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Proceedings of RIKEN BNL Research Center Workshop Volume 15

QCD PHASE TRANSITIONS

November 4–7, 1998

Organizers

Thomas Schäfer and Edward Shuryak

RIKEN BNL Research Center
Building 510, Brookhaven National Laboratory, Upton, NY 11973, USA

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Volume 5 - Color Superconductivity, Instantons and Parity (Non?)-Conservation at High Baryon Density - BNL-65105

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Volume 1 - Open Standards for Cascade Models for RHIC - BNL-64722

DISCLAIMER

Preface to the Series

The RIKEN BNL Research Center was established this April at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkysho" (Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including hard QCD/spin physics, lattice QCD and RHIC physics through nurturing of a new generation of young physicists.

For the first year, the Center will have only a Theory Group, with an Experimental Group to be structured later. The Theory Group will consist of about 12-15 Postdocs and Fellows, and plans to have an active Visiting Scientist program. A 0.6 teraflop parallel processor will be completed at the Center by the end of this year. In addition, the Center organizes workshops centered on specific problems in strong interactions.

Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form a proceedings, which can therefore be available within a short time.

T.D. Lee
July 4, 1997

CONTENTS

Introduction

The title of the workshop, "The QCD Phase Transitions", in fact happened to be too narrow for its real contents. It would be more accurate to say that it was devoted to different phases of QCD and QCD-related gauge theories, with strong emphasis on discussion of the underlying non-perturbative mechanisms which manifest themselves as all those phases.

Before we go to specifics, let us emphasize one important aspect of the present status of non-perturbative Quantum Field Theory in general. It remains true that its studies do not get attention proportional to the intellectual challenge they deserve, and that the theorists working on it remain very fragmented. The efforts to create Theory of Everything including Quantum Gravity have attracted the lion share of attention and young talent. Nevertheless, in the last few years there was also a tremendous progress and even some shift of attention toward emphasis on the unity of non-perturbative phenomena. For example, we have seen some efforts to connect the lessons from recent progress in Supersymmetric theories with that in QCD, as derived from phenomenology and lattice. Another example is Maldacena conjecture and related development, which connect three things together, string theory, super-gravity and the (N=4) supersymmetric gauge theory. Although the progress mentioned is remarkable by itself, if we would listen to each other more we may have chance to strengthen the field and reach better understanding of the spectacular non-perturbative physics.

That is why the workshop was an attempt to bring together people which normally belong to different communities and even cultures (they use different tools, from lattice simulations to models to exact solutions), in order to discuss common physics. It was a very successful, eye-opening meeting for many participants, as some of them said in the last round of discussions. Even organizers (who of course have contacted many speakers in advance) were amazed by completely unexpected things which were popping out of one talk after another. Extensive afternoon discussion, in which we always return back to the morning talks, has helped to clarify many issues.

One specific issue which appeared in many talks at the workshop was the surprisingly dominant role of instantons. We hear about that from lattice practitioners, people who make models for vacuum and extreme QCD, and even in the discussion of the physical origin of the now famous anti-deSitter 5-d space. In these (admittedly very different situations) people find that restricting ourselves to instanton-induced effects, one can actually reproduce results known from other methods (or experiment).

The enclosed copies of some main transparencies were reordered compared to the workshop schedule and divided into 5 major subjects, (i) **High density QCD**, (ii) **High temperature QCD**, (iii) **Lattice instantons**, (iv) **QCD at large number of flavors**, (v) **The lessons from Supersymmetry**, (vi) **Topological effects in Applications** which we now discuss subsequently.

Finally, we are grateful to RIKEN/BNL Center for its support, and to all the speakers for their inspiring work.

High density QCD

Color-Flavor Locking vs. 'Classic' Color Superconductivity

- F. Wilczek

Summary: A new ordering for QCD at high density is proposed. It takes advantage of the equality ~~and~~ of flavor and color number (both = 3). In this 'color-flavor locked' state, both global chiral~~ity~~ $SU(3)^L \times SU(3)^R$ and local c $SU(3)^c$ are broken, as is global baryon numbe $U(1)^B$; a diagonal $SU(3)^\Delta$ global vector symmetry remains. The state is strictly distinguishable, using gauge invariant local order parameters, from all previously contemplated states for QCD. In the color-flavor locked state all elementary excitations have integral charge. It is a true superfluid.

A more detailed account is in hep-ph/9804

Two Flavors: Quick Review of Color Superconductivity

2.1 Motivations

Asymptotic freedom + BCS instability; attractive channel

2.2 Condensations + Symmetry

Primary: $\langle q_i^{\alpha} C\gamma_5 q_j^{\beta} \rangle \propto \epsilon_{ij}\epsilon^{\alpha\beta 3}$

(Secondary: $\langle q_i^{\alpha} \sigma_{0a} q_j^{\beta} \rangle \propto \ldots \delta_3^{\alpha}$

$$SU(3)^{color} \times SU(2)^L \times SU(2)^R \times U(1)^B$$

$$\rightarrow SU(2)^{color} \times SU(2)^L \times SU(2)^R \times U(1)$$

- Primary condensation leaves gapless modes.
- No true order parameter.
- Chiral symmetry restored (i.e. unbroken).

Three Flavors: Color-Flavor Locking

hep-ph/9804403

3.1 Motivations

i) $\langle q^{\alpha}_{a} \, q^{\beta}_{b} \rangle \propto \epsilon^{\alpha\beta I} \, \epsilon_{abI}$

generalizes / reinforces 2-flavor form ($I = 3$ only).

ii) Analogy to ^{3}He "B" phase, which locks spin + orbital angular momentum.

3.2 Symmetry

$$SU(3)^{color} \times SU(3)^{L} \times SU(3)^{R} \times U(1)^{B}$$

$$\downarrow$$

$$SU(3)^{\Delta}$$

Note that chiral symmetry is broken.

The residual diagonal symmetry is global.

3.3 Condensations

All the ones consistent with the symmetry should occur at some level.

$$\langle q_a^{\alpha} q_b^{\beta} \rangle = \kappa_1 \delta_a^{\alpha} \delta_b^{\beta} + \kappa_2 \delta_b^{\alpha} \delta_a^{\beta}$$

$$\langle \bar{q}_{\alpha}^{\dot{a}} q_b^{\beta} \rangle = l_1 \delta_{\alpha}^{a} \delta_b^{\beta} + \underbrace{l_2 \delta_b^{a} \delta_{\alpha}^{\beta}}_{\text{"usual" } \chi \text{ cond.}}$$

$\nearrow$ gauge inva

$$\boxed{\langle (qqq)^2 \rangle \neq 0}$$

* This one is especially profound. i) It is gauge

invariant. ii) It rigorously differentiates the state from conventional χSB.

iii) It corresponds to true superfluidity*. iv) It can be used as a strict order parameter even if $m_q \neq 0$.

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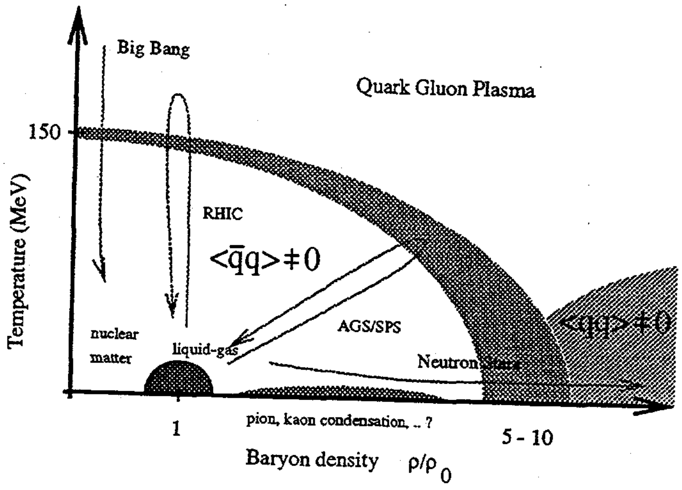
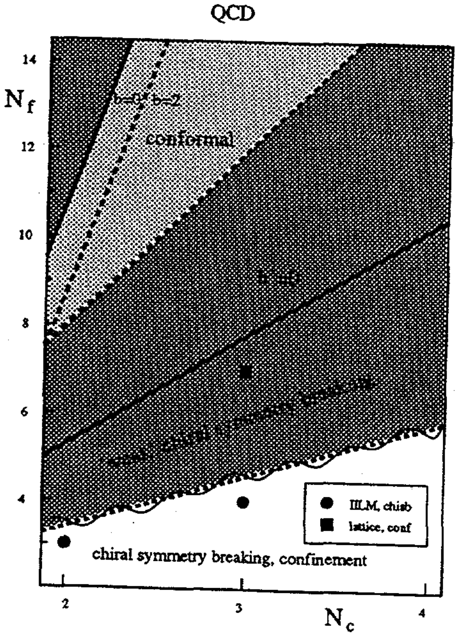
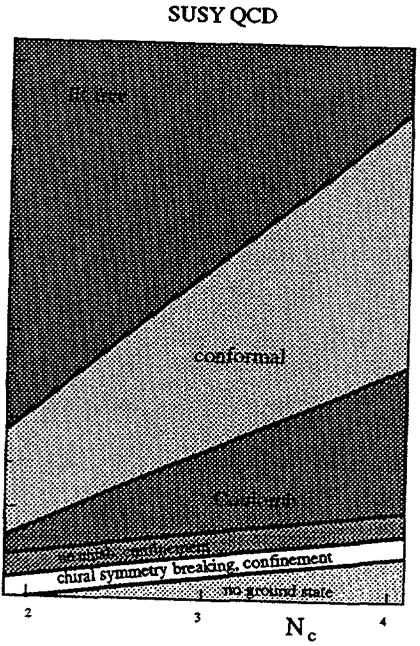
* Not superconductivity - see 3.4.

Extreme QCD in the Instanton Model

Thomas Schäfer
Institute for Advanced Study
Princeton



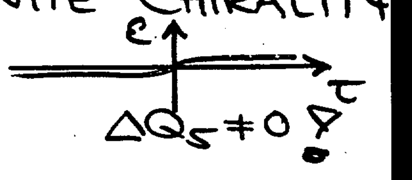




QUARKS IN THE INSTANTON VACUUM

- ONE INSTANTON: DIRAC OPERATOR HAS ZERO MODE

$i\not{D}\psi_0^{I,A}=0 \qquad \gamma_5\psi_0^{I,A}=\pm\psi_0^{I,A}$ ↳ DEFINITE CHIRALITY



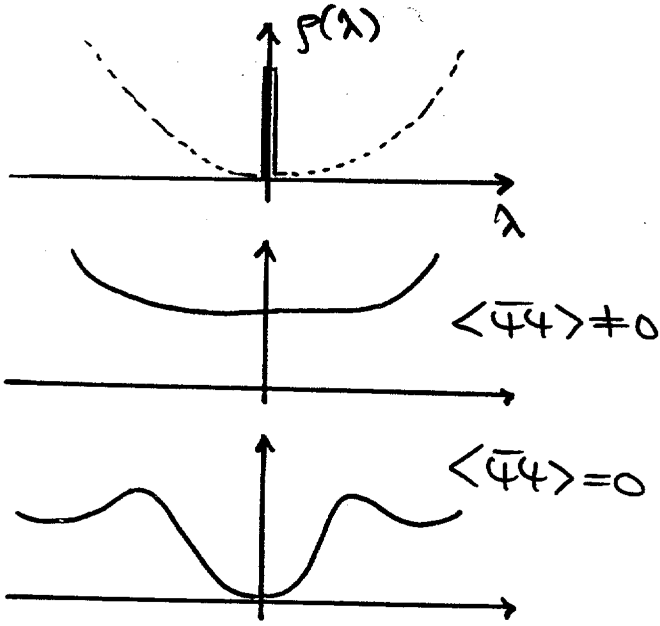
- MANY INSTANTONS: ZERO MODES INTERACT

$$i\not{D}=\begin{pmatrix}0 & T_{IA}\\ T_{IA}^{+} & 0\end{pmatrix} \qquad T_{IA}=\int d^4x\,\psi_I^{+}\,i\not{D}\,\psi_A$$

- QUARK CONDENSATE (CASHER-BANKS)

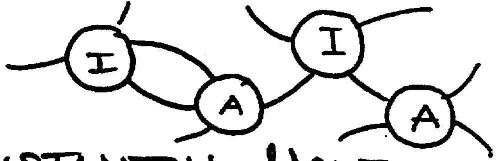
$$\langle\bar{q}q\rangle=\frac{1}{\pi}\rho(\lambda=0)$$

COMPARE:
$\sigma=e^2D\rho(E_F)$
(KONDO)



ONE INSTANTON

INSTANTON LIQUID

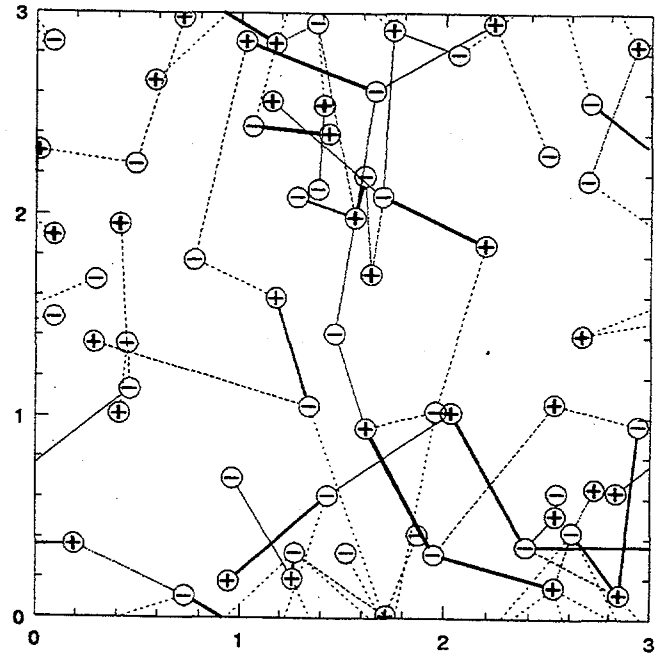


INSTANTON MOLECULES

WHY INSTANTONS?

DOMINATE DIRAC SPECTRUM AT SMALL VIRTUALITY

CHIRAL RESTORATION IN THE INSTANTON LIQUID

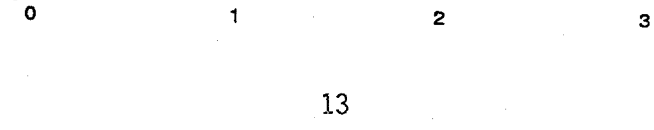


$T^{-1} = 3\ \Lambda_{QC}$

⇝ $T \simeq 75$ Me

$\underline{T < T_c}$

—— - - - MATRIX ELEMENT OF ($i\not{D}$) ⇝ "HOPPING" PROBABILITY

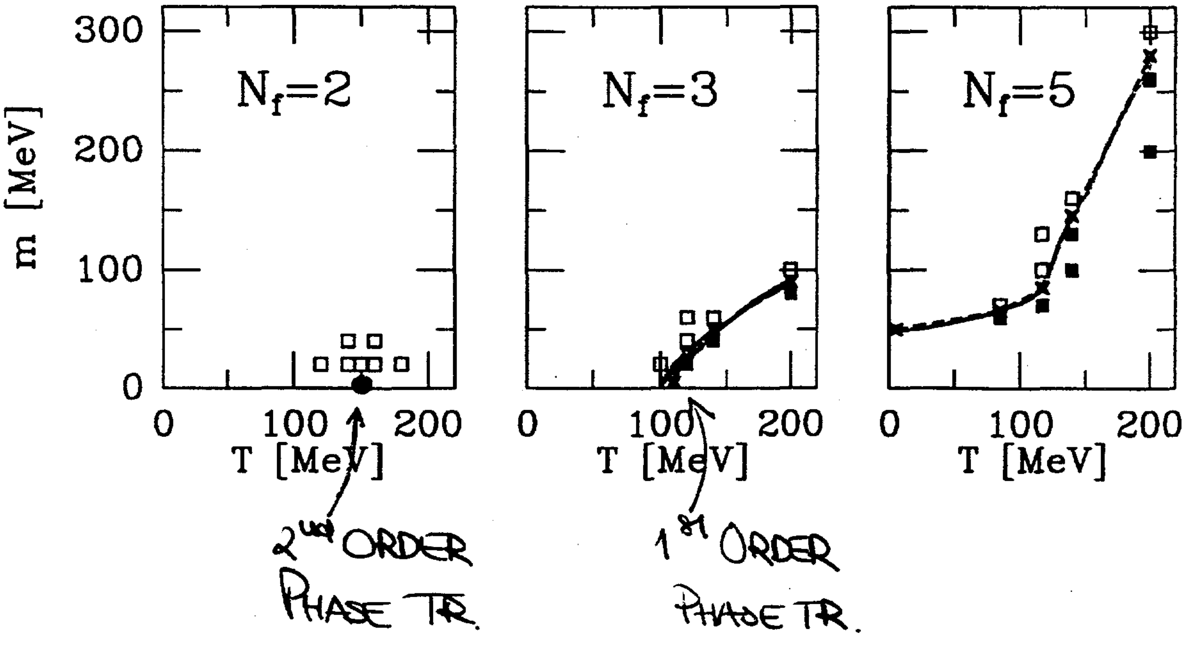


$T^{-1} = 1.4\ \Lambda_{QC}$

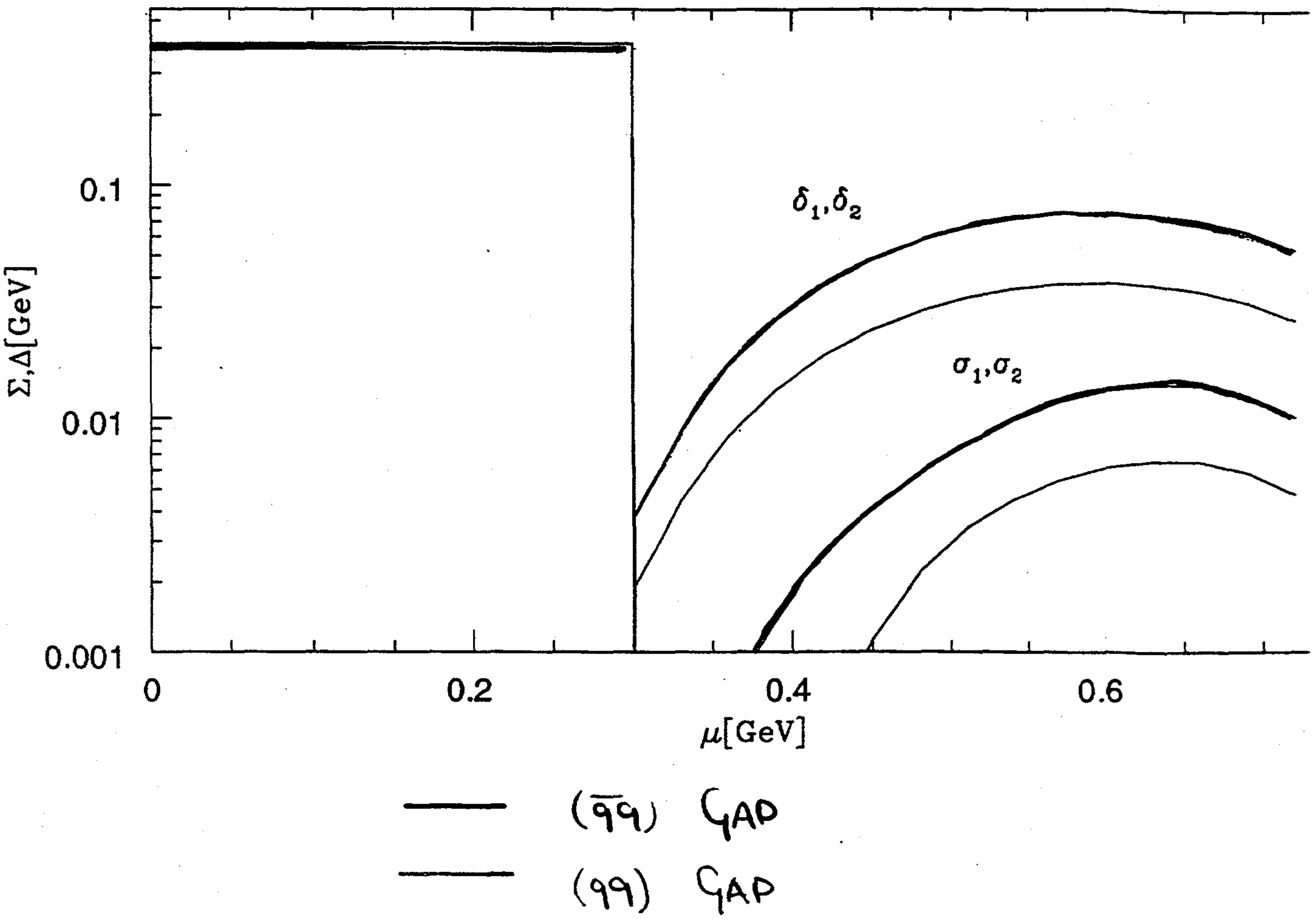
⇝ $T = 160$ Me

$\underline{T > T_c}$

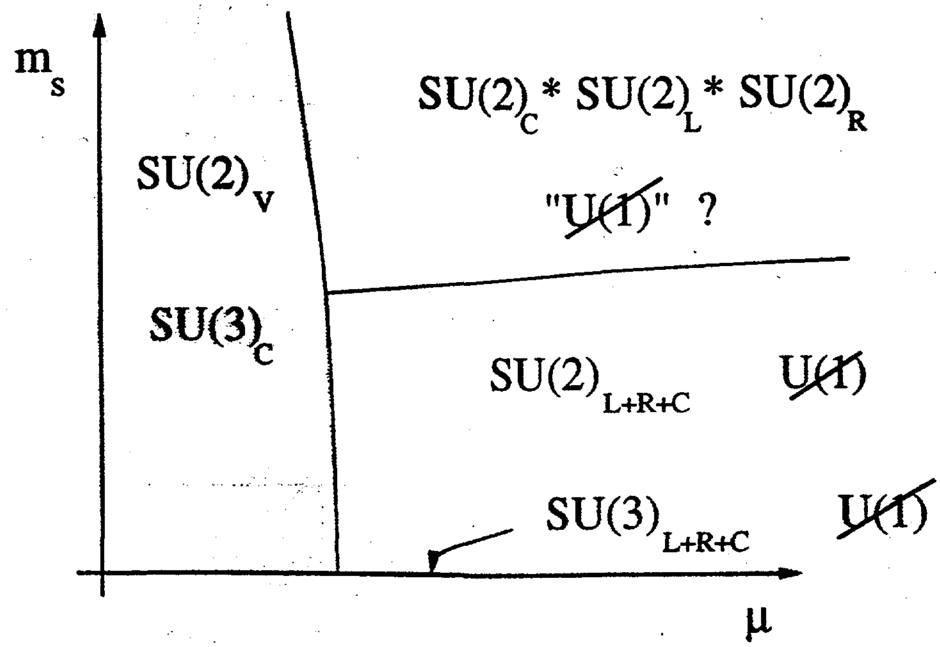
FINITE T PHASE TRANSITION FOR DIFF. NUMBER OF FLAVORS

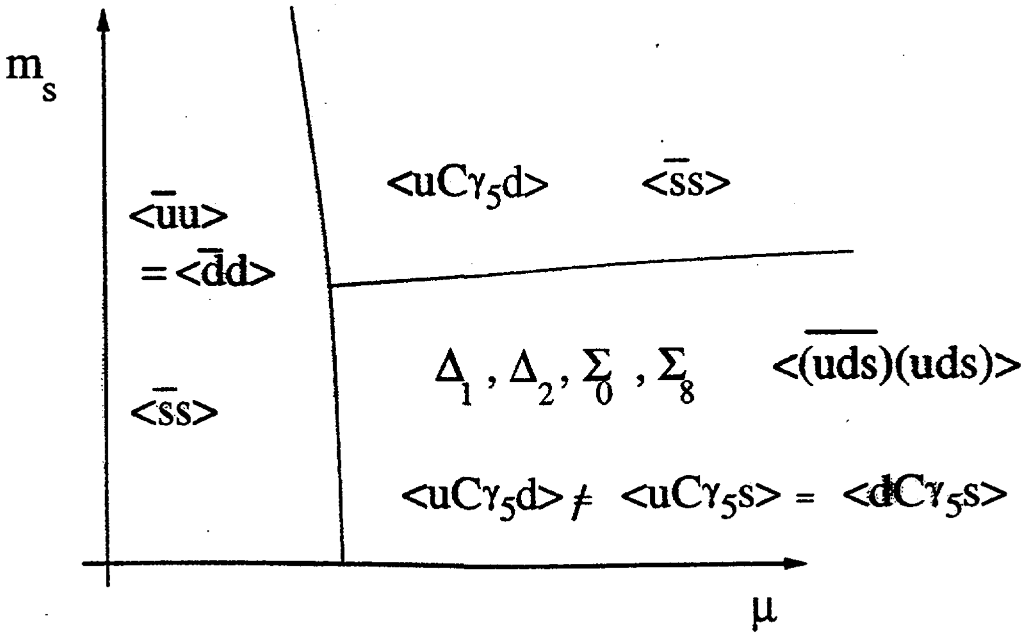


QUARK - DIQUARK CONDENSATION IN THREE FLAVOR QCD

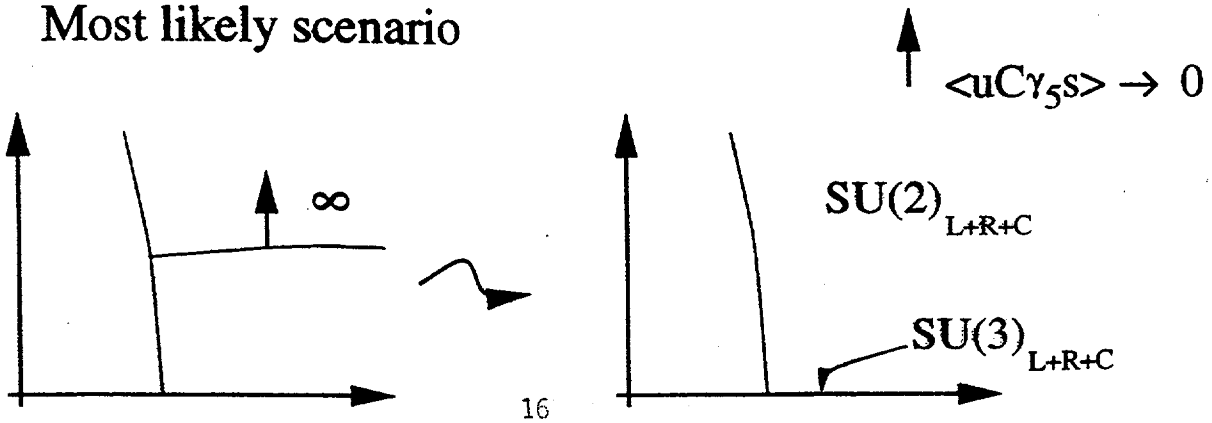


N_f=3 Phase diagram





Most likely scenario



RGE ANALYSIS

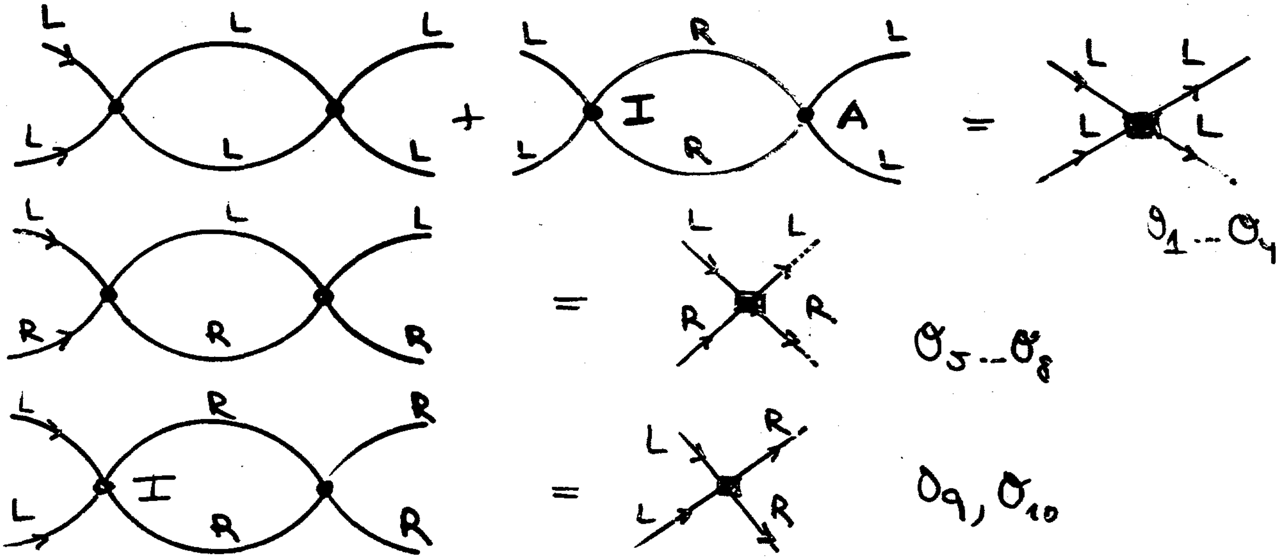
(T.S. + F. WILCZEK, HEP-PH)

- CONSIDER MOST GENERAL 4-FERMION VERTEX

$O_1 - O_8$ $U_A(1)$ SYM. $(\bar{\psi}\gamma_\mu\lambda^a\psi)^2, \ldots$

O_9, O_{10} $U_A(1)$ BR. $\det_F(\bar{\psi}_L\psi_R)$, $\det_F(\bar{\psi}_L\vec{\Sigma}\psi_R)$

- GET RENORMALIZED BY QQ-SCATTERING IN THE VICINITY OF THE FERMI SURFACE



$$\rightarrow \quad \frac{d}{dt} G_i = \Gamma_i^{jk} G_j G_k$$

$$t = \log\left(\frac{\Lambda_{IR}}{\Lambda_{UV}}\right)$$

SUMMARY

- INSTANTONS CAN PLAY A PART IN LARGE T, LARGE N_F, AND LARGE μ TRANSITION
- LARGE T: FORMATION OF MOLECULES ($\rightsquigarrow$ LARGE
  $\rightsquigarrow$ SPECIFIC CORRELATIONS IN PLASMA PHASE
- LARGE μ: SUPERCONDUCTING PHASE
  ORDER PARAMETER $\langle u C \gamma_5 \lambda_A d \rangle$
- LARGE μ, $N_F = 3$: COLOR-FLAVOR LOCKING
  INSTANTONS LEAD TO χSB IN HIGH
  DENSITY PHASE

Symmetry Breaking by Instantons at Finite Density

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We analyze the phases of QCD at zero temperature and finite quark density. An effective action which features instanton-induced interactions is used to consider the possibilites of chiral and diquark condensation. Since these two channels arise from the same interaction, their relative strengths are constrained by a coupling constant, itself dyanamically determined. A coupled set of Schwinger-Dyson-Gorkov equations is constructed and solved to first order in the instanton packing fraction. The resulting mean-field solutions are compared thermodynamically in order to specify the ground state as a function of chemical potential. Although a phase of mixed symmetry breaking is obtained, the primary thermodynamic competition is between a state of exclusively chiral symmetry breaking and one of color breaking alone. At low density, we recover the standard vacuum picture of spontaneously broken chiral symmetry. However, at high density the color superconducting state becomes favored and a first-order phase transition separates the equilibria. This transition occurs at a chemical potential of $\mu_c = 335$ MeV, which corresponds to a jump in quark density from $0.06\,\mathrm{fm}^{-3}$ to $1.14\,\mathrm{fm}^{-3}$. As an intermediate step in the density calculation, we calculate the quark occupation numbers as functions of spatial momentum. This analysis illustrates the different physics at work in each phase, in that chiral symmetry breaking leads to an effective quark mass which reduces the radius of the fermi surface, whereas quark pairing smears the surface itself through a redistribution of the states.

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*Speaker at the workshop, RIKEN BNL Research Center, 04 November 1998

INSTANTONS IN THE QCD VACUUM INDUCE MULTI-QUARK INTERACTIONS OF ORDER $2N_F$.

THEY CAN BE OBTAINED IN THE ZERO MODE APPROXIMATION: ONE WRITES THE EUCLIDEAN QUARK PROPAGATOR

$$S(x,y) = -\sum_{\lambda} \frac{\phi_\lambda(x)\,\phi^{+}_{\lambda}(y)}{\lambda + im}$$

$$\approx -\underbrace{\frac{\Phi_0(x)\,\tilde{\Phi}_0(y)}{im}}_{\text{ZERO MODES FOR LOW MOMENTA}} + \underbrace{S_0(x,y)}_{\text{FREE PROPAGATOR FOR HIGH MOMENTA}}$$

INCORPORATING THE FIRST TERM IN THE PARTITION FUNCTION WITH A LAGRANGE MULTIPLIER, λ, GENERATES

$$\mathcal{L}_{EFF} = \psi^{+}(i\not\partial - i\not{A})\psi + \lambda \prod^{N_F} \int \Big[\psi^{+}(x)(i\not\partial - i\not{A})\Phi_0(x-z)\,\tilde{\Phi}_0(z-y)(i\not\partial - i\not{A})\psi($$

λ IS THEN A DYNAMICALLY DETERMINED COUPLING CONSTANT, SCALING AS $\sqrt{\underbrace{\tfrac{N}{V}}_{\text{INSTANTON DENSITY}}}\;\underbrace{\bar{\rho}}_{\text{INSTANTON SIZE}}$ FOR $N_F = 2$.

A FOURIER TRANSFORMATION SIMPLIFIES CALCULATIONS. FOR 2 MASSLESS FLAVORS THE INTERACTION SEPARATES INTO TWO TERMS OF THE FORM

$$\lambda \int \psi_1^\dagger(p_1)\, \mathcal{F}(p_1,\mu)\, \mathcal{F}(k_1,-\mu)^\dagger\, \psi^1(k_1)\, \psi_2^\dagger(p_2)\, \mathcal{F}(p_2,\mu)\, \mathcal{F}(k_2,-\mu)^\dagger\, \psi^2(k_2)$$

$$\mathcal{F}(p,\mu) = (p+i\mu)^- \varphi(p,\mu)^+ \qquad \mathcal{F}(p,-\mu)^\dagger = \varphi^*(p,-\mu)^- (p+i\mu)^+$$

$$x^\pm = x_\mu \sigma_\mu^\pm \,, \quad \sigma_\mu^\pm = (\pm i\vec{\sigma}, \mathbf{1}) \,, \quad \varphi(p,\mu)^\pm \text{ ARE F.T.'ED } \Phi_0(x,\mu)$$

WHICH SUGGESTS THE POSSIBILITIES:

$$\langle \psi^\dagger \psi \rangle \neq 0 \qquad \text{AND/OR} \qquad \langle \psi \psi \rangle \neq 0$$

THESE CHANNELS WILL COMPETE FOR THE LIMITED RESOURCE λ. THE CONDENSATES ARE COMPUTED WITH ANSATZ OF THREE GREEN FUNCTIONS:

$$\langle \psi^{f\alpha i}(p)\, \psi^{g\beta j}(-p) \rangle = \epsilon^{fg} \epsilon^{ij} \epsilon^{\alpha\beta[\gamma]} F(p)$$

$$\alpha,\beta \neq [\gamma] : \langle \psi^{f\alpha i}(p)\, \psi^\dagger_{g\beta j}(p) \rangle = \delta^f_g \delta^\alpha_\beta S_1(p)^i_j = \begin{bmatrix} G_1(p) & Z_1(p) S_0 \\ Z_1(p) S_0 & G_1(p) \end{bmatrix}$$

$$\alpha,\beta = \gamma : \langle \psi^{f\alpha i}(p)\, \psi^\dagger_{g\beta j}(p) \rangle = \delta^{fg} \delta^\alpha_\beta S_2(p)^i_j = \begin{bmatrix} G_2(p) & Z_2(p) S_0 \\ Z_2(p) S_0 & G_2(p) \end{bmatrix}$$

SOLVING THE GAP EQUATIONS DERIVED FROM THE ONE-LOOP S-D-G EQUATIONS ALLOWS 4 PATTERNS OF SYMMETRY BREAKING (PHASES OF MATTER):

(0)	$g_1 = g_2 = f = 0$	FREE QUARKS; THIS REQUIRES $\lambda \to 0$, WHICH DOES NOT OCCUR.
(1)	$g_1 = g_2 \neq 0,\ f = 0$	STANDARD SPONTANEOUS CHIRAL SYMMETRY BREAKING.
(2)	$g_1 = g_2 = 0,\ f \neq 0$	DIQUARK CONDENSATION; A "COLOR SUPERCONDUCTING" STATE.
(3)	$g_1 \neq g_2 \neq 0,\ f \neq 0$	MIXED SYMMETRY BREAKING.

FOR GIVEN μ, THE PHASE IS DETERMINED BY FINDING THE ABSOLUTE MINIMUM IN

$$\frac{\Omega}{V} = \underbrace{\frac{\Omega_0}{V}}_{\text{FREE MASSLESS FERMIONS}} + \frac{N}{V} \ln \lambda$$

SO THE SOLUTION WITH <u>LOWEST λ</u> IS THE STABLE PHASE.

THE FOLLOWING RESULTS ARE FOR $N_c = 3$.

PHASE (3): MIXED SYMMETRY BREAKING

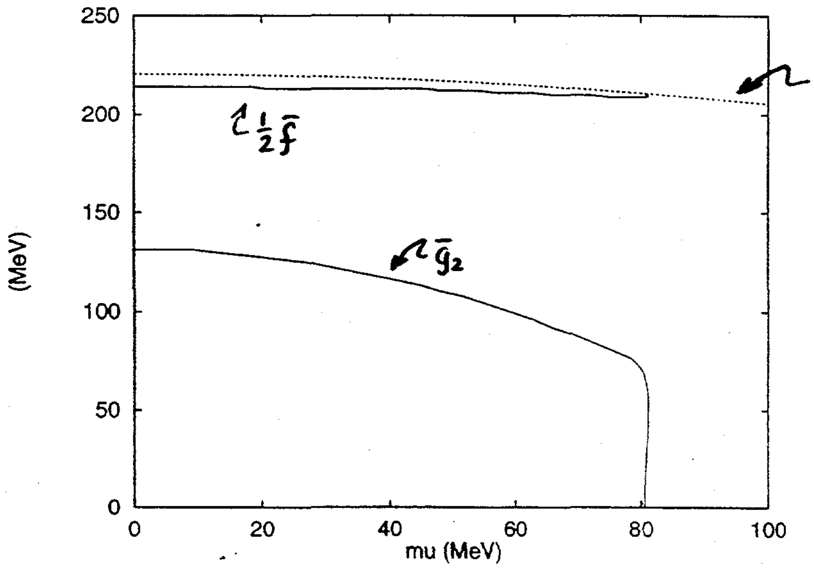
A SOLUTION EXISTS WITH

$g_1 = 0, \ f \neq 0$: DIQUARK CONDENSATION IN COLORS 1,2 (TRANSVERSE)

$g_2 \neq 0$: CHIRAL CONDENSATION IN COLOR 3 (LONGITUDINAL)

f AND g_2 ARE MUTUALLY CONSTRAINED BY

$\lambda = \frac{128}{N_V}\left(g_2^2 + \frac{4}{3} f^2\right)$ AND EVOLVE IN μ AS:



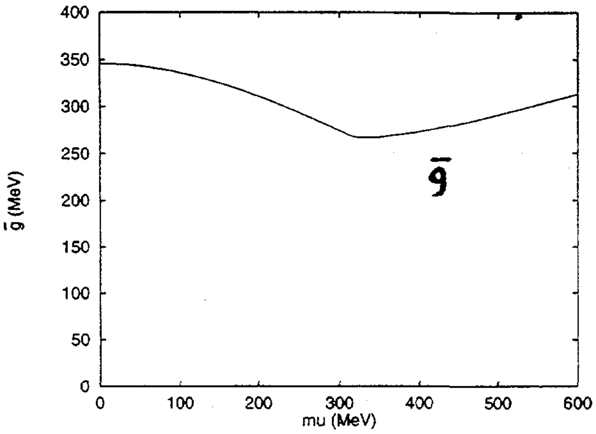
THE DASHED LINE IS THE PHASE (2) SOLUTION

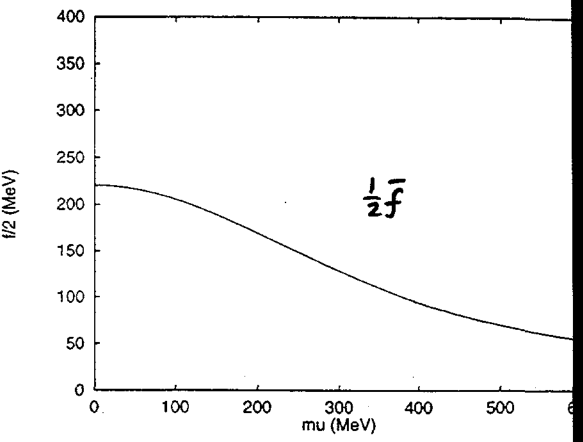
SO THIS PHASE IS SHORT-LIVED IN DENSITY.

IT IS ALSO A STRICTLY LOCAL MINIMUM; NEVER THERMODYNAMICALLY COMPETITIVE.

THE REMAINING THERMODYNAMIC COMPETITION IS

PHASE (1) VS. PHASE (2)



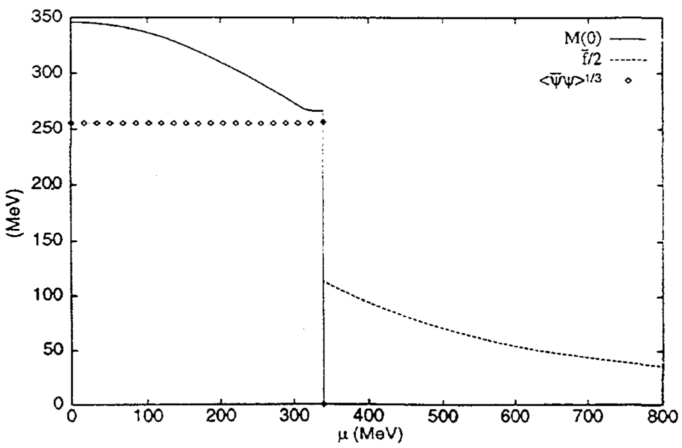


$$\lambda^{(1)} = \frac{2N_c^2}{N/V}\,\bar{g}^2 \qquad\qquad \lambda^{(2)} = \frac{4N_c(N_c-1)}{N/V}\,\bar{f}^2$$

IN THE VACUUM $\lambda^{(1)} < \lambda^{(2)}$, RECOVERING THE USUAL CHIRAL-BREAKING GROUND STATE. BUT $\bar{f}$ DECREASES CONTINUOUSLY, AND THE PHASE TRANSITION OCCURS WHEN $\lambda^{(2)} < \lambda^{(1)}$ AT μ_c

$$\frac{\bar{f}(\mu_c)}{\bar{g}(\mu_c)} = \sqrt{\frac{N_c}{2(N_c-1)}} = \frac{\sqrt{3}}{2}$$

NUMERICALLY THIS FIRST ORDER PHASE TRANSITION OCCURS AT $\mu_c \simeq 335$ MeV



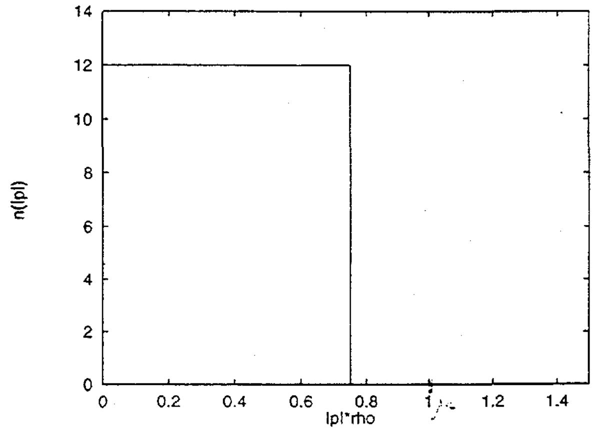
IN TERMS OF DENSITY:

$$n_q = \int d^3p \int dp_4 \; j_4(p_4, p; \mu)$$ ← USING THE CONSERVED CURRENT

$$= \int d^3p \; n(\vec{p})$$

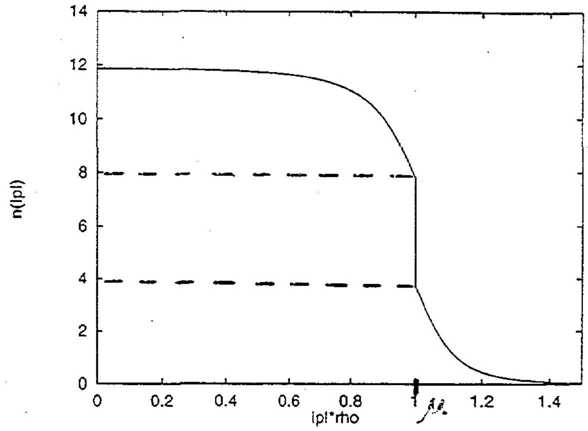
THE FERMION OCCUPATION NUMBERS FOR $\mu\bar{\rho} = 1$:

PHASE (1)



$p_F = \sqrt{\mu^2 - M(p_F)^2}$

PHASE (2)



$p_F = \mu$

INTEGRATING OVER $|\vec{p}|$, ONE SEES THE DISCONTINUITY

IN n_q AT $\mu_c = 335$ MeV.

