheres nospheres := 500 F=power/(no. spheres*surface area of sphere) F = 54.820145594 Calculate y coordinates of each sphere $ysphere_1 = -2.5 + a$ i > 2.. nospheres check calc, of ysphere $ysphere_i = ysphere_1 + \frac{[5-2 \cdot a] \cdot (i-1)}{nosphere_5 - 1}$ ysphere₁ = -2.4619 ysphere3 = -2.442165331 Calculate R, distance from each sphere to each thermocouple zTC = z coordinate of thermocouple xTC=x coord of thermocouple yTC- y coord of thermocouple R(m,s)= distance from thermocouple m (measurement point) to sphere s. s > 1.. nospheres $R_{m,j} = \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere_j)^2 + (zTC_m - 0)^2}$ R_{1, 1} = 2.464921015 R_{1.2} = 2.455065822 Set up function for calculation Check values T(k, K, R_{1,50}, t₁) = 0.000406849 Set up output file $Tout_{m,j} = \sum_{s=1}^{nopheres} T(k, K, R_{m,s}, t_j)$ Tout_{1, 1} = 18.816618252 $Dataout_{1, j+1} = \left(\frac{1}{86400}\right) \cdot t_j$ Dataout_{1,j+1}:= nospheres Dataoutmel, j+1 := Toutm, j mcadout := Dataout

mcadout

Begin calculation for given number of spheres: Case 4= 1000 spheres nospheres := 1000 F=power/(no. spheres*surface area of sphere) F = 27.418072797 Calculate y coordinates of each sphere $ysphere_1 = -2.5 + a$ i > 2.. nospheres

Calculate R, distance from each sphere to each thermocouple

zTC = z coordinate of thermocouple xTC=x coord of thermocouple

yTC- y coord of thermocouple

R(m,s)= distance from thermocouple m (measurement point) to sphere s.

s > 1.. nospheres

$$R_{m,\,1} = \sqrt{(zTC_m - 0)^2 + (yTC_m - ysphere_s)^2 + (zTC_m - 0)^2}$$
 spot check of values: $R_{1,\,1} = 2.464921015$ $R_{1,\,2} = 2.455998319$

Set up function for calculation

$$T(k,K,R,\eta) = \frac{a^2 \cdot F}{k \cdot R} \begin{bmatrix} 1 & \dots & & & \\ & + - \text{ent} \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} \right] & \dots & & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right] \end{bmatrix} \exp \begin{bmatrix} \frac{R - a}{a} & \dots & & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 \end{bmatrix} \begin{bmatrix} 1 & \dots & -1 & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 \end{bmatrix} \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 \end{bmatrix} \end{bmatrix}$$

Check values $T(k, K, R_{1.50}, t_1) = 0.00007933$

Set up output file

$$Tout_{m,j} = \sum_{s=1}^{nopheres} T(k, K, R_{m,s}, t_j)$$
 Check Values
$$Tout_{1,j+1} = \left(\frac{1}{86400}\right) \cdot t_j$$
 Dataout_{m+1,1} = m

Dataout_{1,j+1}:= nospheres Dataoutmel, j+1 := Toutm, j

mcadout := Dataout

1000 18.83544

mcadout

Begin calculation for given number of spheres: Case 5= 1312 spheres nospheres := 1312 F=power/(no. spheres*surface area of sphere) F = 20.891823778

Calculate y coordinates of each sphere

i > 2.. nospheres

$$ysphere_i = ysphere_1 + \frac{[5-2 \cdot a] \cdot (i-1)}{nosphere_5 - 1}$$

check calc, of ysphere

ysphere 1 = -2.4619

ysphere3 = -2.454388482

Calculate R, distance from each sphere to each thermocouple

zTC = z coordinate of thermocouple xTC=x coord of thermocouple

yTC- y coord of thermocouple

R(m,s)= distance from thermocouple m (measurement point) to sphere s.

s > 1.. nospheres

$$R_{m,1} = \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere_s)^2 + (xTC_m - 0)^2}$$

R_{1, 1} = 2.464921015

R_{1.2} = 2.461169866

Set up function for calculation

$$T(k,K,R,\eta) = \frac{a^2 \cdot F}{K \cdot R} \begin{bmatrix} 1 & \dots & & & \\ & + - \cot \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} \right] & \dots & & \\ & + \left[\frac{R - a}{\sqrt{\pi} \cdot \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]} \right] \exp \left[\frac{R - a}{a} & \dots & & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 & \dots & \\ & + \left[\frac{R - a}$$

Check values

T(k, K, R_{1,50}, t₁) = 0.000047888

Set up output file

$$Tout_{m,j} = \sum_{s=1}^{nespheres} T(k, K, R_{m,s}, t_j)$$
 Check Values
$$Tout_{1,j+1} = \left(\frac{1}{16400}\right) \cdot t_j$$
Dataout_{n+1,j+1} = ti

Dataout_{1,j+1}:= nospheres

Dataoutmel, j+1 := Toutm, j

mcadout := Dataout

1312 18.83992

mcadout

Begin calculation for given number of spheres: Case 6= 1500 spheres nospheres := 1500 F=power/(no. spheres*surface area of sphere) F = 18.273381865 Calculate y coordinates of each sphere $ysphere_1 = -2.5 + a$ i > 2.. nospheres check calc, of ysphere $ysphere_i = ysphere_1 + \frac{[5-2 \cdot a] \cdot (i-1)}{nosphere_5 - 1}$ ysphere₁ = -2.4619 ysphere3 = -2.455330554 Calculate R, distance from each sphere to each thermocouple zTC = z coordinate of thermocouple xTC=x coord of thermocouple yTC- y coord of thermocouple R(m,s)= distance from thermocouple m (measurement point) to sphere s. s > 1.. nospheres $R_{m,j} = \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere_j)^2 + (zTC_m - 0)^2}$ R_{1, 1} = 2.464921015 R_{1.2} = 2.461640323 Set up function for calculation

$$T(k,K,R,\eta) = \frac{a^2 \cdot F}{K \cdot R} \cdot \left[\frac{1}{1 - erf} \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} \right] - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} \right] - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} \right] - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a^2} \right] - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a^2} \right] - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 - erf \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 \right]$$

$$Check values$$

$$T(k, K, R_{1,50}, t_0) = 0.000038101$$

Set up output file

$$\begin{split} Tout_{m,j} &= \sum_{s=1}^{nepheres} T(k,K,R_{m,s},t_j) & \text{Check Values} \\ & \text{Tout}_{1,j+1} = 18.841718942 \\ & \text{Dataout}_{1,j+1} = \left(\frac{1}{86400}\right) \cdot t_j \\ & \text{Dataout}_{m+1,j} = m \end{split}$$

$$Dataout_{m+1,j+1} = m \\ Dataout_{m+1,j+1} = Tout_{m,j} \end{split}$$

mcadout := Dataout

0 1 1500 18.84172

mcadout

Begin calculation for given number of spheres: Case 7= 2000 spheres nospheres := 2000 F=power/(no. spheres*surface area of sphere) F = 13.705036399 Calculate y coordinates of each sphere $ysphere_1 = -2.5 + a$ i > 2.. nospheres check calc, of ysphere $ysphere_i = ysphere_1 + \frac{[5-2 \cdot a] \cdot (i-1)}{nosphere_5 - 1}$ ysphere 1 = -2.4619 ysphere3 = -2.456973737 Calculate R, distance from each sphere to each thermocouple zTC = z coordinate of thermocouple xTC=x coord of thermocouple yTC- y coord of thermocouple R(m,s)= distance from thermocouple m (measurement point) to sphere s. s > 1.. nospheres $R_{m,j} = \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere_j)^2 + (zTC_m - 0)^2}$ R_{1, 1} = 2.464921015 R_{1.2} = 2.462460905 Set up function for calculation Check values T(k, K, R_{1,50}, t₁) = 0.000024189 Set up output file $Tout_{m,j} = \sum_{s=1}^{nopheres} T(k, K, R_{m,s}, t_j)$ Tout_{1, 1} = 18.844856502 $Dataout_{1, j+1} = \left(\frac{1}{86400}\right) \cdot t_j$ Dataout_{1,j+1}:= nospheres Dataoutmel, j+1 := Toutm, j mcadout := Dataout

mcadout

Begin calculation for given number of spheres: Case 8= 2500 spheres nospheres := 2500 F=power/(no. spheres*surface area of sphere)

F = 10.964029119

Calculate y coordinates of each sphere

i > 2.. nospheres

$$ysphere_i = ysphere_1 + \frac{|5 - 2 \cdot a| \cdot (i - 1)}{nosphere_i - 1}$$

check calc, of ysphere

ysphere3 = -2.457959384

Calculate R, distance from each sphere to each thermocouple

zTC = z coordinate of thermocouple xTC=x coord of thermocouple

yTC- y coord of thermocouple

R(m,s)= distance from thermocouple m (measurement point) to sphere s.

