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Cold Energy Integration

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Disclaimer

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The following is based on [1], and uses somewhat similar notation. The table below has some definitions for clarity.

Table 1: Notation

symbol	meaning
η	compression measure, $\eta = \rho/\rho_o$
μ	compression measure, $\mu = \eta - 1$
ε	specific internal energy, $\varepsilon = \varepsilon_\vartheta + \varepsilon_c$; $E = \varepsilon\rho_o$;
ε_c	specific cold energy
ε_ϑ	thermal part, $\varepsilon_\vartheta = \varepsilon - \varepsilon_c = c_\eta\vartheta$
p_b	bulk pressure; total pressure for elastically isotropic materials; in [1] this is p_e, the "equilibrium pressure"

We use a bulk pressure of the form

$$p_b(\mu, \varepsilon) = p_g(\mu) + \rho_o \cdot (\Gamma_o + a\mu) \cdot \varepsilon \tag{1}$$

with a common form of p_g being

$$p_g(\mu) = \frac{K_o\mu\left[1 + (1 - \frac{\Gamma_o}{2})\mu - \frac{a\mu^2}{2}\right]}{\left[1 - (S-1)\mu\right]^2} \tag{2}$$

in compression and

$$p_g(\mu) = K_o\mu \tag{3}$$

in tension. To satisfy the Mie-Grüneisen formalism as described by Drumheller [1], let the cold pressure be

$$p_c(\mu) = p_g(\mu) + \rho_o \cdot (\Gamma_o + a\mu) \cdot \varepsilon_c(\mu). \tag{4}$$

This gives

$$\begin{aligned} p_b &= p_c + \rho_o \cdot (\Gamma_o + a\mu) \cdot (\varepsilon - \varepsilon_c) && (5) \\ &= p_c + \rho \cdot \Gamma \cdot (\varepsilon - \varepsilon_c) \quad \text{where} \quad \Gamma = \frac{\Gamma_o + a\mu}{\eta} && (6) \end{aligned}$$

so that we recover Equation 4.72 in [1]. We can then use Equation 4.73 in [1], which reads

$$p_{\mathrm{c}} = -\rho_{\mathrm{o}} \frac{d\varepsilon_{\mathrm{c}}}{dF} \quad \text{where} \quad F = 1/\eta, \tag{7}$$

to integrate the cold energy. After some manipulation, we end up integrating

$$\frac{\mathrm{d}\varepsilon_{\mathrm{c}}}{\mathrm{d}\mu} = \frac{p_{\mathrm{c}}}{(\rho_{\mathrm{o}})(\mu+1)^2} \tag{8}$$

starting at $\varepsilon_{\mathrm{c}}(\mu = 0) = -c_\eta \vartheta_{\mathrm{ref}}$ so that $\varepsilon(\mu = 0, \vartheta = \vartheta_{\mathrm{ref}}) = 0$.

All of this merrily neglects contributions from non-bulk strains.

References

[1] D. S. Drumheller. *Introduction to wave propagation in nonlinear fluids and solids.* Cambridge University Press, 1998.