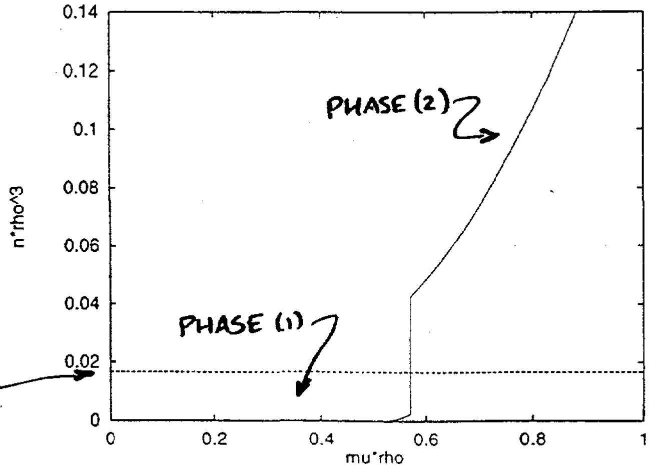
IT JUMPS FROM

$n^{(1)} = 0.06 \text{ fm}^{-3}$

TO

$n^{(2)} = 1.14 \text{ fm}^{-3}$

NOTE $3n^{SAT}_{NM} = 0.45 \text{ fm}^{-3}$ →



Chiral Restoration at Finite Density:

Instanton- versus Cooper-Pairs

Ralf Rapp (Stony Brook) (*), BNL 04.11.98

1.) Introduction

2.) Instanton Liquid Model (ILM) in Vacuum

3.) ILM at Finite μ_q

   3.1. Instanton Interactions: I-A-Molecules

   3.2. Quark Interactions: Cooper Pairs

   3.3. Statistical Mechanics of the ILM

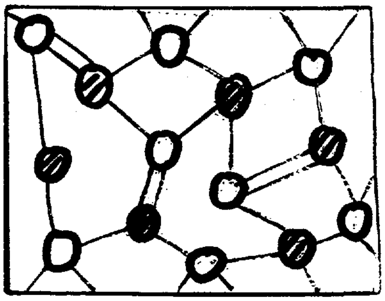
4.) Summary + Conclusions

(*) Collaborators:

E.V. Shuryak (SB), T. Schäfer (IAS), M. Velkovsky (BNL)

1. Introduction - An Instanton Point of View

QCD vacuum: **χS** broken due to 'random' instantons ⇒ delocalized quark-zero-modes ⇒ $\langle \bar{q} q \rangle \neq 0$



Finite T/μ_q: thermal/fermionic excitations 'melt' $\langle \bar{q} q \rangle$

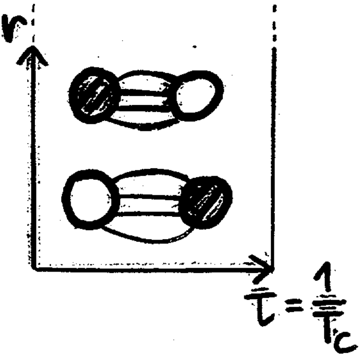
How exactly?

(1) Debye Screening of **I**-/**A**- Fields?

→ no, sets in only above T_c (Chu + Schramm PRD '95)

(2) **I**nstanton-**A**ntiinstanton Molecules?

→ yes at finite T (Schäfer, Shuryak, Verbaarschot '95): due to enhanced **I-A-** interaction (also present at finite μ_q!)



(3) Cooper Pairs at Finite μ_q?

random instanton liquid ⇒ color superconductor ($\mu_q^c \simeq 340$ MeV, Carter, Diakon
Bielefeld '98)

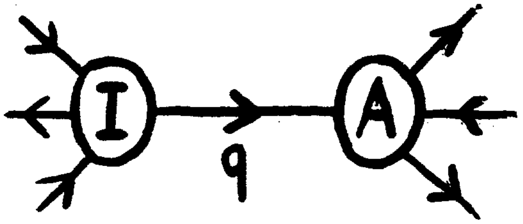
⇒ **I-A-Molecules** and/or **Cooper Pairs** ?

3) ILM at Finite μ_q

3.1. Instanton - Antiinstanton Interactions

⇝ through quark-zero-mode exchange

$T_{IA} \equiv$



quark-zero-mode wave function at finite μ_q from Dirac-equation in the instanton field: (Abrikosov Jr. '83

$$(i\not{D}_I - i\mu_q\gamma_4)\,\Psi_{0,I}(x;\mu_q) = 0$$

$$i\not{D}_I = i\not{\partial} + \not{A}_I^a\frac{\lambda^a}{2}$$ covariant derivative

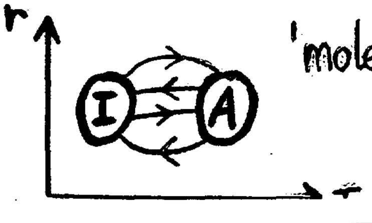
adjoint solution: $\Psi^{+}_{0,I}(x;\underline{\underline{-}}\mu_q)\,(i\not{D}_I - i\mu_q\gamma_4) = 0$

$$\boxed{\begin{aligned} T_{IA}(z,u;\mu_q) &= \int d^4x\,\Psi^{+}_{0,I}(x;-\mu_q)\,(i\not{D}_{I+A} - i\mu_q\gamma_4)\,\Psi_{0,A}(x-z;\mu_q) \\ &= i u_4 f_1(\tau,r;\mu_q) + i\frac{\vec{u}\cdot\vec{r}}{r} f_2(\tau,r;\mu_q) \end{aligned}}$$

relative
distance/color orientation

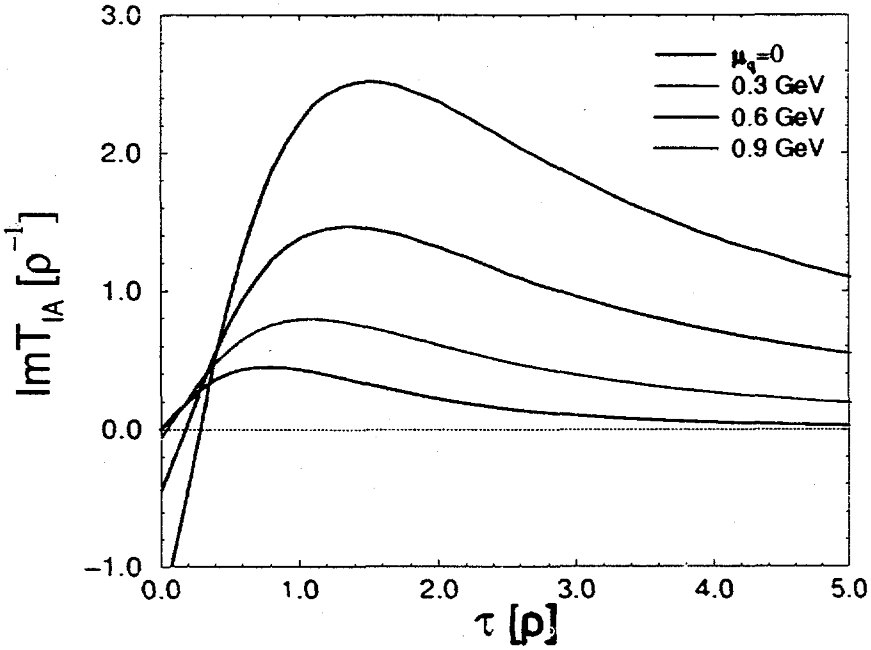
$f_1, f_2 \in \mathbb{R}$; $u_\mu \in \mathbb{C}$ for $SU(3)_c$

oscillating in r-direction
enhanced in τ-direction } ⇒ clustering tendency :

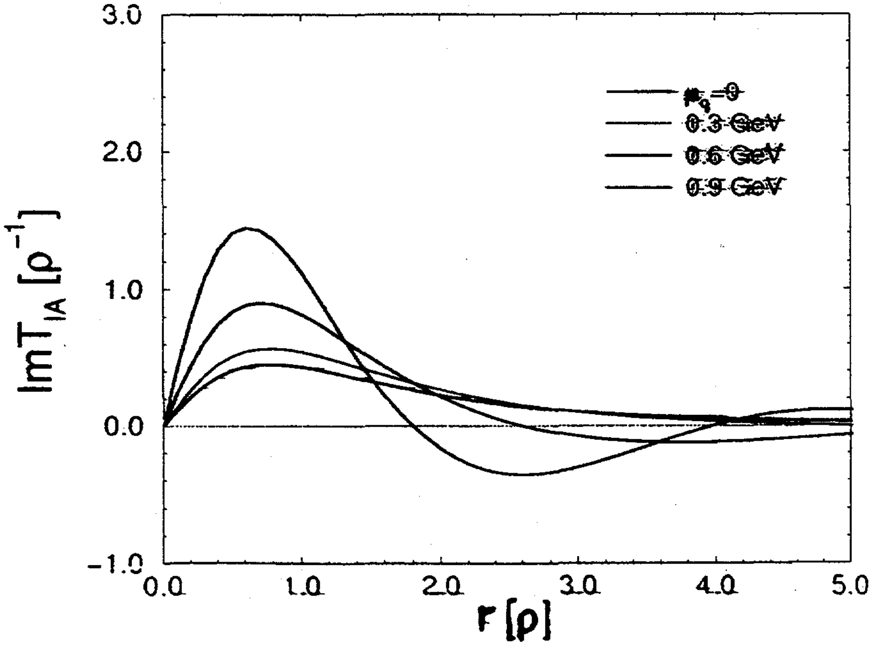


'molecule

Quark-Induced I-A-Interaction at Finite μ_q



Temporal Dependence: $f_1(\tau)$



Spatial Dependence: $f_2(r)$

can perform color-averaging in $\langle T_{IA} T_{AI} \rangle$ analytically

$\Rightarrow$ result $\in \mathbb{R}$:

with $T_{IA} \equiv i u_4 f_1 + i \frac{\vec{u} \cdot \vec{r}}{r} f_2$ obtain:

$$z_a \propto \int du \, T_{IA}(\mu_q) \, T_{IA}^{+}(-\mu_q) = \frac{1}{2N_c} \left[f_1(\mu_q) \, f_1(-\mu_q) + f_2(\mu_q) \, f_2(-\mu_q) \right] \quad \in \mathbb{R}$$

$$z_m \propto \int du \left[T_{IA}(\mu_q) \, T_{IA}^{+}(-\mu_q) \right]^{N_f=2} = \frac{1}{4N_c(N_c^2-1)} \left[(2N_c-1) \left\{ f_1^+ f_1^- + f_2^+ f_2^- \right\}^2 + \left\{ f_1^+ f_2^- - f_1^- f_2^+ \right\}^2 \right] \in \mathbb{R}$$

$\rightarrow$

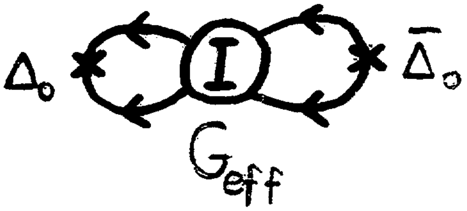
Pressure $P_{inst}(\mu_q) = -\Omega_{inst}(\mu_q) = n_a \left[1 + \log\left(\frac{z_a(\mu_q)}{n_a} \right) \right] + n_m \left[1 + \log\left(\frac{z_m(\mu_q)}{n_m} \right) \right] \quad \in \mathbb{R}$

(2) Quark Contribution : Fermi Sphere

$$\Omega_{quark}(M_q, \Delta_0 ; \mu_q) = \underset{\text{quark propagator}}{\mathrm{tr} \log(D_q)} - \underset{\text{selfenergy}}{\mathrm{tr}(D_q \Sigma_q)} + G_{eff} \, \mathrm{tr}(F) \, \mathrm{tr}(\bar{F})$$

quark propagator ↗ selfenergy ↗ Gorkov propagators ↗ ↗

'kinetic'



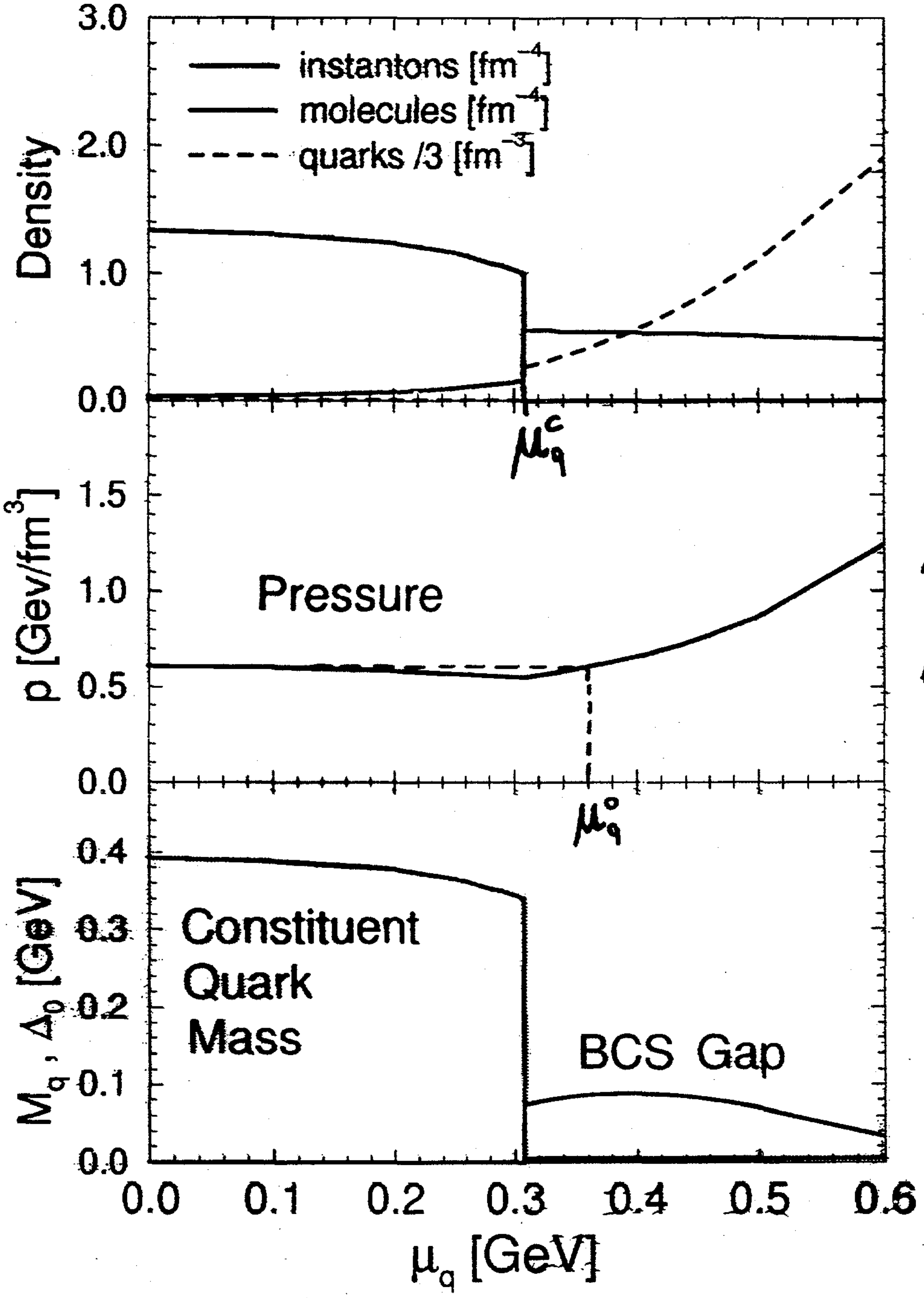
MFA:
constituent quark mass $M_q \propto -g^2 \langle \bar{q} q \rangle$
chiral condensate $\langle \bar{q} q \rangle = -\frac{1}{\pi g} \left(\frac{3}{2} n_a \right)^{1/2}$

~~~

$$\boxed{\Omega(n_a, n_m, \Delta_0 ; \mu_q) = \Omega_{inst} + \Omega_{quark}}$$

minimize w.r.t.
n_a, n_m, Δ_0
~~~

ILM at Finite Density: $N_f=2$, $N_c=3$



$\mu_q^c = 310$ MeV

$\mu_q^0 = 360$ MeV

4.) Summary + Conclusions

- Chiral Restoration in the **ILM** at finite density dominantly driven by **I-A-Molecule Formation**

- **MFA:** complex fermionic determinant eliminated after color-averaging

  $N_f=2$: $\mu_q^c = 310$ MeV , $\mu_q^o = 360$ MeV , 1st order ,

  $\Delta_0 \simeq 50-100$ MeV

- larger N_f accelerate Chiral Restoration

  ($N_f=3$: $\mu_q^c = 260$ MeV)

- <u>next step :</u> mesonic correlation functions

M. STEPHANOV
(SUNY, Stony Brook)
PHASE DIAGRAM OF QCD
AND RANDOM MATRICES

$T=0$:

$$Z=\sum_\alpha \exp\left(-\frac{E_\alpha-\mu N_\alpha}{T}\right).$$

$$\mu_0 \equiv \min_\alpha{}' \frac{E_\alpha}{N_\alpha}\;;$$

$$n(\mu)\equiv 0 \quad \text{for} \quad \mu<\mu_0$$

Free fermion: $p_F=\sqrt{\mu^2-m^2}$

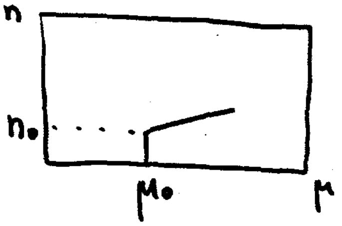
$$n(\mu)=\frac{1}{3\pi^2}(\mu^2-m^2)^{3/2}\quad,\ \mu>m=\mu_0.$$

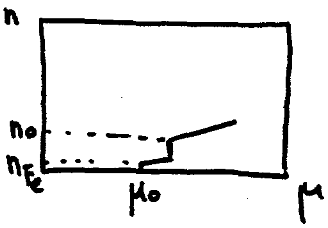
$$\frac{E}{N}=m_N+\frac{E-m_N N}{N}=m_N-\epsilon. \qquad A=56,\ Z=26,\ \epsilon\simeq 8\,\text{MeV}$$

QCD: $\epsilon(A)\simeq a_1-a_2A^{-1/3}$, $a_1\simeq 16\,\text{MeV}$

$A=\infty$, $\epsilon=16\,\text{MeV} \Rightarrow \mu_0=m_N-16\,\text{MeV}$, $n_0\approx 0.16\ \text{fermi}^{-3}$

QCD+: $\mu_0=m_N-8\,\text{MeV}$





$T\neq 0,\ \mu\neq 0$

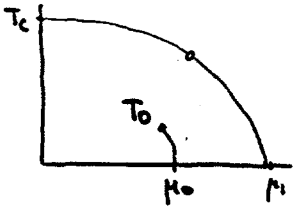
$\langle\bar\psi\psi\rangle$, n

$$n\simeq \frac{\mu}{T}\left(\frac{2m_NT}{\pi}\right)^{3/2}e^{-m_N/T}\quad,\ \mu,T\ll\Lambda_{QCD}$$

$$dp=s\,dT+n\,d\mu$$

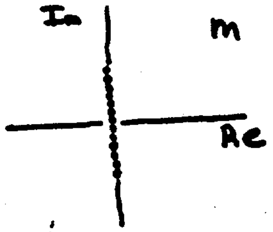
$$\frac{dT}{d\mu}=-\frac{\Delta n}{\Delta s}<0 \quad;\quad T=0,\ \Delta S=0$$

$T_0\sim 10$ MeV, 3d Ising / liquid-gas;
multifragmentation



$\langle\bar\psi\psi\rangle$: $T,\mu\gg\Lambda_{QCD}$; $g\ll 1$

$\mu=0$: $\langle\bar\psi\psi\rangle=0$ for $T>T_c\simeq 160\,\text{MeV}$: $O(4)$



$$\langle\bar\Psi\Psi\rangle = \pi\rho(0).$$

$$D = \not\partial + \not A + \frac{\mu}{N_c}\gamma_0$$

$$Z(m) = \int \mathcal{D}X\, e^{-\frac{N\,\mathrm{Tr}XX^\dagger}{\sigma^2}} \det{}^{N_f}\left(\begin{array}{cc} \overbrace{\qquad}^{N} & \overbrace{\qquad}^{N} \\ m & iX+iC \\ iX^\dagger+iC & m \end{array}\right)$$

$$\langle\bar\Psi\Psi\rangle = \frac{1}{N_f V_4}\frac{\partial \ln Z}{\partial m} = \frac{N}{V_4}\cdot\frac{1}{\sigma}\cdot 2 = \frac{2\,n_{inst}}{\sigma}$$

$$\langle\bar\Psi\Psi\rangle \approx 2\,\mathrm{fermi}^{-3}\ ;\ n_{inst} \approx 0.5\,\mathrm{fermi}^{-4} \Rightarrow \sigma \approx 100\,\mathrm{MeV}$$

$$C = \mathrm{diag}\Big(\overbrace{a\pi T + b\frac{\mu}{iN_c}\,;\ldots}^{N/2}\,;\overbrace{-a\pi T + b\frac{\mu}{iN_c}\,;\ldots}^{N/2}\Big).$$

$$\frac{a\pi T}{\sigma} \to T\ ,\quad \frac{b\mu}{\sigma N_c} \to \mu\ ,\quad \frac{m}{\sigma} \to m$$

$$Z(m) = \int \mathcal{D}\phi\, e^{-N\,\mathrm{Tr}\,\phi\phi^\dagger} \det{}^{N/2}\begin{pmatrix} \phi+m & \mu+iT \\ \mu+iT & \phi^\dagger+m \end{pmatrix} \det{}^{N/2}(T\to -T).$$

$$\phi : N_f \times N_f$$

$$\lim_{N\to\infty}\frac{1}{N}\ln Z = \min_{\phi}\,\Omega(\phi)$$

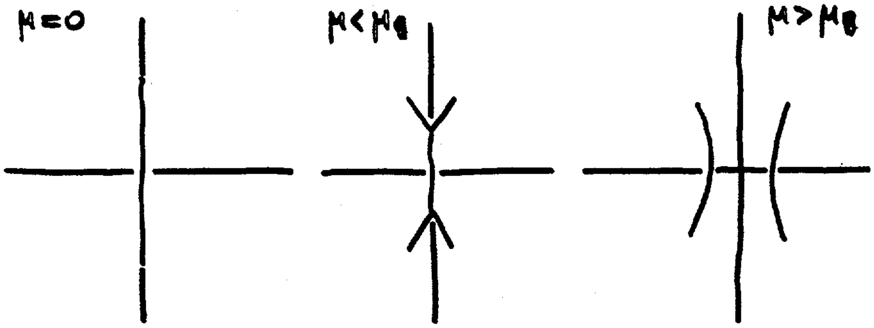
$$\Omega(\phi) = \mathrm{Tr}\Big(\phi\phi^\dagger - \frac{1}{2}\ln[\phi\phi^\dagger - (\mu+iT)^2][\phi\phi^\dagger - (\mu-iT)^2]\Big)$$

$\mu \neq 0$ λ_i – complex

How to calculate $\rho(\lambda)$?

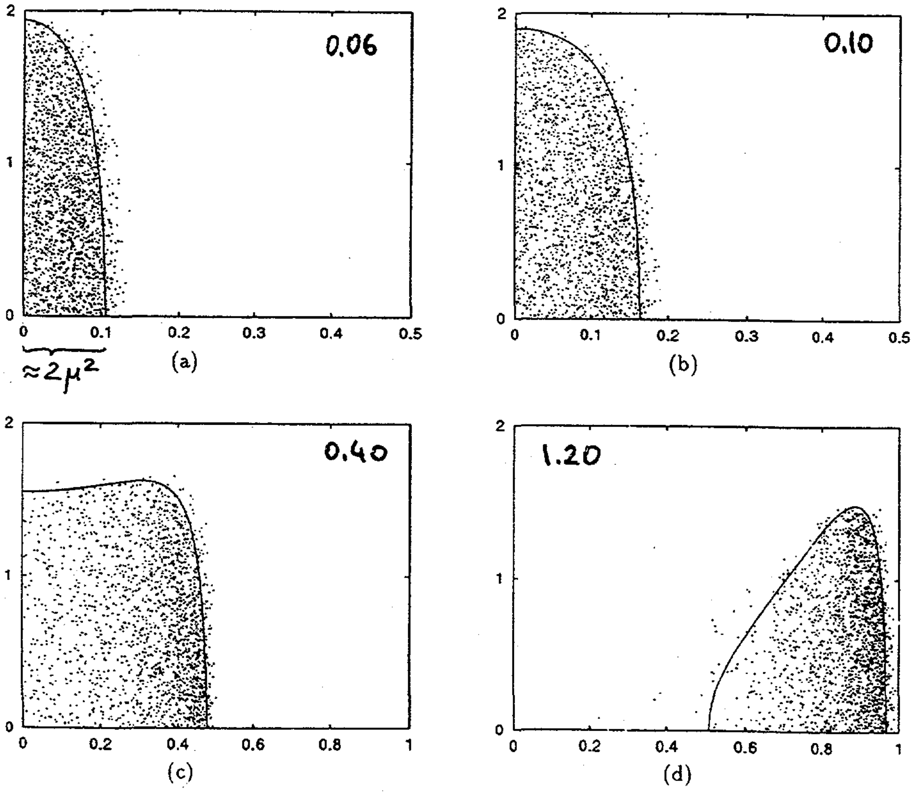
Recipe: calculate $Z(m)$ at $N_f \neq 0$ and take $N_f \to 0$. Worked at $T \neq 0$. Replica trick

At $\mu \neq 0$ we get at finite N_f and as $N_f \to 0$:



$\mu_1 \approx 0.53$

Is this really the distribution of eigenvalues (as it was in $T \neq 0$ case?)



(a) (b)

(c) (d)

Quenched QCD at $\mu \neq 0$ is a limit of another theory:

$$Z = \Big\langle \underbrace{\det{}^{N_f}(m-D)}_{\substack{\text{quarks}\\ \psi}} \cdot \underbrace{\det{}^{N_f}(m^* - \underset{\substack{\uparrow\\ \mu \to -\mu}}{D^\dagger})}_{\substack{\text{conj. quarks}\\ \chi}} \Big\rangle$$

$\langle \bar{\chi}\psi \rangle \neq 0$ baryonic condensate

Remarkable: $SU(2)$, $N_f \neq 0$

quarks are conj. of themselves

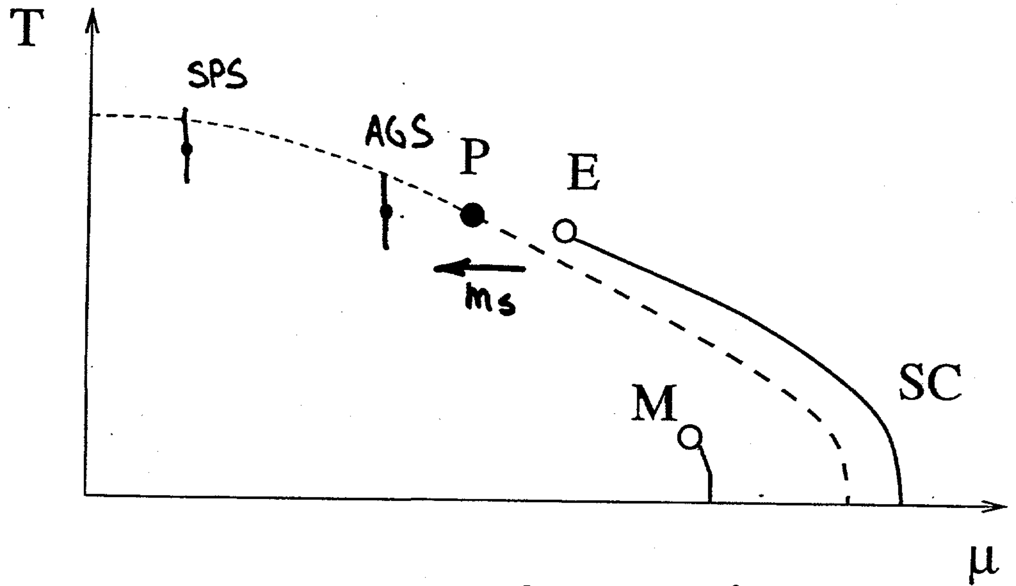
condensate observed

diquark condensate

- QCD, N_f quarks, $N_f \to 0$ $\neq$ quenched QCD

  N_f quarks + N_f conj. quarks, $N_f \to 0$ $=$ quenched QCD

- The phase of the det is important!



$m_q = 0$: $\Omega(\phi) = a\phi^2 + b(\phi^2)^2 + c(\phi^2)^3$

$a = 0,\ b > 0$

$a = 0,\ b = 0$ - tricritical

$m_q \neq 0$: E : only σ massless → Ising

Estimates: 2 flavors: P: $T \sim 100$ MeV, $\mu \sim 600$-700 MeV

Strange quark → $\mu_P \downarrow$ ($\mu_P = 0$ at $m_s = m_{s3}$)

SPS 200 GeV·A : $\mu \sim 200$ MeV

AGS 14.6 GeV·A : $\mu \sim 500$-600 MeV

The phase diagram of two colors QCD

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We discuss the phase diagram of two colours QCD in the temperature-chemical potential-mass space, using lattice results for bulk thermodynamics, susceptibilities / condensates, interquark potential and spectrum. We derive the level ordering at $\mu = 0$ showing that pion, scalar diquark and antidiquark are mutually degenerate, and so are the sigma, pseudoscalar diquark and antidiquark. We carry out a finite density spectroscopy calculation in analogy with what done in past quenched SU(3) studies and we discuss the pattern of chiral symmetry using either susceptibilities in the relevant channels (pseudoscalar and scalar mesons and diquarks) and masses. On a cold lattice our exploratory calculations give hints of deconfinement at $\mu = m_\pi/2$, diquarks appear to condense as expected of phenomenological models, and we find four nearly degenerate, bound states for $\mu > m_\pi/2$, in particular the a_0 particle and the pion seem degenerate even at non–zero mass. On a warmer lattice, close to the chiral deconfinement transition, the rotation of the chiral condensate in the chiral sphere is still evident, however the number density follows μ^3, suggesting either that the critical temperature for diquark condensation is (slightly) below T_c, or that the diquark condensate has little impact on the equation of state. The observed particle spectrum is significantly distinct from that of quenched models : in the quenched case the energies of particles carrying baryon number always equate the Fermi level, here we find significant deviations. We further assess the role of the chemical potential in the dynamics by carrying out a partial quenched calculation. We observe that diquarks propagators for $\mu > m_\pi/2$ resemble that of quenched SU(3), in agreement with random matrix models results. We speculate that, even if spectrum and symmetries of finite density QCD are dramatically N_c dependent, some features of real QCD might be obtained by "extrapolating" from the limiting case $N_c = 2$..

These exploratory results call for a detailed investigation of the vacuum structure at nonzero density, e.g. instanton distributions, gluonic condensates and correlation of the topological charge, which are being planned, while related studies and refinements of the work presented here are in progress. I would like to thank my collaborators: Ian Barbour, Peter Crompton, Simon Hands, John Kogut and Susan Morrison.

$N_c = 2$ vs $N_c = 3$, full vs quenched

Temperature

	$N_c=2$	$N_c=3$
quenched	Ising $Z(2)$ 3-d	Potts 3st $Z(3)$ 3-d
↓ $\cancel{Z}(N_c)$ String breaking $T=0$		
full	$Z_A(N_f) \times SU(N_f) \times SU(N_f)$	

From quenched to full:

from colour to flavour

string breaking at $T=0$: smaller T_c

Density

	$N_c=2$	$N_c=3$
quenched	Similar degrees of freedom : qq : $\mu_c = M_{qq}/2$	
↓ gauge dynamics deconfinement!!!		
full	qq $\mu_c = M_{qq}/2$	~~qq~~ qqq $\mu_c = M_{qqq}/3$

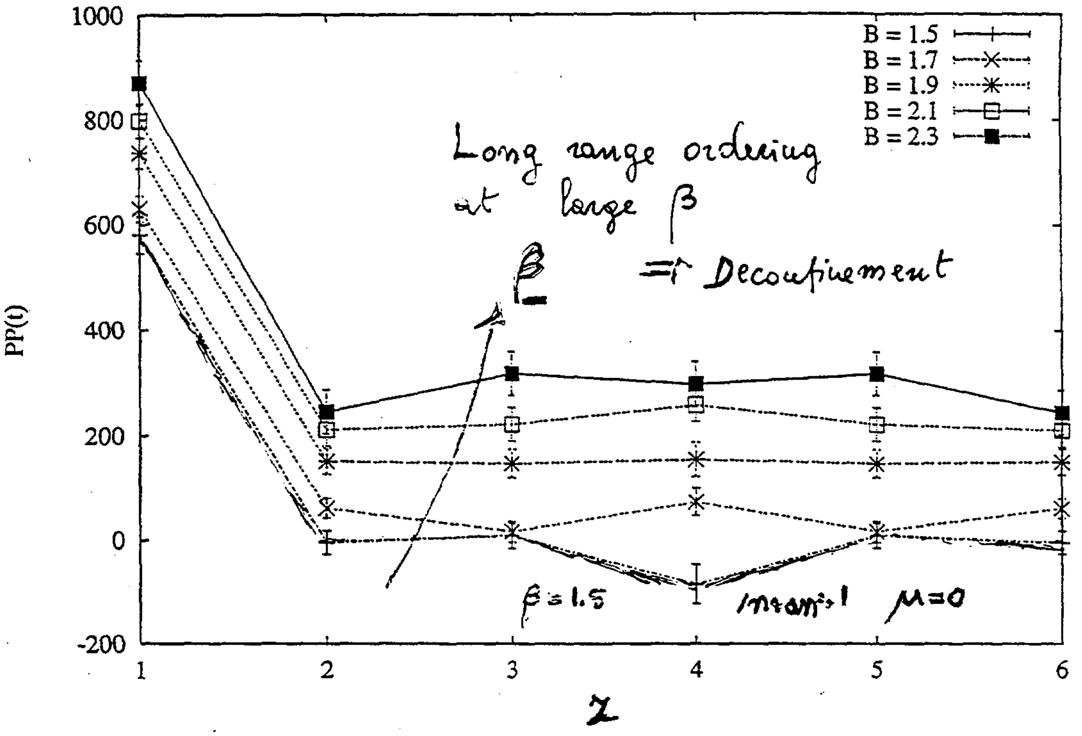
From quenched to full at $\mu \neq 0$

- $SU(2)$ and $SU(3)$ relevant d.o.f: all different!

But:

- Important, common dynamical effects

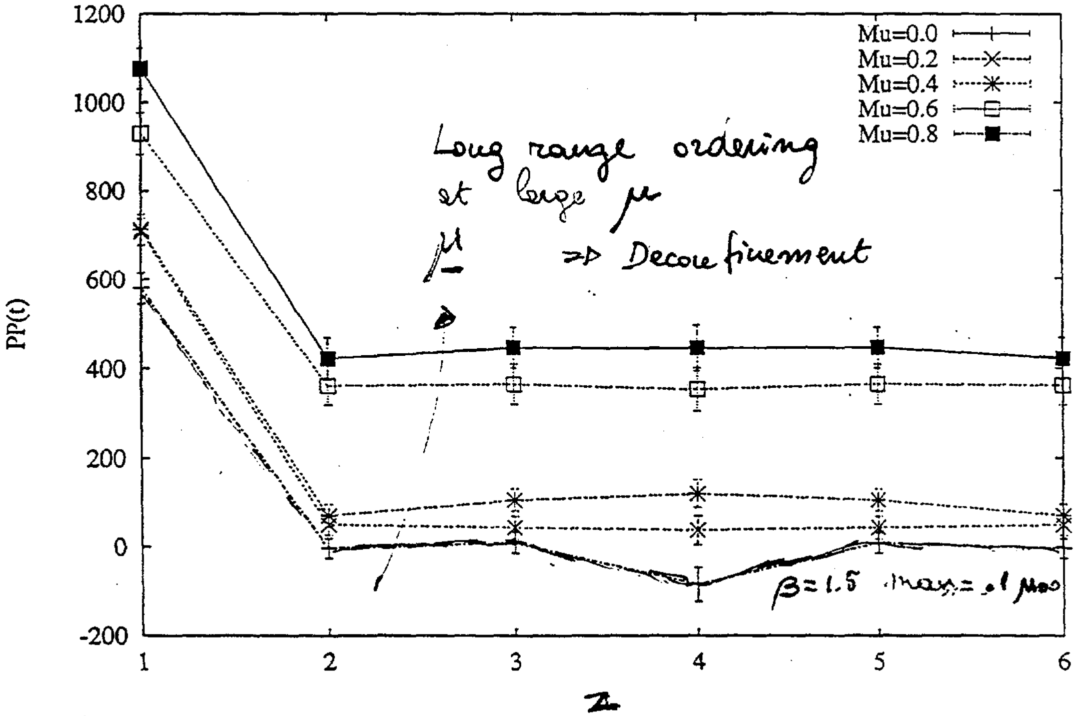
[0 momentum] Polyakov loop correlations : $P(0)P(z) \propto e^{-\sigma z}$



Long range ordering at large β

β ⇒ Deconfinement

β=1.5 ma=.1 μ=0



Long range ordering at large μ

μ ⇒ Deconfinement

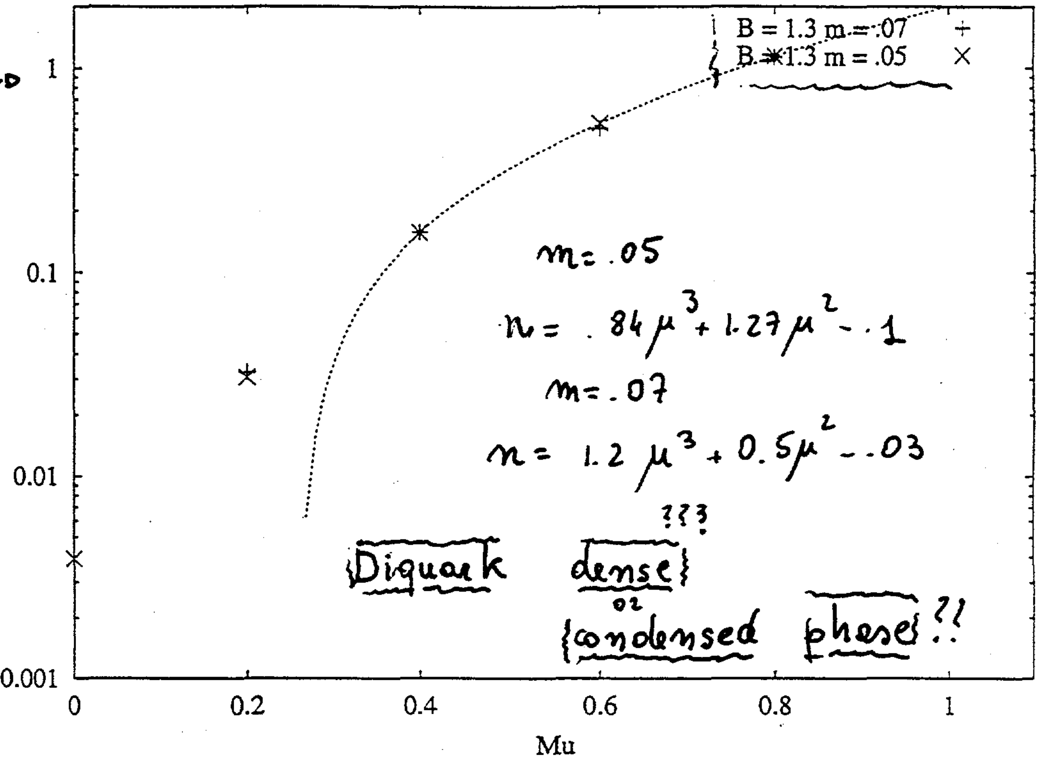
β=1.5 ma=.1 μ=0

- New! → Effects of the chemical potential on the gauge fields

- However: no qualitative difference high T / high μ
- Needed: "Better" studies [instanton, $F_{\mu\nu}F^{\mu\nu}$, Gluonic Condensate...

The number density: searching for 'new' vacua

COLD → $T \ll T_c$



$m = .05$

$n = .84\mu^3 + 1.27\mu^2 - .1$

$m = .07$

$n = 1.2\mu^3 + 0.5\mu^2 - .03$

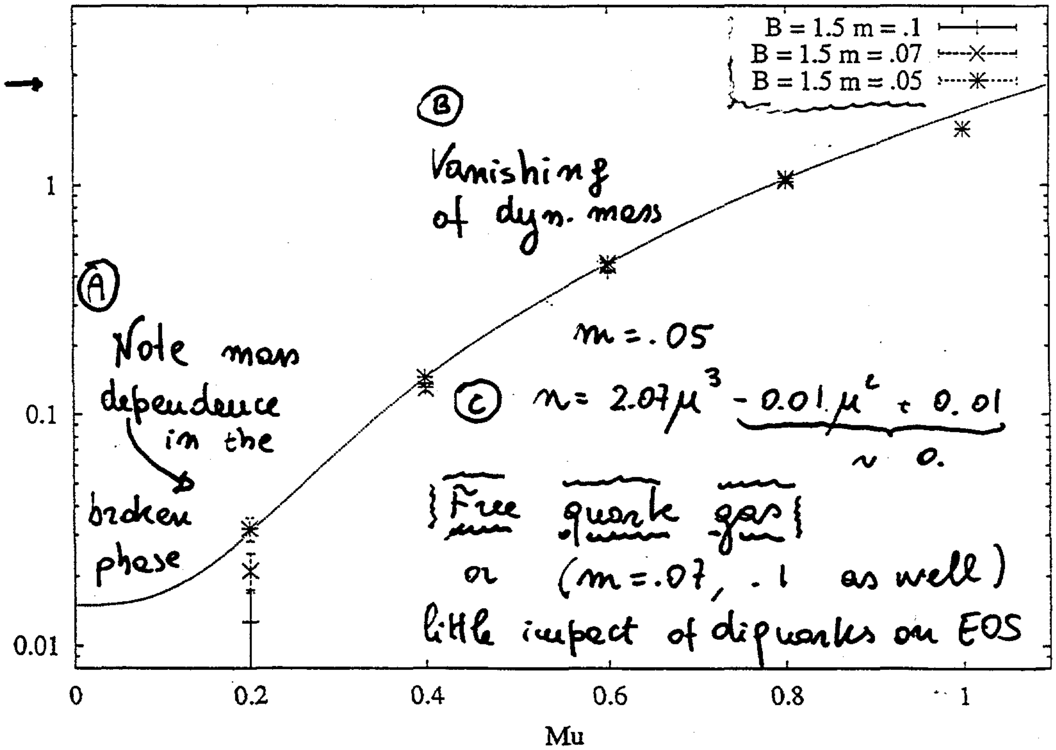
???

{Diquark dense}

or

{condensed phase ?!