s > 1.. nospheres

$$R_{m, j} := \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere)^2 + (zTC_m - 0)^2}$$

R_{1, 1} = 2.464921015

R_{1, 2} = 2,462953124

Set up function for calculation

$$T(k,K,R,\eta) = \frac{a^2 \cdot F}{K \cdot R} \begin{bmatrix} 1 \dots \\ + - \text{errf} \left[\frac{R - a}{2 \cdot (k \cdot \eta^{0.5})} \right] \dots \\ + \left[\frac{R - a}{\sqrt{\pi} \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right] \right]} \right] \exp \left[\frac{R - a}{a} \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \right] \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a} \right]^2 \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta^{0.5})} + \frac{(k \cdot \eta^{0.5})}{a$$

Check values

 $T(k, K, R_{1,50}, t_1) = 0.000017495$

Set up output file

$$Tout_{m,j} = \sum_{s=1}^{nespheres} T(k, K, R_{m,s}, t_j)$$

$$Dataout_{1,j+1} = \left(\frac{1}{16400}\right) t_j$$

$$Dataout_{m+1,1} = m$$

$$Dataout_{m+1,1} = m$$

Dataout_{1,j+1}:= nospheres

Dataoutmel, j+1 := Toutm, j

mcadout := Dataout

2500 18.84674

moadout

Begin calculation for given number of spheres: Case 9= 5000 spheres

nospheres := 5000

$$F = \frac{Power}{\left(no. spheres surface area of sphere\right)}$$

$$F = 5.482014559$$

Calculate y coordinates of each sphere

i > 2.. nospheres

$$ysphere_i = ysphere_1 + \frac{[5-2 \cdot a] \cdot (i-1)}{nosphere_1 - 1}$$

check calc, of ysphere

ysphere₁ = -2.4619

ysphere3 = -2.459930086

Calculate R, distance from each sphere to each thermocouple

zTC = z coordinate of thermocouple xTC=x coord of thermocouple

yTC- y coord of thermocouple

R(m,s)= distance from thermocouple m (measurement point) to sphere s.

s > 1.. nospheres

$$R_{m,1} = \sqrt{(xTC_m - 0)^2 + (yTC_m - ysphere_s)^2 + (zTC_m - 0)^2}$$

R_{1, 1} = 2.464921015

R_{1.2} = 2.463937266

Set up function for calculation

$$T(k,K,R,\eta) = \frac{a^2 F}{k \cdot R} \begin{bmatrix} 1 \dots \\ + - \text{erf} \left[\frac{R - a}{2 \cdot (k \cdot \eta)^{0.5}} \right] \dots \\ + \left[\frac{R - a}{\sqrt{\pi} \cdot \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta)^{0.5}} \right] + \frac{(k \cdot \eta)^{0.5}}{a} \right]} \right] \exp \left[\frac{R - a}{a} \dots \\ + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta)^{0.5}} \right]^2 \dots \right] + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta)^{0.5}} \right]^2 \dots \right] + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 \right] \dots \right] + \left[\frac{R - a}{2 \cdot \left[(k \cdot \eta)^{0.5}} + \frac{(k \cdot \eta)^{0.5}}{a} \right]^2 \dots \right]$$

Check values

 $T(k, K, R_{1.50}, t_1) = 0.000007135$

Set up output file

$$Tout_{m,j} = \sum_{s=1}^{neigheres} T(k, K, R_{m,s}, t_j)$$
 Check Values
$$Tout_{1,j+1} = \left(\frac{1}{16400}\right) \cdot t_j$$
Dataout, $t_{i,j+1} = T_{i,j+1}$

Dataout_{1,j+1}:= nospheres

Dataoutmel, j+1 := Toutm, j

mcadout := Dataout

18.8505 5000

moadout

ATTACHMENT II

ESTIMATION OF ROCK MASS THERMAL CONDUCTIVITY

Rock mass thermal conductivity is affected by of the presence of lithophysae. The following analysis provides an estimate of rock mass thermal conductivity that accounts for air-filled lithophysal porosity in the Tptpll unit.

Air has a thermal conductivity of 0.028 W/(mK) at 53°C (Holman 1997, p. 646). Since the rock will be heated, a nominal value of $\pm 50^{\circ}\text{C}$ was chosen to represent the average rock temperature. Mongano et al. (1999, p. 77, Fig. 13) present lithophysal percentages, which are shown as a function of stationing. This reference shows the range for percent of lithophysae expected in the repository host horizon. These data are extracted from the plot and presented in Table II-1. An average lithophysal porosity of 0.125 is calculated from these data.

The rock mass thermal conductivity is estimated from core samples from the Tptpll unit (only three samples were available with the necessary data qualification) and the lithophysal porosity (Tables II-2 and II-3). The open literature presents a number of thermal conductivity models (Crane 1977, pp. 109-123). In each case, the conductivity of the solid material, the fluid (air or liquid) material, and the volume fraction expressed as a porosity are inputs. The models use simplifications of geometric arrangements to predict the conductivity for the matrix (core size) or the rock mass. In situations where there are measured properties for the matrix porosity and the lithophysal porosity, the models can be applied to develop distributions of thermal conductivity that capture variability.

Hadley (1986, Equation 16, p. 914) presents one formula for the equivalent thermal conductivity (K_e) for series flow:

$$\frac{K_e}{K_g} = \frac{\kappa}{\phi \cdot \kappa + 1 - \phi}$$

Equation II-1

Where:

 K_e = Equivalent thermal conductivity (W/(m•K))

 κ = Ratio of the solids thermal conductivity (K_s) to the fluids thermal

conductivity (K_o)

 K_s = Solids thermal conductivity (W/(m•K))

 K_{σ} = Fluid thermal conductivity (W/(m•K)) and

 ϕ = Porosity

This represents the lowest equivalent thermal conductivity for a porous media. On the other hand, Hadley (1986, Equation 18, p. 914) presents the parallel formula as

$$\frac{K_e}{K_g} = \phi + (1 - \phi) \cdot \kappa$$

Equation II-2

This formula represents the mixture for the highest possible equivalent thermal conductivity for a given ratio of solids to fluid thermal conductivity (κ) and porosity (ϕ). Consequently, any mixture model that depends only upon ϕ , and κ should fall between these two bounds. Other models would provide a relationship of equivalent thermal conductivity to porosity (ϕ) that is intermediate to these bounding relationships.

The dry rock mass thermal conductivity (K_{rmdry}) is given by the parallel model (Equation II-2) by a nested application in which the large-scale lithophysal pores always remain free of water. Substituting in the thermal conductivity of the air (K_a) for the fluid thermal conductivity (K_g) ; the lithophysal porosity (ϕ_L) for the porosity (ϕ) ; the dry matrix thermal conductivity of the rock matrix (K_{mdry}) for the solids thermal conductivity (K_s) , the ratio K_{mdry}/K_a for the ratio κ , and the dry rock mass thermal conductivity K_{rmdry} for the equivalent thermal conductivity (K_e) and simplifying the equation:

$$\frac{K_{rmdry}}{K_a} = \phi_L + (1 - \phi_L) \cdot \frac{K_{mdry}}{K_a}$$

Equation II-3

$$K_{rmdry} = (1 - \phi_L) \cdot K_{mdry} + K_a \cdot \phi_L$$

Equation II- 4

where:

 K_{rmdry} = Dry rock mass thermal conductivity (W/(m•K)) K_{mdry} = Dry matrix thermal conductivity (W/(m•K)) K_a = Thermal conductivity of the air (W/(m•K)) and Φ_L = Lithophysal porosity

A second relation can be developed for the saturated case. In this case, the large-scale voids are free of water due to their negligible retention characteristics in the vadose zone. The saturated rock mass thermal conductivity is given by the parallel model (Equation II-2) by a nested application in which the large-scale voids are free of water:

$$K_{\it rmwet} = (1 - \phi_{\it L}) \cdot K_{\it mwet} + K_{\it a} \cdot \phi_{\it L}$$

Equation II-5

where:

 K_{rmwet} = Saturated rock mass thermal conductivity(W/(m \bullet K)) K_{mwet} = Saturated matrix thermal conductivity(W/(m \bullet K)) K_a = Thermal conductivity of the air (W/(m \bullet K))

These relationships are used in Tables II-2 and Tables II-3 for the saturated rock mass thermal conductivity and the dry rock mass thermal conductivity respectively.

In situ thermal conductivity requires an estimate of the degree of matrix saturation. The following discusses the calculation of the average matrix saturation for the Tptpll unit from selected samples.

Flint (1998, p. 1) characterized hydrologic units using matrix properties from core samples. About 30 hydrogeologic units in the unsaturated zone have been defined and include the Tptpmn and Tptpll units. Physical properties of particle density, matrix bulk density, matrix saturation and matrix porosity were determined from 4,892 rock samples (Flint 1998, p. 1) from the coring of 23 shallow and 8 deep boreholes. The deep boreholes that intersected the Tptpmn and Tptpll units included USW SD-7, USW UZ-7A, USW NRG 7/7A, USW NRG-6, USW SD-9, and USW UZ-14 (Flint 1998, Table 4, p. 14). The borehole and the DTN Core Data source for the calculation of the mean saturation are presented in Table II-4 and the saturation data from the source DTNs are presented in Table II-5 for the Tptpmn and Tptpll units. A saturation value of 0.76 is calculated from the core data.

