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# Benchmark Calculation Of Inclusive Responses In The Four-Body Nuclear System

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# Benchmark calculation of inclusive responses in the four-body nuclear system*

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This paper reports on a recent benchmark calculation in the four-nucleon system, aimed at investigating the reliability of the no-core shell model (NCSM) approach to the description of inclusive response functions via the Lorentz integral transform (LIT) method.

## 1. Introduction

There is much interest in obtaining a more microscopic approach to nuclear reaction theory. In the past decade a new approach, based on an integral transform with a Lorentzian kernel [1], has opened a way to evaluate rigorously reaction observables, reducing the continuum problem to a bound-state problem [2]. This technique, known as the Lorentz integral transform (LIT) method, combined with an accurate bound-state technique such as the effective-interaction hyper-spherical harmonics (EIHH) [3] has allowed the calculation of electromagnetic reaction cross sections beyond break-up thresholds of nuclei up to A=7 [4].

The EIHH and NCSM [5] approaches are rather similar bound-state techniques. However, only the latter has made use of realistic interactions in calculations with $A > 4$. Indeed, the NCSM has the advantage that one can use an equivalent Slater determinant

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basis, allowing description of a larger range of masses. Therefore, it is of great interest to investigate the possibility of combining the NCSM and LIT approaches.

In this regard, we have recently performed a benchmark test in the four-nucleon system [6], consisting of a calculation of the $^4$He response functions for two different excitation operators within both the EIHH and the NCSM techniques. The input interaction for these test calculations is the semi-realistic Minnesota (MN) [8] potential.

After a brief overview of the LIT, NCSM and EIHH approaches, we present and discuss some of the results obtained.

## 2. The LIT approach

In terms of the transition matrix elements to the various allowed final states, the inclusive response of a system to a perturbation-induced reaction takes the form:

$$R(\omega) = \int d\Psi_f \left| \langle \Psi_f | \hat{O} | \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega) \, , \tag{1}$$

where $\omega$ represents the energy transferred by the probe, and $\hat{O}$ the excitation operator. Wave functions and energies of the ground and final states of the perturbed system are denoted by $\left|\Psi_{0/f}\right\rangle$ and $E_{0/f}$, respectively.

While in conventional approaches one usually starts from Eq. (1), in the LIT method [1] one obtains $R(\omega)$ after the inversion of its integral transform with a Lorentzian kernel

$$L(\sigma_R, \sigma_I) = \int d\omega \frac{R(\omega)}{(\omega - \sigma_R)^2 + \sigma_I^2} = \langle \widetilde{\Psi} | \widetilde{\Psi} \rangle \, . \tag{2}$$

Indeed, in order to calculate such a transform it is sufficient to solve the inhomogeneous "Schrödinger-like" equation

$$(H - E_0 - \sigma_R + i\sigma_I) | \widetilde{\Psi} \rangle = \hat{O} | \Psi_0 \rangle . \tag{3}$$

Because of the presence of an imaginary part $\sigma_I$ in Eq. (3) and the fact that the right-hand side of this same equation is localized, one has an asymptotic boundary condition similar to a bound state. Thus, one can apply bound-state techniques for its solution, and, in particular, expansions over basis sets of localized functions. Morover, the solution of Eq. (3) is unique. For inversion methods we refer the reader to Ref. [9].

## 3. NCSM versus EIHH

The NCSM and EIHH approaches are bound-state techniques based on an expansion of the Schrödinger wave function in terms of a complete set of localized states. In both techniques, one works in a finite subset of the Hilbert space, called the model space (or $P$-space). In order to account for many-body correlations left out by the truncation of the Hilbert space, one then builds an effective interaction by means of a unitary transformation [7] in the so-called cluster approximation [5]. The main differences between the NCSM and EIHH techniques originate from the choice of the localized $A$-body basis states: the harmonic oscillator (HO) basis function in the NCSM case, and the hyper-spherical-harmonics (HH) functions in the EIHH case. The latter choice results