WARM → $T \lesssim T_c$



Ⓑ Vanishing of dyn. mass

Ⓐ Note mass dependence in the broken phase

$m = .05$

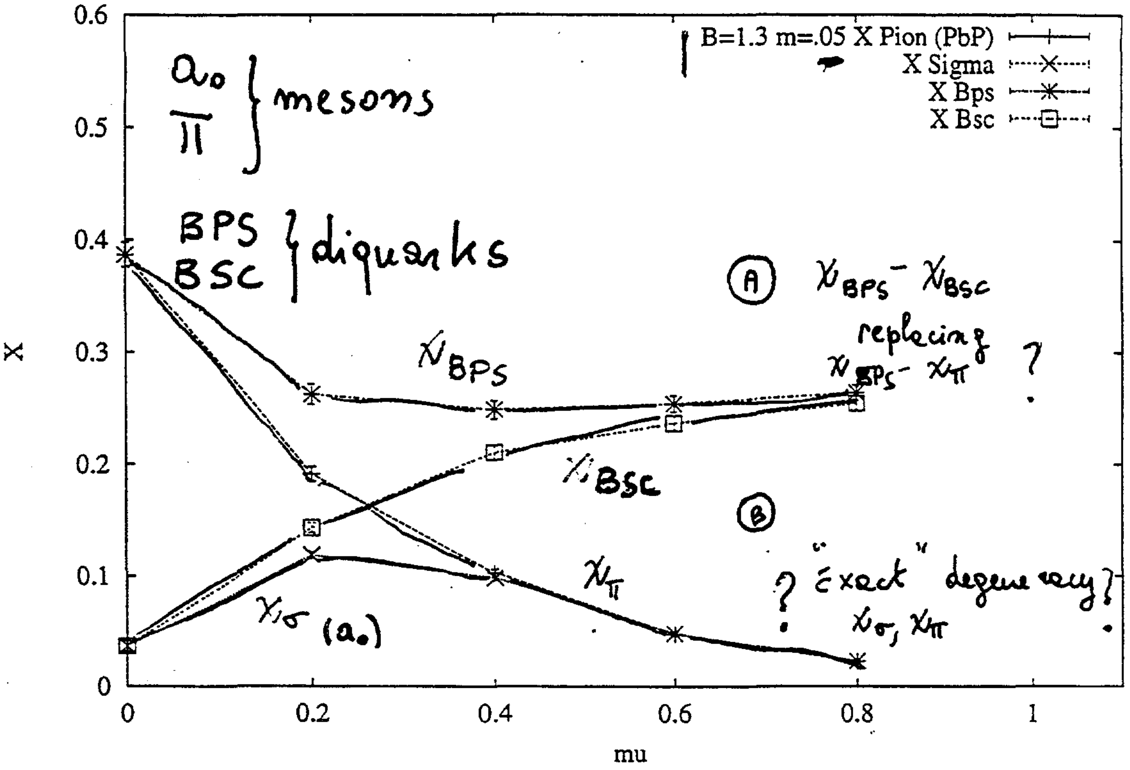
Ⓒ $n = 2.07\mu^3 \underbrace{- 0.01\mu^2 + 0.01}_{\sim 0.}$

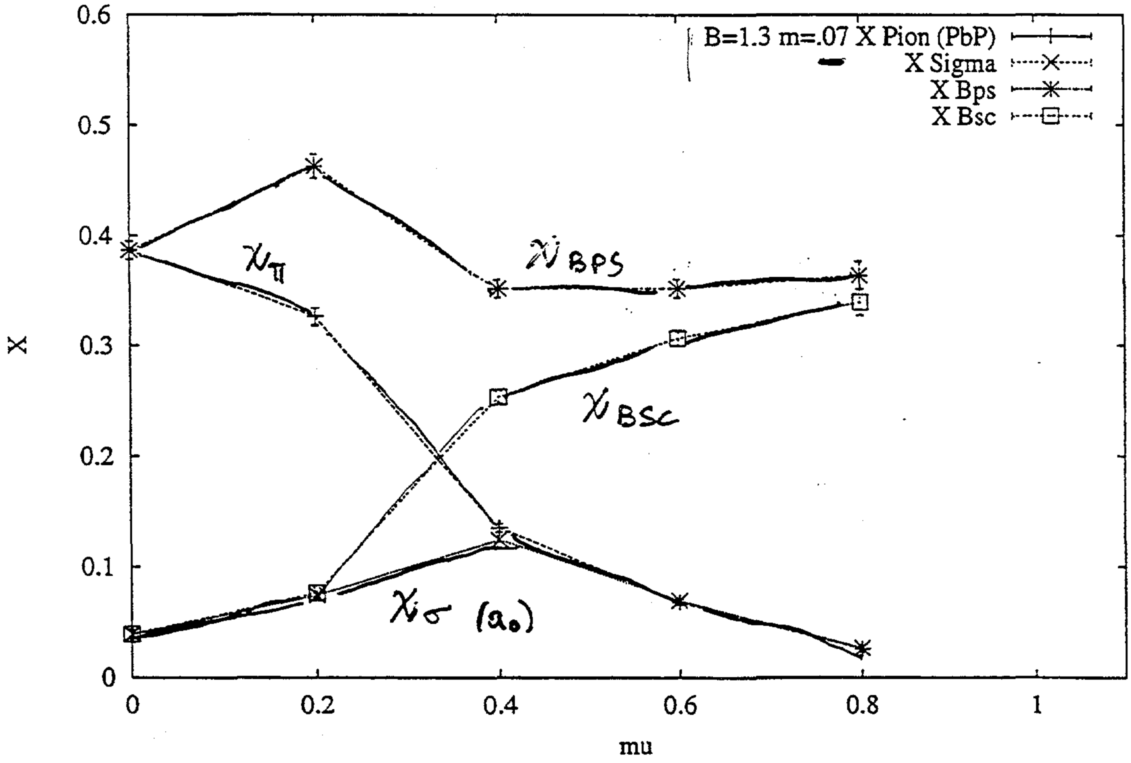
{Free quark gas}

or (m=.07, .1 as well)

little impact of diquarks on EOS

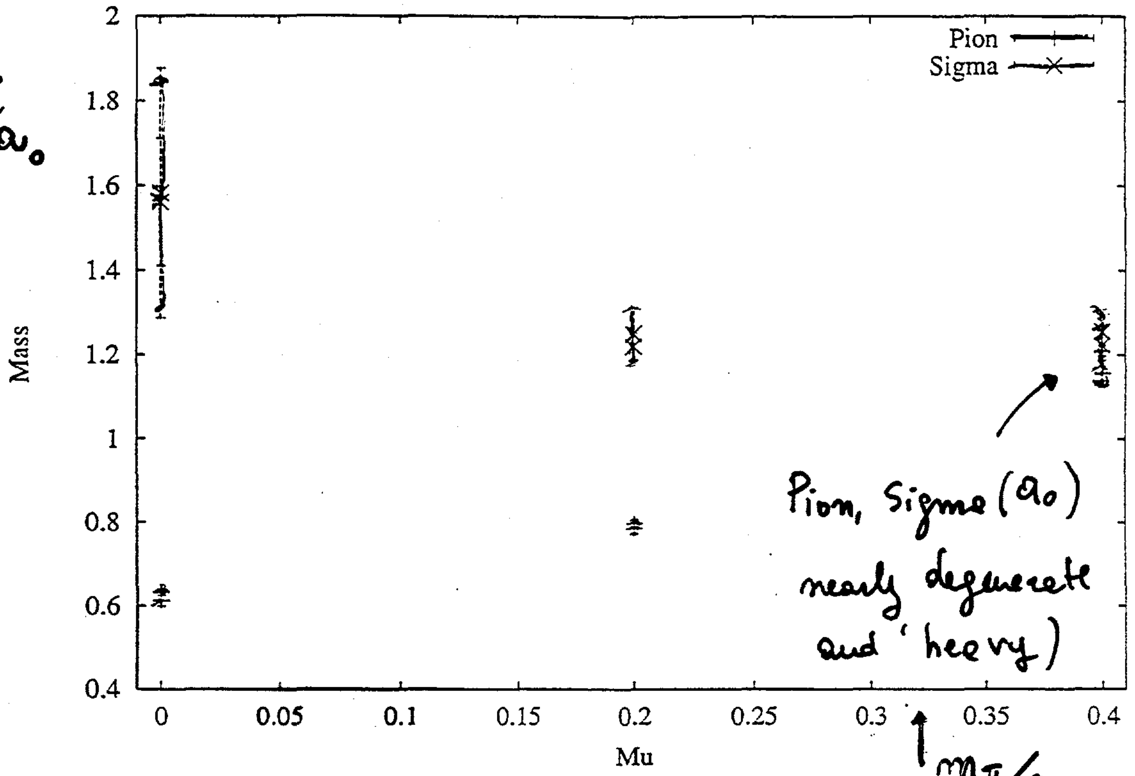
Susceptibilities vs μ : Rotation of condensates





Masses vs μ - Summary dynamical SU(2)

π, σ/a_0

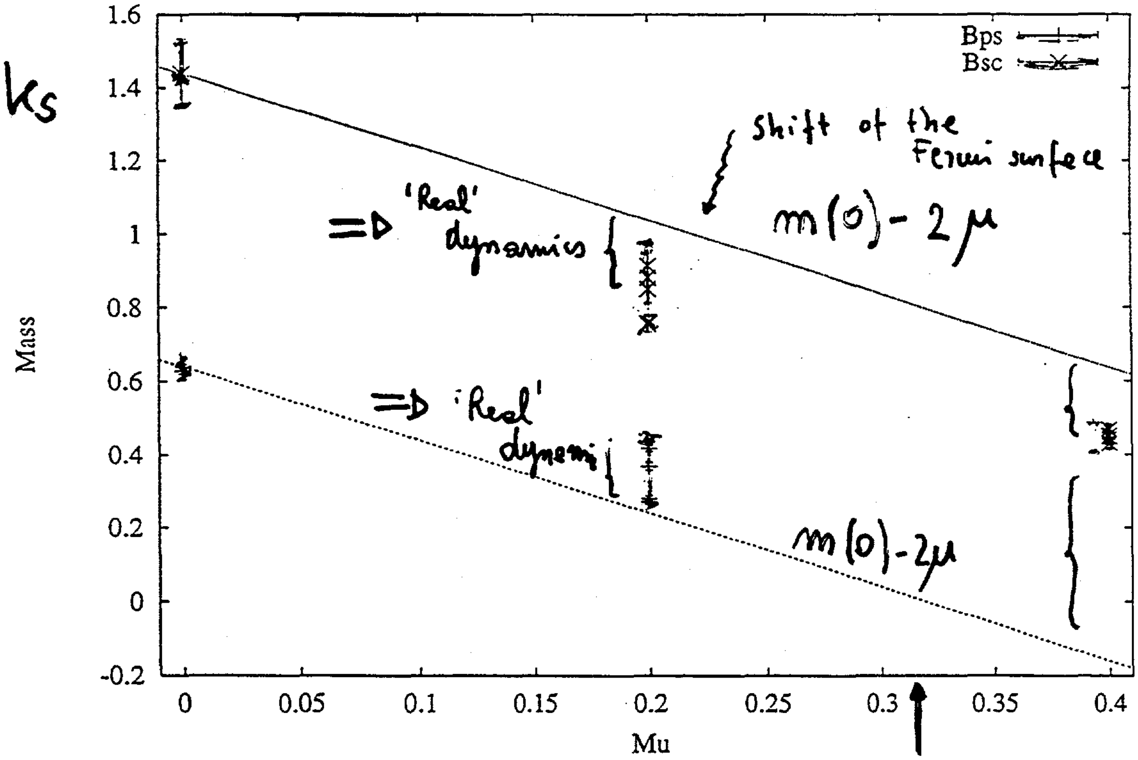


Pion, Sigma (a_0) nearly degenerate and 'heavy)

$m_\pi/2$

Diquarks

BPS - BSC



Shift of the Fermi surface

$m(0) - 2\mu$

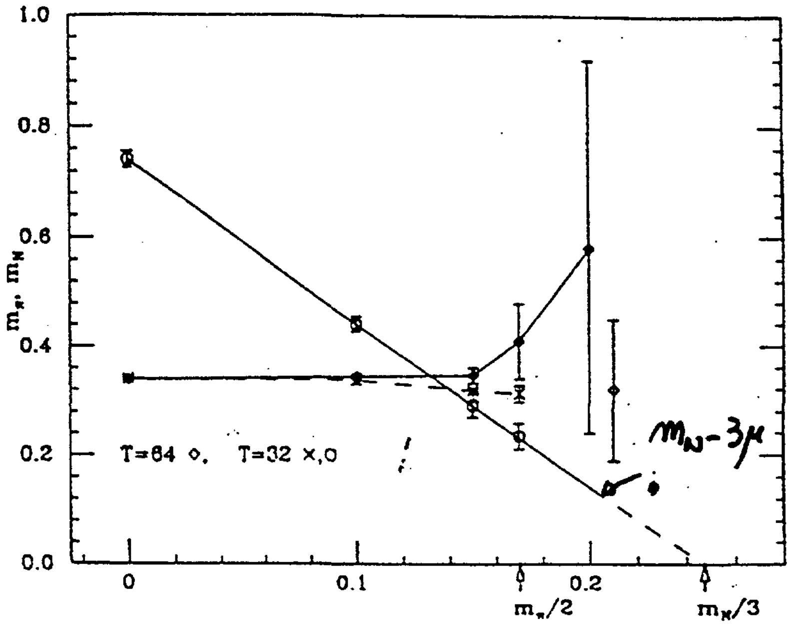
$\Rightarrow$ 'Real' dynamics

$\Rightarrow$ 'Real' dynamics

$m(0) - 2\mu$

$\frac{m_\pi}{2}$

Comparison with quenched SU(3)



$$\text{Nucleon}\,(\mu) = \text{Nucleon}\,(0) - 3\mu \Longrightarrow$$ mean field-type relationship

Dynamical SU(2):

Back to the same 'O' level

at $\boxed{\mu = .4 > m_\pi/2}$

$\boxed{m_\pi \sim m_{a_0}} \sim \boxed{1.2}$

$\boxed{m_{BPS} = m_{BSc}} \sim .4 + 2\mu \sim \boxed{1.2}$

} four nearly degenerate states

bound?

$\boxed{m_\rho = 1.8}$

Imaginary chemical potential as a tool for lattice QCD

M. Alford, A. Kapustin, F. Wilczek

Standard lattice gauge theory algorithms run into the well-known "sign problem" at real chemical potential, since they try to weight configurations by $\det M(\mu)$, which is complex. The Glasgow method uses the $\mu = 0$ ensemble, which is then reweighted with a factor $\det M(\mu)/\det M(0)$. Unfortunately it has not been able to reproduce the simplest feature of QCD at finite density, namely the onset of baryon density at $\mu = m_B/3$. This problem is believed to be caused by measure-mismatch: the $\mu = 0$ ensemble is dominated by quark number $N = 0$ sectors, and has very little overlap with the finite quark number sectors. The Glasgow method therefore needs very high statistics, since it slowly builds up the correct results from rare but large fluctuations. At moderate statistics, before the large fluctuations begin to occur, the Glasgow method can give very misleading results with apparently small statistical error bars.

Recently, we have investigated the use of *imaginary* chemical potential $\mu = i\nu$. This avoids the measure-mismatch problem, since $\det M(i\nu)$ is always real. Monte-Carlo methods can be used to compute ratios of the partition function, so one can calculate $Z(i\nu)/Z(i\nu_0)$, using the $\mu = i\nu_0$ measure, for some range of imaginary chemical potential ν close to ν_0. By chosing several "patches", each centered on a different ν_0, one can calculate ratios of the partition function at different ν without encountering any overlap problems. The canonical partition functions Z_N can then be obtained by a Fourier transform. This is where the main limitation of the imaginary chemical potential method arises: the Fourier transform will become very sensitive to errors in $Z(i\nu)$ for large N. Unlike Glasgow, it will be clear when this method is not working, since the error bars from the Fourier transform will become large.

As a feasibility study, we performed a Monte-Carlo calculation of the partition function of the two-dimensional Hubbard model with imaginary chemical potential. The results for $Z(i\nu)$ and its Fourier transform Z_N are given in the transparencies. It is encouraging that we were able to obtain the first few Z_N with reasonable errors. The imaginary chemical potential ensemble is not biased toward finite baryon number, and relies, like Glasgow, on fluctuations to explore $N \neq 0$ sectors. However, for QCD, we only need to measure the first two canonical partition functions Z_0 and Z_3 in order to obtain the onset chemical potential μ_o, since $Z_3/Z_0 = \exp(-3\mu_o/T)$. Moreover, at temperatures close to the phase transition, the baryon becomes light (at least in the 2 flavor case), and so thermal fluctuations will explore the $N \neq 0$ sectors.

II. Lattice QCD at finite density

$$\begin{aligned} Z(\mu) &= \sum_N Z_N \mathrm{e}^{-\mu N} \\ &= \sum_{U(x)\ \text{configs}} \underbrace{\det M\, \mathrm{e}^{-S_{\text{gauge}}[U]}}_{\text{sampling weight}} \end{aligned}$$

$$S_{\text{ferm}} = \int_x \bar{\psi} M \psi$$

$$M = \gamma^\mu D_\mu + r D^2 + m + \mu\gamma_0$$

When can we guarantee that the sampling weight is positive?
If we have N_F flavors, each with matrix M,

$$\underbrace{\det M \in \mathbb{R}}_{\substack{M^\dagger \sim M \\ M^* \sim M}} \quad \text{and} \quad N_F \text{ even}$$

"Sign Problem:" for $\mu \neq 0$, $M \not\sim M^\dagger$,
so $\det M$ is not necessarily real (let alone positive).

III. Imaginary chemical potential

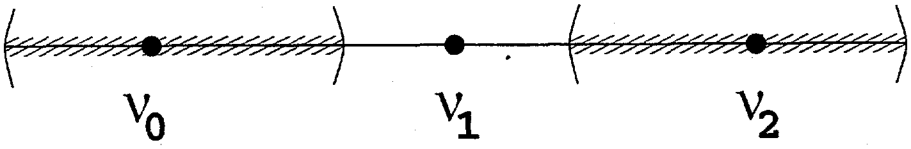
$$\begin{aligned} M &= \gamma^\mu D_\mu + r D^2 + m + i\nu\gamma_0 \\ M^\dagger &= -\gamma^\mu D_\mu + r D^2 + m - i\nu\gamma_0 = \gamma_5 M \gamma_5 \end{aligned}$$

So with imaginary chemical potential $\mu = i\nu$, $\det M$ is real.

$$\begin{aligned} \frac{Z(i\nu)}{Z(i\nu_{\text{update}})} &= \left\langle \frac{\det M(i\nu)}{\det M(i\nu_{\text{update}})} \right\rangle_{\mu = i\nu_{\text{update}}} \\ &= \sum_{U(x)} \underbrace{\mathrm{e}^{-S[U]} \det M(i\nu_{\text{update}})}_{\text{sampling weight}} \frac{\det M(i\nu)}{\det M(i\nu_{\text{update}})}. \end{aligned}$$

Unlike the Glasgow algorithm, we don't have to use $\mu = 0$ measure, which requires high statistics as the correct expectation value builds up from rare fluctuations.

We can make the fluctuations arbitrarily small, using patches $\nu_{\text{update}} = \nu_0, \nu_1 \ldots$



What do we do with $Z(i\nu)$?

Fourier transform to get

$$Z_N = \frac{\beta}{2\pi} \int_0^{2\pi/\beta} d\nu\, Z(i\nu) \mathrm{e}^{-i\beta\nu N}.$$

This is the difficult part.
If $Z_1 \ll Z_0$ then it will be hard to extract.

But some physics may be visible directly in $Z(i\nu) \ldots$

The Hubbard model

Can we do this with real Monte-Carlo data? The simplest system with a "sign problem" is the repulsive Hubbard model: non-relativistic electrons on a lattice with a hopping term and on-site repulsion.

$$\mathcal{H} = -K \sum_{\langle i,j\rangle,\sigma} a^\dagger_{i\sigma} a_{j\sigma} - \frac{U}{2} \sum_i (a^\dagger_{i\uparrow} a_{i\downarrow} - a^\dagger_{i\downarrow} a_{i\uparrow})^2 + \mu \sum_{i,\sigma} a^\dagger_{i\sigma} a_{i\sigma}$$

By a particle-hole transformation, $\mu = 0$ gives half-filling.

$$Z(\mu) = \sum_{A(x)} \mathrm{e}^{-A^2/2} \det M(\mu) \, \det M(-\mu)$$

For real μ, M are real matrices.
For μ zero or imaginary, this is $|\det M|^2$, and is positive.

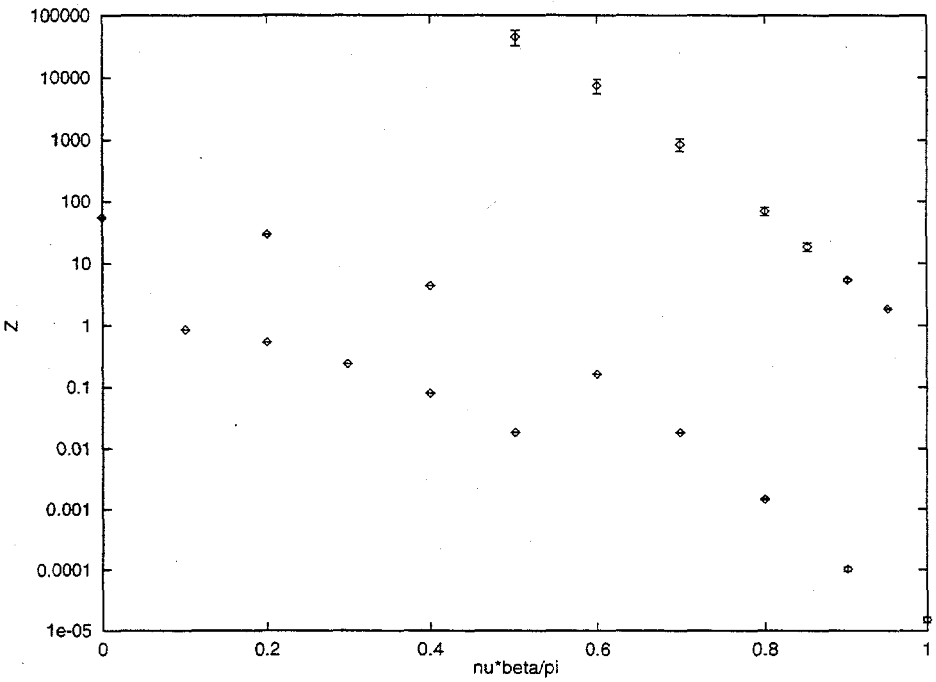
Again, we can calculate ratios of partition functions

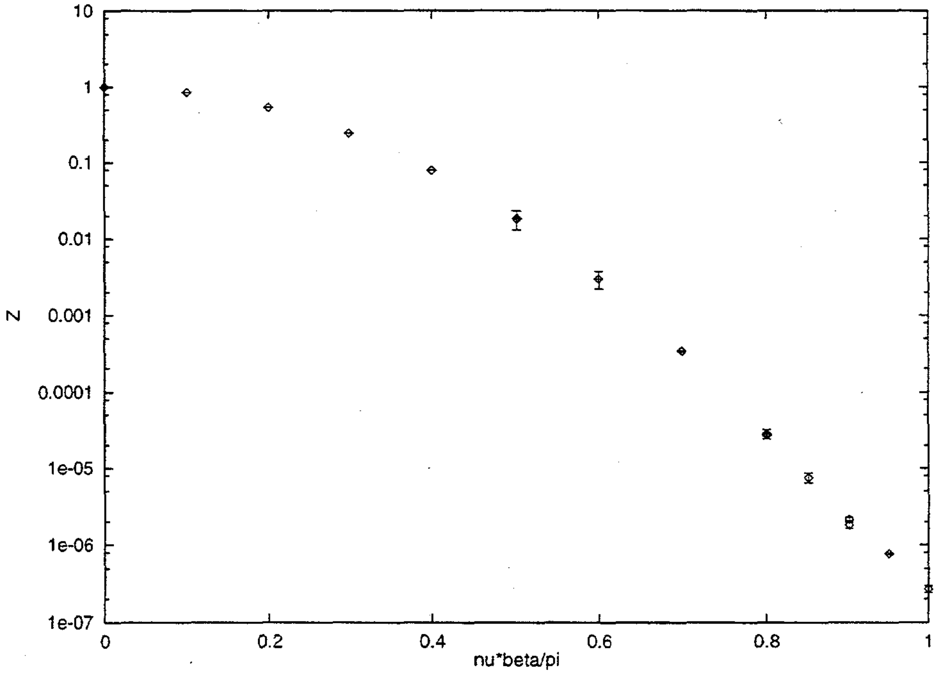
$$\frac{Z(i\nu)}{Z(i\nu_0)} = \sum_{A(x)} \mathrm{e}^{-A^2/2} \det |M(i\nu_0)|^2 \, \frac{|\det M(i\nu)|^2}{|\det M(i\nu_0)|^2}$$

and use several values of ν_0 to reduce measure-mismatch.

Hubbard model $Z(i\nu)$

$4^2 \times 10, K = 1,$
$\beta = 1.5, U = 1.0$



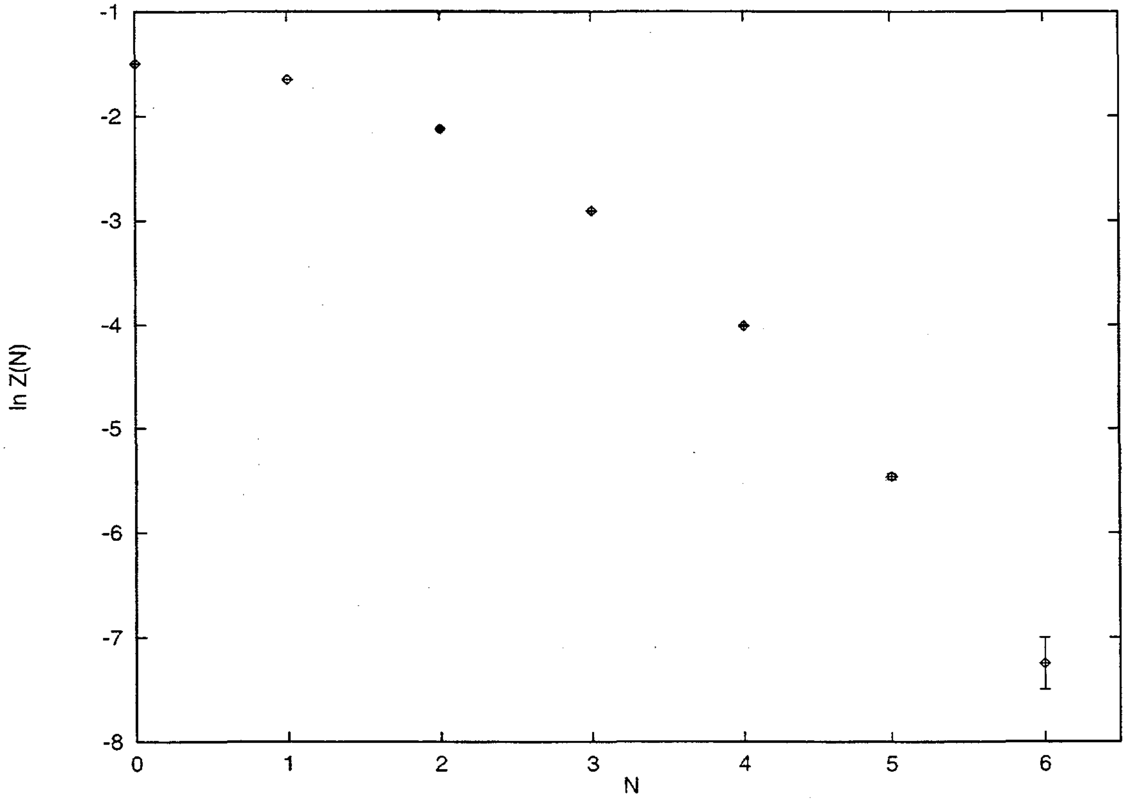


Z_N for Hubbard model

We fit our preliminary $Z(i\nu)$ data to

$$\exp(-a\nu^2) \times \text{spline},$$

and Fourier transform it:



High temperature QCD

Results on Deconfinement and Chiral Symmetry Restoration from Lattice QCD

Frithjof Karsch

Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany

The QCD phase transition from a low temperature hadronic phase to the high temperature quark gluon plasma phase does show features related to deconfinement (liberation of many new degrees of freedom, sudden change in the asymptotic behavior of the heavy quark potential...) as well as chiral symmetry restoration (vanishing of the chiral condensate, degeneracy of thermal screening masses of scalar and pseudoscalar mesons...).

The order of the transition has been analyzed in the heavy quark mass (pure gauge) limit as well as for light quarks with different numbers of light flavors (n_f). To a large extent it has been found to agree with predictions based on general universality arguments which relate the 4-d gauge theories to 3-d spin models with the same global symmetries. In particular, in the pure gauge sector ($m_q \rightarrow \infty$) it has been verified that in the case of second order transition the critical behaviour is controlled by the relevant universality class of 3-d spin models with the same global symmetry.

In the case of QCD with two and three light flavors the situation, however, is still not completely clarified. In the $n_f = 3$ case it seems to be clear that the transition is first order. However, on the quantitative level conflicting results from Wilson and staggered fermion calculations exist. While the former suggests the existence of a first order transition up to rather large values of the quark mass the latter suggest a first order transition only for rather small quark masses, $m_q/T \lesssim 0.1$. These latter findings have been confirmed in recent calculations with improved staggered fermion actions. They suggest that also in the "real world", i.e. for the case of two light quarks and a heavier strange quark, $m_s/T_c \sim 1$, the transition will be continuous.

For $n_f = 2$ the chiral transition seems to be continuous. At least there is no evidence for a first order transition for all values of the quark mass analyzed so far, $m_q/T \geq 0.04$. On the other hand the current analyses did not reproduce the expected critical behavior for a system in the universality class of the 3-d, $O(4)$-symmetric spin models. As different observables yield inconsistent results for critical exponents it may, however, be speculated that theses analyses are still influenced by finite lattice artifacts and/or the use of too large quark masses. One thus may hope that the situation will improve with the use of improved discretization schemes for the fermions. Moreover, it has to be clarified in more detail, in how far the approximate restoration of the $U_A(1)$ symmetry can influence the transition. The current analysis of screening masses and susceptibilities suggests that the $U_A(1)$ remains broken at T_c. However, the amount of breaking does seem to be strongly reduced and may even influence the order of the transition for small quark masses.

So far lattice studies of the QCD phase transition have successfully been performed only at vanishing baryon number density. At non-zero baryon number density most calculations so far explored the formulation of QCD with a non-vanishing chemical potential. This leads to a complex fermion determinant and prohibits the application of conventional Monte Carlo techniques. It therefore may be worthwhile to explore other approaches, which have been around since quite some time, however have not been explored in any greater detail. In particular, one may look again at the formulation of QCD with a non-vanishing baryon number. This formulation has the advantage of leading to a real determinant. However, the difficulties enter again through the need of performing an additional Fourier integration which again introduces a strongly oscillating integrant. It remains to be seen in how far such an approach is applicable also on large lattices.

Results on

Deconfinement and Chiral Symmetry Restoration

from Lattice QCD

- deconfinement ⇒ SU(N_c) gauge theory
- χ-sym. restoration ⇒ QCD with n_f light quarks
- deconf + χ-sym rest. ⇒ SU(3) + adjoint fermions
- QCD at finite density ⇒ $\mu > 0$, $B > 0$, $g^2 \to \infty$

F. Karsch
RIKEN workshop
on QCD phase trans.
Nov. 4-7, 98

$\boxed{m_q \rightarrow \infty}$ pure SU(N_c) gauge theory

detailed lattice calculations with control over finite size and finite cut-off effects (extrapolations to continuum limit, improved actions)

$N_c = 2$ **Z(2) , Ising model**
2^{nd} order for d=3, 4

$N_c = 3$ **Z(3) , Potts model**
2^{nd} order for d=3
1^{st} order for d=4

$N_c \geq 3$, $d = 4$: 1^{st} order SU(4) ⇒ Wingate

$$T_c/\sqrt{\sigma} = \begin{cases} 0.69\ (2) & ,\ SU(2),\ d=4 \\ 0.63 - 0.66 & ,\ SU(3),\ d=4 \\ 0.98\ (2) & ,\ SU(3),\ d=3 \end{cases}$$

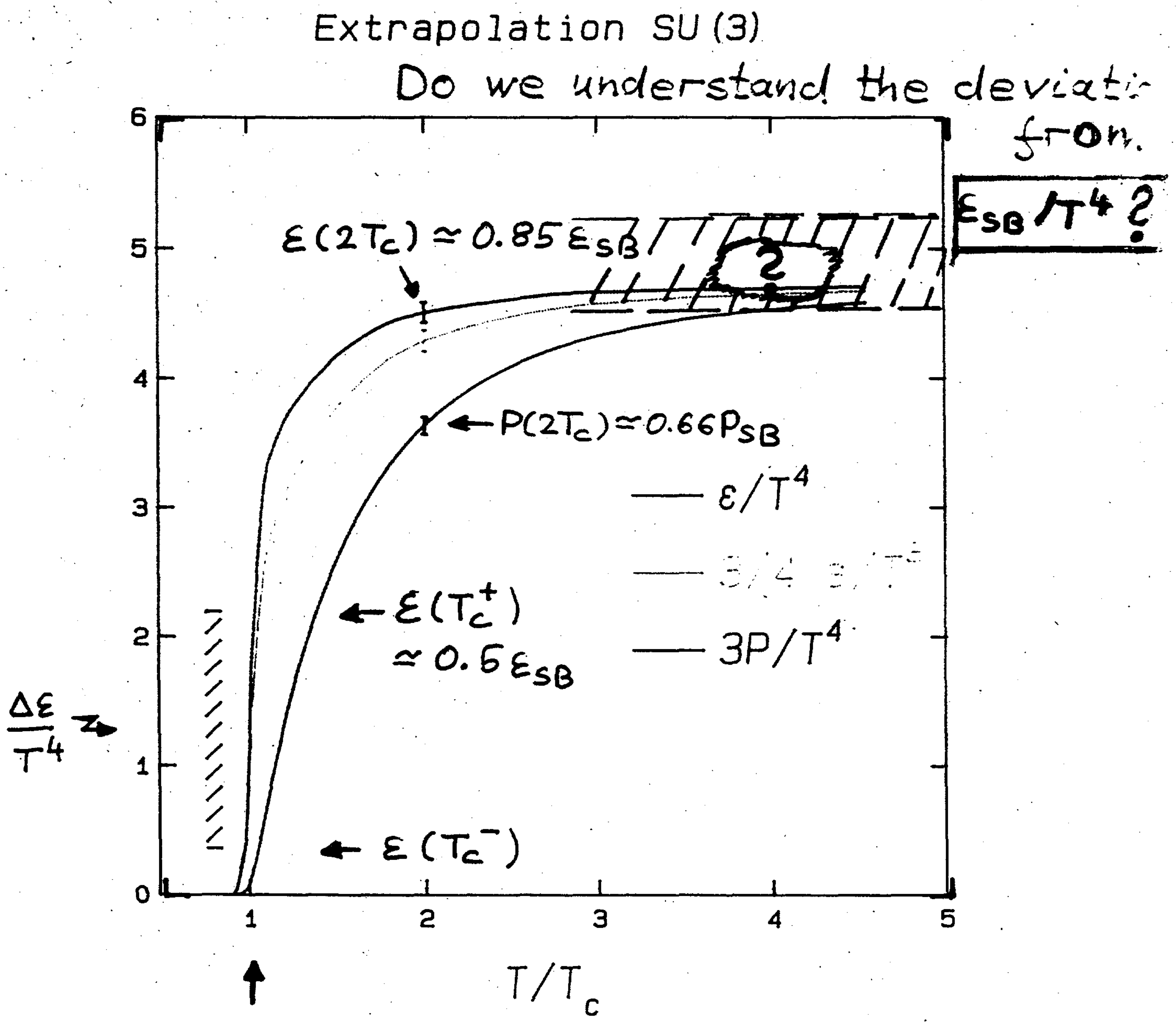
$$\Uparrow \quad \sim \quad \sqrt{3/(d-2)\pi} \quad \text{string models}$$

SU(3) , **$d = 4$:** equation of state

- small latent heat $\epsilon_c^+/T_c^4 = 1.4\ (1)$
- low critical energy density $\epsilon_c^+/T_c^4 = 2.5\ (5)$
- significant deviations from ideal gas even for $T >> T_c$:

$$\epsilon/T^4 \lesssim 0.9 \Rightarrow \text{electric and magnetic screening}$$

Equation of State

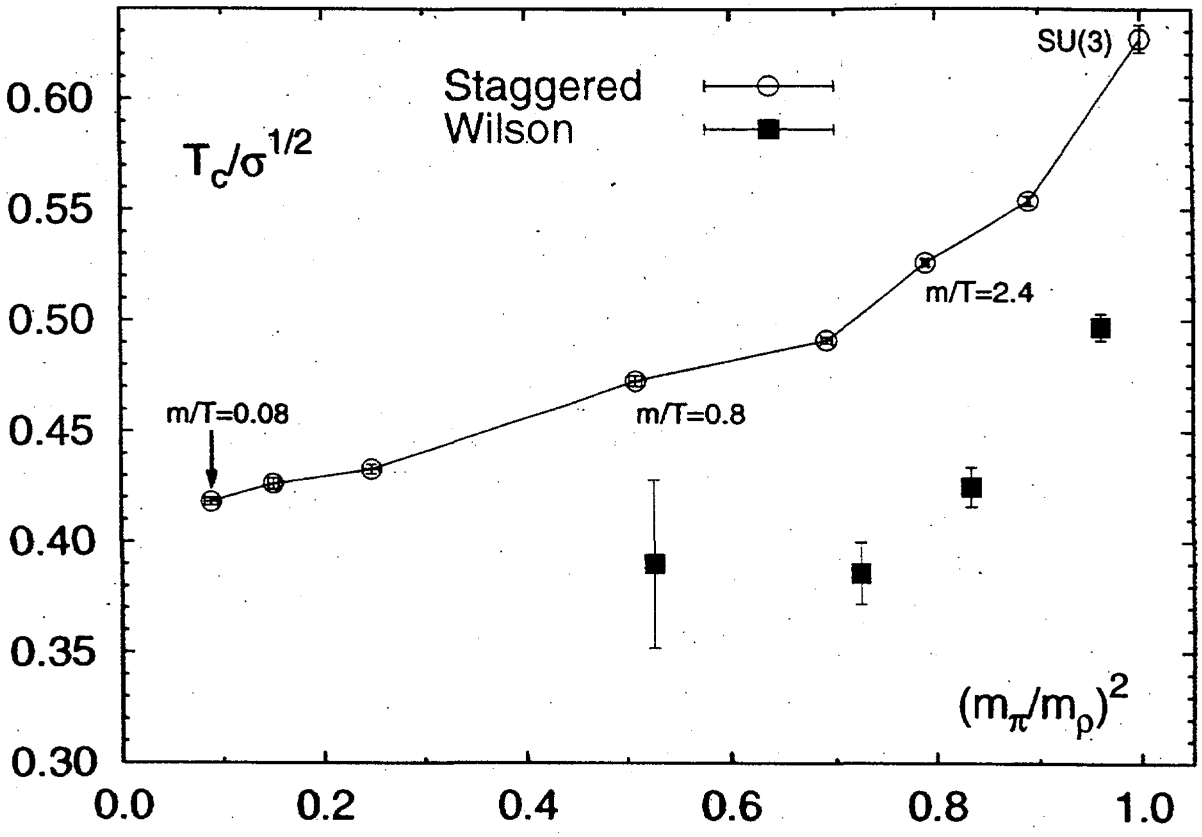


Do we understand the deviation from ε_{SB}/T^4?

$T_c \simeq 0.63(1)\sqrt{\sigma}$
$\simeq 260\,\mathrm{MeV}$

G. Boyd, J. Engels, FK, E. Laermann,
C. Legeland, M. Lütgemeier, B. Petersson,
PRL75('95)4169, NPB469('96)419

Critical Temperature in units of the String Tension for 2-flavour QCD with varying quark mass



SU(3) gauge theory: $\frac{T_c}{\sqrt{\sigma}} \simeq 0.635 - 0.655 \simeq 270$ MeV

2-flavour QCD: $\frac{T_c}{\sqrt{\sigma}} \lesssim 0.4 \qquad \lesssim 170$ MeV

$\boxed{m_q \to 0}$ QCD with light quarks

lattice calculations still struggle with strong cut-off / volume dependence

⇒ Wilson vs. staggered formulation

(domain wall fermions ⇒ Mawhinney)

$\mathbf{n_f \geq 3\ ,\ N_c = 3}$ **1^{st} order for $m_q/T \lesssim 0.1$**

disappearance of χSB for $n_f \geq$?

$\mathbf{n_f = 4\ ,\ N_c = 3}$ **reduction of χSB at $T = 0$,**

(R.D. Mawhinney. NPB (Proc.Suppl.) 63 (98) 212)

$\mathbf{N_c = 2}$ **no indication for a 1^{st} order transition**

- at least for $m_q/T > 0.06$

$\mathbf{n_f = 2\ ,\ N_c = 3}$ **continuous transition (for $m_q/T \gtrsim 0.04$)**

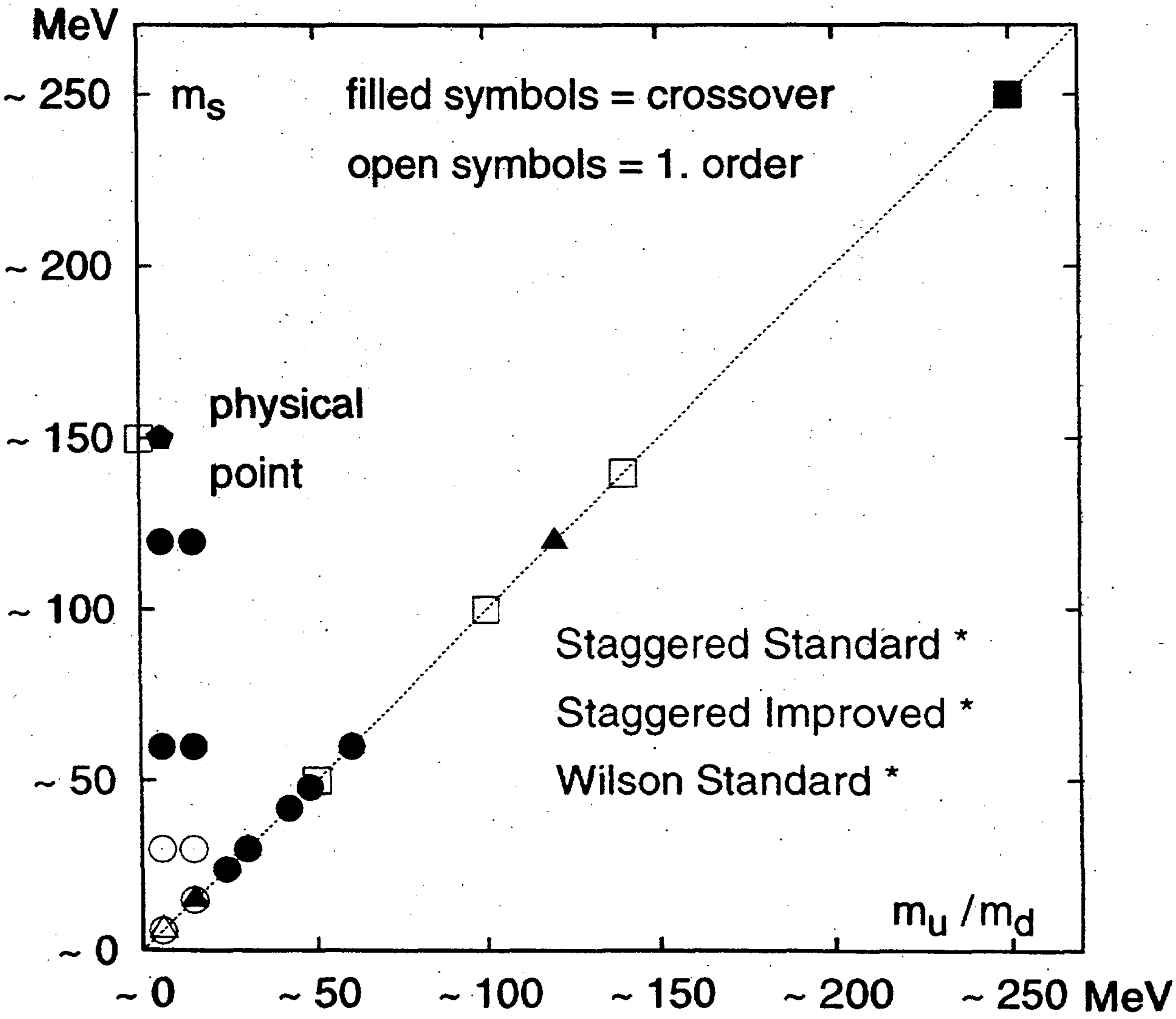
O(4) exponents ?

$U_A(1)$ breaking small (non-zero?)

$T_c/\sqrt{\sigma}$ drops significantly with decreasing m_q

$$T_c/\sqrt{\sigma} \simeq 0.4$$

$$(\mathbf{E.o.S})_{QCD} \sim (\mathbf{E.o.S})_{SU(3)} \frac{\#\text{d.o.f.} - \text{QCD}}{\#\text{d.o.f.} - \text{SU(3)}}$$



* JLQCD, hep-lat/9809102; Columbia, Phys.Rev.Lett 65 (1990) 2491

* Bielefeld

* Y. Iwasaki et al., Phys.Rev. D54 (1996) 7010

QCD at finite density

- non-zero chemical potential

$$S_F \Rightarrow S_F(\mu) = m_q \bar{\psi}_x \psi + \sum_{i=1}^{3} (\bar{\psi}_x \not{D}_+ \psi_{x+\hat{i}} + \bar{\psi}_x \not{D}_- \psi_{x-}$$

$$+ e^{\mu} \bar{\psi}_x \not{D}_+ \psi_{x+\hat{0}} + e^{-\mu} \bar{\psi}_x \not{D}_- \psi_{x-\hat{0}}$$

$$\Rightarrow Z(\mu) = \int \prod_{x,\hat{\nu}} dU_{x,\hat{\nu}} \det Q_F(\mu) e^{-S_G}$$

complex determinant;
algorithmic problems

I. Barbour et al., NPB (Proc. Suppl.) 60A (98) 220

- alternative approach: non-zero baryon numbe

$B > 0$

$$Z_B = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \, e^{-iB\varphi/T} Z(i\varphi)$$

oscillating integral

positive determina

A. Roberge, N. Weiss, NPB275 (86) 734

D.E. Miller, K. Redlich, PRD35 (87) 2524

Critical Behavior at the High Temperature QCD Phase Transition – Summary of Talk

Carleton DeTar

University of Utah

- Question: At the chiral phase transition with two zero mass quark flavors, zero baryon density, high temperature, and two flavors, we expect O(4) critical behavior.