Using this value and the linear relationship between saturation and thermal conductivity proposed in *Laboratory Measurements of Thermal Conductivity as a Function of Saturated State for Welded and Nonwelded Tuff Specimens* (SNL 1998, p. 21) yields:

$$[(1.87-1.27) \times 0.76] + 1.27 = 1.726 \text{ or } 1.73 \text{ W/(m·K)}.$$
 Equation II-6

Table II-1 Summary of Lithophysal Porosity

Unit	ECRB Station	Lithophysal Porosity	Unit	ECRB Station	Lithophysal Po Porosity	Unit	ECRB Station	Lithophysal Porosity
Tptpll	1450	0.070	Tptpll	1740	0.200	Tptpll	2030	0.100
Tptpll	1460	0.070	Tptpll	1750	0.200	Tptpll	2040	0.100
Tptpll	1470	0.150	Tptpll	1760	0.200	Tptpll	2050	0.100
Tptpll	1480	0.150	Tptpll	1770	0.200	Tptpll	2060	0.100
Tptpll	1490	0.150	Tptpll	1780	0.200	Tptpll	2070	0.100
Tptpll	1500	0.150	Tptpll	1790	0.200	Tptpll	2080	0.100
Tptpll	1510	0.150	Tptpll	1800	0.150	Tptpll	2090	0.100
Tptpll	1520	0.150	Tptpll	1810	0.150	Tptpll	2100	0.100
Tptpll	1530	0.150	Tptpll	1820	0.150	Tptpll	2110	0.100
Tptpll	1540	0.150	Tptpll	1830	0.150	Tptpll	2120	0.100
Tptpll	1550	0.150	Tptpll	1840	0.150	Tptpll	2130	0.100
Tptpll	1560	0.150	Tptpll	1850	0.150	Tptpll	2140	0.070
Tptpll	1570	0.150	Tptpll	1860	0.150	Tptpll	2150	0.070
Tptpll	1580	0.150	Tptpll	1870	0.150	Tptpll	2160	0.070
Tptpll	1590	0.100	Tptpll	1880	0.150	Tptpll	2170	0.070
Tptpll	1600	0.300	Tptpll	1890	0.150	Tptpll	2180	0.070
Tptpll	1610	0.300	Tptpll	1900	0.150	Tptpll	2190	0.070
Tptpll	1620	0.300	Tptpll	1910	0.150	Tptpll	2200	0.070
Tptpll	1630	0.300	Tptpll	1920	0.150	Tptpll	2210	0.070
Tptpll	1640	0.100	Tptpll	1930	0.150	Tptpll	2220	0.070
Tptpll	1650	0.100	Tptpll	1940	0.150	Tptpll	2230	0.070
Tptpll	1660	0.100	Tptpll	1950	0.150	Tptpll	2240	0.070
Tptpll	1670	0.100	Tptpll	1960	0.100	Tptpll	2250	0.070
Tptpll	1680	0.100	Tptpll	1970	0.100	Tptpll	2260	0.070
Tptpll	1690	0.100	Tptpll	1980	0.100	Tptpll	2270	0.070
Tptpll	1700	0.100	Tptpll	1990	0.100	Tptpll	2280	0.070
Tptpll	1710	0.050	Tptpll	2000	0.100	Tptpll	2290	0.070
Tptpll	1720	0.050	Tptpll	2010	0.100	Tptpll	2300	0.070
Tptpll	1730	0.200	Tptpll	2020	0.100	Tptpll	2310	0.070
						Tptpll	2320	0.070
The	average	or mean value	lithophy	sal poros	ity for these 8	3 meas	urements	s is 0.125

Source: Mongano et al., p 77, Figure 13

Table II-2. Saturated Rock Mass Thermal Conductivity for the Tptpll Unit

Borehole	Depth (ft)	Matrix Saturated Thermal Conductivity (W/(m•K)) ⁽¹⁾	Lithophysal Porosity ⁽²⁾	Rock Mass Saturated Thermal Conductivity (W/(m•K))
NRG-6	900.3	2.06	0.125	1.81**
NRG-6	926.2	2.25	0.125	1.97**
NRG-6	986.9	2.09	0.125	1.83**
Ave	rage Value	2.13*		1.87*

Sources: (1): DNT: SNL01A05059301.005; (2): Table II-1.

Table II-3. Dry Rock Mass Thermal Conductivity for the Tptpll Unit

Borehole	Depth(ft)	Matrix Dry Thermal Conductivity (W/(m•K)) ⁽¹⁾	Lithophysal Porosity ⁽²⁾	Rock Mass Dry Thermal Conductivity (W/(m•K))
NRG-6	900.3	1.45	0.12	25 1.27**
NRG-6	926.2	1.46	0.12	25 1.28**
NRG-6	986.9	1.43	0.12	25 1.25**
Ave	erage Value	1.45*		1.27*

Sources: (1): DNT: SNL01A05059301.005; (2): Table II-1.* & ** See Table II-2.

Table II-4. Data Sources for Core Properties (used in Table II-5)

Borehole	DTN Core Data
USW SD-7	GS951108312231.009
USW UZ-7A	GS951108312231.011
USW NRG-7/7A	GS951108312231.010
USW NRG-6	GS000508312231.006
USW SD-9	GS950408312231.004
USW UZ-14	GS950408312231.005

^{*} (2.06 + 2.25 + 2.09)/3 = 2.1333 round to 2.13 (same formula for averaging 1.87) ** $[(1.0-0.125) \times 2.06] + (0.125 \times 0.028) = 1.806$ round to 1.81