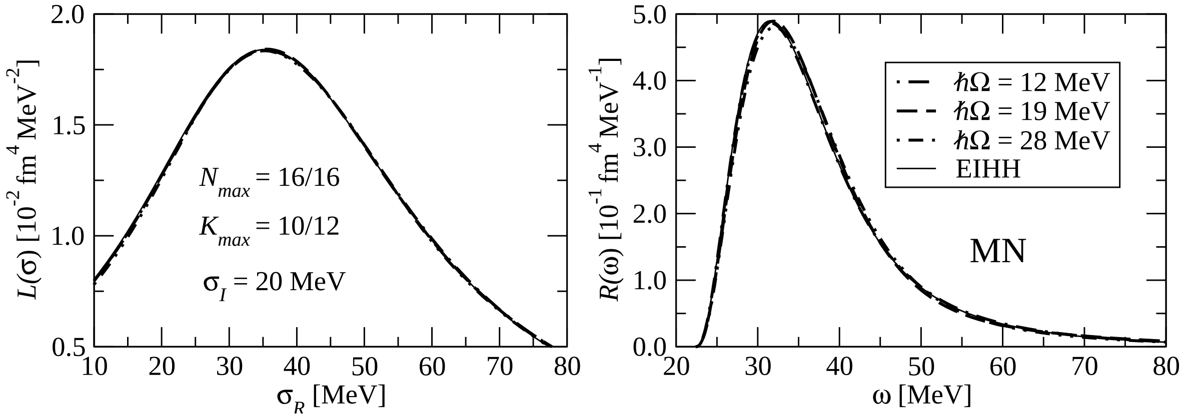


Figure 1. $^4$He: dependence on the HO frequency $\Omega$ of (left panel) the NCSM LIT for $\sigma_I = 20$ MeV, and (right panel) inclusive response for the isoscalar quadrupole transition. Note that $N_{max} = 16/16$ is not a complete model space (see text for details). The solid lines represent the EIHH results for $K_{max} = 10/12$.

in a different asymptotics for the wave functions and a different definition for the model space. In particular, in the NCSM the $P$-space is spanned by states with the total number of HO quanta $N \leq N_{max}$ above the ground state configuration, whereas in the EIHH approach the model space is defined by the HH functions with $K \leq K_{max}$. In view of the above mentioned differences, it is clear that the reliability of the NCSM approach for the description of inclusive response functions via the LIT method, especially concerning the problem of convergence, is not obvious *a priori* and a benchmark calculation is needed in order to assess it.

## 4. Isoscalar quadrupole inclusive response

We present results for the isoscalar quadrupole transition operator. We start the discussion with the NCSM and EIHH results for the LIT (2). Two model spaces are involved in this calculation: the $J^\pi, T = 0^+, 0$ $P$-space for the $\alpha$-particle ground state $|\Psi_0\rangle$ [entering the source term of Eq. (3)] and the $J^\pi, T = 2^+, 0$ for the LIT state $|\tilde{\Psi}\rangle$. In our largest model space NCSM calculations ($N_{max} = 16/16$), we have omitted the contributions coming from the transitions to the $J^\pi T = 2^+, 0$ with $N_{max} = 18$.[2] In the left panel of Fig. 1, the NCSM LIT's in the largest model space for three different HO frequencies ($\hbar\Omega = 12, 19,$ and 28 MeV) are compared with the EIHH converged calculation. One sees that the omitted transitions do not cause a loss of strength in the NCSM curves. Indeed, frequency independent results in good agreement with the EIHH are obtained for $N_{max} = 16/16$.

The right panel of Fig. 1 shows the NCSM and EIHH results for the inclusive response to the isoscalar quadrupole excitation obtained by inverting the LIT's presented in the left panel. The HO frequency value of $\hbar\Omega = 12$ MeV leads to the best agreement with the EIHH response (within 5% in the energy interval from threshold to $\sim 50$ MeV), and the

[2]Note that $N_{max} = N/N'(K_{max} = K/K')$ means that the NCSM(EIHH) calculations were performed for a model space of size $N_{max} = N(K_{max} = K)$ for the $0^+, 0$ and $N_{max} = N'(K_{max} = K')$ for the $2^+, 0$ states, respectively.

discrepancy among the four curves never exceeds the 10% in the range 25 MeV $\leq \omega \leq$ 60 MeV, where the response presents a resonant shape. Indeed, the numerical inversion procedure [9] is very sensitive to the accuracy in the calculation of the LIT, especially in the low-$\sigma_R$ region, where (for a fixed model space) smaller HO frequencies provide a better sampling of the complex-energy continuum. More delicate is the region of the high-energy tail, where the response is very small.

In conclusion, we find that the NSCM can be successfully applied to the solutions of the bound-state equations required by the LIT method. However, although the level of precision reached by the NCSM in this benchmarking calculation with a semi-realistic interaction is encouraging, in the perspective of LIT investigations on heavier nuclei, the large model spaces needed in the NCSM calculations in order to achieve the required accuracy suggest that a more substantial numerical effort will be necessary. In this respect, effective filed-theory two- and three-body potentials, which present a rather soft core, may be convenient input interactions. Their application in the evaluation of the $^4$He total photo-disintegration via the NCSM and LIT approaches is currently under investigation.

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