- Studies at $N_t = 4$ with staggered fermions by the JLQCD, Bielefeld, and MILC collaborations have raised doubts. The MILC collaboration has extended its studies to larger lattices: $24^3 \times 4$ and smaller quark masses: $am = 0.008$ and continues to find strong deviations from O(4), O(2) and mean field predictions for critical exponents and the scaling equation of state.

- At $N_t = 6$, 8, and 12 we see somewhat better agreement, suggesting that lattice artifacts may play a role in the $N_t = 4$ discrepancy. However, all of these weaker coupling studies were done at $N_s/N_t = 2$, so not as exhaustively as $N_t = 4$.

- Nonetheless, we may view the $N_t = 4$ lattice theory as a chiral model without reference to continuum limits. In that case we would still expect O(2) critical behavior, barring additional complications.

- Recent simulations of an $N_t = 4$ chiral QCD model by Kogut, Lagaë, and Sinclair find a first order chiral/deconfining phase transition at zero quark mass when a small four-fermion coupling is turned on. These results suggest the possible proximity of a first-order phase transition at $N_t = 4$ that could spoil the expected O(2) critical behavior of the $N_t = 4$ system.

- We hope improved actions will lead us out of these difficulties.

Critical Behavior at the High Temperature QCD Phase Transition

C. DeTar

Univ. of Utah

MILC:

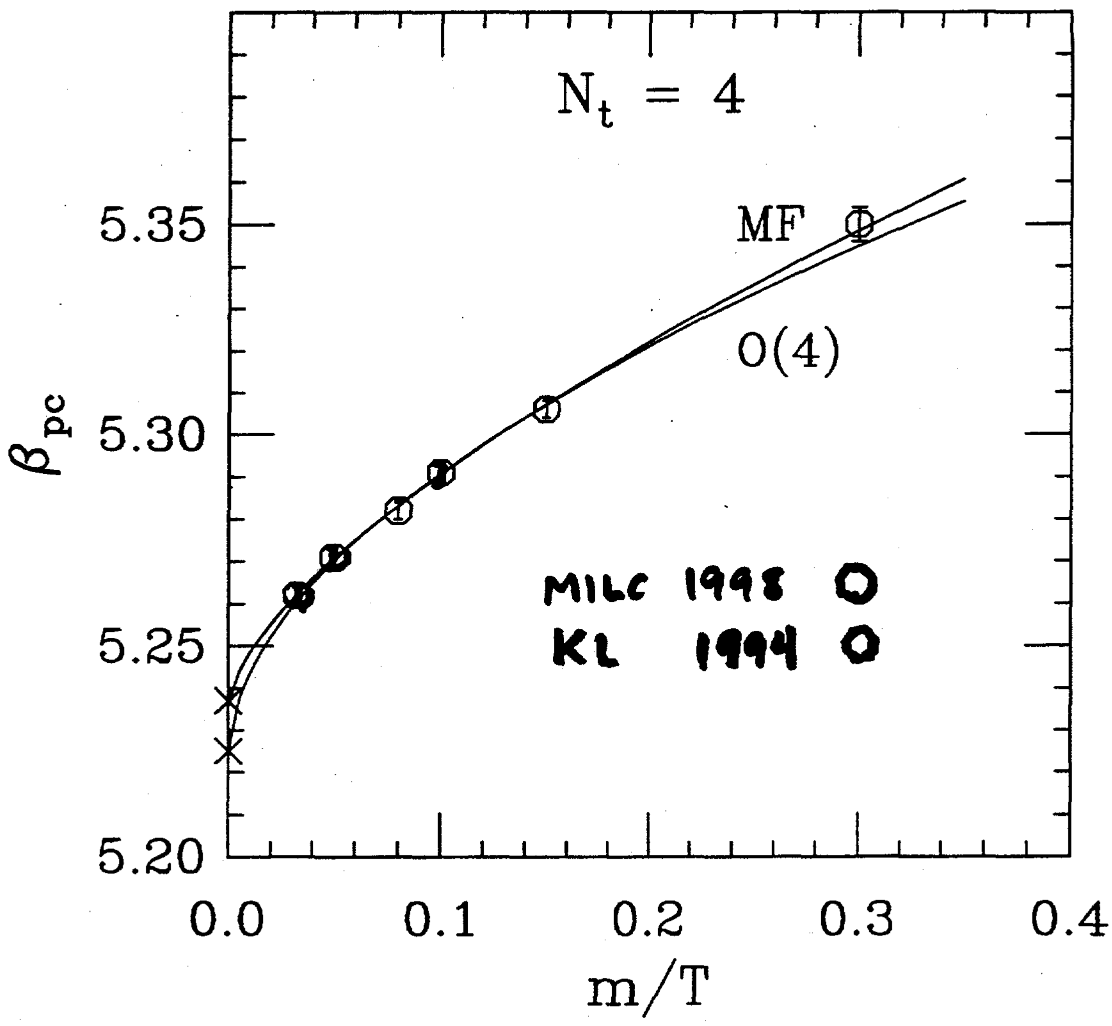
Bernard, DeGrand, DeTar,
Gottlieb, Heller, Hetrick,
Sugar, Toussaint

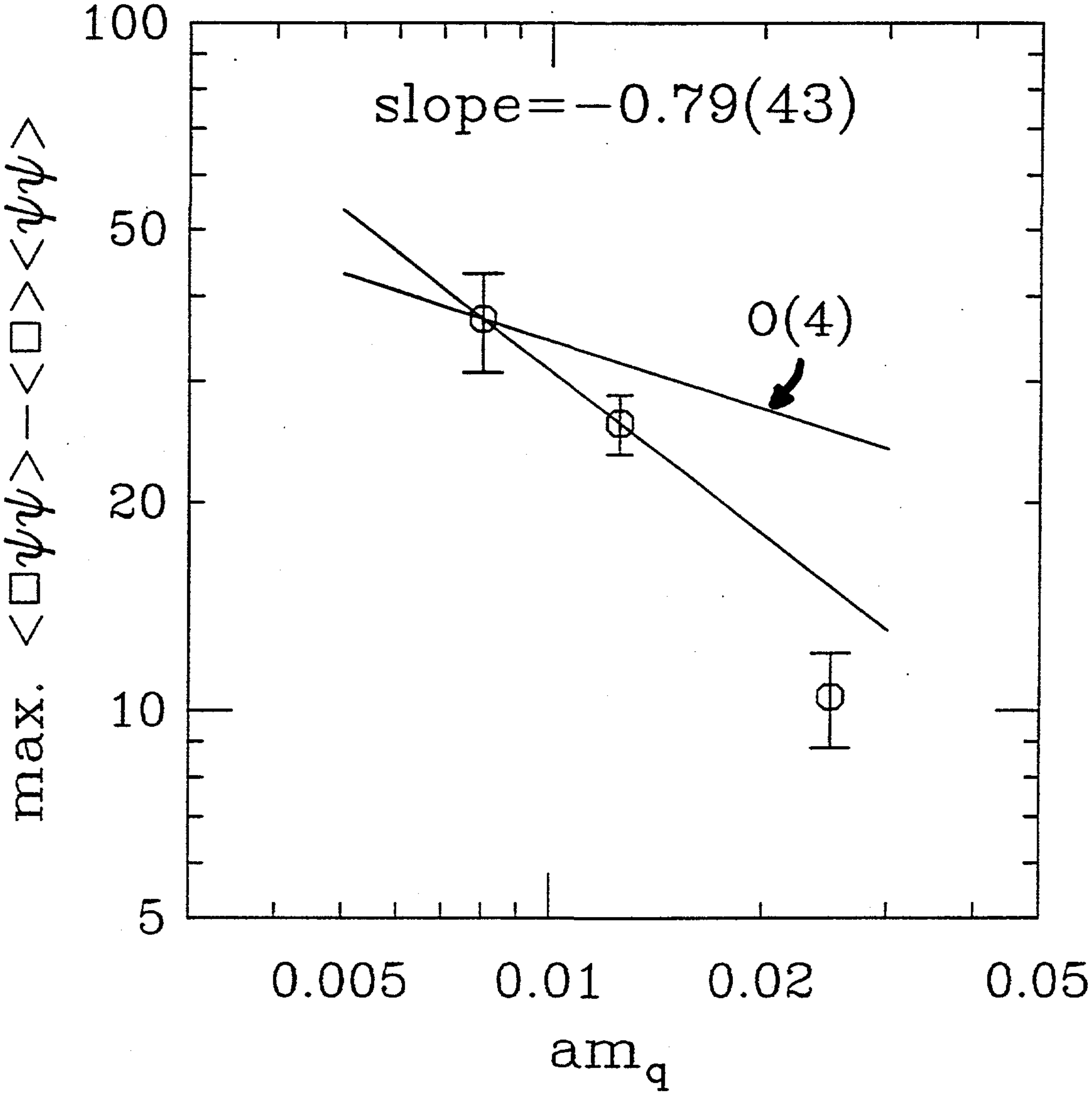
Pisarski Wilczek '84: PhysRev D $\underline{29}$, 338

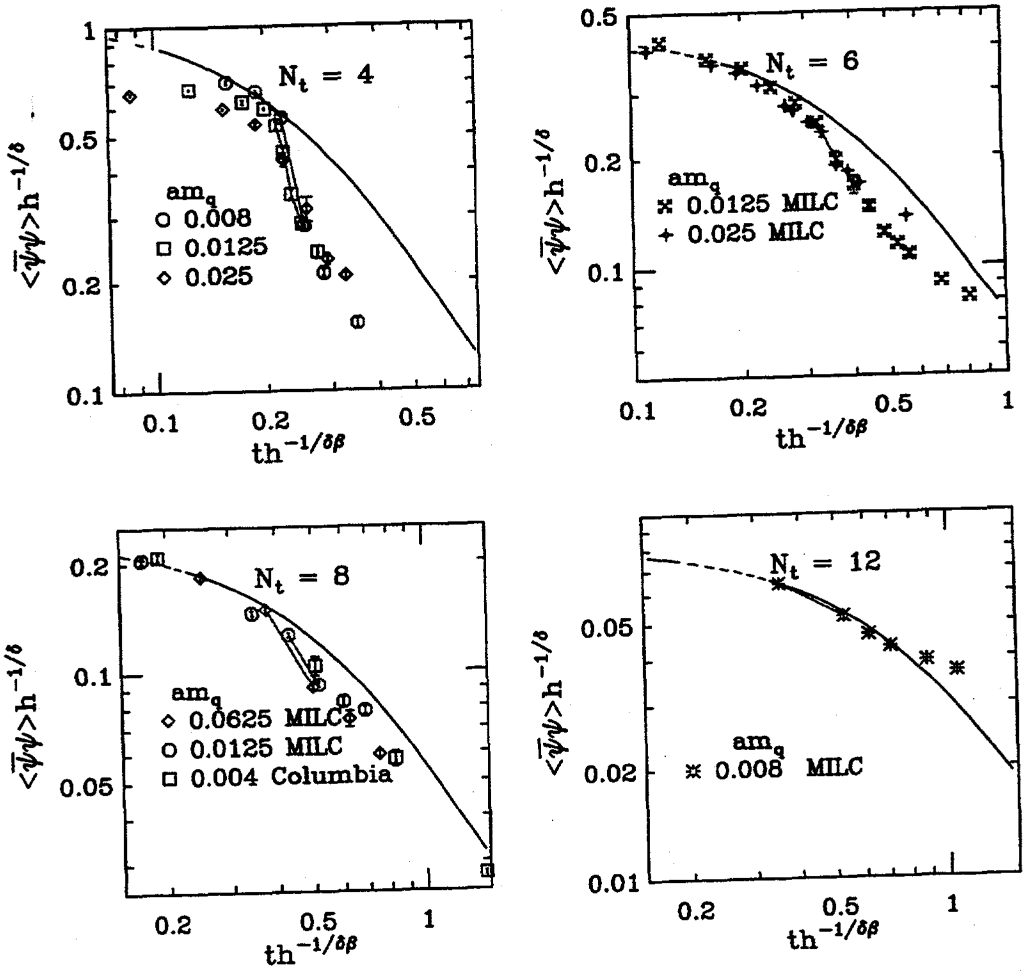
	$U_A(1)$ broken	exact
$N_f = 2$	$O(4)$	1st order
$N_f \geq 3$	1st order	1st order

Why do we care?

1. Understand field theory
2. Possible phenomenological consequences
3. Extrapolation to small m_q

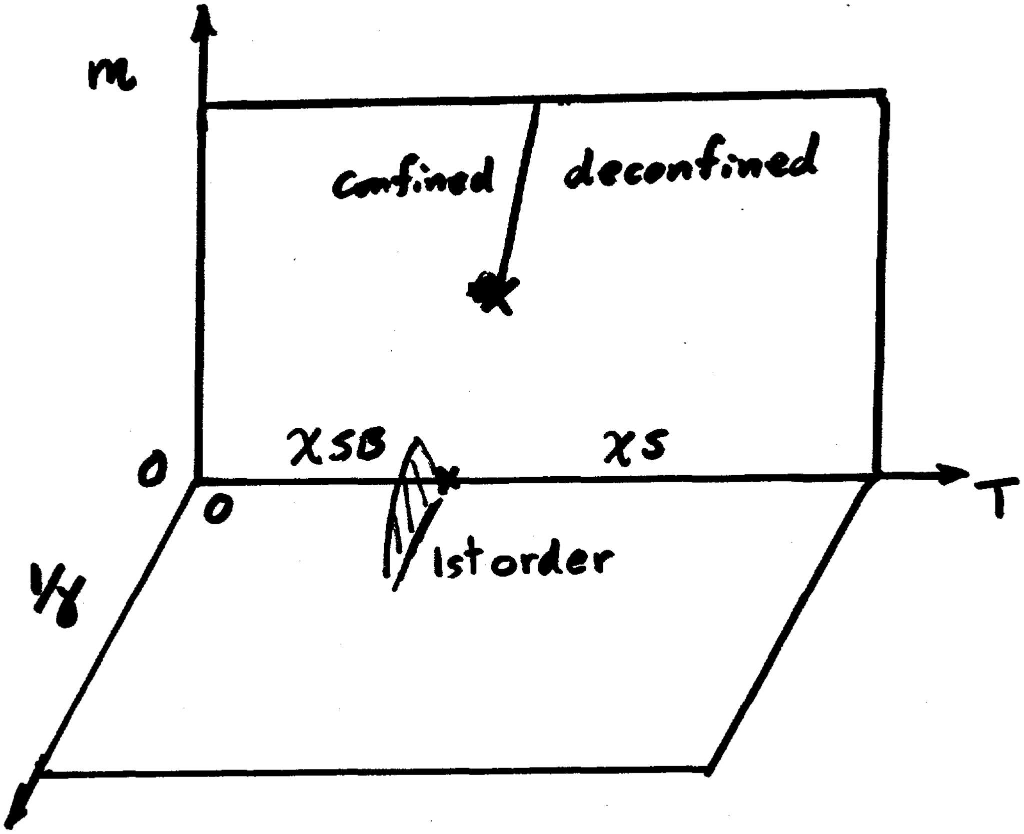

$N_t = 4$
MF
O(4)
β_{pc}
5.35
5.30
5.25
5.20
0.0
0.1
0.2
0.3
0.4
m/T
MILC 1998
KL 1994


slope=−0.79(43)
O(4)
max. $\langle\Box\bar{\psi}\psi\rangle - \langle\Box\rangle\langle\bar{\psi}\psi\rangle$
100
50
20
10
5
0.005
0.01
0.02
0.05
am_q



$N_t = 4$
$<\bar{\psi}\psi>h^{-1/\delta}$
$th^{-1/\delta\beta}$
1
0.5
0.2
0.1
0.1
0.2
0.5
am_q
○ 0.008
□ 0.0125
◇ 0.025
$N_t = 6$
$<\bar{\psi}\psi>h^{-1/\delta}$
$th^{-1/\delta\beta}$
0.5
0.2
0.1
0.1
0.2
0.5
1
am_q
⌘ 0.0125 MILC
+ 0.025 MILC
$N_t = 8$
$<\bar{\psi}\psi>h^{-1/\delta}$
$th^{-1/\delta\beta}$
0.2
0.1
0.05
0.2
0.5
1
am_q
◇ 0.0625 MILC
○ 0.0125 MILC
□ 0.004 Columbia
$N_t = 12$
$<\bar{\psi}\psi>h^{-1/\delta}$
$th^{-1/\delta\beta}$
0.05
0.02
0.01
0.2
0.5
1
am_q
⋇ 0.008 MILC

Is the phase diagram more complex?
(Strong coupling artifact)



Conclusions

1. $N_t = 4$ staggered fermions $N_f = 2$
   critical behavior not confirmed
   lattice artifact?

2. $N_t = 4$ Wilson fermions $N_f = 2$
   surprisingly good
   need larger volume to check

3. $N_t > 4$ improving, perhaps
   need larger volume to check

4. Improved actions may help

Domain Wall Fermion Thermodynamics

Robert D. Mawhinney[1]

We report on simulations of QCD (both quenched and full) using domain wall fermions (DWF). This fermion formulation uses an extra, fifth dimension for the fermions and has the full global symmetry of continuum QCD when the extent of the fifth dimension is infinite. In particular, the formulation has a $U_A(1)$ symmetry, which should be anomalously broken by the dynamics.

The first slide shows the value the chiral condensate $\langle\bar{\psi}\psi\rangle$ as a function of the quark mass for smooth lattice instanton configurations. The expected $1/m$ divergence, due to zero modes, is clearly seen for small masses. (The symbols which do not diverge, are for very small lattice extents in the fifth dimension where true zero modes are not expected.)

On quenched ensembles of lattices, the $1/m$ term is also present in the quenched chiral condensate, since there is no fermionic determinant to suppress zero modes. The second slide shows the chiral condensate on a zero temperature $8^3 \times 32$ lattice. For a $16^3 \times 32$ volume with the rest of the parameters the same, the coefficient of the $1/m$ term drops by a factor of 6.

The third slide shows the situation just above T_c, once again quenched. Here the $1/m$ term is visible, but it is independent of the volume for the two volumes studied. Also notice that there is a constant (mass independent) term, indicating chiral symmetry breaking, even though we are above T_c, as can seen by the change in value for the Wilson line. As the temperature is increased, we have seen this constant term decrease.

For full QCD, we have done a simulation with DWF to determine whether $U_A(1)$ is broken. The fourth slide shows the difference in the susceptibilities for pion (π: isovector, pseudoscalar) and delta (δ: isovector, scalar) screening masses. (The susceptibilities are proportional to $\int d^4x\langle\pi(x)\pi(0)\rangle$ and $\int d^4x\langle\delta(x)\delta(0)\rangle$.) One sees the expected quadratic dependence on the quark mass and a statistically non-zero value for $m \to 0$.

The fifth slide shows a similar result for the screening masses themselves. Once again there is a non-zero value for $m \to 0$, although it is less than 10% of the value of either screening mass individually. We have seen that $U_A(1)$ is broken above T_c for domain wall fermions, although the size of the breaking is small.

---

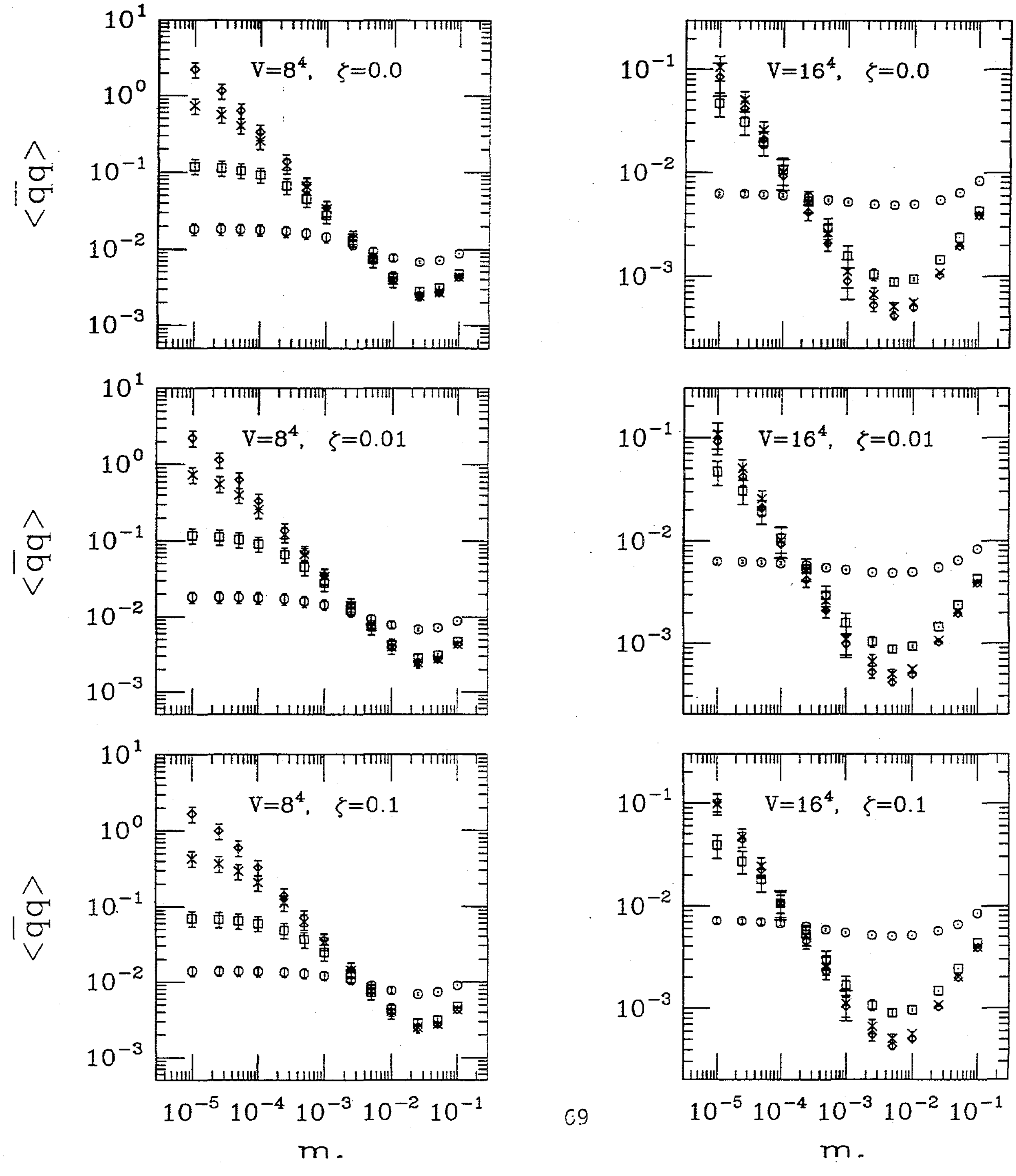
[1] This work is done in collaboration with Ping Chen, Norman Christ, George Fleming, Adrian Kaehler, Catalin Malareanu, Gabriele Siegert, ChengZhong Sui and Pavlos Vranas. It is supported in part by the DOE.

$$\langle \bar{\psi}\psi \rangle \sim m \int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}$$

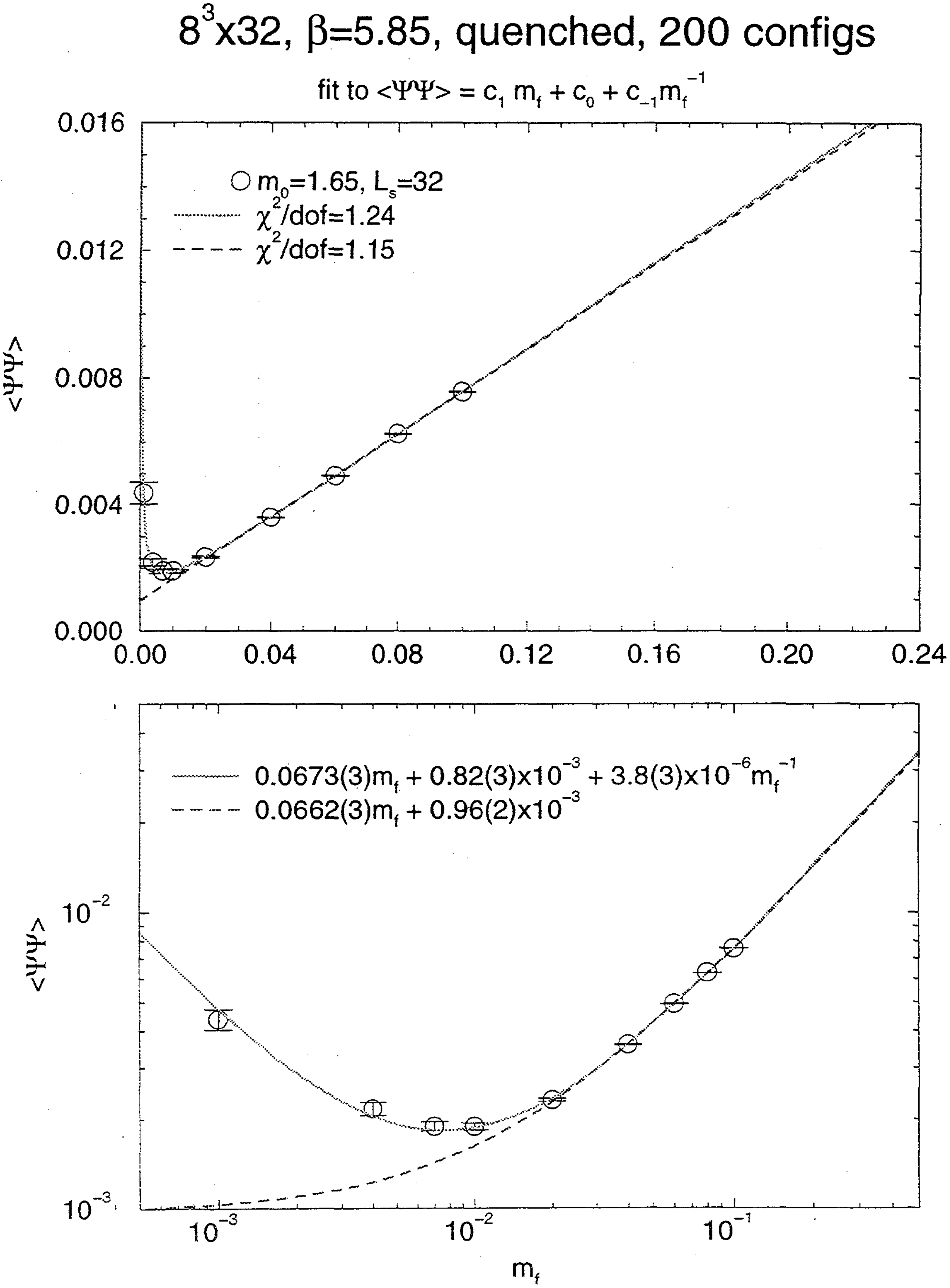
DWF - Smooth instanton configuration on lattice

ρ/a small

ρ/a large

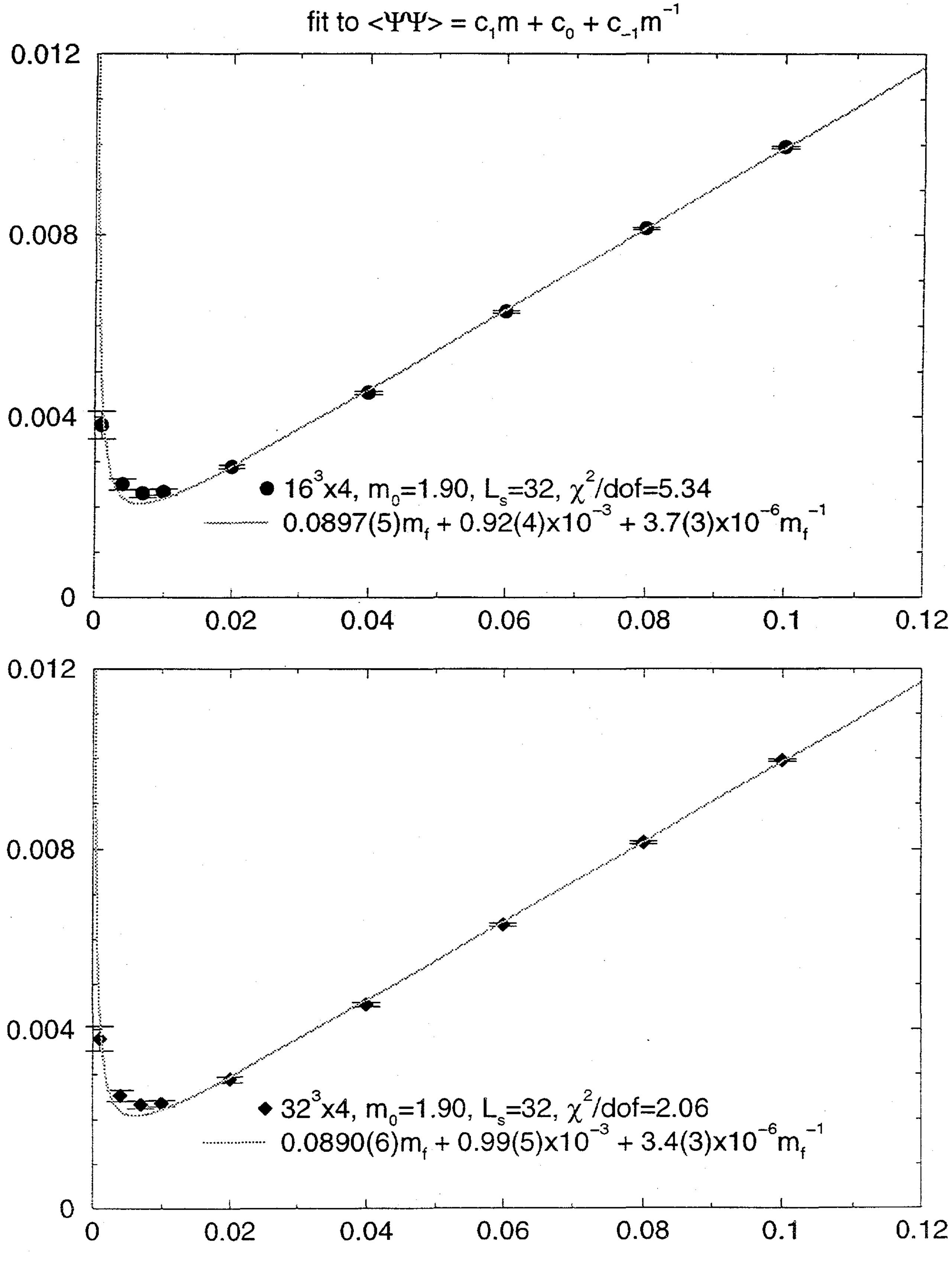


add random noise

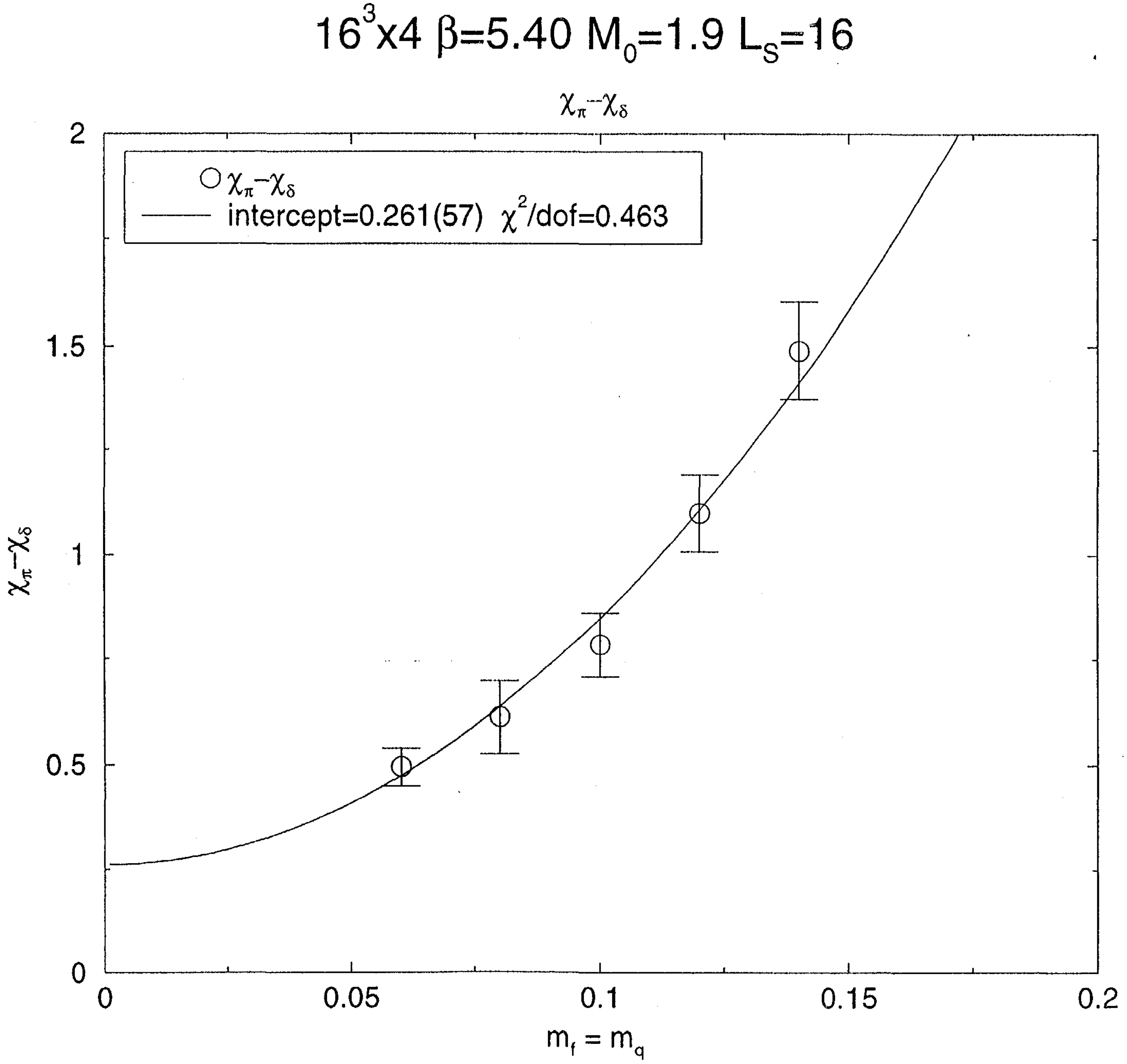


Sun Jul 12 14:56:07 1998

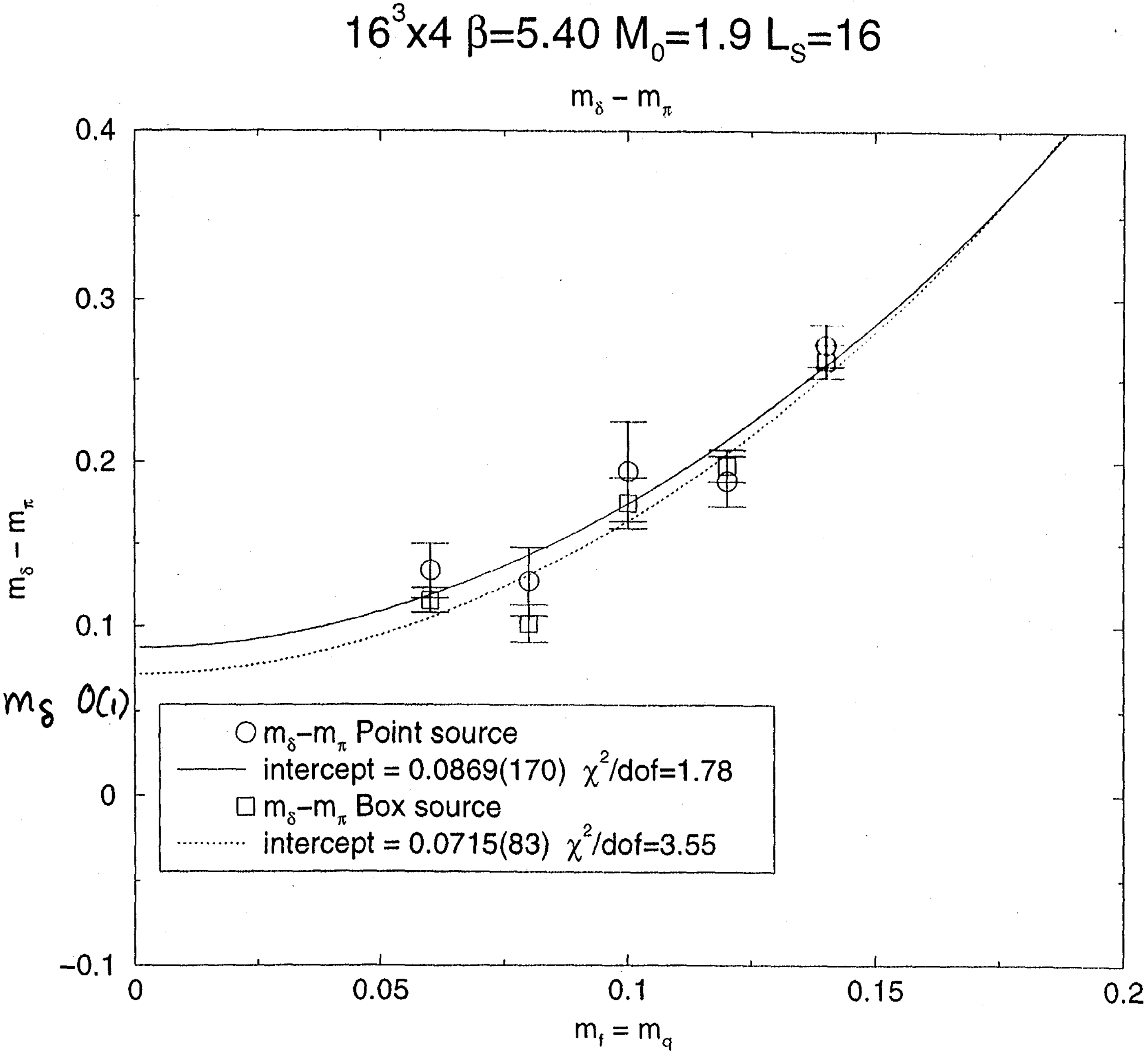
β=5.71, quenched, 128/20 configs



Thu Dec 10 11:37:17 1998



Wed Nov 25 11:35:15 1998



m_π, m_δ $O(1)$

Mon Nov 30 15:30:38 1998

SU(4) Yang–Mills: Phase Transition and String Tensions

Matthew Wingate[1], in collaboration with Shigemi Ohta[1,2]

[1]RIKEN BNL Research Center and [2]KEK

In this talk I summarize the motivation for and results of our exploratory SU(4) Yang–Mills simulations [1]. Ref. [2] suggests that a second–order SU(∞) deconfinement transition would reconcile simple large N_c arguments with lattice SU(3) results: the cubic term in the effective 3–d Lagrangian for SU(3) could be solely responsible for the first–order behavior. Also of interest are ratios of string tensions [3]. $N_c = 4$ is the smallest number of colors where one expects string tensions to be different between fundamental and diquark representations at large separations. For this study, we work at $N_t = 6$ and compute the plaquette, the Polyakov loop in different representations, and the corresponding Polyakov loop correlators.

Slide 1 shows the magnitude of the fundamental Polyakov loop L_4 and the plaquette as one increases the gauge coupling $\beta \equiv 8/g^2 \sim T$. The deconfinement transition occurs at $\beta_c = 10.76 \pm 1$, which should be well separated from the artificial bulk transition around 10.3. Slide 2 is a plot of the deconfinement fraction which also indicates a breaking of $Z(4)$ symmetry at β_c. In slide 3 we plot histograms of $z - \arg(L_4)$, the angle between $\arg(L_4)$ and its nearest $Z(4)$ symmetry axis, as well as histograms of $|L_4|$. The latter would show a two state signal if there is a first–order transition at β_c. With $\sim$ 4000 measurements on the 12^3 lattice, we cannot locate such a two state signal. For SU(3) larger volumes were necessary to locate such a signal. Therefore the only conclusion we draw presently is that a very strong first–order SU(4) deconfinement transition is unlikely.

In slide 4 we turn to our 16^3 lattice calculations at $\beta = 10.65$ and 10.7 where we compute σ_4 and σ_6, the string tensions between static fundamental and anti-symmetric diquark charges, respectively. From our best fits, we find $1 < \sigma_6/\sigma_4 < 2$. However, this result should be taken with care since our volume may not be large enough to be sure that only the string tension remains at our largest separation.

These results, while not definitive, are exciting and encouraging. An understanding of SU(N_c) Yang–Mills with $N_c > 3$ would shed some light on real world QCD.

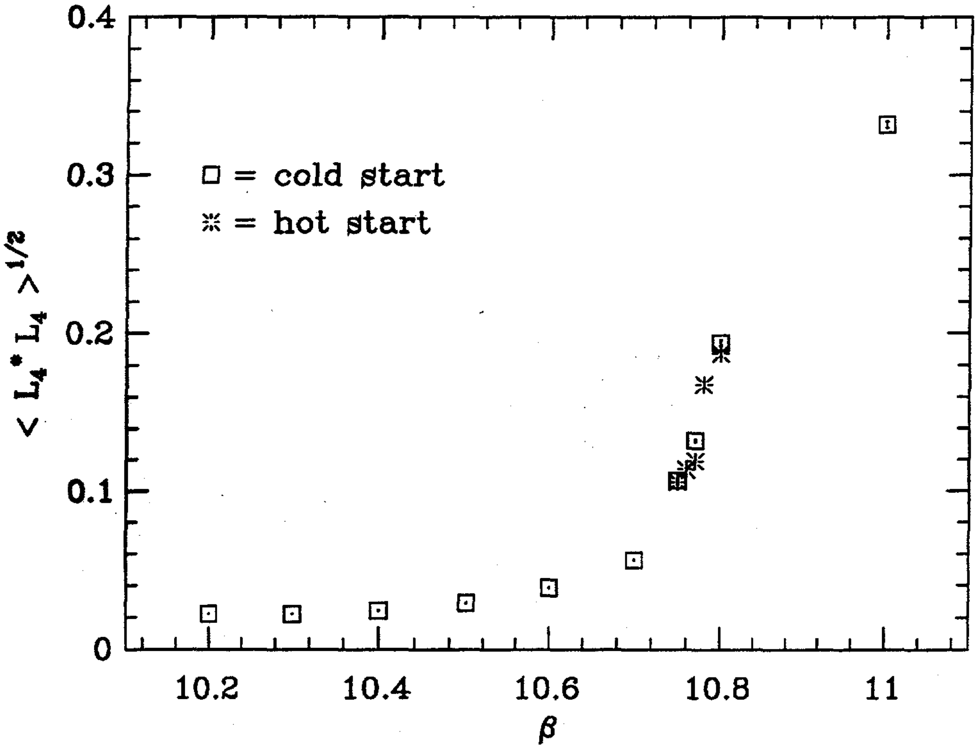
References

[1] S. Ohta and M. Wingate, poster at "International Symposium on Lattice Field Theory 1998", Boulder, CO, USA, `hep-lat/9808022`.

[2] R. Pisarski and M. Tytgat, proceedings of the XXV Hirschegg Workshop on "QCD Phase Transition", Jan. 1997, `hep-ph/9702340`.

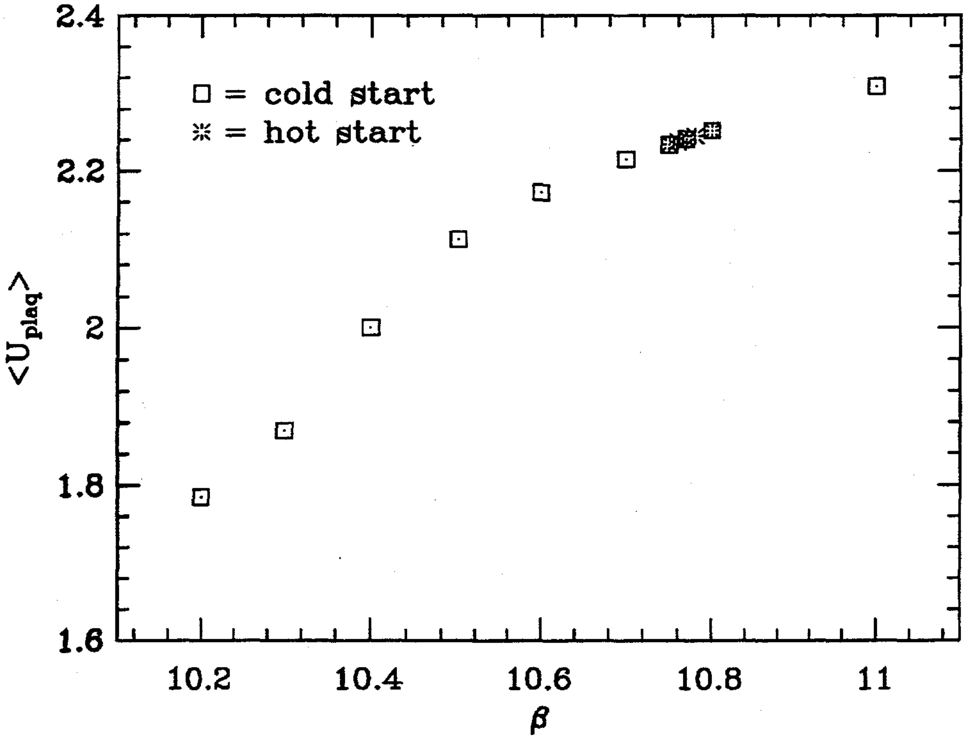
[3] M. Strassler, plenary talk at "International Symposium on Lattice Field Theory 1998", Boulder, CO, USA, `hep-lat/9810059`; and these proceedings.