Table II-5. Saturation Values from Selected Boreholes

Borehole/ Depth	Depth *	Matrix Sat.									
SD7-809.2	809.2	0.862	SD9-1055.8	1055.8	0.909	USW NRG-6	1079.1	0.72	USW UZ-7a	221.7	0.814
SD7-819	819	0.904	SD9-1064.8	1064.8	0.958	USW NRG-6	1081.9	0.75	USW UZ-7a	222.9	0.723
SD7-824.7	824.7	0.911	SD9-1068.1	1068.1	0.798	USW NRG-6	1084.2	0.77	USW UZ-7a	224.4	0.779
SD7-835.4	835.4	0.874	SD9-1070.4	1070.4	0.821	USW NRG-6	1087.1	0.8	USW UZ-7a	225.4	0.499
SD7-836.8	836.8	0.698	SD9-1076.7	1076.7	0.92	USW NRG-6	1090.3	0.86	USW UZ-7a	227.7	0.664
SD7-842.5	842.5	0.891	SD9-1080.1	1080.1	0.837	USW NRG-6	1096.6	0.8	USW UZ-7a	228.7	0.769
SD7-847.6	847.6	0.862	SD9-1086.4	1086.4	0.918	USW NRG-7a	269.1	0.8	USW UZ-7a	230.1	0.762
SD7-848.4	848.4	0.775	SD9-1091.1	1091.1	0.863	USW NRG-7a	271.9	0.84	USW UZ-7a	231.2	0.784
SD7-856.9	856.9	0.794	SD9-1095.4	1095.4	0.84	USW NRG-7a	272.8	0.71	USW UZ-7a	234.2	0.579
SD7-857.7	857.7	0.845	SD9-1098.4	1098.4	0.712	USW NRG-7a	274.1	0.61	UZ14-829.5	829.5	0.611
SD7-862.3	862.3	0.863	SD9-1101.3	1101.3	0.757	USW NRG-7a	276.7	0.67	UZ14-832.8	832.8	0.61
SD7-864.9	864.9	0.778	SD9-1104.1	1104.1	0.596	USW NRG-7a	280	0.57	UZ14-836.2	836.2	0.613
SD7-867.4	867.4	0.942	SD9-1106.4	1106.4	0.761	USW NRG-7a	285.7	0.71	UZ14-839.4	839.4	0.484
SD7-872	872	0.72	SD9-1110.3	1110.3	0.729	USW NRG-7a	287.5	0.64	UZ14-847.6	847.6	0.818
SD7-874.4	874.4	0.772	SD9-1113.5	1113.5	0.706	USW NRG-7a	288.3	0.61	UZ14-851.3	851.3	0.881
SD7-875.5	875.5	0.835	SD9-1116	1116	0.749	USW NRG-7a	290.2	0.62	UZ14-861	861	0.612
SD7-878.8	878.8	0.844	SD9-1119.2	1119.2	0.755	USW NRG-7a	291.1	0.63	UZ14-866.4	866.4	0.837
SD7-884.2	884.2	0.821	SD9-1125.1	1125.1	0.806	USW NRG-7a	292.1	0.56	UZ14-867.9	867.9	0.934
SD7-885	885	0.879	SD9-1128.6	1128.6	0.877	USW NRG-7a	293.9	0.66	UZ14-878	878	0.774
SD7-887.6	887.6	0.888	SD9-1133.6	1133.6	0.799	USW NRG-7a	295.7	0.6	UZ14-881.2	881.2	0.728
SD7-891	891	0.843	SD9-1139.6	1139.6	0.84	USW NRG-7a	296.4	0.57	UZ14-887.2	887.2	0.811
SD7-894	894	0.864	SD9-1142	1142	0.903	USW NRG-7a	297.2	0.56	UZ14-917	917	0.726
SD7-897.3	897.3	0.904	SD9-1146.1	1146.1	0.863	USW NRG-7a	298.2	0.7	UZ14-923	923	0.907
SD7-899.5	899.5	0.924	SD9-1149	1149	0.865	USW NRG-7a	300.3	0.69	UZ14-926.3	926.3	0.981
SD7-904.9	904.9	0.793	SD9-1152.7	1152.7	0.855	USW NRG-7a	301.1	0.78	UZ14-928.8	928.8	0.652
SD7-910.7	910.7	0.855	SD9-1158.5	1158.5	0.69	USW NRG-7a	304.8	0.6	UZ14-931.6	931.6	0.72
SD7-914.7	914.7	0.854	SD9-1161.1	1161.1	0.864	USW NRG-7a	306.7	0.97	UZ14-938	938	0.894
SD7-916.2	916.2	0.902	SD9-1163.8	1163.8	0.828	USW NRG-7a	313	0.46	UZ14-943.5	943.5	0.718
SD7-919.1	919.1	0.818	SD9-1166.6	1166.6	0.862	USW NRG-7a	314.1	0.55	UZ14-947.1	947.1	0.758
SD7-920.4	920.4	0.831	SD9-1170.5	1170.5	0.813	USW NRG-7a	314.9	0.5	UZ14-953.1	953.1	0.907
SD7-924.1	924.1	0.847	SD9-1172.8	1172.8	0.88	USW NRG-7a	316.9	0.4	UZ14-956.4	956.4	0.69
SD7-928.4	928.4	0.903	SD9-1179	1179	0.868	USW NRG-7a	317.4	0.65	UZ14-958.5	958.5	0.755
SD7-929.7	929.7	0.871	USW NRG-6	816.6	0.24	USW NRG-7a	318.5	0.56	UZ14-961.1	961.1	0.895
SD7-932.8	932.8	0.798	USW NRG-6	817.9	0.71	USW NRG-7a	319.4	0.34	UZ14-967.6	967.6	0.833
SD7-936.7	936.7	0.781	USW NRG-6	820.8	0.8	USW NRG-7a	322.1	0.72	UZ14-971	971	0.864
SD7-940.7	940.7	0.903	USW NRG-6	823	0.87	USW NRG-7a	323.1	0.74	UZ14-973.8	973.8	0.872
SD7-941.5	941.5	0.879	USW NRG-6	826.1	0.63	USW NRG-7a	324.9	0.57	UZ14-977.2	977.2	0.743
SD7-946.4	946.4	0.825	USW NRG-6	829.2	0.78	USW NRG-7a	326.7	0.72	UZ14-981.4	981.4	0.728

Table II-5. Saturation Values from Selected Boreholes

Borehole/	Depth	Matrix	Borehole/	Depth	Matrix	Borehole/	Depth	Matrix	Borehole/	Depth	Matrix
Depth	*	Sat.	Depth	*	Sat.	Depth	*	Sat.	Depth	*	Sat.
SD7-951.2			USW NRG-6								
SD7-954.5	954.5	0.847	USW NRG-6	835.4	0.54	USW NRG-7a	331.3	0.69	UZ14-985.1	985.1	0.81
SD7-957	957	0.778	USW NRG-6	838.6	0.71	USW NRG-7a	332.2	0.73	UZ14-989	989	0.887
SD7-961.4	961.4	0.762	USW NRG-6	841.7	0.39	USW NRG-7a	334.2	0.58	UZ14-992.2	992.2	0.872
SD7-962.5	962.5	0.956	USW NRG-6	844.8	0.89	USW NRG-7a	334.9	0.53	UZ14-996.3	996.3	0.866
SD7-966.9	966.9	0.839	USW NRG-6	851.9	0.75	USW NRG-7a	336.9	0.63	UZ14-998.3	998.3	0.893
SD7-968.9	968.9	0.881	USW NRG-6	854.9	0.83	USW NRG-7a	337.8	0.68	UZ14-1001.3	1001.3	0.866
SD7-971.4	971.4	0.905	USW NRG-6	857.8	0.12	USW NRG-7a	338.4	0.51	UZ14-1003.6	1003.6	0.82
SD7-974.5	974.5	0.85	USW NRG-6	861	0.08	USW NRG-7a	340	0.52	UZ14-1009.9	1009.9	0.771
SD7-978.1	978.1	0.918	USW NRG-6	862.7	0.28	USW NRG-7a	342.4	0.38	UZ14-1013.1	1013.1	0.79
SD7-981	981	0.846	USW NRG-6	865.8	0.64	USW NRG-7a	344	0.79	UZ14-1016	1016	0.667
SD7-983.8	983.8	0.831	USW NRG-6	867.7	0.7	USW NRG-7a	346	0.59	UZ14-1018.8	1018.8	0.832
SD7-986.2	986.2	0.965	USW NRG-6	871.5	0.83	USW NRG-7a	348	0.53	UZ14-1024	1024	0.843
SD7-990.2	990.2	0.918	USW NRG-6	873.8	0.64	USW NRG-7a	348.8	0.57	UZ14-1027.2	1027.2	0.926
SD7-993.1	993.1	0.995	USW NRG-6	877.6	0.58	USW NRG-7a	353.2	0.48	UZ14-1032.4	1032.4	0.83
SD7-994.3	994.3	0.985	USW NRG-6	879.7	0.77	USW NRG-7a	354.3	0.39	UZ14-1036.9	1036.9	0.849
SD7-999	999	0.878	USW NRG-6	886	0.86	USW NRG-7a	355	0.52	UZ14-1039.2	1039.2	0.784
SD7-1005	1005	0.901	USW NRG-6	890.7	0.66	USW NRG-7a	357	0.39	UZ14-1043	1043	0.809
SD7-1008.2	1008.2	0.909	USW NRG-6	892.8	0.31	USW NRG-7a	357.9	0.5	UZ14-1045.8	1045.8	0.871
SD7-1013.3	1013.3	0.955	USW NRG-6	898.6	0.72	USW NRG-7a	358.9	0.38	UZ14-1048.4	1048.4	0.893
SD7-1017.6	1017.6	0.952	USW NRG-6	901.6	0.75	USW NRG-7a	359.6	0.66	UZ14-1053.3	1053.3	0.889
SD9-847.2	847.2	0.852	USW NRG-6	904.8	0.85	USW NRG-7a	360.5	0.42	UZ14-1055.2	1055.2	0.799
SD9-849.6	849.6	0.775	USW NRG-6	910.7	0.69	USW NRG-7a	361.5	0.53	UZ14-1057.7	1057.7	0.856
SD9-853.4	853.4	0.843	USW NRG-6	912.8	0.8	USW NRG-7a	362.6	0.53	UZ14-1061	1061	0.772
SD9-859	859	0.974	USW NRG-6	917.1	0.78	USW NRG-7a	363.2	0.35	UZ14-1063.8	1063.8	0.807
SD9-865.3	865.3	0.907	USW NRG-6	920.4	0.71	USW NRG-7a	366	0.76	UZ14-1070.2	1070.2	0.775
SD9-879.6**	879.6	1.02	USW NRG-6	928.8	0.8	USW NRG-7a	366.9	0.56	UZ14-1072.9	1072.9	0.756
SD9-888.8	888.8	0.774	USW NRG-6	932	0.72	USW NRG-7a	367.8	0.7	UZ14-1075.9	1075.9	0.747
SD9-897	897	0.898	USW NRG-6	936	0.67	USW NRG-7a	368.9	0.75	UZ14-1082.8	1082.8	0.78
SD9-899.5	899.5	0.854	USW NRG-6	942.7	0.81	USW NRG-7a	370.6	0.68	UZ14-1085.1	1085.1	0.875
SD9-905.8	905.8	0.886	USW NRG-6	949.3	0.49	USW NRG-7a	373.2	0.56	UZ14-1090.3	1090.3	0.845
SD9-921.9	921.9	0.794	USW NRG-6	952.5	0.65	USW UZ-7a	184.2	0.606	UZ14-1093.8	1093.8	0.863
SD9-924.2	924.2	0.717	USW NRG-6	955.4	0.75	USW UZ-7a	185.3	0.702	UZ14-1100.7	1100.7	0.839
SD9-936.1	936.1	0.728	USW NRG-6	959	0.71	USW UZ-7a	186.3	0.669	UZ14-1102.7	1102.7	0.848
SD9-938.9	938.9	0.812	USW NRG-6	962	0.77	USW UZ-7a	188.6	0.636	UZ14-1108.7	1108.7	0.927
SD9-944.6	944.6	0.787	USW NRG-6	968.2	0.69	USW UZ-7a	189	0.715	UZ14-1114	1114	0.893
SD9-948	948		USW NRG-6		0.65				UZ14-1117.9		
SD9-954			USW NRG-6						UZ14-1121.3		
SD9-958.1			USW NRG-6						UZ14-1123.3		
2 000.1		30		J.,	0.52			50			5.02