Polyakov Loop (Fund. rep.)

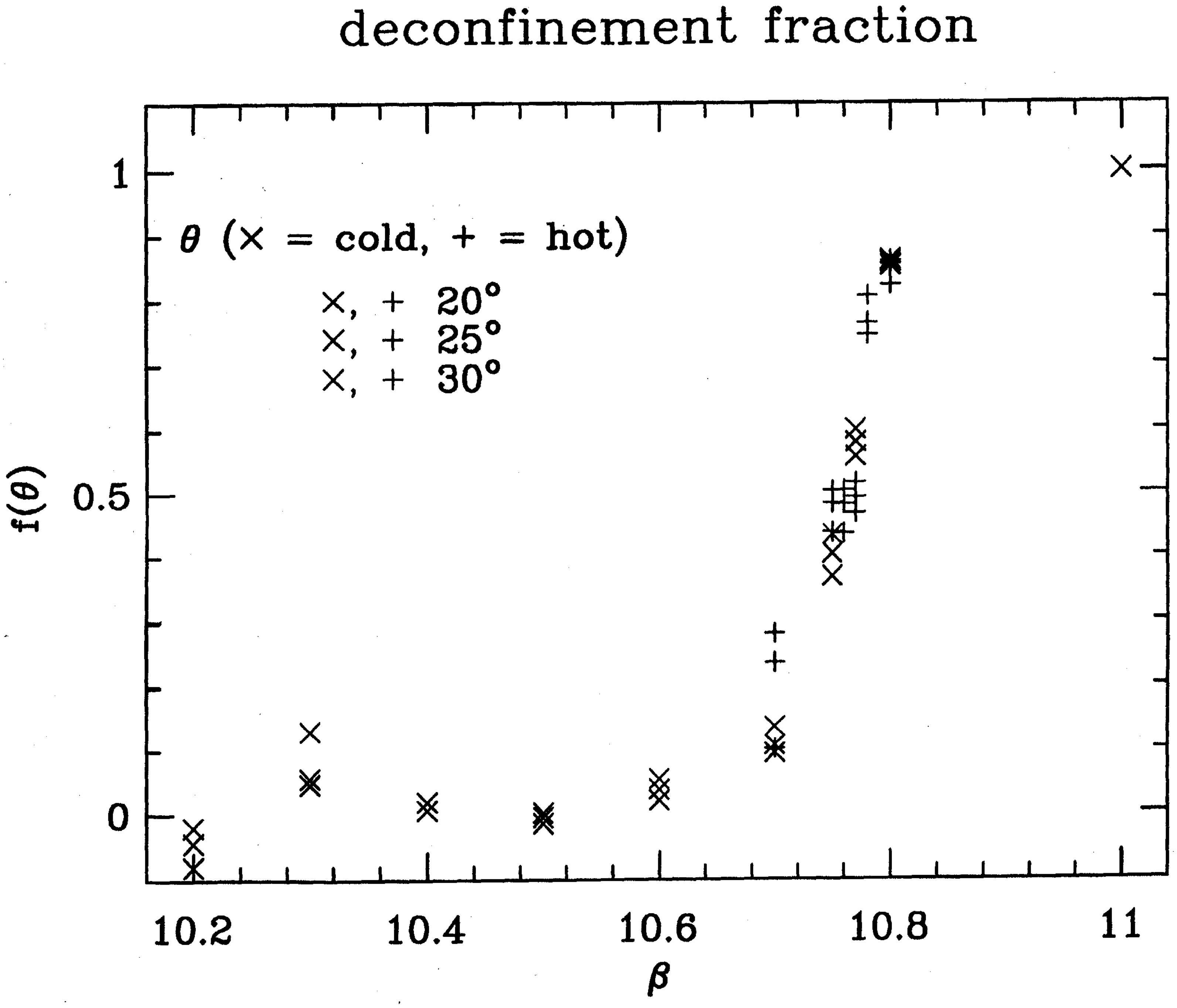


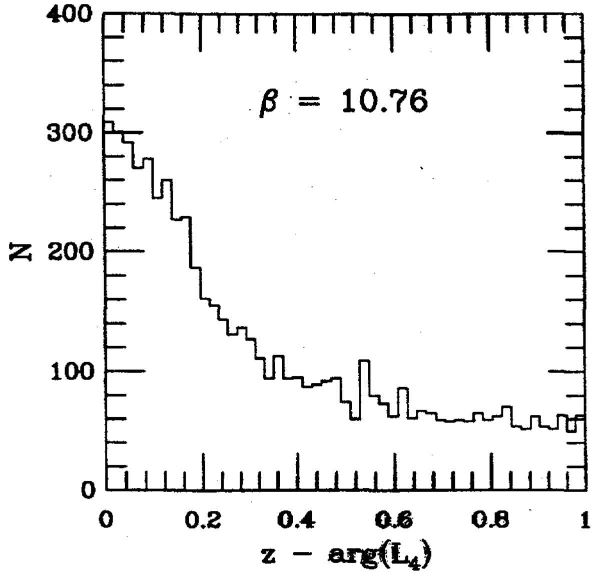
$12^3 \times 6$

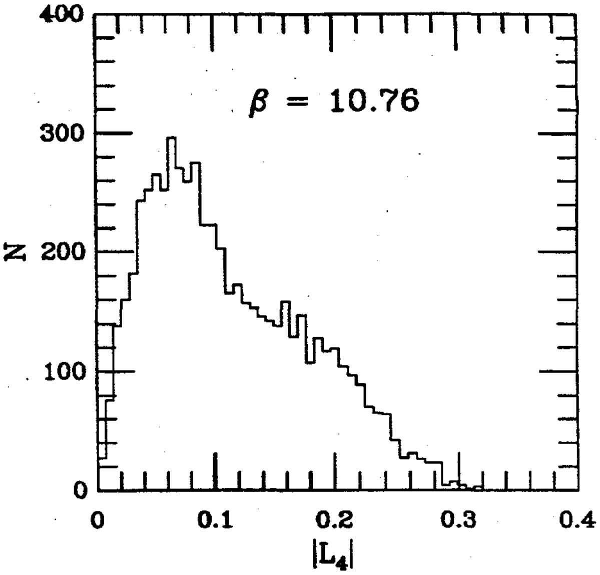
Plaquette



$12^3 \times 6$


deconfinement fraction
θ (× = cold, + = hot)
×, + 20°
×, + 25°
×, + 30°
f(θ)
1
0.5
0
10.2
10.4
10.6
10.8
11
β


β = 10.76
N
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
z − arg(L₄)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N
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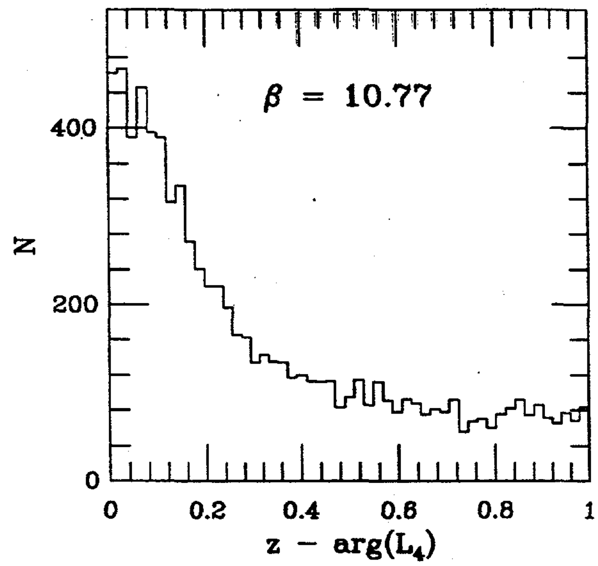
300
200
100
0
0
0.1
0.2
0.3
0.4
|L₄|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200
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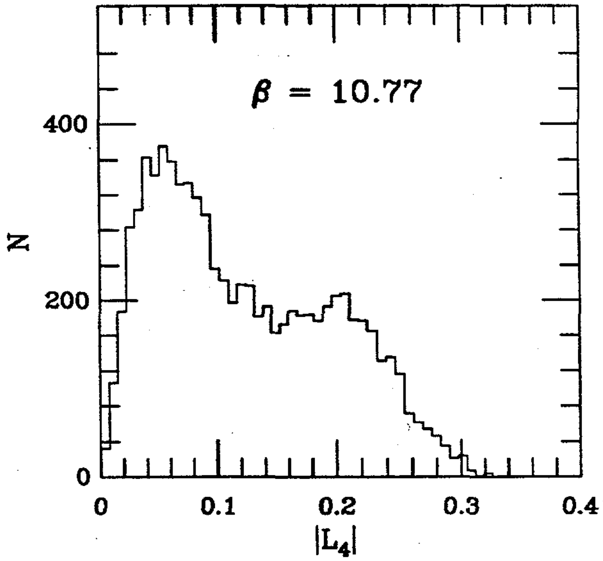
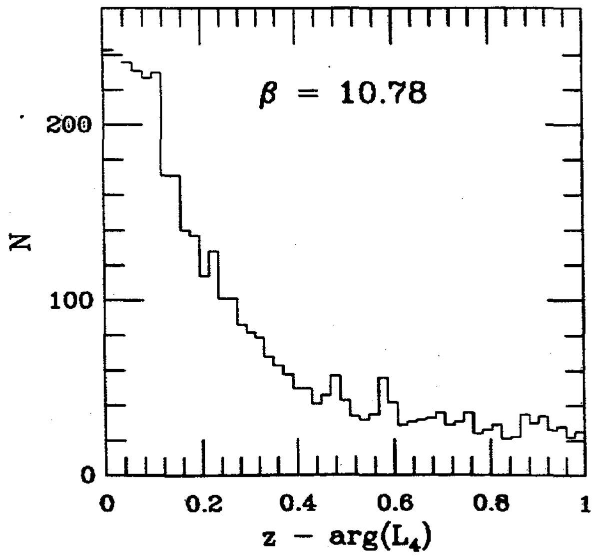
0.8
1
z − arg(L₄)


β = 10.77
N
400
200
0
0
0.1
0.2
0.3
0.4
|L₄|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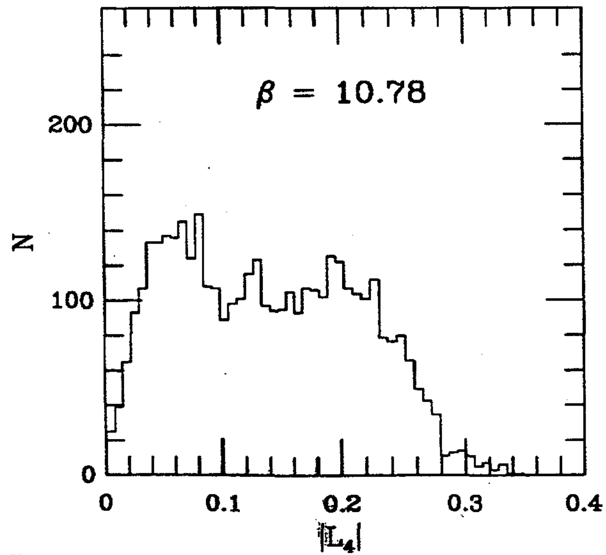
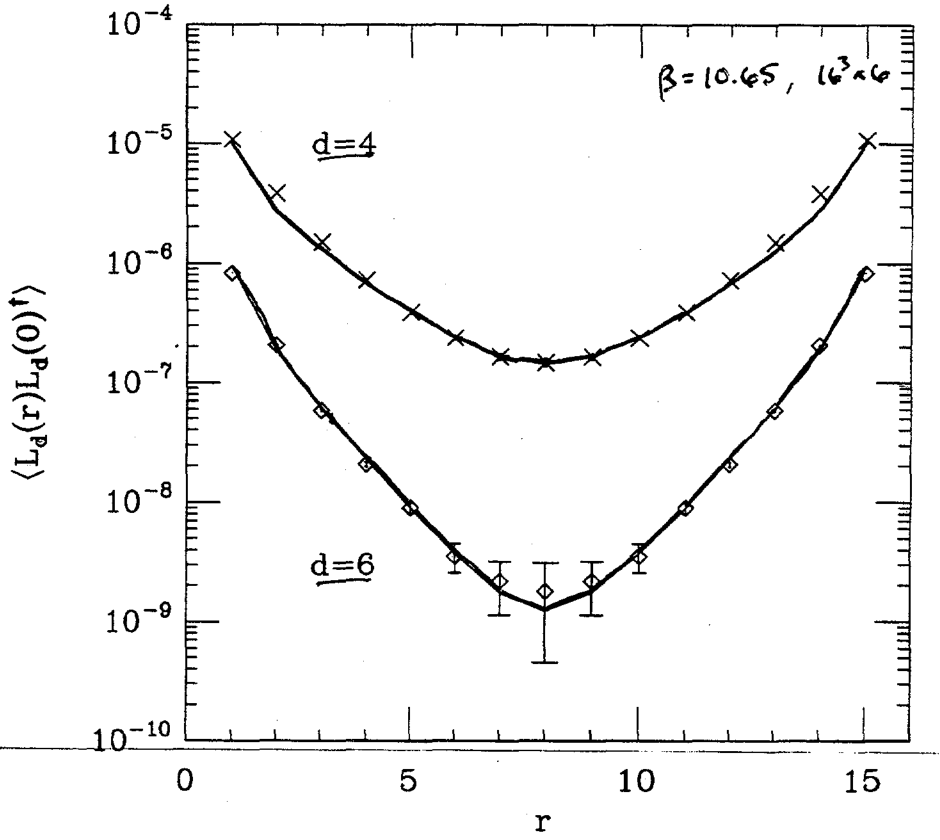
100
0
0
0.2
0.4
0.6
0.8
1
z − arg(L₄)


β = 10.78
N
200
100
0
0
0.1
0.2
0.3
0.4
|L₄|

Polyakov loop correlation



r = 5-8

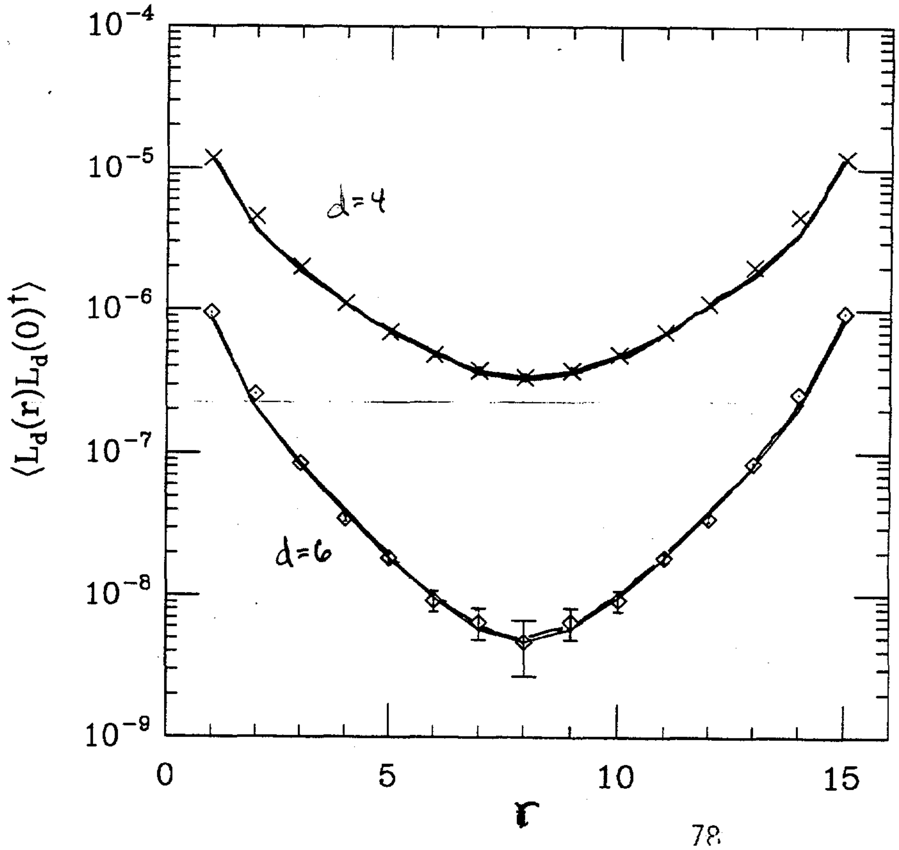
r = 5-7

$\sigma_4 = 0.086(3)$

$\sigma_6 = 0.142(57)$

$\frac{\sigma_6}{\sigma_4} = 1.7(6)$

Polyakov loop correlation



$\beta = 10.7$
$16^3 \times 6$

r = 6-8

r = 5-8

$\sigma_4 = 0.068(4)$

$\sigma_6 = 0.108(17)$

$\frac{\sigma_6}{\sigma_4} = 1.6(2)$

Statistical errors only!

String model (Pisarski & Alvarez '82) ⇛

$$\frac{T_c}{\sqrt{\sigma}} = \sqrt{\frac{3}{(d-2)\pi}}$$

depends only on dimension.

From F. Karsch's talk yesterday

$$\frac{T_c}{\sqrt{\sigma(T=0)}} = \begin{cases} 0.69(2) & SU(2) \\ 0.63-0.66 & SU(3) \end{cases}$$

This work in SU(4)

$$\frac{a_c}{a} \times \frac{T_c}{\sqrt{\sigma(T\neq 0)}} = \begin{cases} 0.50 & \beta = 10.6 \\ 0.57 & \beta = 10.65 \\ 0.64 & \beta = 10.7 \end{cases}$$

Really need $T=0$ string tension.

Deconfinement in SU(2) Yang-Mills theory as a center vortex percolation transition[*]
(Summary)

M. Engelhardt

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Auf der Morgenstelle 14, 72076 Tübingen, Germany

Summary of talk presented at the RIKEN BNL Workshop on "QCD Phase Transitions", Brookhaven National Laboratory, USA, November 4-7, 1998. Work performed in collaboration with K. Langfeld, H. Reinhardt and O. Tennert. For a preliminary account of some of this work and pertinent references, see eprint hep-lat/9805002.

The center vortex picture of confinement generates an area law for the Wilson loop by invoking the presence of vortices in typical configurations entering the Yang-Mills functional integral. These vortices are closed two-dimensional surfaces in four-dimensional space-time. They carry flux such that they contribute a factor -1 (for the SU(2) color case considered here) to any Wilson loop whenever they pierce its minimal area. Random intersections of vortices with Wilson loop areas generate an area law (cf. 1st transparency). The present work investigates whether and how this picture can generate a deconfinement transition. It is observed that randomness in the above sense demands that vortex clusters percolate in space-time. Imposing a maximal cluster length leads to a perimeter law (cf. 1st transparency). It is conjectured that vortices cease to percolate in the deconfined phase.

The tools which allow to test this conjecture have only become available recently through the work of Faber, Greensite, and collaborators. In analogy to 't Hooft's Abelian gauges and Abelian projection, they define maximal center gauges and center projection to localize vortices on the dual lattice (cf. 2nd transparency). It has been verified that the vortices defined thus generate the full Yang-Mills string tension (so-called center dominance) at zero and at finite temperatures.

Using these techniques, one can measure planar densities of points at which vortices pierce two-dimensional planes in the lattice. One observes a certain polarization of the vortices into the time direction as temperature is increased (cf. 3rd transparency). More importantly, vortices indeed cease to percolate at the deconfinement transition, cf. the middle plots on the 3rd transparency, which show the percentage of available vortex material concentrated in clusters of a given extension (measured in units of the size of the universe). These plots apply to the vortex lines obtained in a cut of the universe in which one space coordinate is fixed. If one instead considers a time slice, percolation persists in the deconfined phase, cf. the bottom plot on the 3rd transparency. This in particular explains the presence of a spatial string tension above the deconfinement temperature.

One obtains an intuitive picture of the dominant configurations in the respective regimes which is summarized on the 4th transparency. The short vortices which dominate the deconfined phase wind around the lattice in time direction. These configurations are not available in the low temperature regime due to the different shape of (Euclidean) space-time. It is observed that the picture obtained here is dual to the electric flux tube picture, in which electric flux percolates in the deconfined phase and does not percolate in the confined phase.

In a lattice model of vortices as random surfaces, one can understand the percolation transition in terms of an action-entropy competition. One of the questions deserving further inquiry (cf. 5th transparency) is whether such a random surface model indeed emerges as a low-energy effective theory from Yang-Mills theory. Also, besides generalizing the present work to SU(3) color, continuum formulations of the random surface model need to be investigated, along with the inclusion of fermions in such a model.

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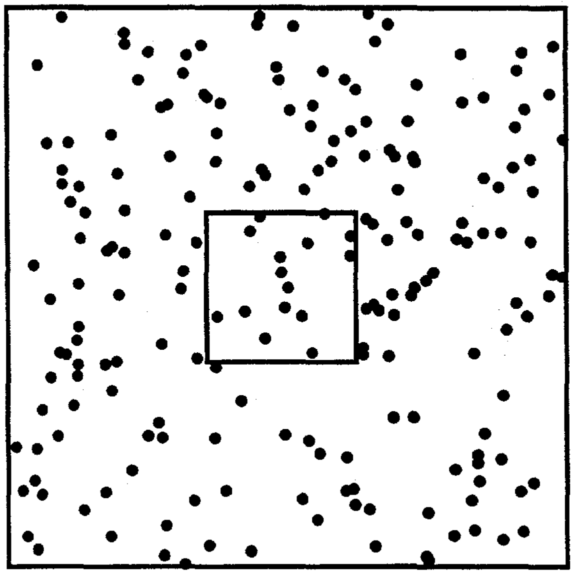
[*] Supported in part by DFG under contract Re 856/1-3.

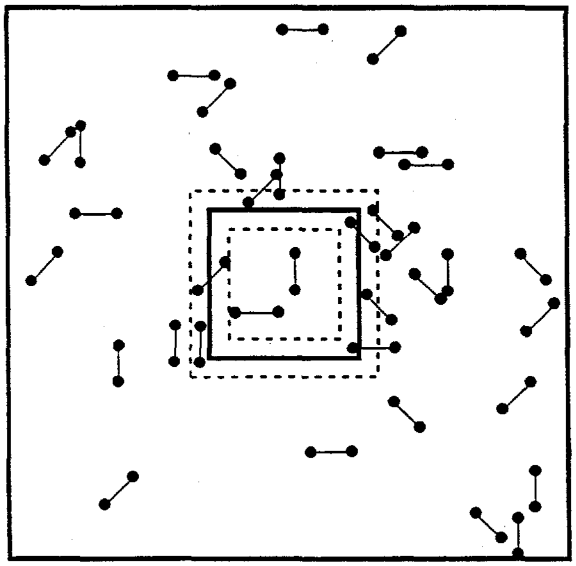
Center vortex picture – Heuristics

Vortices: Closed 2-D world-sheets in 4-D space-time
Contribution (-1) to Wilson loop when minimal area pierced



Universe L^4
Slice of universe L^2
Wilson loop area A
Number of points N
Density of points $\rho = N/L^2$

$$\langle W\rangle = \sum_{n=0}^{N}(-1)^n \begin{pmatrix} N \\ n \end{pmatrix}\left(\frac{A}{L^2}\right)^n\left(1-\frac{A}{L^2}\right)^{N-n} = \left(1-\frac{2\rho A}{N}\right)^N \xrightarrow{N\to\infty} \exp(-2\rho A)$$



Wilson loop perimeter P
Number of pairs N
Extension of pairs d
Density of points $\rho{=}2N/L^2$
Linking probability p

$$\langle W\rangle = \sum_{n=0}^{N}(-1)^n \begin{pmatrix} N \\ n \end{pmatrix}\left(\frac{pPd}{L^2}\right)^n\left(1-\frac{pPd}{L^2}\right)^{N-n} = \left(1-\frac{\rho pPd}{N}\right)^N \xrightarrow{N\to\infty} \exp(-\rho pPd)$$

Deconfinement Transition ⟷ Percolation Transition?

Tools: Locating vortices in lattice configurations

Idea: 1. Choose gauge such that relevant physical information concentrated optimally on collective degrees of freedom under investigation

2. Project onto collective degrees of freedom

E.g. Maximal Abelian gauge, Abelian projection
⟶ Dual superconductor picture

Here: 1. **Maximal center gauge**

$$\max \sum_i |\mathrm{tr}\, U_i|^2$$

Links as close as possible to center elements

2. **Center projection**

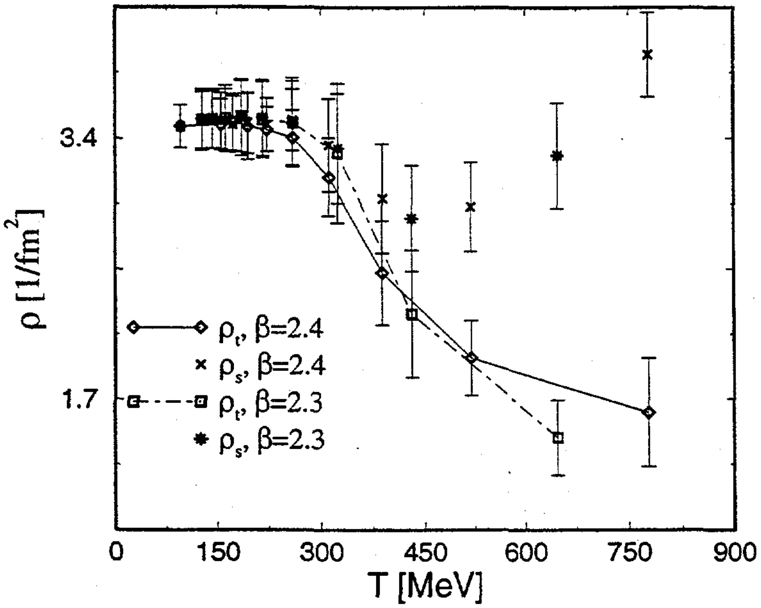
$$U \longrightarrow \ \mathrm{sign}\ \mathrm{tr}\, U$$

⟶ Lattice of center elements
⟶ Vortices on dual lattice

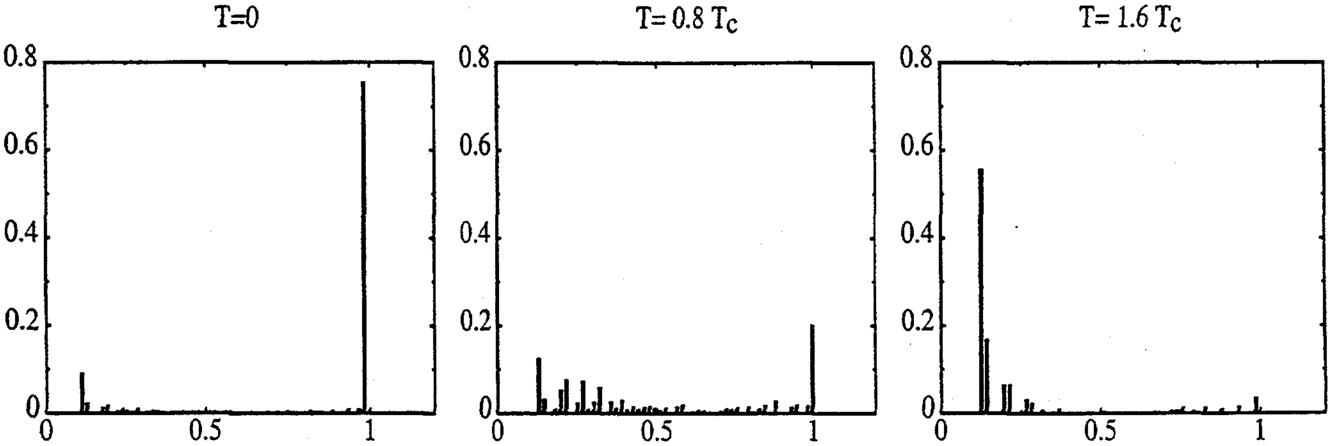
Question: Is information relevant for confinement indeed concentrated on vortices?

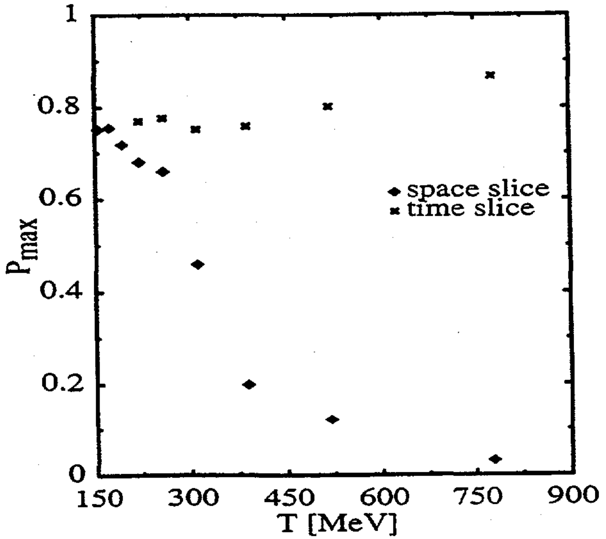
Answer empirically: ⟶ **Center dominance**

Vortex properties: Density

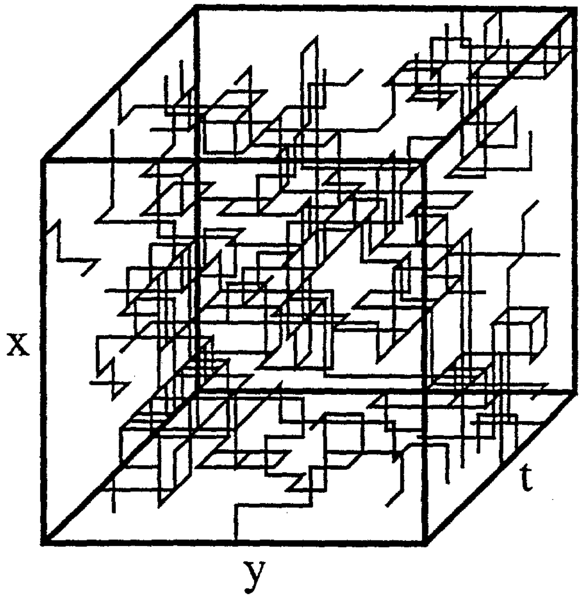


Vortex material distribution (cluster size)

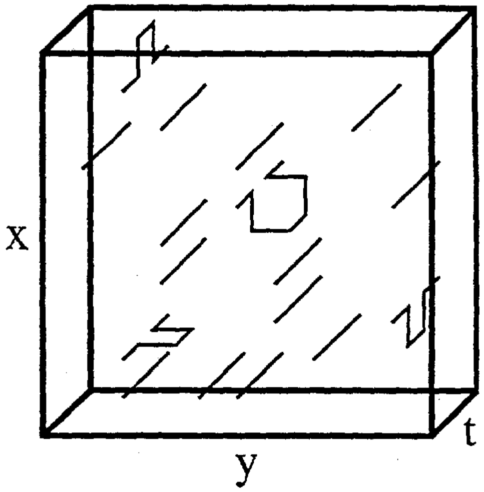




Typical configurations in the confined and deconfined phases



confined phase



deconfined phase

⟶ understand also the spatial string tension

Comparison to electric flux picture

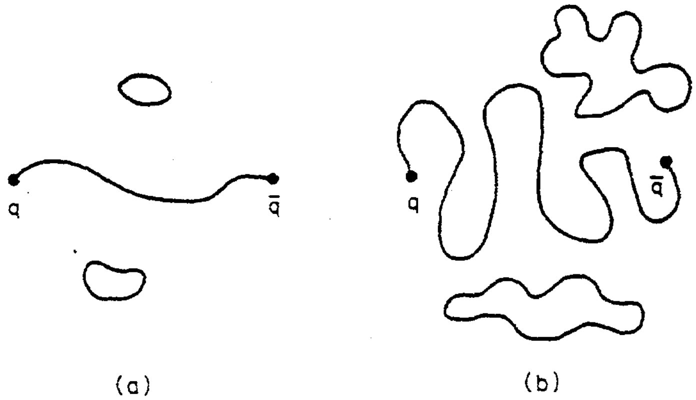


Fig. 3. Flux tubes between static quarks in SU(2) pure gauge theory: (a) at low temperatures: (b) at high temperatures.

(From: A. Patel, Nucl. Phys. B243 (1984) 411)

Outlook: Dynamics of vortices

Model: Vortices as random surfaces on the dual lattice

- Bianchi constraint organizes magnetic flux into vortices
- In all other respects, expect magnetic degrees of freedom to weakly interact at large distances

Deconfinement understandable in such a model in terms of an action-entropy competition

Further investigation:

- Emergence of random surface model as low-energy effective theory from Yang-Mills theory?
- Generalization to SU(3) color
- Continuum version of random surface model
- Including fermions (Bianchi constraint does not rigidly couple vortices and sources!)

Insight from the Lattice into the Role of Instantons

J.W. Negele

RIKEN BNL Workshop on QCD Phase Transitions, November 1998

Lattice QCD provides strong evidence that instantons play a major role in quark propagation in the vacuum and in light hadron structure.

- Two-point vacuum correlation functions of hadron currents display behavior expected from the 't Hooft interaction and instanton liquid models.
- Calculations with all gluons and only instantons agree for vacuum correlation functions, hadron density-density correlation functions, and the nucleon axial change.
- Quark zero modes are observed in quenched and full QCD.
- Truncation of the quark propagator to the zero mode zone accounts for the ρ, π, and η' contributions to vacuum correlation functions and for the topological charge.
- Quark localization is observed at instantons in uncooled configurations.

Consistent results for the instanton content of the QCD vacuum have been obtained by a variety of methods.

- Average instanton size:

  $\bar{\rho} \sim 0.39 \pm 0.05$ fm when extrapolated to the uncooled vacuum
  $\bar{\rho} \sim 0.54 \pm 0.05$ fm when cooled to $N/V \sim 3\ \mathrm{fm}^{-4}$

- Methods for measuring topological susceptibility agree and are consistent with the Veneziano-Witten formula.

The heavy quark potential has been measured in an instanton liquid to ~ 3 fm.

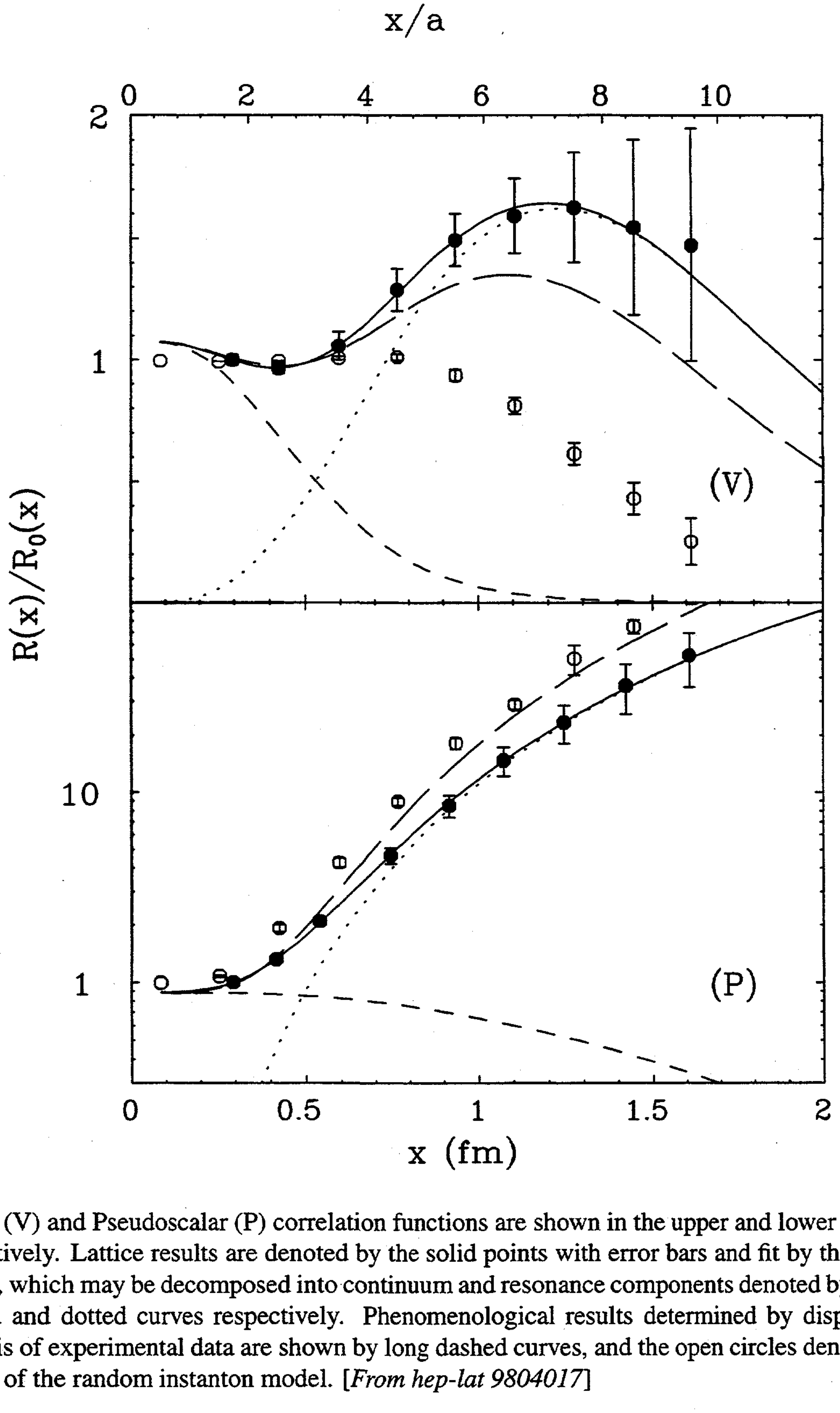
- Linear for small ρ, constant for large ρ.
- Slope $\sim \frac{1}{10}\sigma$ at $N/V = 1\ \mathrm{fm}^{-4}$ and $\bar{\rho} = 0.33$ fm
  $\sim \sigma$ at $N/V \sim 10\ \mathrm{fm}^{-4}$, $\bar{\rho} = 0.33$ fm or $N/V \sim 1\ \mathrm{fm}^{-4}$, $\bar{\rho} = 0.59$ fm

The topological susceptibility has been measured at nonzero T for quenched and full QCD.

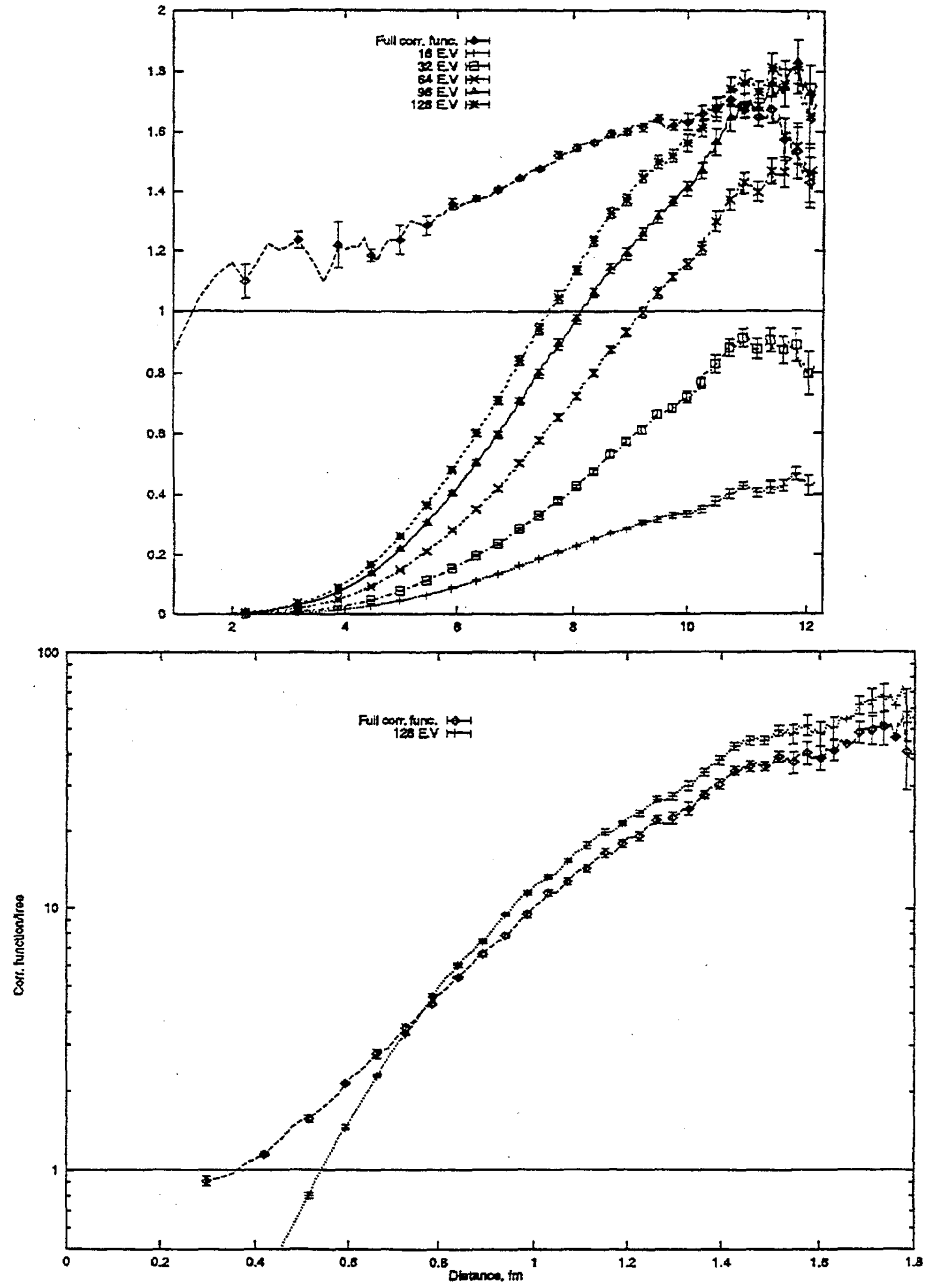
- $\bar{\rho}$ decreases $\sim 25\%$ by $1.3\ T_c$
- Qualitative agreement with Debye screening observed.

A new class of calorons comprised of monopoles has been discovered for $T \neq 0$ and Polyakov loop $\neq \pm 1$.

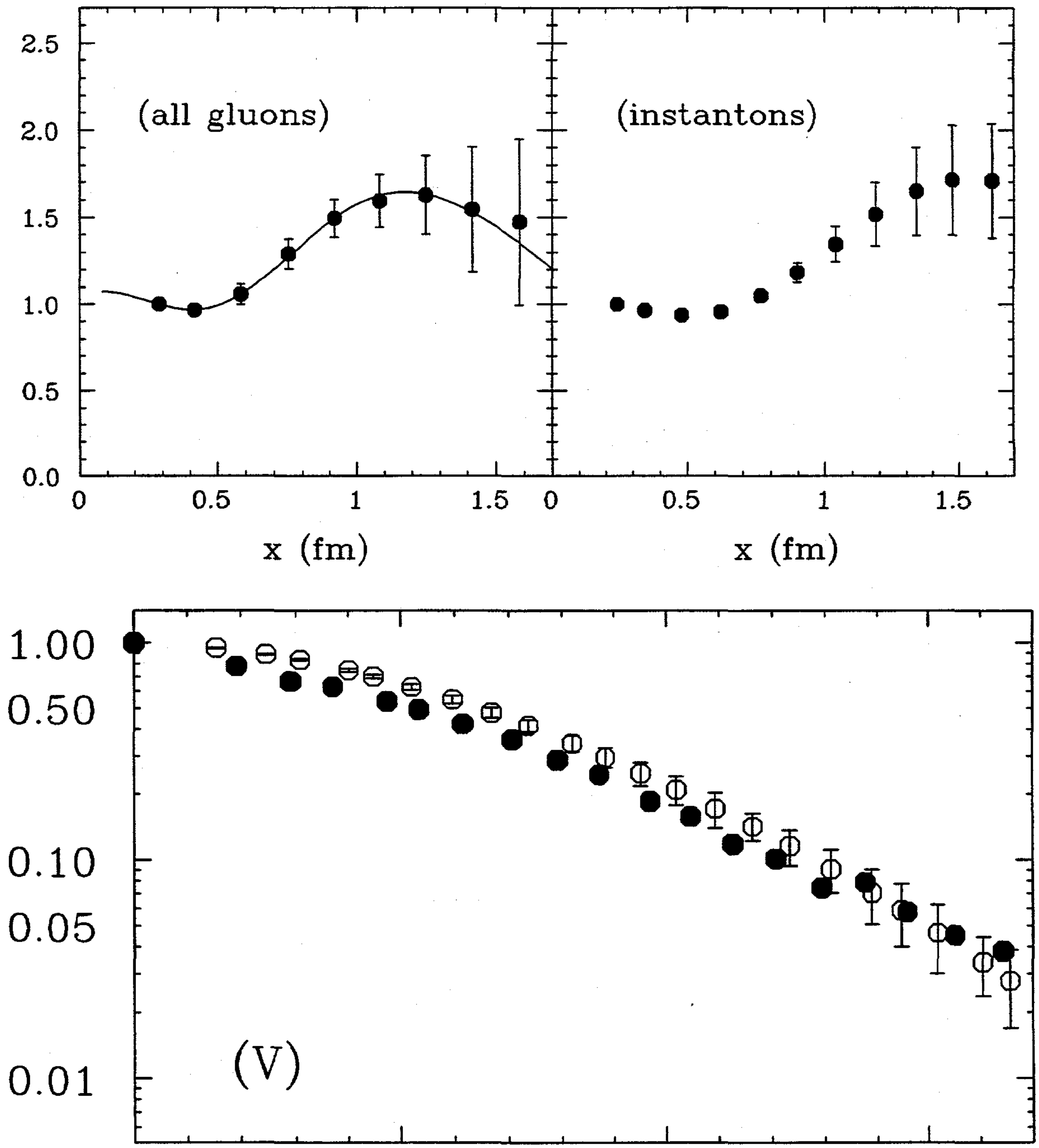
Details are given in hep-lat/9810053.



Vector (V) and Pseudoscalar (P) correlation functions are shown in the upper and lower panels respectively. Lattice results are denoted by the solid points with error bars and fit by the solid curves, which may be decomposed into continuum and resonance components denoted by short dashed and dotted curves respectively. Phenomenological results determined by dispersion analysis of experimental data are shown by long dashed curves, and the open circles denote the results of the random instanton model. [*From hep-lat 9804017*]



Contributions of low Dirac eigenmodes to the vector (upper graph) and pseudoscalar (lower graph) vacuum correlation functions. The upper graph shows the contributions of 16, 32, 64, 96, and 128 eigenmodes compared with the full correlation function for an unquenched configuration with a 63 MeV valence quark mass. The lower graph compares 128 eigenmodes with the full correlation function for a quenched configuration with a 23 MeV quark mass.



Comparison of rho observables calculated with all gluon configurations and only instantons. The upper left-hand plot shows the vacuum correlator in the rho channel calculated with all gluons and the upper right-hand plot shows the analogous result with only instantons. The lower plot shows the ground state density-density correlation function for the rho with all gluons (solid circles) and with only instantons (open circles). Error bars for the solid circles are comparable to the open circles and have been suppressed for clarity. [*From hep-lat 9804017*]

Studies of the instanton content of the SU(3) Vacuum

β	Lattice	Method	$\bar{\rho}_{\mathrm{N/V}}$ (fm)	$\bar{\rho}_{\mathrm{extrap}}$ (fm)	$\frac{N}{V}$ (fm^{-4})	Reference
6.0 6.2 6.4	$16^3 \times 48$ $32^3 \times 64$ $24^3 \times 48$ $32^3 \times 64$	Underrelaxed Cooling	$0.60(5)^a$	$0.37(5)^b$	55–3.2	Smith, Teper
5.85 6.0 6.1	12^4 12^4, 16^4 16^4	APE Smearing		0.32^c	1.1	A. Hasenfratz, Nieter
5.85 6.0	12^4 16^4	Improved Cooling	$< 0.53(5)^d$		3.3–0.38	de Forcrand, Pérez Hetrick, Stamatescue
5.7	$16^3 \times 24$	Cooling		$> 0.39^e$	$0.59\text{–}0.28^f$	Chu, Grandy, Huang, Negele
5.85	16^4	Relaxation	$0.50(5)^g$	$0.43(5)^{b,g}$	$5.3\text{–}1.4^g$	Ivanenko, Negele
unquenched, κ=0.16						
5.5	16^4		$0.52(5)^g$	$0.42(5)^{b,g}$	$6.5\text{–}1.8^g$	
Summary			0.54(5)	0.39(5)		

[a] Value 0.56(5) at N/V= 8.5 for $\beta = 6.4$ evolved to N/V=3.2 using $\beta = 6.0$ data.

[b] Extrapolation sketched in the figure on p. 6.

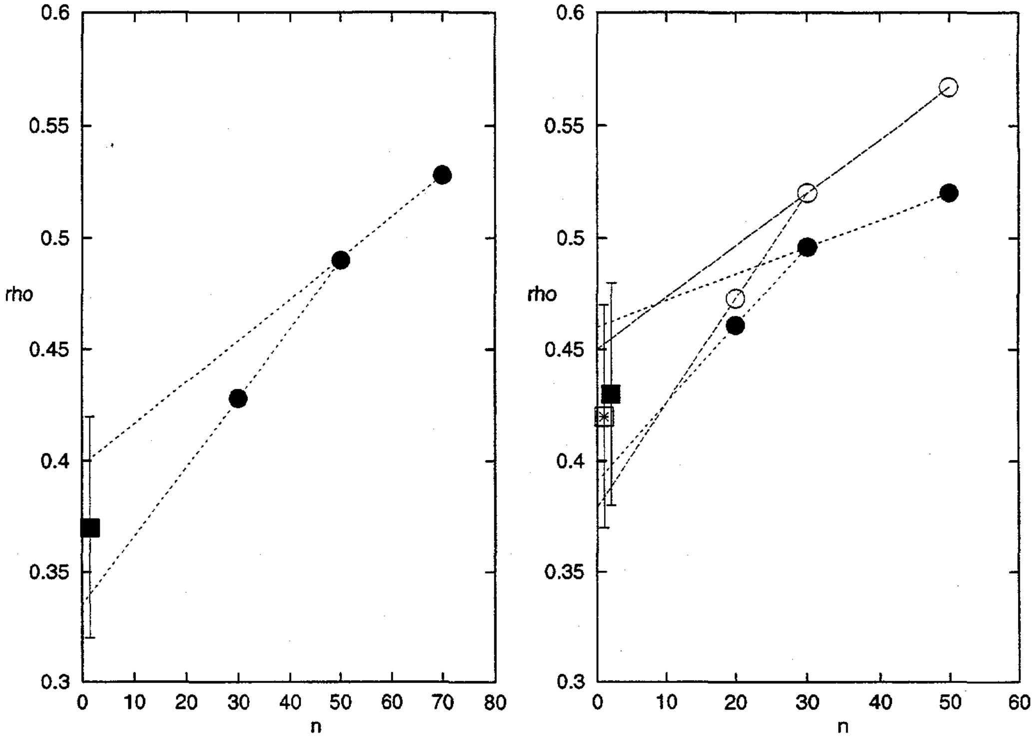
[c] Value 0.3 scaled 5.6% using $a(\sqrt{\sigma} = 440\,\mathrm{MeV})$.

[d] From graphs of $N/V = 1.81$ and 1.43 data. Evolution to 3.2 would reduce ρ further.

[e] Value 0.36 scaled 9% using $a(\sqrt{\sigma} = 440\,\mathrm{MeV})$. Correlation function range underestimates average ρ.

[f] Lattice spacing increased 9% using $a(\sqrt{\sigma} = 440\,\mathrm{MeV})$.

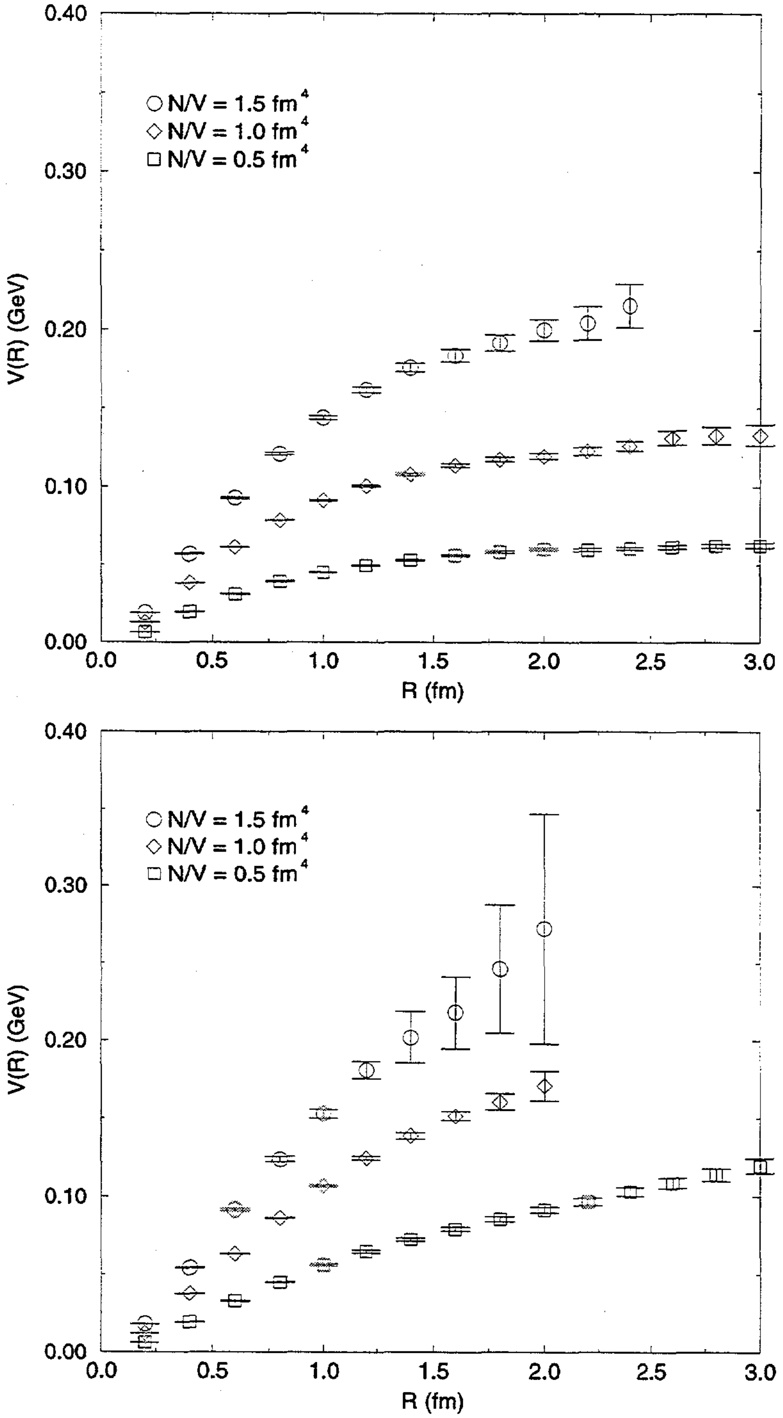
[g] Lattice spacing from hadron masses increased 18% using $a(\sqrt{\sigma} = 440\,\mathrm{MeV})$.



Sketch of extrapolations of the average instanton size to $n_c = 0$ cooling steps for data from Smith and Teper (*left*) and Ivanenko and Negele (*right*). Solid and open symbols denote quenched and full QCD respectively.

Topological Susceptibility

$\chi^{1/4}$ (MeV)	Method	Reference
	SU(2)	
230 (30)	RG Cycling	DeGrand, A. Hasenfratz, Kovács
220 (6)	APE Smearing	DeGrand, A. Hasenfratz, Kovács
200 (15)	Improved Cooling	de Forcrand, Pérez, Stamatescu
198 (8)	APE + Renorm.	
	Geometric + Renorm.	Allés, D'Elia, DiGiacomo, Kerchner
226 (4)	Spectral Flow	Edwards, Heller, Narayan
	SU(3)	
187 (14)	Underrelaxed Cooling	Smith, Tepper
192 (5)	APE Smearing	A. Hasenfratz, Nieter
185 (9)	Improved Cooling	de Forcrand, Pérez, Hetrick, Stamatescu
175 (5)	APE + Renorm.	Allés, D'Elia, DiGiacomo
197 (4)	Spectral Flow	Edwards, Heller, Narayanan
180	Veneziano-Witten	



Heavy quark potential in the instanton liquid model. Upper graph corresponds to fixed instanton size $\rho = 1/3$ fm and lower graph is for the distribution

$$n(\rho) = \frac{\rho^6}{(\rho_0^{3.5} + \rho^{3.5})^{11/3.5}}$$

Instantons and Fermions: Chiral Symmetry and Optimized Actions

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Physics Department, University of Colorado,
Boulder, CO 80309 USA

1 From QCD to Instanton

What features of QCD are responsible for chiral symmetry breaking? Or, to prejudice the question, how do instantons and fermions interact in "real" (aka lattice) QCD? The lattice vacuum is filled with UV noise which swamps most direct tests of these questions. We have investigated a number of techniques for desigining variables which filter out this noise to reveal IR physics. This allows us to see individual instantons in the vacuua and measure their properties. We study the role of instantons by creating multi-instanton background fields which have exactly the same instanton content as the filtered QCD vacuum, but of course lack all other IR physics.

Fig. 1 shows the potential for $SU(2)$ gauge theory from locally smoothed gauge variables and from multi-instanton backgrounds. It also shows the lattice $\langle\bar{\psi}\psi\rangle$ for staggered fermions on smoothed and in instanton background configurations. $\langle\bar{\psi}\psi\rangle$ in the instanton background tracks the value of $\langle\bar{\psi}\psi\rangle$ measured on the smoothed configurations quite closely, down to small quark mass. Fig 2 contrasts the pseudoscalar correlator on the two ensembles. It appears that the instantons, present in equilibrium gauge field configurations of the QCD vacuum generated by Monte Carlo, are breaking chiral symmetry by themselves. However, by themselves, instantons do not produce a spectrum which qualitatively resembles the world as we see it.

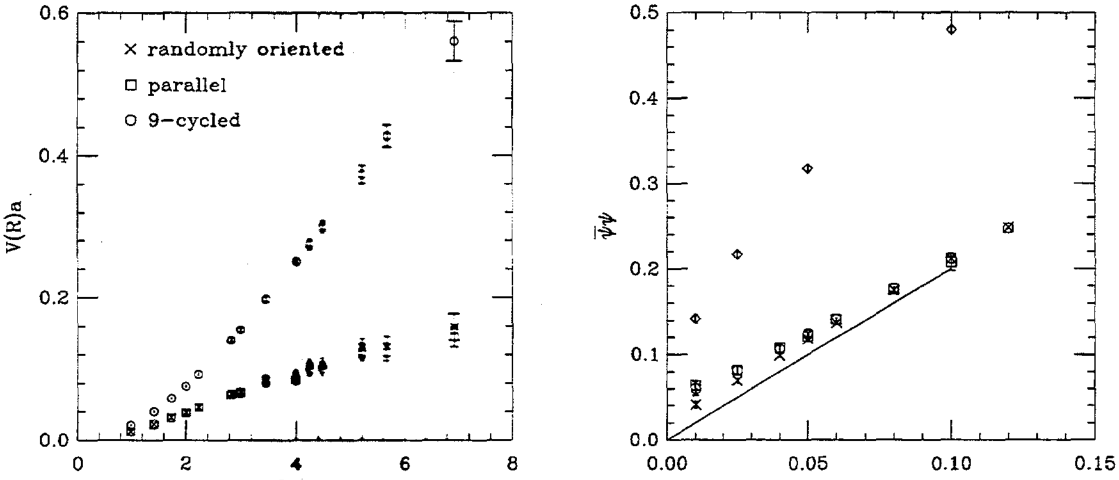


Figure 1: (Left) The heavy-quark potential measured on the 9 times cycled real configurations (octogons), the randomly (crosses) and the parallel (squares) oriented instantons. (Right) $\langle\bar{\psi}\psi\rangle$ from raw configurations (diamonds), 9-cycled configurations (squares), and instanton-background configurations which are parallel (crosses) and randomly oriented (octagons), vs. bare quark mass am_0. The line shows the free-field value, $2m_0$.

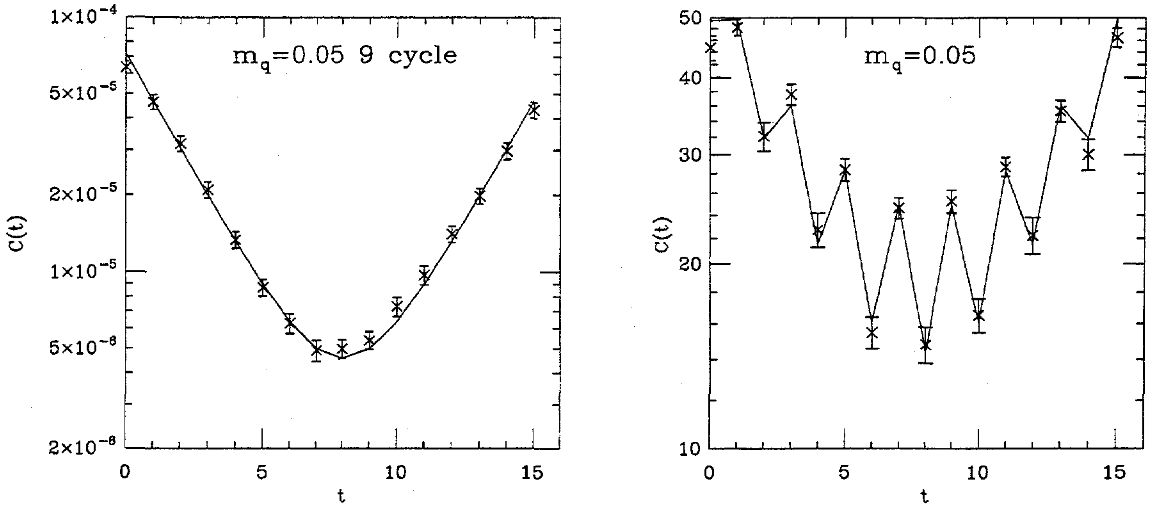


Figure 2: (Left) The pseudoscalar propagator from smoothed configurations, with staggered fermions of bare mass $am_0 = 0.05$. (Right) The pseudoscalar propagator in (randomly rotated) instanton background configurations, with staggered fermions of bare mass $am_0 = 0.05$. The curve is a fit to a single propagating particle plus the $q\bar{q}$ branch cut.

2 From Instanton to QCD

The results of the first section suggest that one might think of constructing actions with good chiral behavior by replacing the usual links by some kind of "fat" link, and that by tuning the action to respect the instantons, we might design an improved action for QCD simulations. Justification for such an approach comes from considering the axial Ward identity and how Wilson fermions break chiral symmetry, explicit studies of instantons and the low-lying real eigenmodes of the lattice Dirac operator, and fixed point fermions and how they implement the Ginzsparg-Wilson relation.

We optimized actions by tuning them to have a good spectrum of real eigenvalues on background instanton configurations and in the QCD vacuum. Along the way, we made two discoveries: First, if the lattice spacing is too large, it is not possible to tune the actions–the fermions cannot see the instantons. This represents a fundamental barrier of a maximum lattice spacing for improvement, of about $a \simeq \langle\rho\rangle \simeq 0.2$ fm. Second, we think we know why the commonly-used nonperturbatively-improved clover action has s many exceptional configurations: for large values of the clover term the real eigenmodes of the Dirac operator which are accessible to fermions at physical values of the

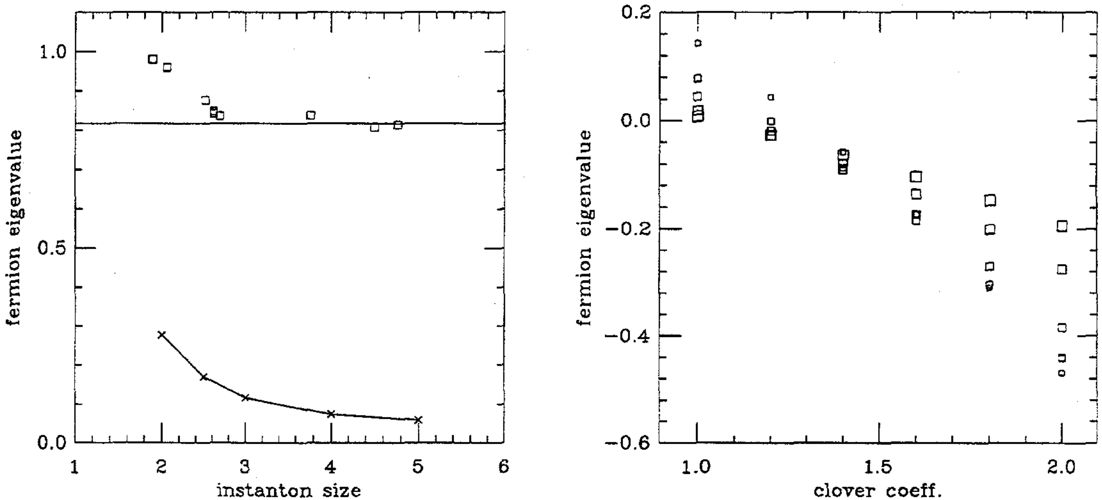


Figure 3: (Left) The instanton size (in units of the lattice spacing) vs. the eigenvalue of the corresponding (Wilson action) fermionic real mode on smooth instantons (crosses) and on real Monte Carlo generated configurations at $\beta = 6.0$ (squares). The horizontal line indicates $-m_c$ for the $\beta = 6.0$ quenched ensemble. (Right) The real fermionic eigenvalue of the clover action versus the clover coefficient on instantons of sizes $\rho/a = 1.2, 1.4, 1.6, 2.0, 2.5$. Bigger symbols correspond to larger instantons.

pion mass are connected not with large instantons, (as they are with the Wilson action) but with small ones.

We have tested a number of UV insensitive fermion actions. They all have small additive quark mass renormalization. The renormalization factors for their vector and axial vector currents are quite close to unity. Their scaling behavior appears to be quite good. Wilson-like versions of these actions appear to be largely free of exceptional configurations down to small π/ρ msas values.

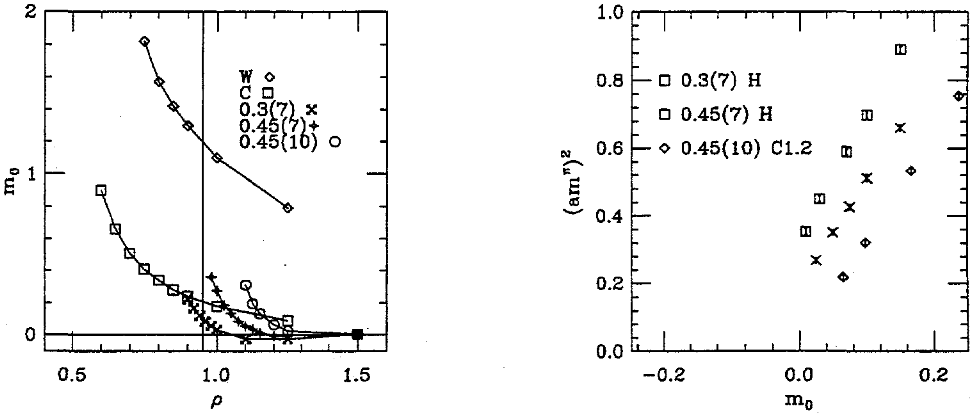


Figure 4: (Left) Eigenmode spectrum vs. instanton size of the Wilson, clover, and various hypercube fat link actions. (Right) Squared pion mass vs bare quark mass for various UV insensitive actions, at $a = 0.18$ fm.

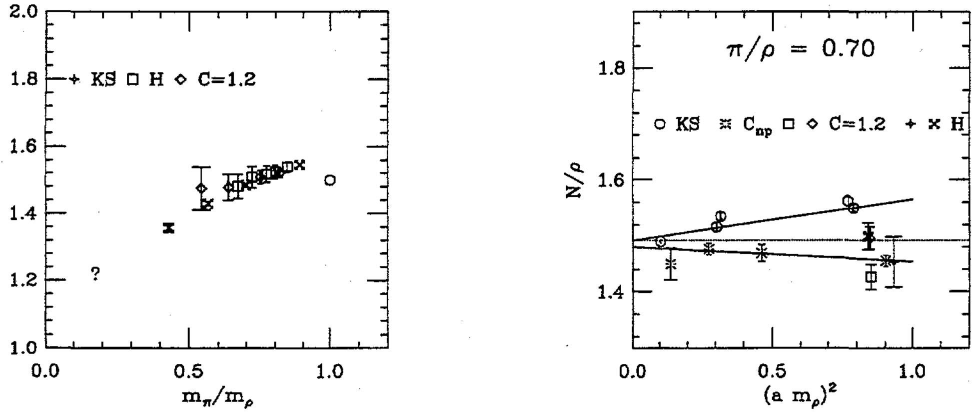


Figure 5: (Left) Edinburgh plot comparing various fat link actions at $a = 0.18$ fm with staggered fermions at small lattice spacing. (Right) N/ρ mass ratio at fixed $\pi/\rho = 0.7$ comparing staggered fermions, nonperturbatively improved clover fermions, and various fat link fermions.

Instantons and Quarks in QCD

Anna Hasenfratz

The QCD vacuum is filled with instantons. Their role in solving the U(1) anomaly problem is long known. They are also considered to be essential for spontaneous chiral symmetry breaking and the low energy hadron spectrum. Instanton Liquid Models provide a successful phenomenological description of the instanton vacuum. At this time only lattice QCD calculations can describe the vacuum from first principles. There are two basic approaches to study instanton effects in lattice QCD. One can study instantons directly using a topological charge operator constructed from the gauge field. Alternately, one can consider the effect of instantons on the quarks and study them through the spectrum of the Dirac operator. Several methods based on the first approach have been developed and used over the last few years. In the first part of this talk I describe one such method and some results obtained with it. The second part of the talk considers the second approach. I describe how quarks can reveal the topological structure of the vacuum. I compare the two methods and argue that the latter one gives information more directly, without distorting the vacuum. That is important if we want to study the spatial distribution of instantons, like formation of molecules and spatial ordering of molecules at finite temperatures in dynamical configurations.

Instantons and Quarks in QCD

Introduction:

- why do we care about instantons?
- what properties of the instanton vacuum are well defined and what properties are not?

Instantons through gauge operators:

- method and results for pure gauge SU(3)
- finite temperature dynamical config. (where are the pairs?)

Instantons through fermions:

→ the spectrum of the Dirac operator and instantons

- example I: a smooth pair
  example II: MC configurations
- where are the pairs in dynamical configurations?

Instanton density, size distibution:

- Our instanton finder will not identify objects closer than 80% of combined radii

$$d \gtrsim 0.8\,(\rho_1 + \rho_2)$$

Challange:

Too little smoothing will leave behind vacuum fluctuations that can look like instantons.

Too much smoothing will kill too many real topologica objects.

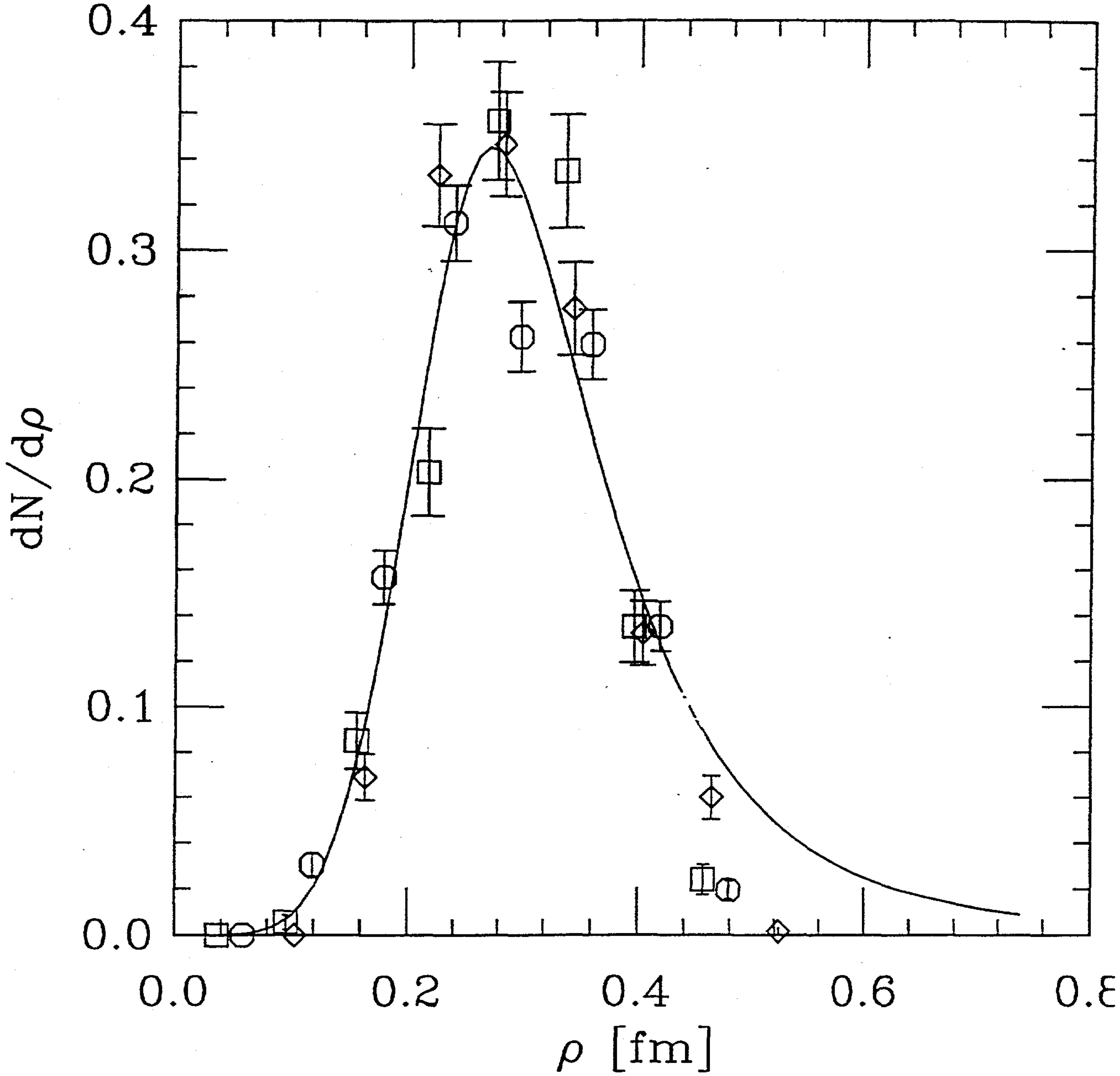
The instanton density after

12 ape steps is $\sim 6\ \mathrm{fm}^{-4}$
24 ape steps is $\sim 3\ \mathrm{fm}^{-4}$

expected $\sim 1\ \mathrm{fm}^{-4}$

How to distinguish real topological objects from vacuum fluctuations without destroying too many instantons?

Instanton size distibution
SU(3) pure gauge



◇ : β = 5.85   a = 0.12 fm

○   β = 6.0   a = 0.093 fm

□   β = 6.1   a = 0.079 fm

Instantons through fermions

If the near zero eigenmodes of the Dirac operator are dominated by instantons and are localized on instantons, they can be used to study the topology of the vacuum.

Eigenmodes: $\not{D}\,\phi_n = i\lambda_n\,\phi_n$ $\qquad\lambda$: real

Consider an arbitrary source

$$\eta(x) = \sum_n \alpha_n\,\phi_n(x)$$

Then

$$\chi(x) = (\not{D}-m)^{-1}\,\eta(x) = \sum_n \frac{\alpha_n}{i\lambda_n - m}\,\phi_n(x)$$

as $m \rightarrow 0$, only modes with $|\lambda| \lesssim m$ are important in χ. If those modes are dominated by instantons, we expect

$$\chi(x) \sim \sum_n c_n^+\,\varphi_n^+(x) + c_n^-\,\varphi_n^-(x)$$

where $\varphi_n^\pm(x)$ are individual I/A zero modes.

IF the $\varphi_n^{\pm}$ modes are localized

$$\Omega(x) := \mathrm{Tr}\left(\chi^{+}(x)\ \gamma_5\ \chi(x)\right) \sim \sum |c_n^{+}|^2 |\varphi_n^{+}(x)|^2 \ - \\ - \ |c_n^{-}|^2\, |\varphi_n^{-}(x)|^2$$

a combination of zero mode densities (with sign)

Peaks in $\Omega(x)$ signal instantons / antiinstantons.

The peak height varies with m as

$$\omega = \frac{1}{\Omega_{\text{peak}}} \sim m^2$$

Real HC configurations

$\beta = 5.9$ ($a \sim 0.12$ fm) 12^4 pure gauge conf.

#1: $Q = -2$ from gauge operator.

Location and size of identified instantons with gauge method, S_1 and S_5 fermion actions:

	gauge		S_1		S_5	
A	9, 2, 1, 8	$\rho = 2.5$	10, 3, 2, 9	$\rho = 3.5$	9, 1, 1, 8	$\rho = 4$
A	6, 2, 8, 1	4.0	5, 2, 10, 1	4.0	6, 2, 8, 2	4
A			0, 5, 8, 8	4.5		
I			3, 3, 2, 11	1.5		

Only objects that were identified at several mass values with peak value 6-8x the background were kept.

The surface plot of $\Omega(x)$ shows that even S_1 action separates instantons and vacuum fluctuations.

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"Topological & Chiral properties of QCD from lattice studies"

- **Improved cooling** preserves Inst. of size $> 2.3a$

Problem with I-A pairs exists also in the continuum

Here: cool until $\sim 90\%$ of top. charge dens. can be described by linear superposition of continuum Inst.

Yang-Mills, $T \approx 0$:

- dense ensemble of largish instantons
- consistent with homogeneous spatial distribution

QCD, $N_f = 2, m_\pi/m\rho \sim 0.6, T \sim 0.9 - 1.2T_c$:

- precise check of Banks-Casher
- Good agreement between Dirac eigenmodes (original config.) and Instantons (cooled config.), at $a \sim 0.1fm$, even on I-A pairs after long cooling $\Longrightarrow$ Validation of improved cooling
- Puzzle: what happens above T_c ?
  - instanton fit prefers zero instantons
  - $Q = 0$ $\approx$ always → neutral excitations
  - some evidence for space-time asymmetry, fragile under cooling. Geometric factor only ??
  - no asymmetry in Fourier top. charge density
  - other excitations than Schaefer-Shuryak ?

- Improved cooling -

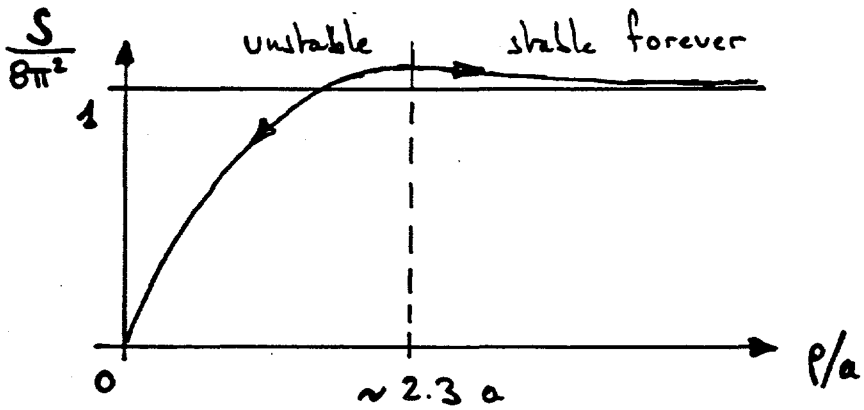
(Ph.de F et al., hep-lat/950906

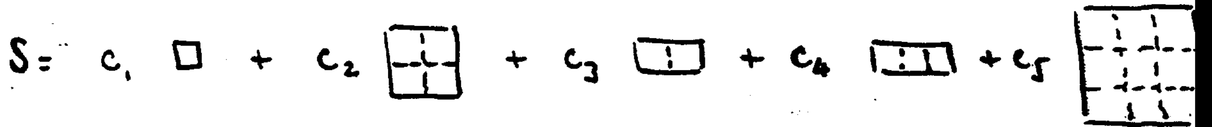
iterative, local minimization of improved action

$$\frac{S(\rho)}{8\pi^2} = 1 + \cancel{\frac{a^2}{\rho^2}} + \cancel{\frac{a^4}{\rho^4}} + \mathcal{O}\left(\frac{a^6}{\rho^6}\right)$$

tune to minimize

(instantons are classical objects → classical improvement





Measure $q(x) \equiv \varepsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$ with improved operator

Q stable under arbitrary amount of cooling

Remaining problem: A-I pair annihilation

Properties invariant under cooling (sizes? spatial distribution?) are physical

Instanton pair problem (continuum)

Note ambiguity:

is this an I-A pair to keep

or a trivial fluctuation to discard ?

Here: cool until ~ 90% of $q(x)$ accounted for

by linear superposition of classical instantons:

$$\sum \frac{6}{\pi^2} \frac{1}{\rho^4 \left(1 + \sum_{\mu=1}^{4} \left(\frac{x_\mu - x_\mu^0}{\rho}\right)^2\right)^4}$$

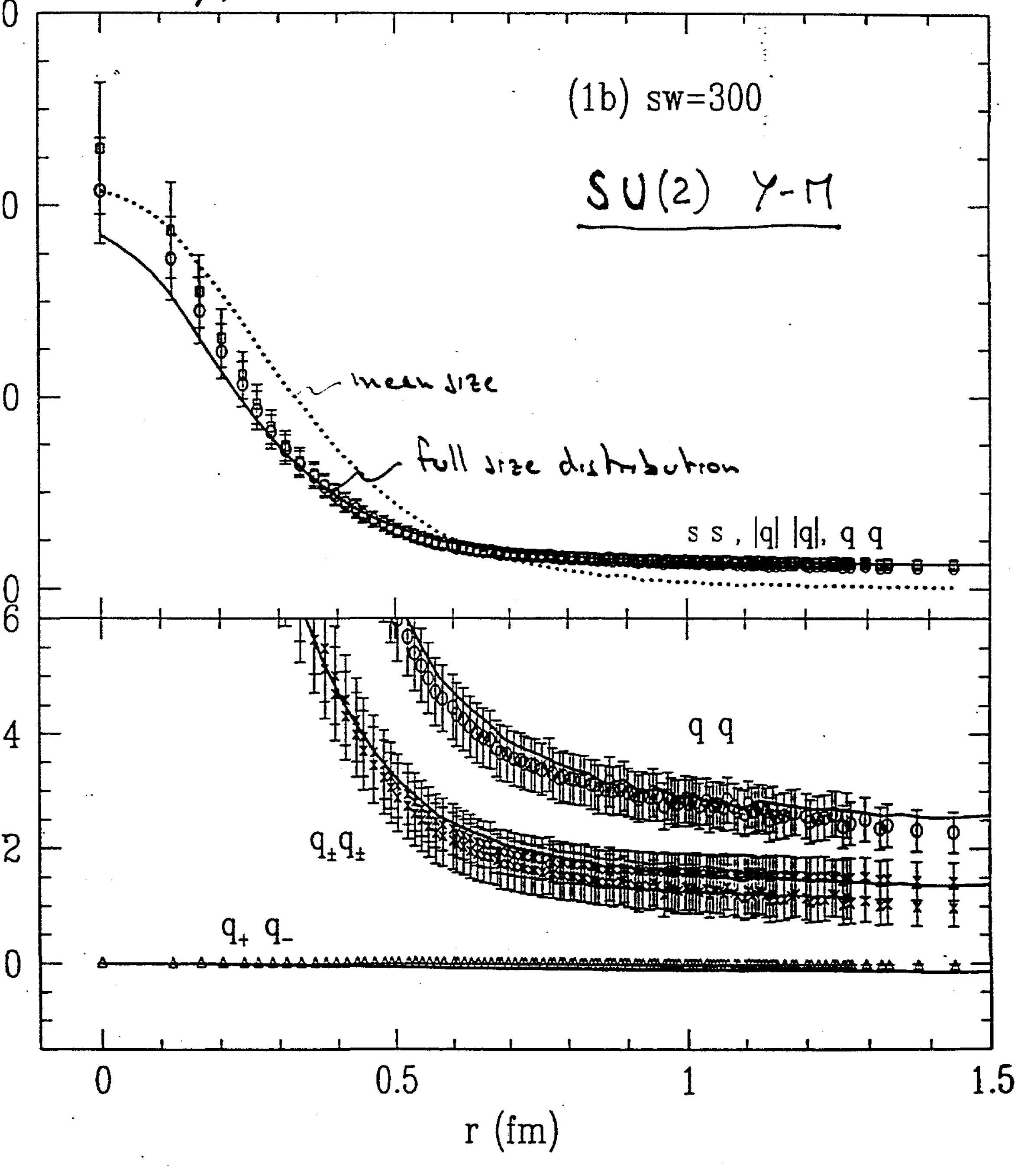
ie $\frac{\| q - q_{fit} \|^2}{\| q \|^2} \sim 10\%$ ($\longleftrightarrow$ de Grand ...)