Table II-5. Saturation Values from Selected Boreholes

Borehole/	Depth	Matrix	Borehole/	Depth	Matrix	Borehole/	Depth	Matrix	Borehole/	Depth	Matrix
Depth	*	Sat.	Depth	*	Sat.	Depth	*	Sat.	Depth	*	Sat.
SD9-962.6	962.6	0.791	USW NRG-6	978.9	0.72	USW UZ-7a	198.3	0.76	UZ14-1127.1	1127.1	0.712
SD9-968.7	968.7	0.837	USW NRG-6	985.1	0.77	USW UZ-7a	198.9	0.84	UZ14-1129.9	1129.9	0.867
SD9-971.9	971.9	0.716	USW NRG-6	989	0.73	USW UZ-7a	203.6	0.705	UZ14-1132.7	1132.7	0.958
SD9-975.5	975.5	0.91	USW NRG-6	991.6	0.75	USW UZ-7a	205.1	0.818			
SD9-981	981	0.793	USW NRG-6	995.6	0.41	USW UZ-7a	205.4	0.839			
SD9-984.7	984.7	0.742	USW NRG-6	1004.1	0.71	USW UZ-7a	206.6	0.779			
SD9-986.6	986.6	0.744	USW NRG-6	1010.2	0.62	USW UZ-7a	207	0.803			
SD9-995.7	995.7	0.809	USW NRG-6	1015.7	0.84	USW UZ-7a	208.1	0.85			
SD9-1003	1003	0.672	USW NRG-6	1018.5	0.88	USW UZ-7a	210	0.846			
SD9-1007.3	1007.3	0.781	USW NRG-6	1024.1	0.5	USW UZ-7a	210.7	0.844			
SD9-1012.3	1012.3	0.731	USW NRG-6	1033.8	0.41	USW UZ-7a	211.7	0.876			
SD9-1017.2	1017.2	0.886	USW NRG-6	1036	0.62	USW UZ-7a	212.8	0.749			
SD9-1023.8	1023.8	0.89	USW NRG-6	1040.1	0.84	USW UZ-7a	213.2	0.776			
SD9-1028.9	1028.9	0.81	USW NRG-6	1042.7	0.87	USW UZ-7a	213.7	0.744			
SD9-1033.1	1033.1	0.903	USW NRG-6	1049	0.66	USW UZ-7a	214.8	0.784			
SD9-1035.1	1035.1	0.922	USW NRG-6	1054.8	0.37	USW UZ-7a	215.7	0.678			
SD9-1038.8	1038.8	0.95	USW NRG-6	1058.3	0.87	USW UZ-7a	216.7	0.751			
SD9-1041	1041	0.895	USW NRG-6	1060.9	0.83	USW UZ-7a	217.5	0.852			
SD9-1044.2	1044.2	0.874	USW NRG-6	1063.5	0.81	USW UZ-7a	218.2	0.719			
SD9-1047.2	1047.2	0.932	USW NRG-6	1067	0.52	USW UZ-7a	219.2	0.706			
SD9-1050.2	1050.2	0.871	USW NRG-6	1069.8	0.82	USW UZ-7a	220.2	0.756			
SD9-1053.6	1053.6	0.985	USW NRG-6	1076.1	0.68	USW UZ-7a	220.9	0.678			

Sources: See Table II-4.

The calculated mean value for the saturation is 0.76 from the data in Table II-5.

^{*} Depth is shown in both feet and meters. Depths below 400 are in meters and depths greater than 800 are in feet.

ATTACHMENT III

ESTIMATION OF ROCK MASS THERMAL DIFFUSIVITY

Calculation of pretest rock mass thermal diffusivity

Grain Density (kg/m3)	2526	Finley et al. 1998, p. 148
Grain Specific Heat (J/kg-K)	928	Finley et al. 1998, p. 148
Matrix Porosity	0.11	Finley et al. 1998, p. 148

Lithophysal Porosity 0.125 Attachment Π Saturation 0.76 Attachment Π

Water Specific Heat at 25 C (J/kg-K) 4180 Weast (1972, p. D128) Water Density at 25 C (kg/m3) 997 Weast (1972, p. F11)

$$sat := 0.76$$

$$matrix_por := 0.11$$

$$K := 1.7$$

 $litho_por := 0.125$

 $water_heat := 4180$

$$roe_Cp = 2.13 \times 10^6$$

$$diffusivity := \frac{K}{roe_Cp}$$

diffusivity =
$$7.98 \times 10^{-7}$$

ATTACHMENT IV

PRINTOUT OF MATHCAD FILE "MCAD_2HOLEFWD_PRETEST.MCD"

This file calculates temperatures at thermocouple coordinates near 5-m heater.

The coordinate system origin is the center of the heater; the y-axis is coincident with the heater axis.

Inputs:

Thermocouple coordinates

Times at which temperatures should be calculated

$$k := 8.0 \cdot 10^{-07}$$

nospheres := 1312

$$F \coloneqq \frac{Power}{\left(nospheres \cdot 4 \cdot \pi \cdot a^2\right)}$$
 F=power/(no. spheres*surface area of sphere)

$$F = 20.891823778$$

Calculate y coordinates of each sphere

$$ysphere_1 := -2.5 + a$$

$$i := 2... 1312$$

$$ysphere_i := ysphere_1 + \frac{(5-2 \cdot a) \cdot (i-1)}{nospheres - 1}$$

check calc, of ysphere

$$ysphere_1 = -2.4619$$

 $ysphere_2 = -2.458144241$

ysphere3 = -2.454388482

 $ysphere_4 = -2.450632723$

 $ysphere_5 = -2.446876964$

Read in TC coordinates and times at which temperatures are to be predicted

Inputfile :=

TCs	1	2	3	4	5	6	7	8
×	-3	-2.666	-2,333	-2	-1.584	-1.146	-0.882	-0.696
Υ	0	0	0	0	0	0	0	0
Z	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122
0.1	0	0	0	0	0	0	0	0
0.25	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0

Inputfile := Inputfile I

ir = 1... num TCs

ic := 1... tempc = 4

 $Meastemp_{ir,ic} := Inputfile_{ir+1,ic+4}$

maximes := numtimes

j := 1... rumtimes

t_j := 86400 · Inputfile _{1, j+4}

Check values

numfimes = 17

numTCs = 30

tempc = 21

maxtimes = 17

 $t_d = 86400$

Spot check input values

	1	2	3	4	5	6	7	8	9	10
1	"TCs"	.X.		"Z"	0.1	0.25	0.5	1	2	3
2	1	-3	0	0.122	0	0	0	0	0	0
3	2	-2.666	0	0.122	0	0	0	0	0	0
4	3	-2.333	0	0.122	0	0	0	0	0	0
5	4	-2	0	0.122	0	0	0	0	0	0
6	5	-1.584	0	0.122	0	0	0	0	0	0
7	6	-1.146	0	0.122	0	0	0	0	0	0
8	7	-0.882	0	0.122	0	0	0	0	0	0
9	8	-0.696	0	0.122	0	0	0	0	0	0
10	9	-0.556	0	0.122	0	0	0	0	0	0
11	10	-0.446	0	0.122	0	0	0	0	0	0
12	11	-0.358	0	0.122	0	0	0	0	0	0
13	12	-0.285	0	0.122	0	0	0	0	0	0

Inputfile =

Read in X, Y, and Z coordinates for each thermocouple

m = 1... mmTCs

xTCm := Inputfilem+1, 2

yTCm := Inputfilem+1.3

zTCm := Inputfilem+1.4

Input values were checked

Calculate R. distance from each sphere to each thermocouple

zTC = z coordinate of thermocouple

xTC-x coord of thermocouple

yTC= y coord of thermocouple R(m,s)= distance from thermocouple m (measurement point) to sphere s.

s > 1.. nospheres

$$R_{m,3} = \sqrt{(\pi T C_m - 0)^2 + (y T C_m - y sphere_3)^2 + (\pi T C_m - 0)^2}$$

spot check of values:

R_{17.1} = 2.464921015 R_{29.656} = 1.588692395

R_{17, 2} = 2.461169866 R_{28, 1} = 2.718299397

R_{17, 3} = 2.457418731 R_{27, 2} = 2.614437054

R_{17, 4} = 2.453667611 R_{30, 656} = 2.003718425

Set up function for calculation

$$T(k,K,R,t) := \frac{a^2 \cdot F}{K \cdot R} \begin{bmatrix} 1 \dots \\ + -ett \begin{bmatrix} \frac{R-a}{2 \cdot (k \cdot t)^{0.5}} \end{bmatrix} \dots \\ + -1 \cdot \begin{bmatrix} \frac{1}{\sqrt{\pi} \begin{bmatrix} \frac{R-a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \end{bmatrix}} \end{bmatrix} \exp \begin{bmatrix} \frac{R-a}{a} \dots \\ + \frac{t}{a^2} \dots \\ + \frac{1}{2 \cdot \left[(k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1 \dots \\ + 1 - \frac{1}{2 \cdot \left[\frac{R-a}{2 \cdot \left[(k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 \right]} \\ + \frac{1}{2 \cdot \left[\frac{R-a}{2 \cdot \left[(k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 \right]} \end{bmatrix} \\ + -1 - \frac{1}{2 \cdot \left[\frac{R-a}{2 \cdot \left[(k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 \right]} \end{bmatrix} \end{bmatrix}$$

Check values

T(k, K, R_{17, 656}, t₁) = 0.047133698

Set up output file

$$Tout_{m,j} := \sum_{s=-1}^{mospheres} T(k, K, R_{m,s}, t_j)$$

Check Values

 $Tout_{17, 1} = 2.69762732$

$$Dataout_{1,\ j+1} \coloneqq \left(\frac{1}{86400}\right) \cdot t_j$$

Dataout_{m+1, 1} := m

 $Dataout_{m+1, j+1} := Tout_{m, j}$

 $mcadout := Dataout^T$

0	1	2	3	4	5	6	7
0.1	-1E-140	-3E-111	-2.3E-85	-5.8E-63	-7.4E-40	-2.4E-21	1.93E-13
0.25	-1.7E-57	-1.1E-45	-2.9E-35	-3E-26	2.67E-17	1.7E-09	5.49E-06
0.5	-1.1E-29	-1E-23	-1.7E-18	5.58E-14	3.77E-09	3.54E-05	0.002576
1	1.22E-15	1.39E-12	7.15E-10	1.71E-07	5.57E-05	0.007216	0.075891
2	2.46E-08	9.28E-07	2.38E-05	0.000424	0.009352	0.136798	0.530145
3	7.94E-06	9.6E-05	0.000903	0.006698	0.059415	0.412309	1.128854
4	0.000154	0.001052	0.005966	0.028481	0.159257	0.754663	1.725244
5	0.000955	0.004609	0.019268	0.070446	0.297575	1.116173	2.281742
6	0.003305	0.01266	0.043134	0.131809	0.460804	1.474579	2.791713

mcadout

ATTACHMENT V

PRINTOUT OF "PREDICT_INTO_MCAD.XLS"

 $predict_into_mcad.xls: Rows 1 - 21, Columns A - K$

TCs	1	2	3	4	5	6	7	8	9	10
x	-3	-2.666	-2.333	-2	-1.584	-1.146	-0.882	-0.696	-0.556	-0.446
Y	0	0	0	0	0	0	0	0	0	0
Z	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122
0.1	0	0	D	0	D	0	0	D	0	D
0.25	0	0	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	D	0	0	0	0	D
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	D
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	D	0	D	0	0	0	0	D
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	D	0	D	0	0	0	0	D
10	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	D

predict_into_mcad.xls: Rows 1 – 21, Columns L - U

11	12	13	14	15	16	17	18	19	20
-0.358	-0.285	-0.224	-0.172	-0.125	-0.08	0	0.08	0.129	0.172
0	0	0	0	0	0	0	0	0	0
0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

predict_into_mcad.xls: Rows 1 – 21, Columns V – AE

21	22	23	24	25	26	27	28	29	30
0.224	0.285	0.358	0.446	0.556	0.696	0.882	1.146	1.584	2
0	0	0	0	0	0	0	0	0	0
0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122	0.122
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

ATTACHMENT VI

PRINTOUT OF "PREDICT_OUTOF_MCAD.XLS"

NOTE: For this attachment the x, y, & z rows are the coordinates of the thermocouples.

predict_outof_mcad.xls: Rows 1 - 20, Columns A - H

x coord:	-3	-2.666	-2.333	-2	-1.584	-1.146	-0.882
0	1	2	3	4	5	6	7
0.1	-1.2181E-140	-3.2351E-111	-2.34066E-85	-5.76614E-63	-7.40766E-40	-2.40472E-21	1.92596E-13
0.25	-1.7272E-57	-1.14358E-45	-2.87605E-35	-3.01476E-26	2.67107E-17	1.69526E-09	5.4906E-06
0.5	-1.12813E-29	-9.99703E-24	-1.74109E-18	5.58098E-14	3.7654E-09	3.54365E-05	0.002576223
1	1.21956E-15	1.39472E-12	7.15014E-10	1.70861E-07	5.56852E-05	0.007215989	0.075890583
2	2.45789E-08	9.28353E-07	2.38463E-05	0.000424099	0.009352154	0.136798045	0.530145383
3	7.9422E-06	9.597E-05	0.000902647	0.006697861	0.059414871	0.412309392	1.128853502
4	0.000154278	0.001051564	0.005965598	0.028481496	0.159257455	0.754662758	1.725244306
5	0.000954843	0.004608686	0.019268096	0.070445798	0.29757455	1.116172949	2.281741845
6	0.003304995	0.012660469	0.043133735	0.131808654	0.460804278	1.474578654	2.791713145
7	0.008164783	0.026499192	0.07793795	0.209323356	0.638339115	1.819989868	3.25716625
8	0.016286232	0.046655272	0.122822223	0.299229212	0.822776013	2.148350719	3.682351862
9	0.028112313	0.073055964	0.17636151	0.398088987	1.009168896	2.458411703	4.071796634
10	0.043795837	0.1052478	0.236982973	0.503046183	1.194291653	2.750288235	4.429695823
15	0.173563484	0.329146953	0.599954964	1.0562168	2.051425543	3.970413663	5.85960306
20	0.358944958	0.603579992	0.987819765	1.580054863	2.766487454	4.888618151	6.886726564
25	0.565755535	0.884217832	1.355018634	2.043722121	3.356671851	5.604921975	7.667955773
30	0.774804294	1.15254677	1.689290285	2.44817149	3.849199511	6.181805472	8.287166842

predict_outof_mcad.xls: Rows 1 - 20, Column I - P

-0.696	-0.556	-0.446	-0.358	-0.285	-0.224	-0.172	-0.125
8	9	10	11	12	13	14	15
.72396E-09	5.88269E-06	0.00039317	0.00644584	0.04527341	0.179173	0.4819384	1.01029072
000533833	0.009283351	0.06222008	0.23159366	0.60189582	1.2185126	2.07366098	3.14623163
031058203	0.15371689	0.4619991	1.01513808	1.83722515	2.89429405	4.12813186	5.50618406
312186542	0.800958783	1.57033098	2.58955822	3.82943578	5.22450172	6.7145771	8.2825156
239123247	2.233723643	3.46157877	4.8532623	6.38153389	7.98800925	9.62814753	11.3024972
158490309	3.422858338	4.86014901	6.40558125	8.04517192	9.72966577	11.4235541	13.1352665
2.96769191	4.394090587	5.9496099	7.5784548	9.27689022	11.0019207	12.7234199	14.4541806
672872287	5.206464453	6.83761387	8.51868623	10.2534552	12.0032845	13.7415722	15.4838681
1.29170552	5.9011239	7.58465254	9.30141269	11.0607924	12.8273221	14.5768727	16.3268877
839880862	6.505565355	8.22739976	9.97001861	11.7471237	13.5256338	15.2832429	17.0387713
329792917	7.038744336	8.78972296	10.5518834	12.342325	14.129823	15.893466	17.6531174
5.77108577	7.51423833	9.28806824	11.0654721	12.8662819	14.6607518	16.4290687	18.1919107
171321982	7.942110804	9.73428789	11.5238809	13.3329598	15.1329804	16.905014	18.6703912
731390299	9.585571231	11.4324842	13.2581806	15.0916372	16.9079711	18.690906	20.4637093
823651798	10.71869879	12.592093	14.4351144	16.2802075	18.1043011	19.8924095	21.6687316
642948146	11.56161042	13.4502026	15.303124	17.1548476	18.9833546	20.7744017	22.552721
0.28667073	12.22043223	14.1187001	15.9779071	17.8338331	19.6651339	21.4580399	23.237622