Trade-off: $\left\{ \begin{array}{l} \text{Keep more objects} \\ \Updownarrow \\ \text{Identify them less reliably} \end{array} \right.$

Test of "null hypothesis"

$\langle q(0)\, q(x)\rangle$ vs "synthetic" homogeneous instanton gas with same $n(\rho)$

- parameter-free
- bypass instanton identification

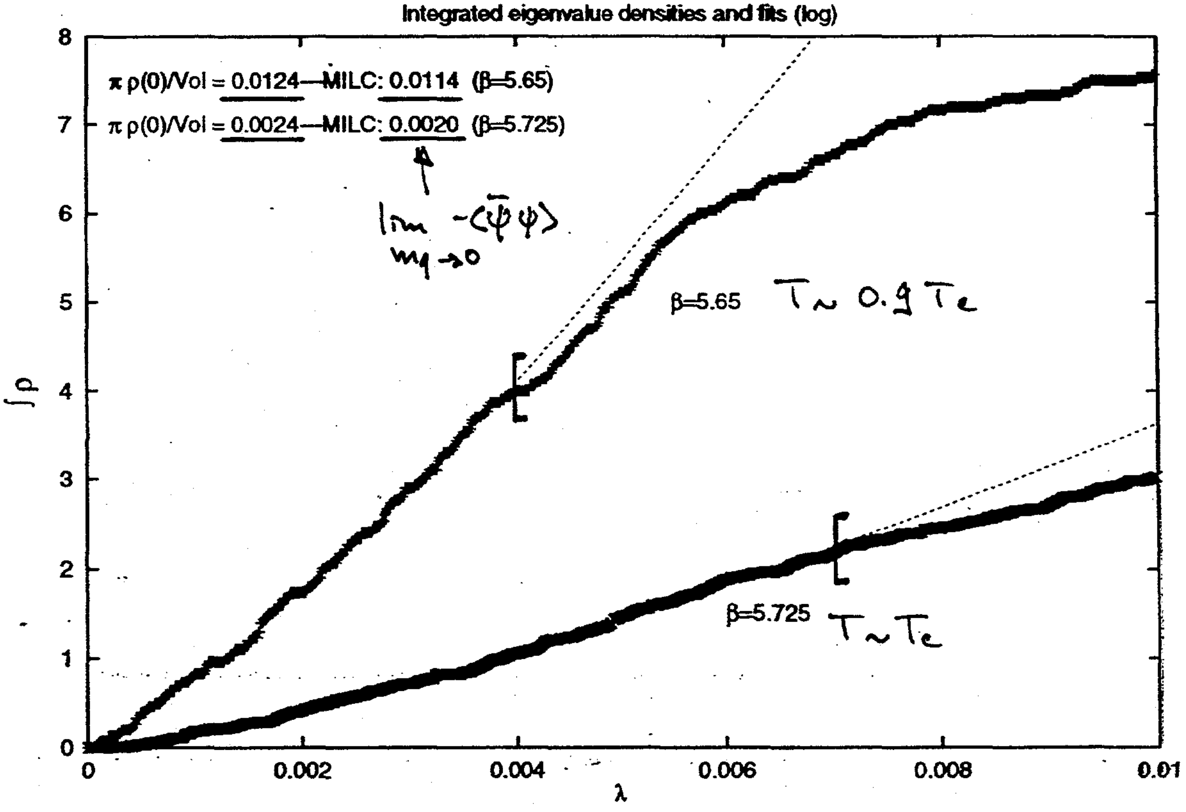


$$q_+ = \max(0, q)$$
$$q_- = \min(0, q)$$

Test of Banks-Casher: QCD, $N_f=2$,

Fit $\int_0^\lambda d\lambda' \rho_{MC}(\lambda)$ to $\int_0^\lambda d\lambda' \rho_{Ansatz}(\lambda)$

ρ_{Ansatz}	$\chi^2 (\beta = 5.65)$	$\chi^2 (\beta = 5.725)$
$\rho(0) + a\lambda$	0.85	0.9
$\rho(0) + a\lambda^2$	1.3	1.9
→ $\rho(0) + a \, \mathrm{Log}(1 + \lambda \frac{V}{\pi}\langle\bar\psi\psi\rangle)$	0.8	0.5

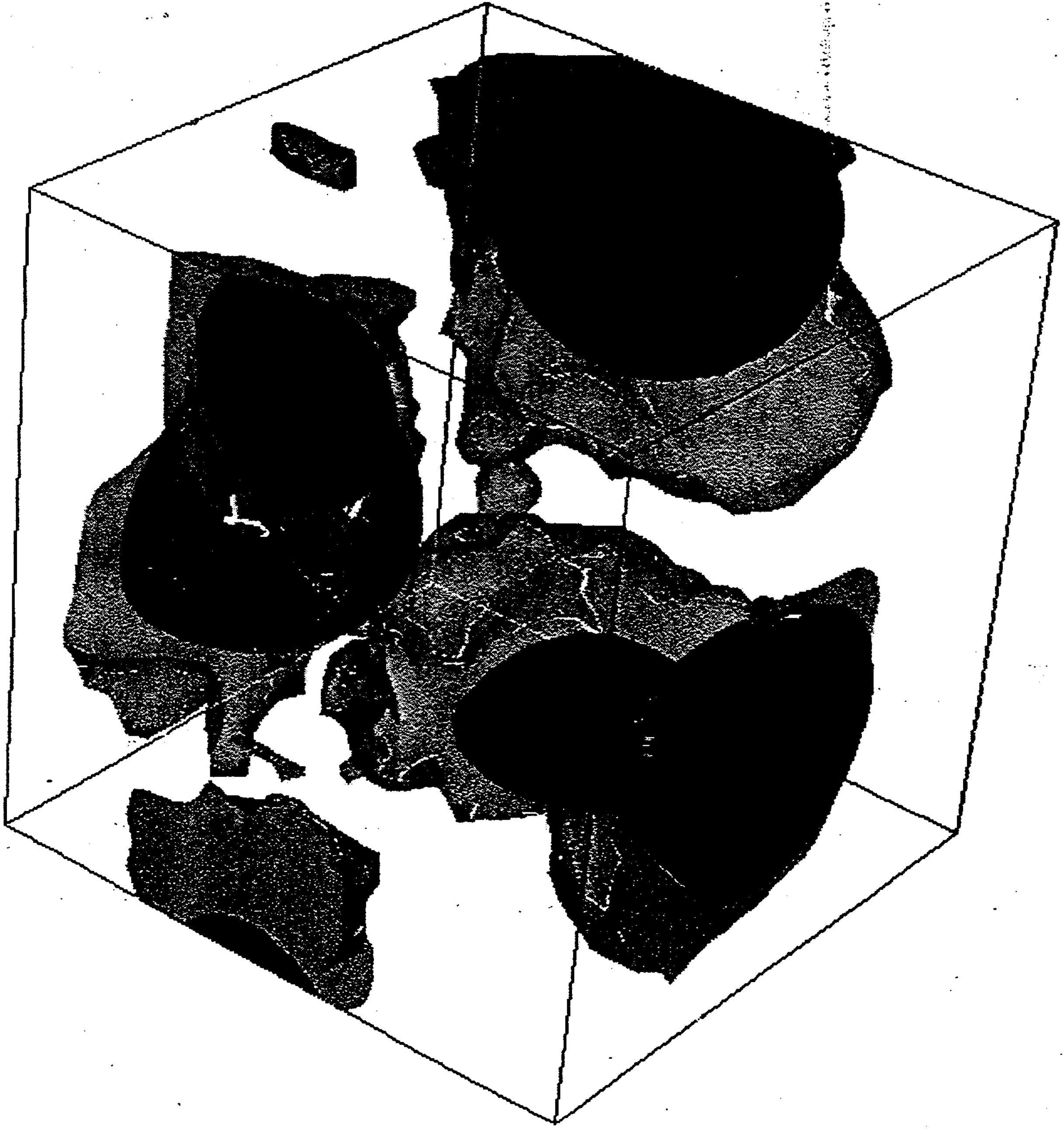


On the right of [, not all eigenvalues have been measured.

Consistent with Smilga-Stern $\rho(\lambda) = \rho(0) + c(N_f^2 - 4)\lambda + \ldots$ up to Log corrections ($a \propto 1/V$?)

- $q(x)$ after 150 cooling sweeps
- $|\psi_1(x)|$ on original configuration

} Correlation ~ 70%



~~Scale down A_μ (Instanton) until $Q = 0$~~

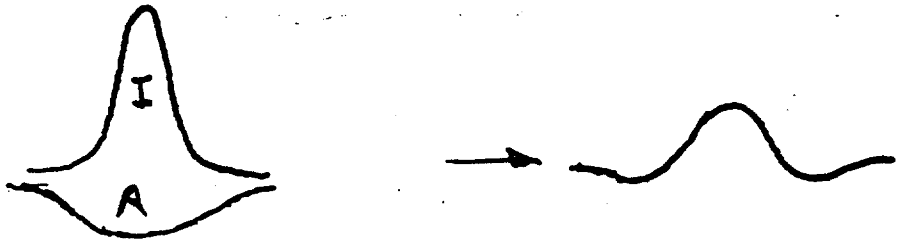
Lesson : low-action $Q=0$ fluctuations (eg. instanton "duds") can give localized eigenmodes. Connection

size of q "lump" ⟷ { size of eigenmod
chirality

lost

"Duds":
- can be spherically symmetric
  isotropic
- must be quickly killed by cooling
- perverse view:

one (or more) I-A pair at same locatio



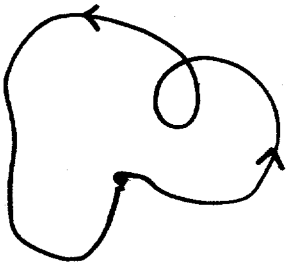
Chiral Disorder in QCD

R. Janik + M. Nowak + G. Papp + I. Zahed

hep-ph-9803289	PRL98'
-9806479	PLB98'
-9807499	PLB98'
-9807550	

● Quark Return Probability

(Anderson 58': e-localization)

$p(t,x,x) = \Big|$  $\Big|^2 \equiv p(t)$

$$\cong e^{-2mt}\left(1 + e^{-4\pi^2 \frac{D}{L^2} t} + \dots\right)$$

$$t_{ergodic} = \frac{L^2}{D} = \frac{\sqrt{V}}{D}$$

For $m \sim 0$ and $t > t_{erg}$: $p(t) \simeq 1$

● Relevant (proper) times

elastic ≃ 

$$= t_e \simeq \frac{1}{2m_Q} \qquad \sim V^0$$

ergodic ≃ 

$$= t_{erg} = \frac{L^2}{D} \qquad \sim \sqrt{V}$$

Heisenberg $= t_H = \frac{1}{\Delta} = \rho V \qquad \sim V$

Finite V :

$$0 \underset{\underset{\text{Ballistic}}{\downarrow}}{<} t_e \underset{\underset{\text{Diffus.}}{\downarrow}}{<} t_{erg} \underset{\underset{\text{ergodic}}{\downarrow}}{<} t_H \underset{\underset{\text{quant.}}{\downarrow}}{<} \infty$$

● Conclusion I ($m \downarrow 0$)

- vacuum : $p(t) \simeq \frac{1}{t^2}$
- 2nd order : $p(t) \simeq \frac{1}{t^{1/8}}$
- $d=1$ percolation : $p(t) \simeq \frac{1}{t^{1/2}}$
- MI (fractal) : $p(t) \simeq \frac{1}{t^{0.943}}$

Lattice ?

Quenching, QCD Dirac spectra and localization transition

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Abstract

Eigenvalues and eigenfunctions of the QCD Dirac operator are studied for gauge field configurations given by a liquid of instantons. We find that the fermion determinant has a strong effect on the value of the chiral condensate. In particular, we find that the spectral density diverges in the quenched limit, and behaves as $(N_f^2 - 4)|\lambda|$ near $\lambda = 0$. The Dirac spectrum follows from a QCD partition function that in addition to the usual degrees of freedom contains a valence quark and its superpartner both with the same mass that is different from the sea quark masses. The corresponding low energy effective partition function is based on the Riemannian symmetric superperspace $Gl(N_f + 1|1)$. The zero momentum sector of this theory reduces to chiral Random Matrix Theory. Taking into account the nonzero momentum modes results in the small-λ behavior found in instanton liquid simulations. This effective partition function also describes disordered two sublattice condensed matter systems. This leads to the question to what extent phenomena observed in disordered systems, such as for example a localization transition, are realized in QCD. We argue that the essential difference between QCD and disordered condensed matter systems is the presence of a fermion determinant. Based on an extension of an argument given by Parisi to the chiral effective partition function, we conclude that an Anderson localization transition is only possible in quenched systems.

[1] J.C. Osborn and J.J.M. Verbaarschot, Phys. Rev. Lett. **81** (1998) 268.

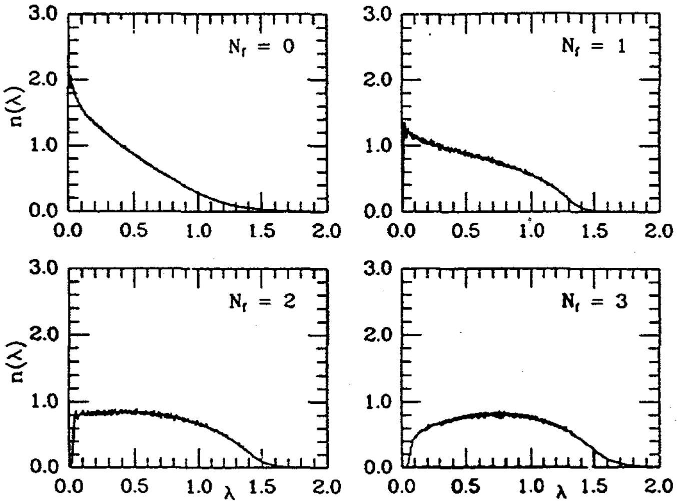
[2] J.C. Osborn and J.J.M. Verbaarschot, Nucl. Phys. **B525** (1998) 738.

[3] J.C. Osborn, D. Toublan and J.J.M. Verbaarschot, hep-th/9806110, Nucl. Phys. **B**) (in press).

[4] P.H. Damgaard, J.C. Osborn, D. Toublan and J.J.M. Verbaarschot, (coming soon).

Effect of fermion determinant on Dirac spectra

Let us study the Dirac spectrum for the Shuryak liquid



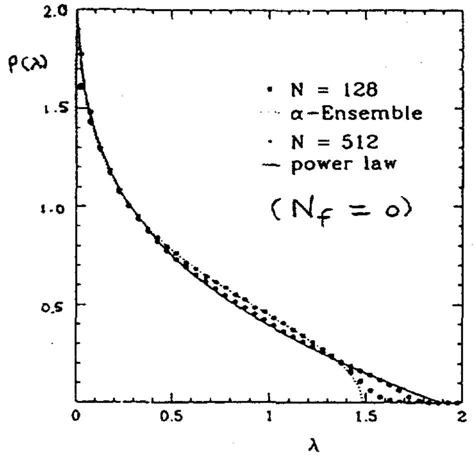
Spectral density of the Dirac operator for a liquid of instantons

$N_c = 3$, $m_q = 0$

$N_I = N_A = 32$

$(N_I + N_A)/V = 1\ fm^{-4}$

JV, NPB 1994



Osborn-JV NPB, 1998

Fit by a power law $(N_f = 0)$

$a - b\lambda^{\beta}$

N	β
128	0.25
256	0.13
512	0.09
∞	0.03 ± 0.04

$\rho(\lambda) \sim \log\lambda$ for $\lambda \to 0$

- the fermion determinant has an order ∞ effect on $\rho(0)$

Our conclusions have been confirmed by Teper and Smith

Lattice 98

Number variance of Dirac eigenvalues for gauge field configurations given by a liquid of instantons

Osborn - JV, PRL 1998

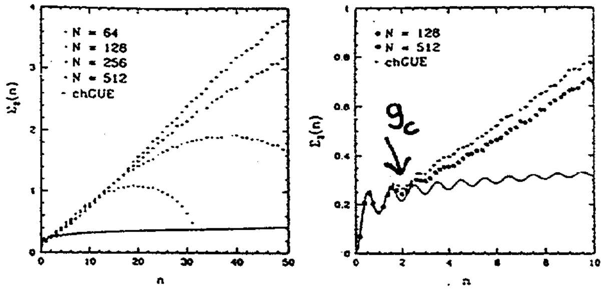


Fig. 1. The number variance $\Sigma_2(n)$ versus n in the quenched approximation for an interval starting at $\lambda = 0$. The total number of instantons is indicated in the legend of the figure.

$$g_c = \frac{E_c}{\Delta} = \frac{F^2 L^2}{\pi}$$

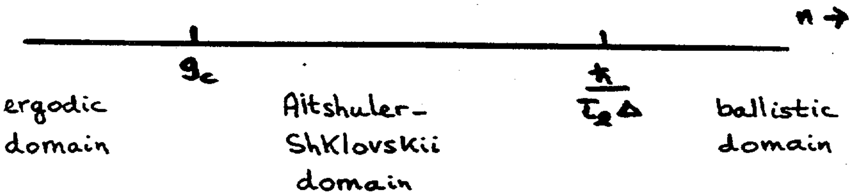
Diffusive motion :

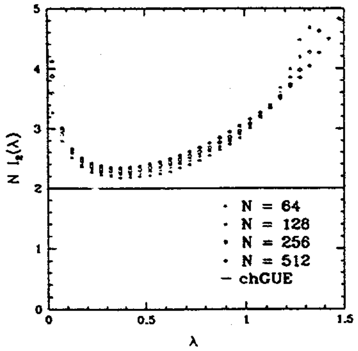
$$\underset{\displaystyle\downarrow\atop\displaystyle L^2}{(\Delta x)^2} = D\,\Delta\tau$$

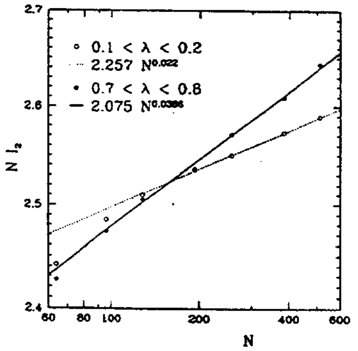
$$\Rightarrow \Delta\tau = \frac{L^2}{D}$$

$$\Rightarrow E_c = \frac{\hbar D}{L^2}$$

↑ Thouless energy

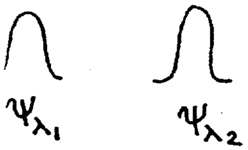






Osborn - JV, PRL 1998

localized states



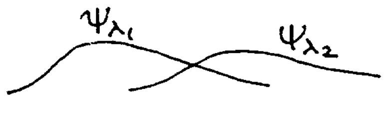
eigenvalues are uncorrelated

no Goldstone modes

$\rho(E) \neq 0$

(Stone's alternative to Goldstone's theorem)

extended states



correlated eigenvalues

Goldstone modes

$\rho(E) \neq 0$

③

Theory for the Dirac spectrum

$$Z^V_{QCD} = \int dA \, \frac{\det(D+z+J)}{\det(D+z)} \prod_f \det(D+m_f) e^{-S_{YM}}$$

Valence quark mass dependence

$$\Sigma(z) = \left. \frac{\partial \log Z^V_{QCD}}{\partial J} \right|_{J=0} = \langle \mathrm{Tr} \frac{1}{z+D} \rangle$$

Spectral density

$$\rho(\lambda) = \frac{1}{2\pi} (\Sigma(i\lambda+\varepsilon) - \Sigma(i\lambda-\varepsilon))$$

Symmetry of Z^V_{QCD}

$$Gl(N_f+1|1) \times Gl(N_f+1|1)$$

Osborn-Toublan-JV, 1998

Damgaard-Osborn-Toublan-JV, 1998

Explicit breaking of $U_A(1)$

Spontaneous breaking according to

$$Sl(N_f+1|1) \times Sl(N_f+1|1) \to Sl(N_f+1|1)$$

- Vafa-Witten theorem
- maximum breaking of chiral symmetry

Partition function for the infrared sector of QCD

$$Z_\nu^{eff} = \int_{U \in Gl(N_f+1|1)} dU \; e^{-\frac{F^2}{4}\int d^4x \; \mathrm{Str}\, \partial_\mu U \, \partial_\mu U^{-1}}$$

$$\times \; e^{\frac{1}{2}\int d^4x \; \Sigma \; \mathrm{Str} \; m(U+U^{-1})}$$

$$\times \; \mathrm{Sdet}^\nu(U)$$

Osborn-Toublan-JV, 1998

$m = (m_1, \ldots, m_{N_f}, z+J, z)$

Golterman-Bernard, 1993

The integration domain is restricted to a maximum Riemannian submanifold of the symmetric supermanifold

$$U = \begin{pmatrix} U_n & \begin{array}{c} \chi_1 \\ \vdots \\ \chi_n \end{array} \\ \psi_1 \ldots \psi_n & e^s \end{pmatrix}$$

Damgaard-Osborn-Toublan-JV, 1998
Osborn-Toublan-JV, 1998

- For $z \ll \frac{F^2}{L^2 \Sigma}$ the nonzero momentum modes decouple and the valence quark mass dependence reduces to that of a chiral Random Matrix Theory

Osborn-JV, PRL 1998
Osborn-Toublan-JV, NPB 1998

- Dirac spectrum

$N_f = 0 \quad \rho(\lambda) \sim \log|\lambda|$

$N_f \neq 0 \quad \rho(\lambda) \sim (N_f^2 - 4)\,|\lambda|$

Osborn-Toublan-JV, 1998
Smilga-Stern, 1994

No localization transition in quenched systems

Parisi's argument

Stone's* alternative to Goldstone's theorem is only possible in the quenched approximation

Based on an instanton calculation by Cardy

*) McKane - Stone

Let us see how this argument works in QCD

$$Z = \int \underbrace{\langle \rho(\lambda_1, \dots, \lambda_n) \rangle}_{\text{regular for any of the } \lambda_K \to 0} \prod_K (\lambda_K^2 + m^2)^{N_f}$$

if the eigenvalues are uncorrelated then

$$\langle \rho(\lambda_1, \dots, \lambda_n) \rangle = \rho_1(\lambda_1) \cdots \rho_1(\lambda_n)$$

then

$$\rho(\lambda) \sim \lambda^{2N_f} \quad \text{for} \quad \lambda \to 0 \quad (m \to 0)$$

i.e. no chiral symmetry breaking

A localization transition is not possible below T_c

⑥

Evidence for correlated eigenvalues for $T < T_c$

Valence quark mass dependence of the chiral condensate

Chandrasekharan-Christ 1995

$16^3 \times 4$
KS fermions
$m_{sea} a = 0.01$
$N_f = 2, \quad N_c = 3$

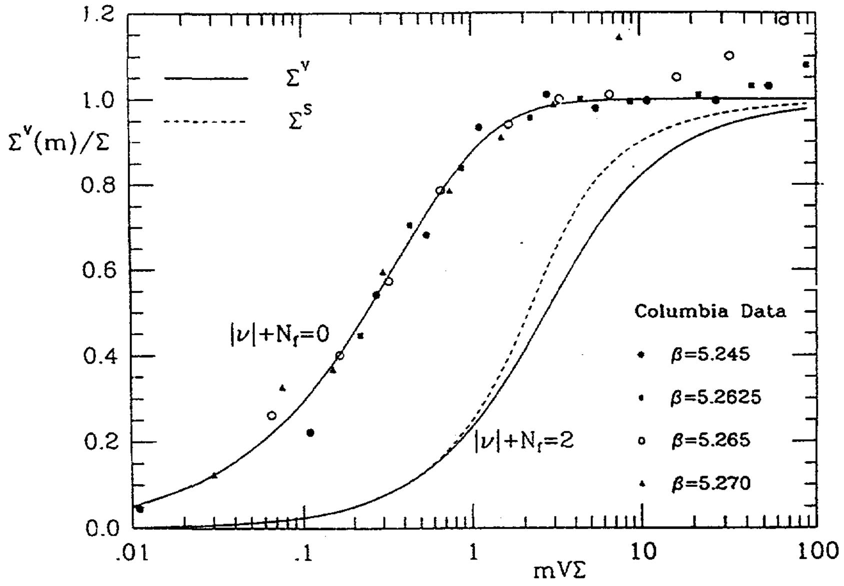


Figure 2: The valence quark mass dependence of the chiral condensate $\Sigma^V(m)$ plotted as $\Sigma^V(m)/\Sigma$ versus $mN\Sigma$. The dots and squares represent lattice results obtained by the Columbia group [1] for values of β as indicated in the label of the figure. The full lines correspond to the random matrix result (eq. (8)) for $\nu = 0$ and N_f as indicated in the label of the figure. Also shown are results (dashed curves) for the quark mass dependence with equal valence and sea quark masses.

(V, PLB 1995)

$\lambda_{min} a \approx 10^{-4} \ll m_{sea} \Rightarrow$ calculation is quenched on the scale of the smallest eigenvalues ($N_f = 0$)

$\nu = 0$ fermionic zero modes are completely mixed with the rest of the states

domain of validity: $m \ll \sqrt{\Lambda \lambda_{min}} \Rightarrow mV\Sigma \ll \sqrt{\frac{\Lambda}{\lambda_{min}}} \sim \Lambda^2 \sqrt{V}$

The lessons from Supersymmetry

AN INSTANTON PHYSICIST'S "PROOF" OF MALDACENA'S CONJECTURE

-M. MATTIS / LOS ALAMOS

BNL WORKSHOP

OF SUSY's

$\mathcal{N}=4$

(finite model, highly nontrivial conformal field theory)

PROGRESS

- SUSY-Multi-inst. action + measure derived
  hep-th/9612231,
  9709072

- BRAND NEW RESULT:
  9810243

MULTI-INST SERIES SUMMED EXACTLY FOR $SU(N)_{gauge}$ IN LIMIT $N \to \infty$

- ANSWER: A MODULAR FORM $f_{16}(\tau)$ OF COMPLEXIFIED COUPLING τ:

$$\tau = \frac{4\pi i}{g^2} + \frac{\Theta}{2\pi}$$

$$SL(2,\mathbb{Z}): \begin{cases} \tau \to \tau + 1 \\ \tau \to -1/\tau \end{cases}$$

- PRECISE AGREEMENT w/ MALDACENA's CONJECTURE:

QUALITATIVELY: SPACE OF COLL. COORDS. $\cong AdS_5 \times S^5$!!

QUANTITATIVELY: $f_{16}(\tau)$ OBTAINED BY BANKS + GREEN FROM IIB SUGRA ON $AdS_5 \times S^5$!!

OF SUSY's

$\mathcal{N} = 2$

(asym. free for $N_F < 2N$)

("SEIBERG-WITTEN MODEL")

PROGRESS

- SUSY-Multi-inst. action + measure derived 9603136, 9607202, 9708036, 9804009
- SEIBERG-WITTEN PREPOTENTIA

$\mathcal{F}(v)$ SOLVED IN QUADRATURES

VEV

FOR ALL $SU(N)$:

$$\mathcal{F}(v)\Big|_{k\text{-inst}} = \text{INTEGRAL OVER}$$

BOSONIC + FERMIONIC k-inst COLLECTIVE COORDINATES

- INTEGRALS DONE FOR $k \leq 2$:

→ AGREE WITH ELLIPTIC CURVE SOLUTIONS FOR $0 \leq N_F < 2N$

→ DISAGREE WITH ELLIPTIC CURVE SOLUTIONS FOR $N_F = 2N$

→ THE LATTER CASE CAN BE FIXED FOR $N \leq 3$ BUT MAYBE NOT FOR $N > 3$.

- MATONE RELATIONS "BUILT IN"

OF SUSY's

$\mathcal{N} = 1$

("standard Model" over in Europe)

$\mathcal{N} = 0$

("standard Model" over here)

PROGRESS

- SUSY-MULTI-inst. action + measure derived

  9607202, 9708036, 9804009

- MULTI-inst. action + measure derived EXCEPT for the 't Hooft det. contribution to the measure

  9709072

THE $\mathcal{N}=4$ MODEL

- $SU(N)_{gauge}$, $N \to \infty$
- All (adjoint) VEV's turned off

$\Rightarrow$ HIGHLY NONTRIVIAL SUPERCONFORMAL FIELD THEORY

($\beta = 0$, FINITE)

MALDACENA'S CONJECTURE:

For $N \to \infty$, this theory is equivalent to type IIB string theory on $AdS_5 \times S^5$!!

THEN GET: $|\vec{q}|^2+|\vec{p}|^2=1$

$\therefore$ THE $6k^2$ VARIABLES

$(\chi_{AB})_{ij}$

LIVE ON S^5 !! $(N\to\infty)$

WHAT ABOUT AdS_5 ?

$$\int d\mu^{(k)} \ni \int d^4x \, \frac{d\rho}{\rho^5}$$

= VOLUME FORM for AdS_5 !

[Witten's initial observation]

IN LEADING ORDER IN $1/N$, THE (ENORMOUS) COLL. COORD. SPACE FOR CHARGE-k MULTI-INSTANTONS COLLAPSES TO

$$AdS_5 \times S^5$$

SUMMARY :

$$\sum_{k=1}^{\infty} \int d\mu^{(k)} e^{-S^{k}_{inst}} \prod_{i=1}^{16} \mathrm{tr}_N \left[F^{(k)}_{\mu\nu} \lambda(x_i) \right] \sigma^{\mu\nu}$$

$$\propto \left[\sum_{k=1}^{\infty} k^{25/2} \cdot \sum_{d|k} \frac{1}{d^2} \cdot e^{-\frac{8\pi^2 k}{g^2}} \right]$$

$$\times \sqrt{N} \int \underbrace{d^4x \frac{d\rho}{\rho^5}}_{AdS_5} \times \underbrace{d^5 \hat{X}_{AB}}_{S^5}$$

$$\times \int \prod_{A=1}^{4} \underbrace{d^2 \xi^A_\alpha}_{SUSY} \underbrace{d^2 \bar{\eta}^A_{\dot{\alpha}}}_{SUCONF.}$$

$$\times \prod_{i=1}^{16} G\left(x_i \, , \, y \in AdS_5 \times S^5\right)$$

IN PERFECT AGREEMENT WITH IIB SUGRA

The conformal window in QCD and supersymmetric QCD*

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Abstract

In both QCD and supersymmetric QCD (SQCD) with N_f flavors there are conformal windows where the theory is asymptotically free in the ultraviolet while the infrared physics is governed by a non-trivial fixed-point. In SQCD, the lower N_f boundary of the conformal window, below which the theory is confining is well understood thanks to duality. In QCD there is just a sufficient condition for confinement based on superconvergence. Studying the Banks-Zaks expansion and analyzing the conditions for the perturbative coupling to have a *causal analyticity structure*, it is shown that the infrared fixed-point in QCD is perturbative in the entire conformal window. This finding suggests that there can be no analog of duality in QCD. On the other hand the infrared fixed-point in SQCD is strictly non-perturbative in the lower part of the conformal window, in agreement with duality. Nevertheless, we show that it is possible to interpolate between the Banks-Zaks expansions in the electric and magnetic theories, for quantities that can be calculated perturbatively in both. This interpolation is explicitly demonstrated for the critical exponent that controls the rate at which a generic physical quantity approaches the fixed-point.

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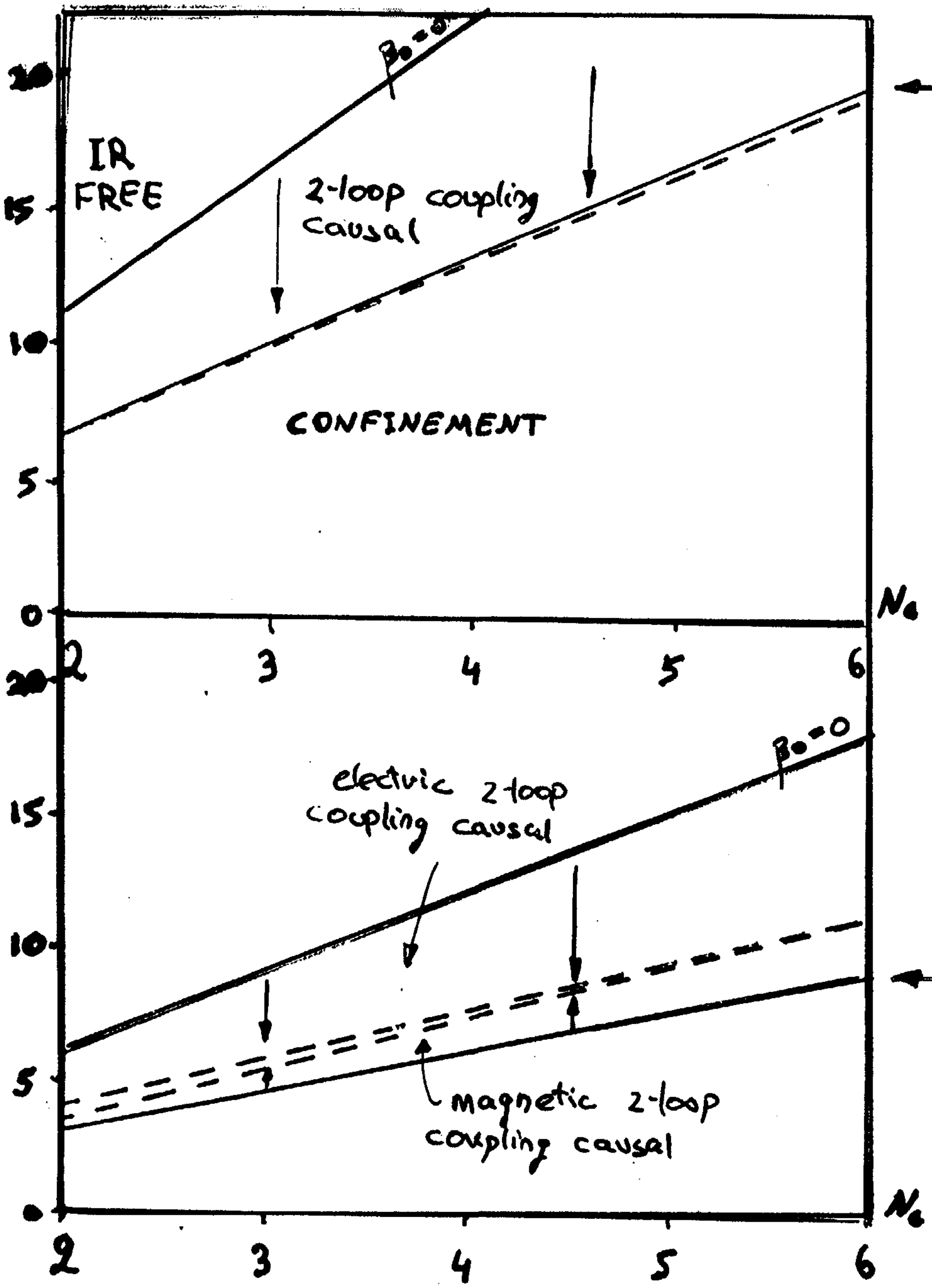
[*] The talk in the Workshop on QCD Phase Transitions is based on hep-th/9810192.
[†] CNRS UMR C7644



$\leftarrow \gamma_0 = 0$

gluon propagator (D)

$\gamma_0 \equiv$ anomalous dim. of gluon (Landau gauge)

$$\gamma_0 = -\frac{1}{4}\left(\frac{13}{6} N_c - \frac{2}{3} N_f\right) \quad , \quad \rho = \frac{1}{\pi} \, \mathrm{Im}(D)$$

If $\gamma_0 < 0 \quad \Rightarrow \int_{0^-}^{\infty} dk^2 \rho(k^2, \mu^2, g) = 0 \Rightarrow$ gluons are not part of physical space of states

superconvergence $\Rightarrow$ confinement (oehme)

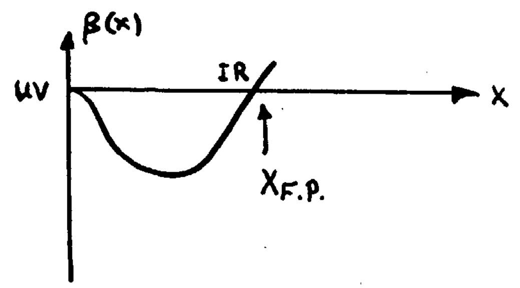
Seiberg Duality

$$SU(N_c),\ N_f \ \overset{IR}{\longleftrightarrow} \left\{ \begin{array}{l} SU(N_f - N_c),\ N_f \\ W \end{array} \right.$$

electric — magnetic

$\leftarrow \gamma_0 = 0 \ ; \ \beta_0^{dual} = 0$

β(x) in the conformal window



Purpose: to describe the infrared using perturbation theory

- When does the theory become strongly coupled?
  comparison between QCD and SQCD
- In SQCD: using the electric and magnetic descriptions together

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Can perturbation theory describe the infrared in asymptotically free theories? ~~confinement~~, conformal?

top of the conformal window: $\beta(x) = -x^2(\beta_0 + \beta_1 x)$
perturbative fixed-point: $x_{F.P.} = -\beta_0/\beta_1 \xrightarrow[\beta_0 \to 0^+]{} 0^+$
higher-orders, non-perturbative effects: small

lower N_f: $\beta(x) = -x^2(\beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \dots) = 0 \Rightarrow x_{F.P}$
Banks-Zaks expansion: $x_{F.P} = z_1 \epsilon + z_2 \epsilon^2 + \dots \quad \epsilon = \frac{11}{2} - \frac{N_f}{N_c}$

How far down in N_f/N_c can perturbation theory be used?

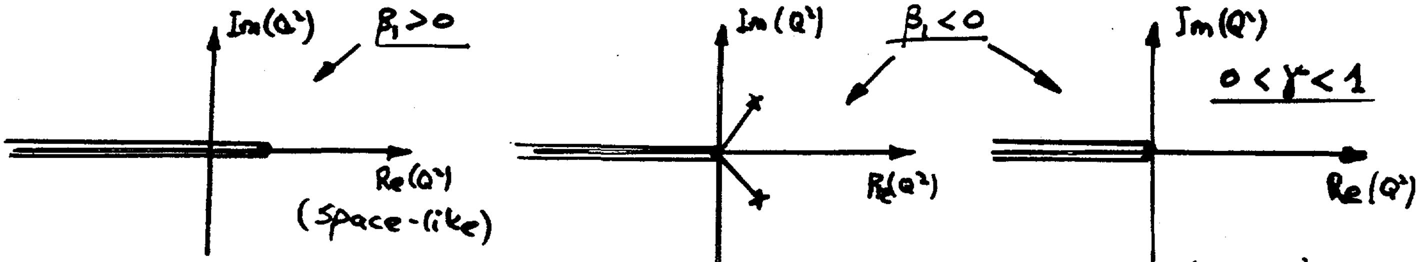
- to trust perturbation theory in the infrared we require "perturbative causality":

a) causal analyticity structure (physical schemes)

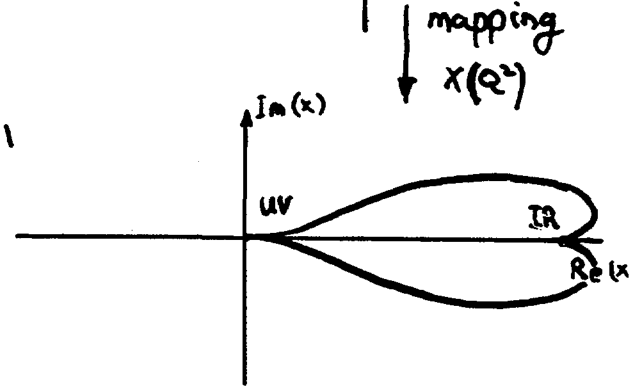
b) stability with respect to higher-orders

Analyticity structure of the perturbative coupling

2-loop $\beta(x) = -x^2(\beta_0 + \beta_1 x)$, $\gamma = \frac{d\beta(x)}{dx}\Big|_{F.P.} \overset{2\text{-loop}}{=} -\frac{\beta_0^2}{\beta_1}$

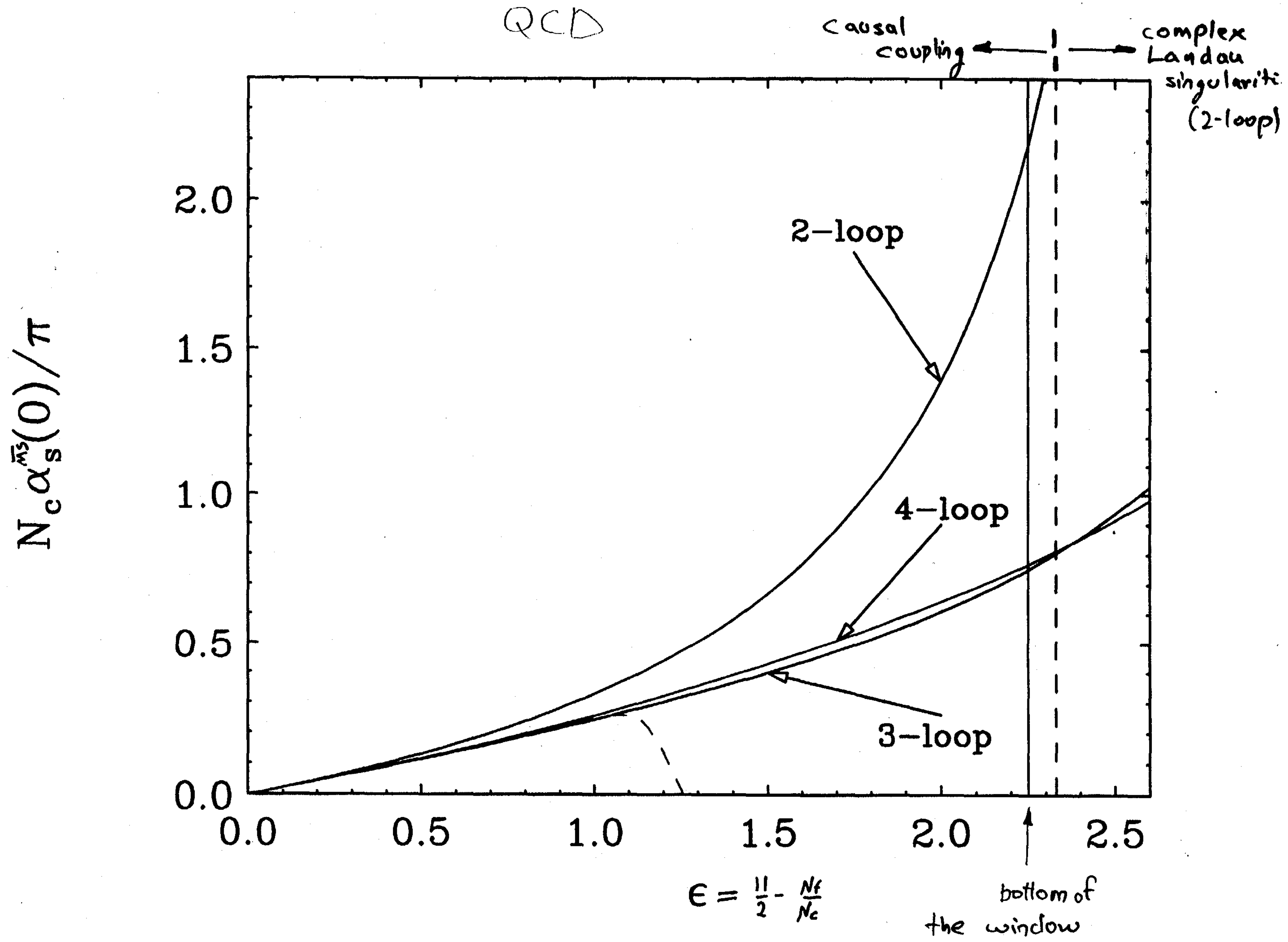


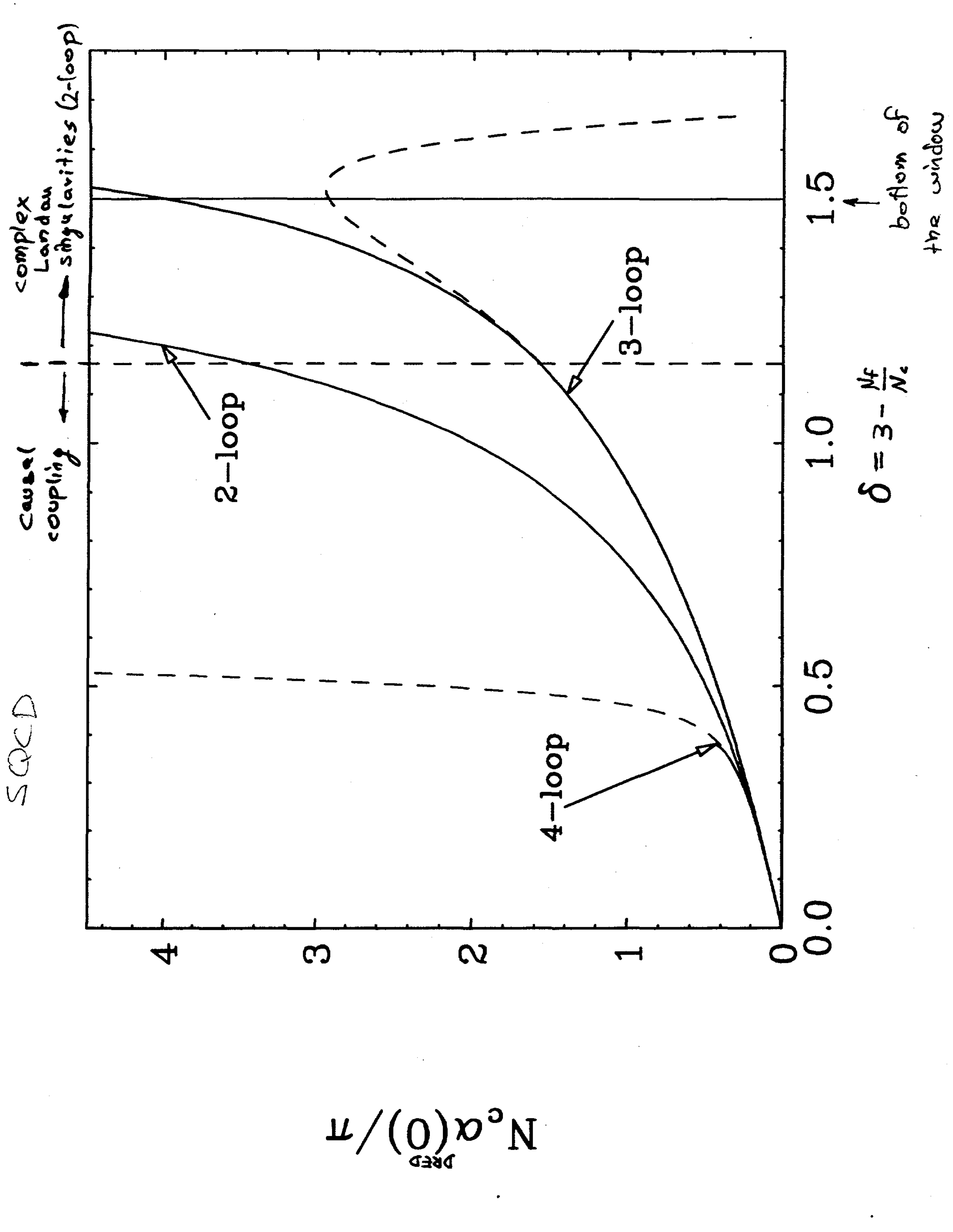
- 2-loop coupling is causal $\Leftrightarrow 0<\gamma<1$
- stability with respect to higher orders : $|\beta_2 x(Q^2)| << |\beta_1|$

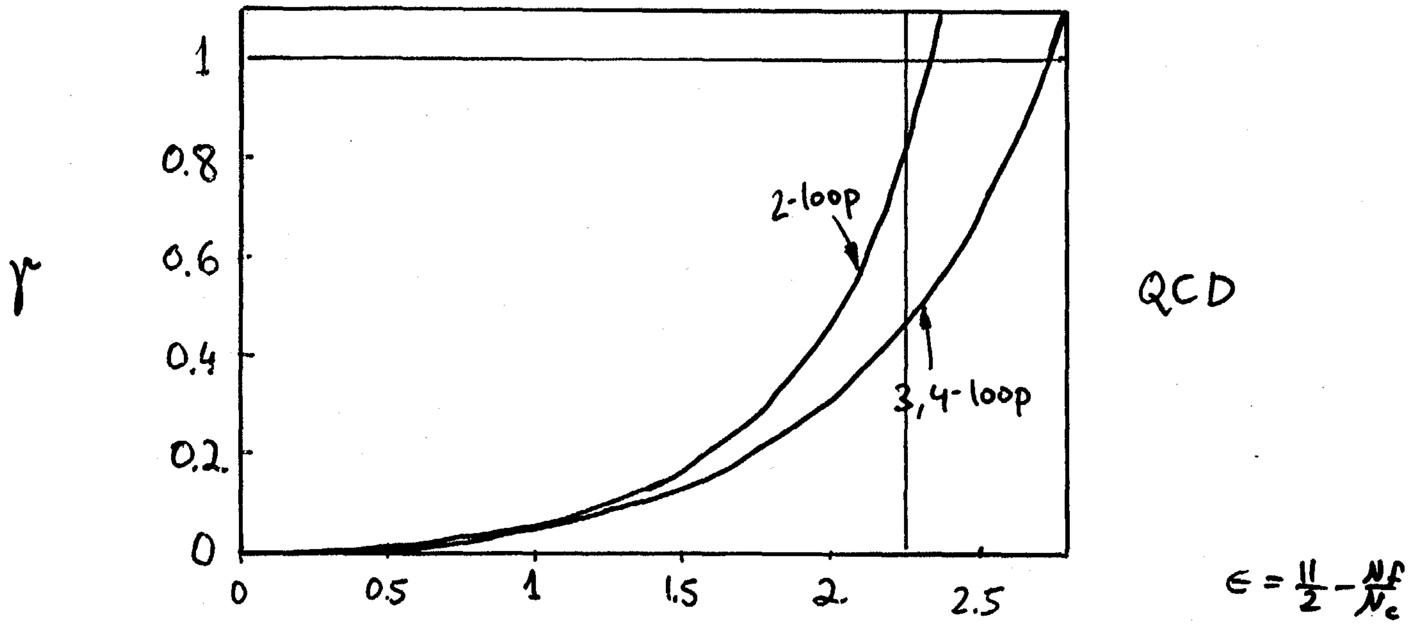


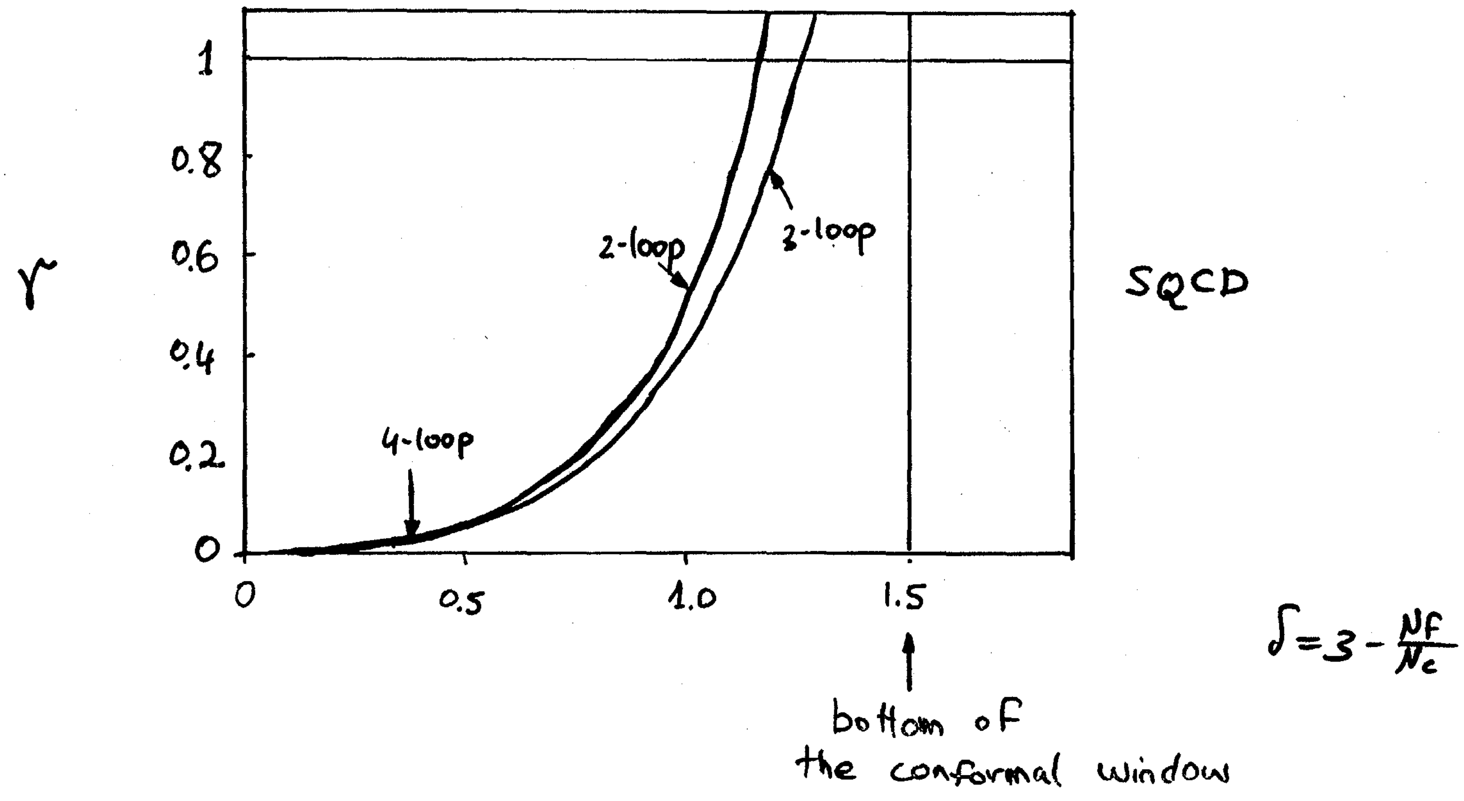
3-loop 3-loop coupling is causal $\Leftrightarrow$ $0<\gamma<1$
if $\underline{\beta_2<0}$

in general causality $\Longrightarrow$ $0<\gamma<1$


QCD
Causal coupling
complex Landau singulariti (2-loop)
$N_c \alpha_s^{\overline{MS}}(0)/\pi$
2.0
1.5
1.0
0.5
0.0
2–loop
4–loop
3–loop
0.0
0.5
1.0
1.5
2.0
2.5
$\epsilon = \frac{11}{2} - \frac{N_f}{N_c}$
bottom of the window


SQCD
causal coupling
complex Landau singularities (2-loop)
2—loop
3—loop
4—loop
0.0
0.5
1.0
1.5
$\delta = 3 - \frac{N_f}{N_c}$
bottom of the window
0
1
2
3
4
$N_c \alpha_{DRED}(0)/\pi$


γ
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
2-loop
3,4-loop
QCD
$\epsilon = \frac{11}{2} - \frac{N_f}{N_c}$


γ
1
0.8
0.6
0.4
0.2
0
0
0.5
1.0
1.5
2-loop
3-loop
4-loop
SQCD
$\delta = 3 - \frac{N_f}{N_c}$
bottom of
the conformal window

critical exponent and causality:

In general: $$\underset{\substack{\uparrow \\ \text{free theory}}}{0} \le \gamma < \underset{\substack{\uparrow \\ \text{non causal theory}}}{1}$$

In perturbation theory: "perturbative causality" – a criterion for reliability of perturbation theory in the infrared.