predict_outof_mcad.xls: Rows 1 – 20, Column Q - X

-0.08	0	0.08	0.129	0.172	0.224	0.285	0.358
16	17	18	19	20	21	22	23
1.74878441	2.69762732	1.74878441	0.95447679	0.4819384	0.179173	0.04527341	0.00644584
4.34750676	5.67595932	4.34750676	3.04492742	2.07366098	1.2185126	0.60189582	0.23159366
6.93408086	8.43216612	6.93408086	5.38116837	4.12813186	2.89429405	1.83722515	1.01513808
9.84351201	11.4375472	9.84351201	8.14321603	6.7145771	5.22450172	3.82943578	2.58955822
12.9359265	14.5811767	12.9359265	11.1553227	9.62814753	7.98800925	6.38153389	4.8532623
14.7937943	16.4566378	14.7937943	12.9853473	11.4235541	9.72966577	8.04517192	6.40558125
16.1254404	17.7971797	16.1254404	14.3028661	12.7234199	11.0019207	9.27689022	7.5784548
17.1628177	18.8399206	17.1628177	15.3317099	13.7415722	12.0032845	10.2534552	8.51868623
18.0109745	19.6916567	18.0109745	16.1741654	14.5768727	12.8273221	11.0607924	9.30141269
18.7265232	20.4097571	18.7265232	16.8856463	15.2832429	13.5256338	11.7471237	9.97001861
19.3436079	21.0287476	19.3436079	17.4996915	15.893466	14.129823	12.342325	10.5518834
19.8845191	21.5711318	19.8845191	18.038252	16.4290687	14.6607518	12.8662819	11.0654721
20.3646817	22.0524641	20.3646817	18.5165476	16.905014	15.1329804	13.3329598	11.5238809
22.1629223	23.8541258	22.1629223	20.3093242	18.690906	16.9079711	15.0916372	13.2581806
23.3702748	25.0630967	23.3702748	21.51409	19.8924095	18.1043011	16.2802075	14.4351144
24.2555862	25.9493258	24.2555862	22.3979339	20.7744017	18.9833546	17.1548476	15.303124
24.9413227	26.6356423	24.9413227	23.082743	21.4580399	19.6651339	17.8338331	15.9779071

predict_outof_mcad.xls: Rows 1 – 20, Columns Y - AE

0.446	0.556	0.696	0.882	1.146	1.584	2
24	25	26	27	28	29	30
0.00039317	5.88269E-06	8.72396E-09	1.92596E-13	-2.40472E-21	-7.40766E-40	-5.76614E-63
0.06222008	0.009283351	0.000533833	5.4906E-06	1.69526E-09	2.67107E-17	-3.01476E-26
0.4619991	0.15371689	0.031058203	0.002576223	3.54365E-05	3.7654E-09	5.58098E-14
1.57033098	0.800958783	0.312186542	0.075890583	0.007215989	5.56852E-05	1.70861E-07
3.46157877	2.233723643	1.239123247	0.530145383	0.136798045	0.009352154	0.000424099
4.86014901	3.422858338	2.158490309	1.128853502	0.412309392	0.059414871	0.006697861
5.9496099	4.394090587	2.96769191	1.725244306	0.754662758	0.159257455	0.028481496
6.83761387	5.206464453	3.672872287	2.281741845	1.116172949	0.29757455	0.070445798
7.58465254	5.9011239	4.29170552	2.791713145	1.474578654	0.460804278	0.131808654
8.22739976	6.505565355	4.839880862	3.25716625	1.819989868	0.638339115	0.209323356
8.78972296	7.038744336	5.329792917	3.682351862	2.148350719	0.822776013	0.299229212
9.28806824	7.51423833	5.77108577	4.071796634	2.458411703	1.009168896	0.398088987
9.73428789	7.942110804	6.171321982	4.429695823	2.750288235	1.194291653	0.503046183
11.4324842	9.585571231	7.731390299	5.85960306	3.970413663	2.051425543	1.0562168
12.592093	10.71869879	8.823651798	6.886726564	4.888618151	2.766487454	1.580054863
13.4502026	11.56161042	9.642948146	7.667955773	5.604921975	3.356671851	2.043722121
14.1187001	12.22043223	10.28667073	8.287166842	6.181805472	3.849199511	2.44817149

ATTACHMENT VII

${\bf PRINTOUT\ OF\ "MCAD_6HOLEFWD_PRETEST.MCD"}$

This file calculates temperatures at locations around 3 heaters.

Inputs: Thermocouple coordinates, times at which temperature will be calculated.

Thermocouple measured temperatures.

a := 0.0381
Power := 500
K := 1.7
k := 0.8 \cdot 10^{(-05)}
nospheres := 1312
F :=
$$\frac{\text{Power}}{\left(\text{nospheres} \cdot 4 \cdot \mathbf{x} \cdot \mathbf{a}^2\right)}$$

F = 20.891823778

Calculate coordinates of each sphere. For ysphere sub a,b; a is the heater no., b is the sphere no. Variable "starty" is the y coordinate of heater at most negative value.

noheater = 1...3

 $heaterlength_{noheater} = 5$

touch sphere
$$s_{noheater} = \frac{heater length_{noheater}}{2 \cdot a}$$
 touch sphere $s = \begin{pmatrix} 65.6167979 \\ 65.6167979 \\ 65.6167979 \end{pmatrix}$

$$a_{on} x_{noheater} := \frac{\left(endx_{noheater} - startx_{noheater}\right)}{2 \cdot touch sphere s_{noheater}}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} endy_{noheater} - starty_{noheater} \\ 2 \cdot touch sphere s_{noheater} \\ 2 \cdot touch sphere s_{noheater} \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

$$a_{on} x = \begin{pmatrix} 0 \\ 0 \\ 0.00079240 \end{pmatrix}$$

```
i:=\,2\,..\,\,1312
        yspherenoheater, 1:= startynoheater + a_on_ynoheater
                                                                                         ysphere_{1,1} = -2.4619
                                                                      \left(\texttt{endy}_{\texttt{noheater}} - \texttt{starty}_{\texttt{noheater}} - 2 \cdot \texttt{a\_on\_y}_{\texttt{noheater}}\right) \cdot (i-1)
              yspherenoheater, i = yspherenoheater, 1 +
                                                                                                 check calc, of ysphere
                                                                                                    ysphere<sub>1,2</sub> = -2.458144241
                                                                                                  ysphere_{1,3} = -2.454388482
                                                                                                    ysphere_{1,4} = -2.450632723
                                                                                                      ysphere_{3,1312} = 2.4619
          xsphere_{noheater, 1} = startx_{noheater} + a_on_x_{noheater}
                                                                                           xsphere_{1,1} = 0.507
                                                                        \left(\texttt{endx}_{\texttt{noheater}} - \texttt{startx}_{\texttt{noheater}} - 2 \cdot \texttt{a\_on\_x}_{\texttt{noheater}}\right) \cdot (i-1)
                xspherenoheater, i := xspherenoheater, 1 +
                                                                                                    nospheres - 1
                                                                                                    check calc, of xsphere
                                                                                                      xsphere_{3,1300} = -0.287729917
               zsphere_{noheater, 1} = startz_{noheater} + a_on_z_{noheater}
                                                                                                zsphere_{1,1} = 0.0283904
                     zsphere_{noheater,\,i} \coloneqq zsphere_{noheater,\,1} + \frac{\left(endz_{noheater} - startz_{noheater} - 2 \cdot a\_on\_z_{noheater}\right) \cdot (i-1)}{\cdot}
                                                                                                        check calc, of zsphere
                                                                                                          zsphere<sub>3, 1300</sub> = 0.012532282
```

Read in TC coordinates and measured data

In	m	utfi	le	×

TCs		1	2	3	4	5	6
X		-2.49735	-1.99788	-1.74814	-1.49841	-1.24867	-0.99894
Y	1 1	-0.936	-0.936	-0.936	-0.936	-0.936	-0.936
Z	1 1	-0.55385	-0.57688	-0.58839	-0.59991	-0.61142	-0.62294
	0.25	0.1	0.1	0.1	0.1	0.1	0.1
	0.5	0.1	0.1	0.1	0.1	0.1	0.1
	- 1	0.1	0.1	0.1	0.1	0.1	0.1
	- 5	0.1	0.1	D.1	0.1	0.1	0.1
	10	0.1	0.1	D.1	0.1	0.1	0.1
	20	0.1	0.1	D.1	0.1	0.1	0.1
	30	0.1	0.1	D.1	0.1	0.1	0.1

Inputfile > Inputfile T

numTCs = rows(Inputfile) - 1 tempc := cols(Inputfile)

Check values

tempc = 11

numtimes = tempc - 4

calc of numtimes is a check numtimes = 7

numTCs = 90

tempc - 11

ir = 1_ numTCs

to:= 1.. tempo - 4

 $Meastemp_{k,ic} := Inputfile_{k+1,ic+4}$

Inputfile =

Read in measurement times

mactimes := numtimes

j = 1_ maximes

η = 86400 Inputfile 1, j+4

t₁ = 21600 maximes = 7

		1	2	3	4	5
	1	"TCs"	"X"	7-	, Z.	0.25
	2	1	-2.497346647	-0.936	-0.553849122	0.1
ı	3	2	-1.997877318	-0.936	-0.576879298	0.1
	4	3	-1.748142653	-0.936	-0.588394385	0.1
	5	4	-1.498407988	-0.936	-0.599909473	0.1
	6	5	-1.248673324	-0.936	-0.611424561	0.1
	7	6	-0.998938659	-0.936	-0.622939649	0.1
=	8	: : : 7	-0.874071326	-0.936	-0.628697193	0.1
	9	8	-0.749203994	-0.936	-0.634454737	0.1
	10	9	-0.649310128	-0.936	-0.639060772	0.1
	11	10	-0.567397158	-0.936	-0.642837721	0.1
	12	11	-0.487482065	-0.936	-0.646522549	0.1
	13	12	-0.407566973	-0.936	-0.650207377	0.1
	14	13	-0.310869923	-0.936	-0.654875231	0.1
	15	14	-0.209777118	-0.936	-0.659327326	0.1
	16	15	-0.079915093	-0.936	-0.665315172	0.1

Read in X, Y, and Z coordinates for each thermocouple

m := 1.. numTCs

 $xTC_m = Inputfile_{m+1, 2}$

 $yTC_m = Inputfile_{m+1,3}$ $zTC_m = Inputfile_{m+1,4}$

Input values were checked

Calculate R, distance from each sphere to each thermocouple

zTC = z coordinate of thermocouple

xTC=x coord of thermocouple

yTC- y coord of thermocouple

R(m,s)= distance from thermocouple m (measurement point) to sphere s.

s := 1... nospheres

$$R_{m, 3} := \sqrt{(xTC_m - xsphere_1,)^2 + (yTC_m - ysphere_1,)^2 + (zTC_m - 0)^2}$$

stot := nospheres + 1.. 2-nospheres

$$R_{m, stat} = \sqrt{(xTC_m - xsphere_{2, stat-nosphere})^2 + (yTC_m - ysphere_{2, stat-nosphere})^2 + (zTC_m - 0)^2}$$

stot := 2-nospheres + 1... 3-nospheres

$$R_{m, stot} = \sqrt{(xTC_m - xsphere_3, stot-2 nosphere_2)^2 + (yTC_m - ysphere_3, stot-2 nosphere_2)^2 + (zTC_m - 0)^2}$$

spot check of values:

R_{16.656} = 1.2558639 $R_{30.656} = 2.334300004$

R_{28.1} = 1.963679769 R_{16.2} = 1.738255761

R_{16.3} = 1.734967891 R_{1.656+1312.1} = 2.723246823

R_{16.4} = 1.731681924 R_{1.656+1312.0} = 3.194594147 Set up function for calculation

$$T(k,K,R,t) > \frac{a^2 \cdot F}{K \cdot R} \begin{bmatrix} 1 \dots \\ + - \cot \left[\frac{R - a}{2 \cdot (k \cdot t)^{0.5}} \right] \dots \\ + \left[\frac{-1}{\sqrt{\pi} \left[\frac{R - a}{2 \cdot [(k \cdot t)^{0.5}]} + \frac{(k \cdot t)^{0.5}}{a} \right] \right]} - \exp \left[\frac{R - a}{a} \dots \\ + \left[\frac{R - a}{2 \cdot [(k \cdot t)^{0.5}]} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 \right] \\ + \left[\frac{R - a}{2 \cdot [(k \cdot t)^{0.5}]} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 - \left[\frac{R - a}{2 \cdot [(k \cdot t)^{0.5}]^2} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 - \left[\frac{R - a}{2 \cdot [(k \cdot t)^{0.5}]^2} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 \right] \\ + \left[\frac{R - a}{2 \cdot [(k \cdot t)^{0.5}]^2} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 \right]$$

 $T(k, K, R_{15, 500}, t_6) = 0.010605609$

Set up output

$$Tout_{1,j} := \left(\frac{1}{86400}\right) \cdot t_j$$

$$Tout_{m+1,j} := \sum_{s=-1}^{nospheres-3} T(k, K, R_{m,s}, t_j)$$

$$Tout := Tout^T$$

0.25	-1.1E-31	-4.9E-20	1.33E-15	1.21E-11	1.87E-08	4.97E-06
0.5	6.07E-17	8.94E-11	2.39E-08	2.67E-06	0.000126	0.002463
1	6.26E-09	7.92E-06	0.000149	0.001837	0.014921	0.079306
5	0.039308	0.235935	0.51824	1.058953	2.008983	3.51457
10	0.467	1.371072	2.23561	3.526183	5.369619	7.84931
20	2.069727	4.100016	5.652361	7.684225	10.28525	13.49192
30	3.705976	6.360264	8.239804	10.59434	13.5007	16.98204

Tout Tout

ATTACHMENT VIII

ERROR ANALYSIS FOR ROUGH APPROXIMATION OF UNCERTAINTY IN PREDICTED TEMPERATURES

Note: This attachment references a superposition solution from Carslaw and Jeager 1959, p. 262.

This worksheet calculates the errors associated with calculated temperature rise some distance from a heater.

A superposition solution from Carslaw and Jaeger is used to calculate the temperature any distance from a spherical heater. This equation can be rearranged to calculate thermal conductivity from thermal diffusivity, temperature, time, heater power, and heater radius.

$$T(k,K,R,t,F,a) \approx \frac{a^2 \cdot F}{K \cdot R} \begin{bmatrix} 1 & \dots & & & \\ & + - \text{erf} \left[\frac{R - a}{2 \cdot (k \cdot t)^{0.5}} \right] \dots & & \\ & + \frac{1}{\sqrt{\pi} \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]} \cdot \exp \left[\frac{(R) - a}{a} \dots & & \\ & + k \cdot \frac{t}{a^2} \dots & \\ & + \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2 - \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots & \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot (k \cdot t)^{0.5}} + \frac{(k \cdot t)^{0.5}}{a} \right]^2} \dots \\ & + \frac{1}{2 \cdot \left[\frac{(R) - a}{2 \cdot ($$

Where:

T - temperature (C)

k = thermal diffusivity (m^2/s)

K = thermal conductivity (W/mK)

R = distance from center of heated sphere to measurement point (m)

F = heat flux at surface of sphere (W/m^2)

a. - radius of sphere (m)

t = time (s)

Calculate radius for spherical heater, equivalent in volume to 5-m heater in 0.0381 m radius borehole

heatervol :=
$$(0.0381)^2 \cdot 5 \cdot 3.14$$

hostomol - 0.023790277

equiv_a :=
$$\sqrt[3]{3 \cdot \frac{\text{heatervol}}{4 \cdot 3.14}}$$

Assign typical input values for variables

Calculate value of temperature increase based on these variables

$$T(k, K, R, t, F, a) = 15.431381211$$

Calculate sensitivity of temperature, T, to each independent variable

Relative sensitivity factors:

$$dTda := \frac{d}{da}T(k,K,R,t,F,a) \qquad dTda = 175.915435932 \\ Sa := dTda \cdot \frac{a}{T(k,K,R,t,F,a)} \\ Sa = 2.005362496$$

$$dTdF = \frac{d}{dF}T(k,K,R,t,F,a) \qquad dTdF = 0.035638294$$

$$SF = dTdF \cdot \frac{F}{T(k,K,R,t,F,a)} \qquad SF = 1$$

$$dTdK = \frac{d}{dK}T(k,K,R,t,F,a) \qquad dTdK = -9.077283065 \\ SK := dTdK \cdot \frac{K}{T(k,K,R,t,F,a)} \qquad SK = -1$$

$$dTdR \approx \frac{d}{dR}T(k,K,R,t,F,a) \qquad dTdR = -49.01785023 \\ SR \approx dTdR - \frac{R}{T(k,K,R,t,F,a)} \qquad SR = -1.270601758$$

$$dTdk = \frac{d}{dk}T(k,K,R,t,F,a) \qquad dTdk = 2558130.10277464 \\ Sk = dTdk \cdot \frac{k}{T(k,K,R,t,F,a)} \qquad Sk = 0.132619631$$

$$dTdt = \frac{d}{dt}T(k, K, R, t, F, a) \qquad dTdt = 0.000001579 \\ St := dTdt - \frac{t}{T(k, K, R, t, F, a)} \\ St = 0.132619631$$

Assigin input measurement errors for variables

$$da = 0.005$$
 $dR = 0.01$ dR and da are in units of meters dF is in units of watts. Plus/minus 4 W is the standard $dF = 4$ $dK = 2 \times 10^{-107}$ $dE = 2010$ $dE = 2010$

Calculate error in temperature, T, resulting from input measurement errors

$$errorinT = \sqrt{\left[dTda\cdot da\right]^2 + \left[dTdF\cdot dF\right]^2 + \left(dTdK\cdot dK\right)^2 + \left[dTdR\cdot dR\right]^2 + \left(dTdk\cdot dk\right)^2 + \left(dTdr\cdot d\eta\right)^2}$$

errorinT = 2.702363742

percenterror
$$\succ 100 \cdot \frac{\text{errorinT}}{T(k, K, R, t, F, a)}$$
 percenterror = 17.512131316