QCD:

→ basing on the superconvergence criterion,

→ assuming perturbation theory is reliable as long as it does not signal its inconsistency

⇒ 1) QCD is weakly coupled in the entire conformal window

2) no place for duality

3) IR quantities may change sharply at the confining phase transit

$V(r) \simeq \frac{\alpha_V}{r}$ potential between heavy (external) quarks

SQCD:

- Electric theory becomes strongly coupled within the window
- This is signaled by Landau singularities in perturbation theory.
- In agreement with duality: no overlap between the regions where perturbation theory is reliable in the IR.
- Padé approximants can be used to "interpolate".

Perturbation theory breaks down (Landau singularities) → non-perturbative infrared physics strong coupling

↗ QCD confinement

↘ SQCD magnetic theory weakly coupled in IR

①

QCD at large number of flavors

T. Appelquist, J. Schechter, F.S. (in progress)

Effective Action

$$\theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F_a^{\mu\nu} F_{\mu\nu, a} \equiv 2bH$$

$$\delta_{U(1)_A} V_{QCD} = N_f \alpha \frac{g^2}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} F_a^{\mu\nu} F_a^{\rho\sigma}$$

$$\equiv 4 N_f \alpha G$$

$$b \equiv -\frac{\beta(g)}{g^3} 16\pi^2$$ 1-loop $$b = \frac{11}{3} N - \frac{2}{3} N_f$$

Let us consider a set of gauge invariant operators

$$\{ O_n \}$$

mass dim $O_n = d_n$

Axial charge $q_n(O_n) \equiv q_n$

$$V = -b\mathcal{F} \sum_n \frac{c_n}{d_n} \ln\left(\frac{O_n}{\Lambda^{d_n}}\right) + h.c.$$

$$\sum_n c_n = 1$$

$$\mathcal{F} = H + i \underset{\rightsquigarrow 1}{\delta} G$$

$$\sum_n c_n \, q_n / d_n = \frac{2 N_f}{b \delta}$$

$$\mathcal{O}_1 \equiv F \longrightarrow \dim F = 4$$

$$\mathcal{O}_2 \equiv \det M \quad \hookrightarrow \begin{cases} \dim \overset{\det}{M} = (3-\underbrace{\gamma}_{\hookrightarrow \text{Anomalous dim of the quark mass operator}}) N_f \\ q_2 = 2 N_f \end{cases}$$

$$V(F, M) = \left[\frac{\beta(g)}{g^3} 16\pi^2 + (3-\gamma) N_f \right] \frac{F}{4} \ln\left[\frac{-F}{\Lambda^4} \right]$$

$$- F \ln\left[\frac{\det M}{\Lambda^{(3-\gamma)N_f}} \right] + AF + h.c.$$

Meson Potential

$$\frac{\delta V}{\delta F} = 0 \implies$$

$$V = -c \left[\frac{\Lambda^{-\frac{\beta(g)}{g^3} 16\pi^2}}{\det M} \right]^{\frac{4}{-\frac{\beta}{g^3} 16\pi^2 - (3-\gamma) N_f}} + h.c.$$

$$c = \frac{1}{4e} \left[-\frac{\beta(g)}{g^3} 16\pi^2 - (3-\gamma) N_f \right] \exp\left[\frac{4A}{-\frac{\beta}{g^3} 16\pi^2 - (3-\gamma) N_f} \right]$$

- What if Conformal and χ- Symmetries are restored Together

conformal $\rightarrow$ $\beta = 0$

χ - Symmetry $\rightarrow$ $\frac{4 N_f}{\frac{\beta}{g^3} 16\pi^2 + (3-\gamma) N_f} < 2$

$\boxed{\gamma < 1}$ Non - Perturbative Result.

- δ - independence $F = H + i \underbrace{\delta} G$

$$\frac{4 N_f}{\frac{\beta}{g^3} 16\pi^2 \cdot \delta + (3-\gamma) N_f} < 2$$

for $\beta = 0$ $\rightarrow$ $\boxed{\gamma < 1}$ δ - independent result.

- GAP (SD Methods)

$$\gamma(2-\gamma) = 1 \rightarrow \gamma = 1$$

critical condition.

Simple Model

$$V_T = |c|\,\rho^{\frac{4}{3-\gamma}} - (\gamma-1)\,\Lambda^{2(\gamma-1)}\rho^2$$

$$\langle\rho\rangle = \Lambda^{3-\gamma}\left[\frac{(\gamma-1)(3-\gamma)}{2|c|}\right]^{\frac{1}{2}\frac{3-\gamma}{\gamma-1}}$$

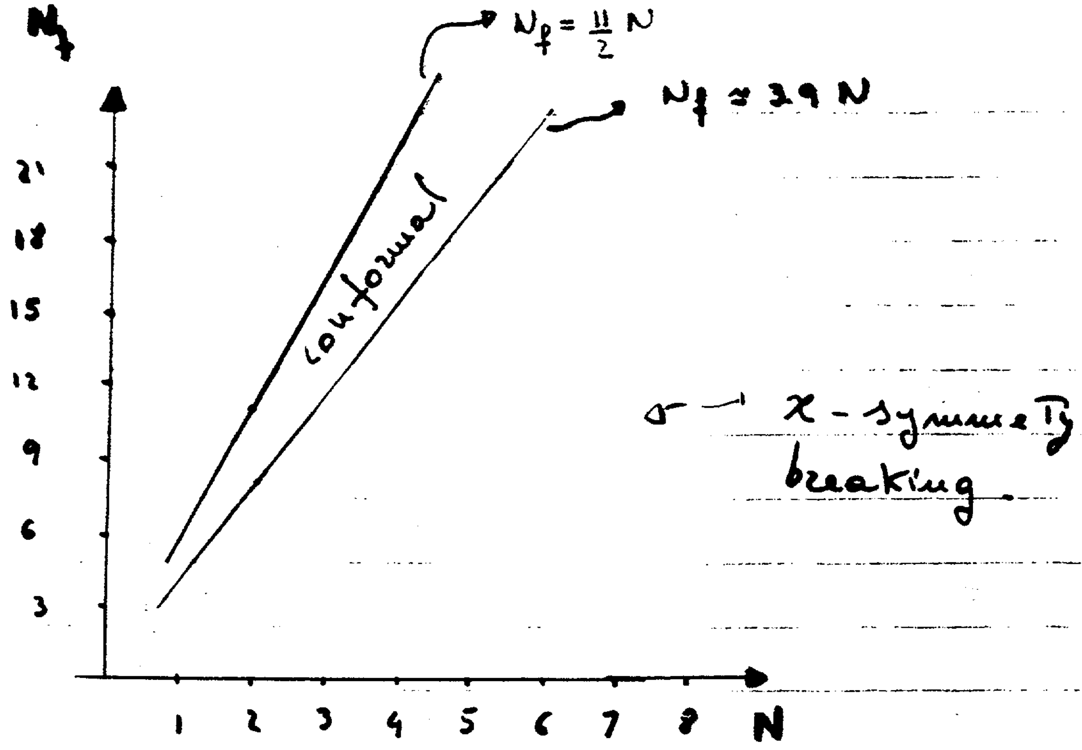
$\gamma \to 1$

$$\boxed{\langle\rho\rangle \approx \Lambda^{2}\left[\frac{\gamma-1}{|c|}\right]^{\underbrace{\frac{1}{\gamma-1}}}}$$

Ginzburg-Landau

$$V_{GL} = D\,\rho^4\Lambda^{4(\gamma-2)} - (\gamma-1)\,\rho^2\Lambda^{2(\gamma-1)}$$

$$\boxed{\langle\rho\rangle_{GL} \approx \Lambda^{\gamma}\left[\frac{\gamma-1}{2D}\right]^{\underbrace{\frac{1}{2}}}}$$



- conformal window (QCD)

$\gamma < 1$ $\quad$ $3.9 < N_{f/N} < \frac{11}{2} N$

- χ - symmetry breaking (QCD)

$\gamma \geqslant 1$ $\quad$ $N_f \leq 3.9 N$

Summary (1)

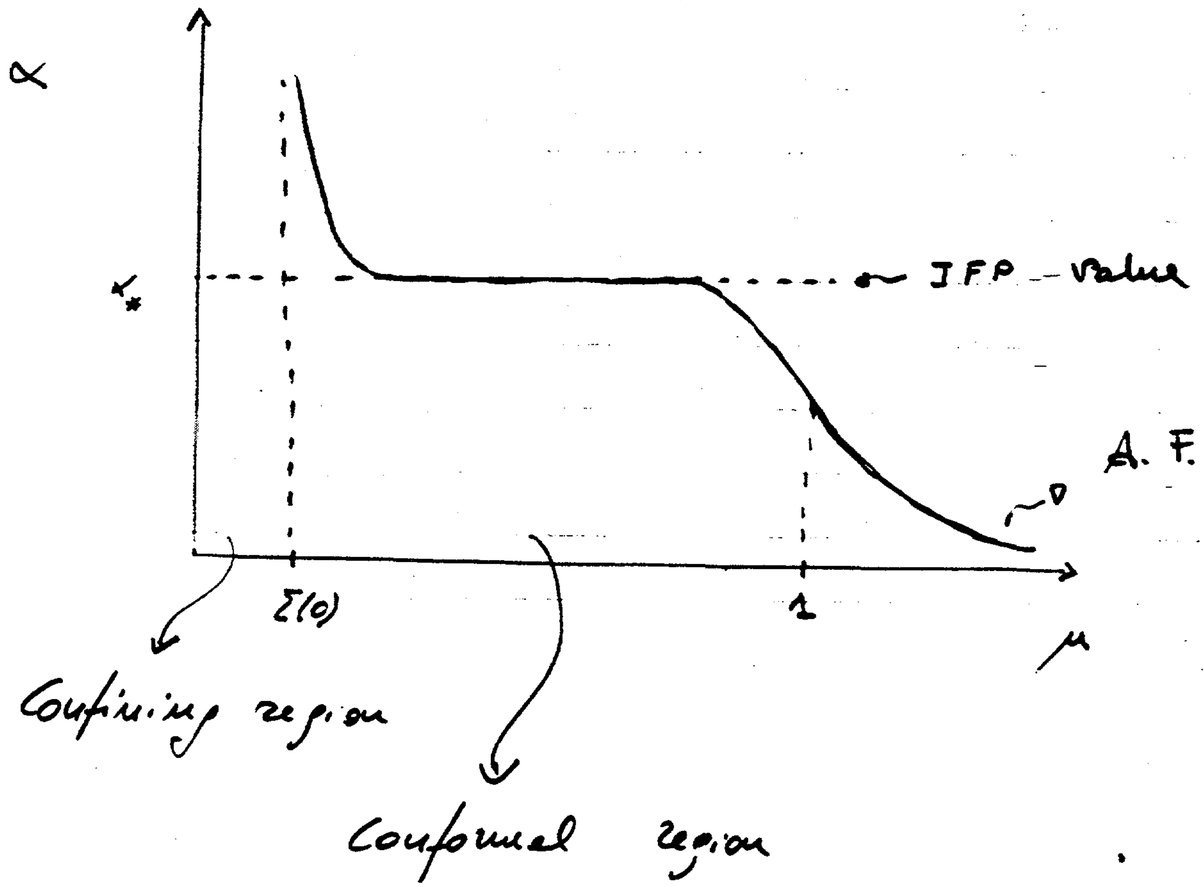
- We study the chiral phase transition for vector like SU(N) gauge theories as functions of the number of quark flavors N_f by making use of an anomaly induced effetive potential.
- The effective potential depends explicitly on the full β-function and the anomalous discussion γ of the quark mass operator.
- Using this potential we argue that chiral symmetry is restored for $\gamma < 1$.
- A perturbative evaluation of γ leads to the conclusion that the transition takes place at a value of $N_f \simeq 4N$.

Physical Spectrum

T. Appelquist, F.S. hep-ph/9806409

$N_f \to N_f^c$ Conf + χ restored sim.

- Implications for Vector - Axial spectrum.



Σ(0) ~ dynamical Quark mass at zero momentum

Λ ~ intrinsic scale : $\alpha_\Lambda := \alpha(\Lambda) \approx 0.78\alpha^*$

- VV - AA Vacuum polarization

$$i\Pi^{ab}_{\mu\nu}(q) \equiv \int d^4x \, e^{-iqx} \left[\langle J^a_{\mu V}(x) J^b_{\nu V}(0) \rangle - \langle J^a_{\mu A}(x) J^b_{\nu A}(0) \rangle \right]$$

$a, b = 1, \dots, N_f^2 - 1$

$$\Pi^{ab}_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2) \delta^{ab} \Pi(q^2)$$

- Dispersion Relation

$$\frac{1}{\pi} \int_0^\infty ds \frac{\operatorname{Im} \Pi(s)}{s + Q^2} = \Pi(Q^2)$$

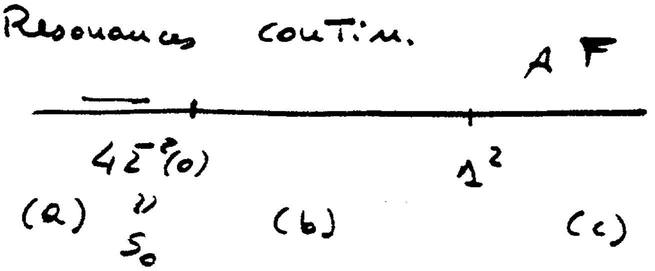
$Q^2 \equiv -q^2 > 0 \qquad - Q^2 \Pi(Q^2) > 0 \qquad 0 < Q^2 < \infty$

$Q^2 \mapsto \infty \qquad \frac{1}{Q^2}$ Expansion

$\Pi(Q^2) \sim \frac{1}{Q^6}$

$$\frac{1}{\pi} \int_0^\infty ds \operatorname{Im} \Pi(s) = 0 \qquad \frac{1}{\pi} \int_0^\infty s \, ds \operatorname{Im} \Pi(s) = 0$$

$\mathrm{Im}\,\Pi(s)$?



(a) $\mathrm{Im}\,\Pi(s) = \pi F_V^2 \delta(s-M_V^2) - \pi F_A^2 \delta(s-M_A^2) - \pi F_\pi^2 \delta(s$

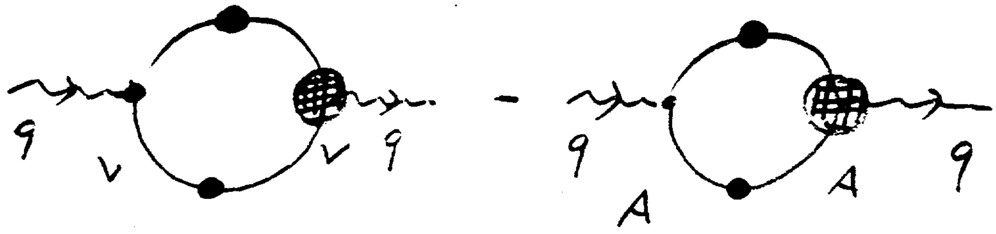
zero width approx.

$2\pi F_\pi \sim \sqrt{N}\,\Sigma(0)$

N.B. $M_V^2, M_A^2, \cdots$ any other mass

$\sim \Sigma^2(0) \to 0$ at CPT.

(b) $S_0 < S < \Lambda^2$ Continuum.



- Dynamical Mass Insertion.

▨ Appropriate Vertex to insure WTI

Schemat. $$\int d^4k \, \frac{\Sigma(k)\,\Sigma(k+q)}{k^2\,(k+q)^2}$$ UV, IR finite

$$M_A^2 - M_V^2 \simeq \frac{F_\pi^2}{F_A^2}\left[M_V^2 - 2a\Sigma^2(0)\right]$$

$$a > 0 \quad O(1)$$

- <u>ESB</u>

"S" - parameter

$$S \equiv 4\int_0^\infty \frac{ds}{s}\, \mathrm{Im}\,\bar{\pi}(s) = 4\pi\left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right]$$

$\bar{\pi}$ (subtracted GB)

$$\boxed{S \simeq 4\pi F_\pi^2\left[\frac{1}{M_V^2} + \frac{1}{M_A^2} - \underbrace{\frac{2a\Sigma^2(0)}{M_V^2 M_A^2}}\right]}$$

$M_V^2,\ M_A^2 \sim \Sigma^2(0)$.

- Conformal Region and Eff. Lag. Appr.

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left[D_\mu M D^\mu M^+\right] - \frac{1}{2}\mathrm{Tr}\left[F_{\mu\nu}^L F^{L\mu\nu} + F_{\mu\nu}^R F^{\mu\nu}\right.$$

$$+ m_0^2\,\mathrm{Tr}\left[A_\mu^L A^{L\mu} + A_\mu^R A^{R\mu}\right] + \underline{h}\,\mathrm{Tr}\left[A_\mu^L M A^{R\mu} M^+\right]$$

$$D_\mu M \equiv \partial_\mu M - ig A_\mu^L M + ig M A_\mu^R$$

$$\boxed{M_A^2 - M_V^2 = \frac{F_\pi^2}{2}\left[g^2 - \underbrace{h}\right]}$$

→ parametrizes conformal region

with

$$\langle M \rangle = 1 \frac{F_\pi}{\sqrt{2}}$$

Summary (2)

- We investigate the physical spectrum of vector-like SU(N) gauge theories with infrared coupling close to but above the critical value for a conformal phase transition.
- We use dispersion relations, the momentum dependence of the dynamical fermion mass and resonace saturation.
- It is shown that the second spectral function sum rule is substantially effected by the continuum contribution, allowing for a reduction of the axial vector-vector mass splitting with respect to QCD-like theories.
- In technicolor theories, this feature can result in a small or even negative contribution to the electroweak S parameter.

Matthew Strassler : More on SUSY and QCD

Summary

- Classify CFTs by the dimensions of their relevant and lowest-dim irrelevant operators [in addition to their symmetries, of course] is an important goal.
- Determine how RG flow from relevant perturbations carries one CFT to another or to a different phase
- Unitarity is an important constraint on operator dimensions in CFTs — can be useful for discovering edge of conformal phase
- SUSY theories suggest many examples of phenomena in 2,3,4 dimensions which are both familiar and unfamiliar. Do these really occur? We can look for them or their analogues in the non-SUSY case: relevant multi-fermion operators, irrelevant fermion masses, gauge beta functions which change sign, quantum enhancement of symmetries in CFTs.
- Applications beyond particle physics?

Simple application of unitarity

- Suppose we have a sequence of similar CFTs that come from similar field theories [perhaps $SU(N)$ QCD for several values of N_f.]
- Suppose that a certain operator [perhaps $(\bar{\psi}\psi)^2$] has a dimension in these theories which decreases along the sequence.
- Since its dimension cannot be less than one, the sequence of CFTs must end before this happens, and a new phase [possibly a different-looking set of CFTs, possibly confinement, possibly something unknown] must kick in instead.
- Thus, a gauge invariant operator of large canonical dimension but with CFT dimension near 1 may indicate that the CFT lies near the edge of a phase boundary.

Dangerous Irrelevant Operators

at $g = 0$, irrelevant
at $g = g_*$, relevant

Temptation to ignore them is ill-advised.

Example: QCD/technicolor

The operators

$$(\bar{\psi}\psi)^2, (\psi)^N \tag{5}$$

may be relevant in IR — but for what range of N, N_f? and what is their effect when added to Lagrangian?

$\mathcal{N} = 1$ $SU(N)$ with N_f

Meson mass: $\mathcal{O} = (Q\tilde{Q})^2$
$d_{\mathcal{O}} = 6(1 - N/N_f) < 3$ for $N_f < 2N$,

Baryon: $\mathcal{O} = Q^N$
$d_O < 3$ for $N_f < N(N_f - N)/2$

Dependence of Phase on Dangerous Relevant Ops

Consider $SU(4)$ with N_f flavors $Q_i, \tilde{Q}^i, \psi_i, \tilde{\psi}^i$. Take $W = hQ_1Q_2Q_3Q_4$ and add dimension-six terms

$$\Delta\mathcal{L} = \left[\sum_{i,j} \frac{\partial^2 W}{\partial Q_i \partial Q_j}\psi_i\psi_j + h.c. + \sum_i \left|\frac{\partial W}{\partial Q_i}\right|^2\right] \qquad (6)$$

- $N_f \geq 8$: perturbation irrelevant, as in classical limit [SCFT unchanged]
- $N_f = 7$: theory driven from expected interacting fixed point to a different one [SCFT(1) → SCFT(2)]
- $N_f = 6$: instead of flowing to free dual $SU(2)$ gauge theory, the theory flows to an interacting fixed point [Free magnetic → SCFT(3)]
- $N_f = 5$: perturbation causes chiral symmetry breaking [Confinement (no χSB) → Confinement (with χSB)]

There are hundreds of other interesting examples.

Harmless Relevant Operators

at $g = 0$, relevant
at $g = g_*$, irrelevant

Temptation to include them is ill-advised.

$D = 3$ Example

$U(1)$ with $\Phi, \tilde{\Phi}, S$ of charge $1, -1, 0$; $W = hS\Phi\tilde{\Phi}$

define $M = \Phi\tilde{\Phi}$; vortex creation operators $V, \tilde{V}$.

Vacuum equations: $M = \Phi\tilde{\Phi} = 0$ in SCFT [redundant]

claim (mirror symmetry [dBOOY,AHISS,dBOO])
the theory is in same universality class as $W_{eff}(V, \tilde{V}) = 0$
(free vorticial phase again) with $S \leftrightarrow V\tilde{V}$

The mass term $\Delta W = mS^2$ is relevant in the free theory. Drives theory toward theory without S.

But at SCFT $S^2 \sim (V\tilde{V})^2$, $d_{S^2} = 2$: *marginal*; in fact marginally irrelevant perturbation of free vorticial phase.

Can check: consistent with previous example.

Quantum Accidental Symmetries

This is a common phenomenon in SCFTs:

Quantum effects cause the operators of the theory to reorganize themselves into multiplets with an enhanced symmetry that could not be guessed classically.

The enhanced symmetry is called quantum [since it is quantum mechanical] and accidental [since it is only obeyed in the far infrared and is not a symmetry of the whole theory.]

A $D = 3$ example with a discrete symmetry:

$U(1)$ with $\Phi, \tilde{\Phi}$ of charge $1, -1$, $W = 0$

at SCFT: $W_{eff} = MV\tilde{V}$
This theory has a quantum accidental triality symmetry permuting
a) positronium $M = \Phi\tilde{\Phi}$
b) the vortex V with magnetic flux $+1$
c) the vortex $\tilde{V}$ with flux -1

Can we find an example using lattice gauge theory?

Vacuum Energy in Large N_f QCD and Instanton Molecules

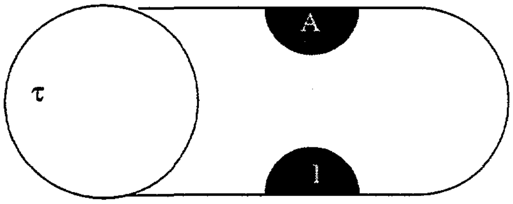
Momchil Velkovsky (BNL)

- The $I\bar{I}$ valley
- "Classical molecules" and finite T
- The instanton ensemble for QCD with N_f flavors
- Vacuum energy of a dilute molecular gas
- Separating the non-perturbative contribution - a complex saddle point + other tricks
- Perturbation series asymptotic for the vacuum energy

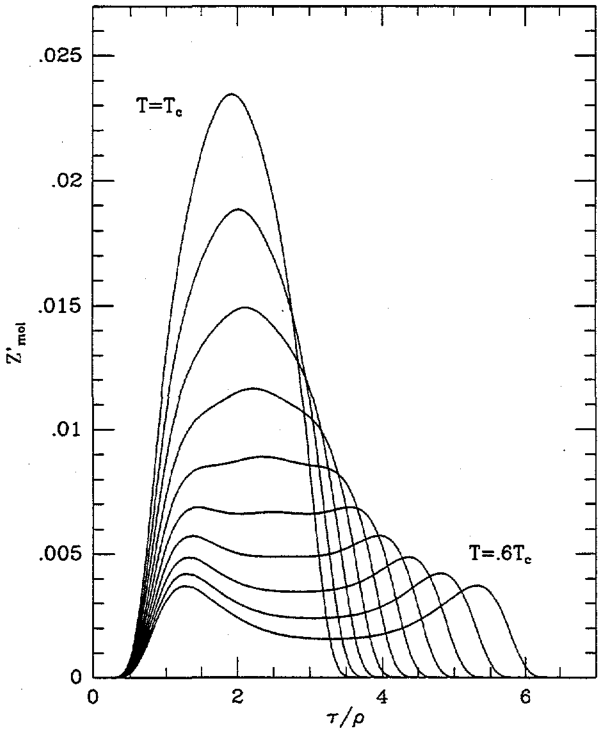
"Classical molecules" and finite T

At sufficiently high temperatures - stable configuration (real saddle point).

$0 \leq \tau \leq 1/T$



Adding fermions makes the molecules stable at even lower temperatures (about 110 MeV for $N_f = 2$).



Vacuum energy of a dilute molecular gas

$$Z_{mol.gas} = \exp(-V^{(4)} E_{mol.gas}) = \exp(Z_{mol}^{non.pert}),$$

$$Z_{mol}^{non.pert} = (Z_{mol} - Z^{pert})/Z^{pert}$$

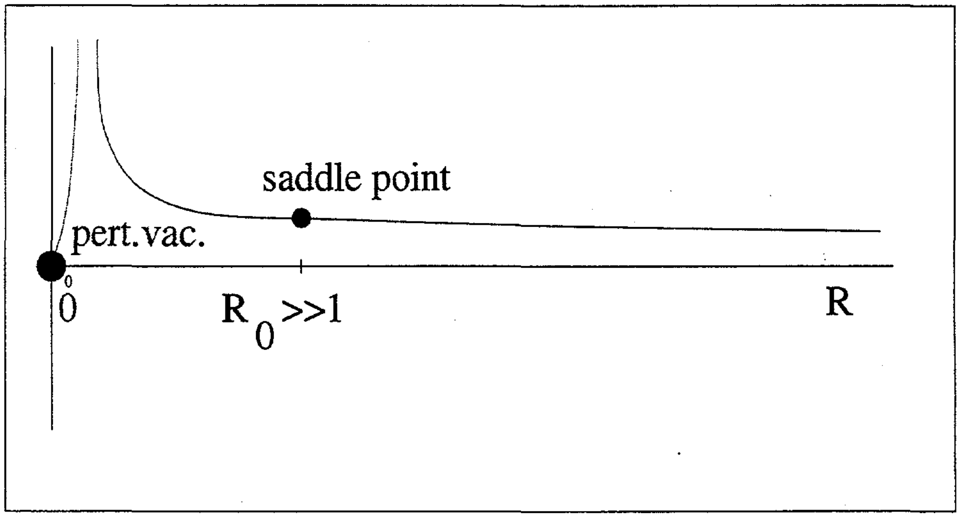
$$\frac{d^2 Z_{mol}}{d\rho_I d\rho_{\bar I}} = V^{(4)} C^2 \rho^4 \int d^4 R d\Omega (-\rho^2 |T_{I\bar I}(R,\Omega)|^2)^{N_f} \exp(-S_{int}(R,\Omega))$$

$$C = \frac{4.6 \exp(-1.86 N_c)}{\pi^2 (N_c-1)!(N_c-2)!} \frac{1}{\rho^5} (S_0)^{2N_c} \exp(-S_0),$$

$$T_{I\bar I}) = \int d^4 x \phi_I^\dagger (x - z_I, \Omega_I) \not{D} \phi_{\bar I}(x - z_{\bar I}, \Omega_{\ barI}),$$

At $T = 0$, semi-classically there is no stable $I\bar I$ configuration .

Still one can separate the non-perturbative contribution for $N_f >> 1$!



Results

Units of $\Lambda, N_c = 3, \rho = \frac{1}{3\Lambda}$

N_f	$Re\frac{d^2 E_{mol.gas}}{d \ln \rho_I d \ln \rho_{\bar{I}}}$	$Im\frac{d^2 E_{mol.gas}}{d \ln \rho_I d \ln \rho_{\bar{I}}}$	$I(N_f)$
2	0.	$-.5098 * 10^{-6}$	$-.2690 * 10^{-5}$
3	$-.3668 * 10^{-7}$	$-.5474 * 10^{-7}$	$-.2833 * 10^{-5}$
4	$-.1917 * 10^{-7}$	$-.5330 * 10^{-8}$	$-.1991 * 10^{-5}$
5	$-.1027 * 10^{-7}$	$.6464 * 10^{-8}$	$.1178 * 10^{-4}$
6	0.	$.1520 * 10^{-7}$	$.7913 * 10^{-4}$
7	$.5067 * 10^{-7}$	$.1127 * 10^{-7}$	$.7147 * 10^{-4}$
8	0.	$-.2714 * 10^{-5}$	$-.2292 * 10^{-2}$
9	$-.1898 * 10^{-3}$	$.1368 * 10^{-4}$	$.2067 * 10^{-2}$
10	0.	$.3365 * 10^{-4}$	.4120
11	$.1093 * 10^{-6}$	$.2421 * 10^{-8}$	.1480
12	0.	$-.6030 * 10^{-11}$	-132.8

with $I(N_f) = Im\frac{d^2 E_{mol.gas}}{d \ln \rho_I d \ln \rho_{\bar{I}}} e^{2S_0} S_0^{-4N_c+(3/2)N_f-1}$

Perturbation series asymptotic for the vacuum energ

$$E_k^{pert} = -\frac{1}{\pi}\int_0^\infty \frac{dg}{g^{k+1}} Im(E^{non.pert}(g))$$

$$\frac{d^2 E_k^{pert}}{d \ln \rho_1 d \ln \rho_2}\rho^4 = -\frac{1}{\pi}\int \frac{dg}{g^{k+1}} \frac{8\pi^2}{g^2}^{4N_c-(3/2)N_f+1} e^{-\frac{16\pi^2}{g^2}} I(N_f)$$

$$= -I(N_f)\frac{1}{2\pi}\left(\frac{1}{8\pi^2}\right)^{k/2}\int_0^\infty d\,(S_0)\,(S_0)^{4N_c-(3/2)N_f+k/2}\, e^{-2S_0},$$

Yet another saddle point integral with a saddle point at $S_0 = 2N_c - 3/4N_f + k/4$.

$$\frac{d^2 E_k^{pert}}{d \ln \rho_1 d \ln \rho_2}\rho^4 = -I(N_f)\sqrt{\frac{1}{4\pi}}\left(\frac{1}{8\pi^2}\right)^{k/2} \times$$

$$\left(\frac{4N_c-(3/2)N_f+k/2}{2}\right)^{4N_c-(3/2)N_f+k/2+1/2} e^{-(4N_c-(3/2)N_f+k/2)}$$

$$= -\frac{I(N_f)}{2^{4N_c-(3/2)N_f}}\left(\frac{1}{4\pi}\right)^{k+1}\Gamma(4N_c-(3/2)N_f+k/2+1).$$

But the renormalons produce bigger coefficients.

Compare to Faleev and Silvestrov:

$$R_{e^+e^-\to hadrons} = \sum -813(3280.5k)^{-35/k}(10+k/2)!\frac{g}{4\pi}^k$$

Padé-Summation Representations of the $\overline{\text{MS}}$ ß-Function for $n_f = 0$ QCD

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Kogan and Shifman have argued that in the absence of matter fields, the exact ß-function for supersymmetric SU(N) gluodynamics exhibits a pole which allows a double-valued coupling, indicative of an additional strong phase in the ultraviolet domain.[1] After reviewing successes of asymptotic Padé-approximant predictions, as applied to the ß-function of massive ϕ^4 scalar-field theory as well as the scalar current correlation function within QCD, we address whether Padé-summations of the $\overline{\text{MS}}$ ß-function for $n_f = 0$ QCD exhibit ß-function structure similar to that of supersymmetric gluodynamics, as opposed to a more conventional picture in which the strong coupling freezes out at low momenta to an infrared fixed-point value. We find that $[2 \mid 1]$, $[1 \mid 2]$, and the entire set of possible $[2 \mid 2]$ Padé-summation expressions whose Maclaurin expansions reproduce the presently-known four-loop ß-function series, regardless of the magnitude of the unknown five-loop term, always exhibit a positive pole prior to the occurrence of their first positive zero, consistent with a double-valued coupling in the ultraviolet domain, and precluding identification of this first positive zero as an infrared fixed point. Moreover, specific $[2 \mid 2]$, $[1 \mid 3]$, and $[3 \mid 1]$ Padé summations obtained from asymptotic Padé-approximant predictions of the unknown 5-loop ß-function term also have a positive pole preceding their first positive zero.

We conclude that there is no evidence for a positive infrared fixed point from Padé-summations of the gluodynamic ($n_f = 0$ QCD) ß-function. All such Padé summations appear to be consistent with the ß-function properties of supersymmetric gluodynamics, in which a weak and strong coupling-constant phase share a common infrared attractor.

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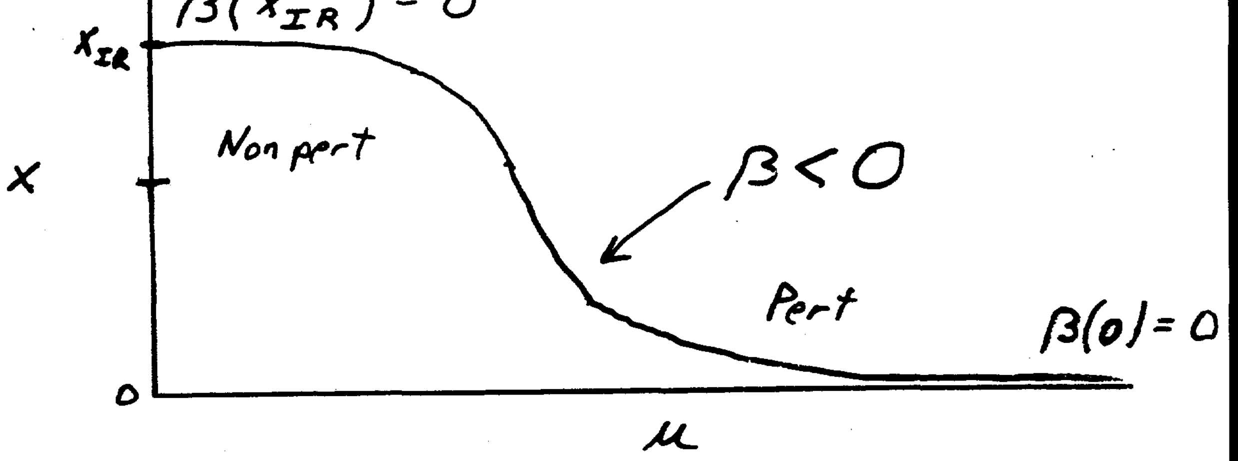
[1] I. Kogan and M. Shifman, Phys. Rev. Lett. 75 (1995) 2085.

INFRARED FIXED POINT IN QCD?

$$\mu^2 \frac{dx}{d\mu^2} = \beta_{\overline{MS}}(x) \qquad (x \equiv \alpha_s/\pi)$$



IRFP: Coupling Freezes Out to nonzero value at low momentum. (Mattingly + Stevenson: $x_{IR} = 0.82/\pi$)

[N/M]-Padé-Approximant within β-function:

$$\beta_{\overline{MS}}(x) \Longrightarrow -\beta_0 x^2 \left[\frac{1 + a_1 x + \ldots + a_N x^N}{1 + b_1 x + \ldots + b_M x^M}\right]$$

For IRFP to occur (as above):

1) First positive zero of numerator is at x_{IR}

<u>and</u> 2) Denominator has no zero in range $0 \leq x \leq x_{IR}$

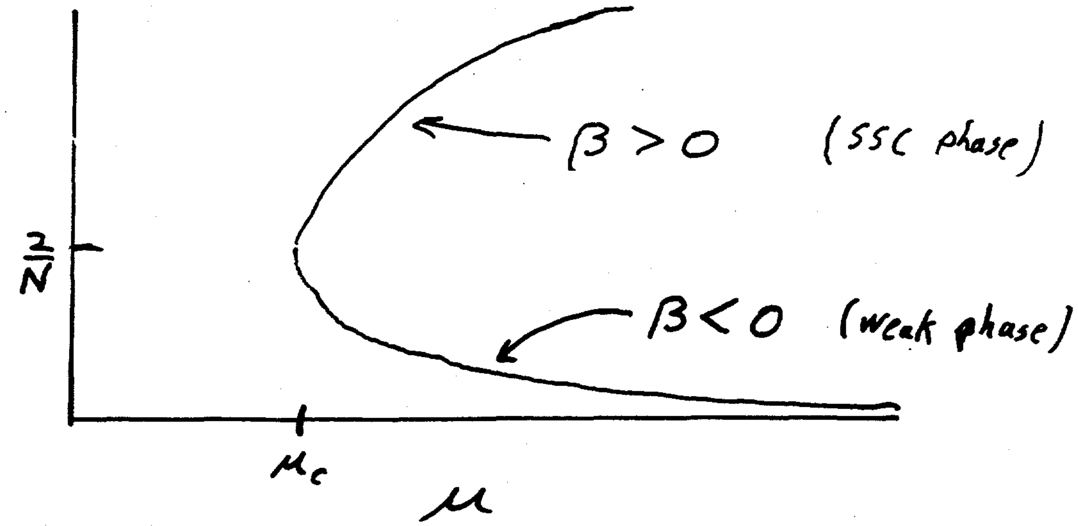
... or else β changes sign!

ELIAS

SU(N) SUSY Gluodynamics (Novikov, Shifman, Vainshtey, Zakharov,

$$\mu^2 \frac{dx}{d\mu^2} = \beta(x) = -\frac{3N}{4} x^2 \left[1 + \frac{N}{2} x + \frac{N^2}{4} x^2 + \cdots \right]$$

$$= -\frac{3Nx^2}{4} \left[\frac{1}{1 - Nx/2} \right] \quad \frac{\text{Exact}}{[0|1]} \text{ Padé}$$



x is double-valued: Two Phases!
(Kogon & Shifman PRL 75 (1995) 2085,

IR-region (of $\mu < \mu_c$) is INACCESSIBLE
(unless α is complex)

QCD? $\quad \beta(x) \Longrightarrow -\beta_0 x^2 \left[\frac{1 + a_0 x + \ldots + a_N x^N}{1 + b_1 x + \ldots + b_M x^M} \right]$

For Kogon-Shifman scenario to occur, First positive zero of the denominator <u>must be</u> <u>smaller</u> than ANY positive zero of the numerator

⑨ ELIAS

Gluodynamics ($n_f = 0$ QCD): $x \equiv \alpha_s/\pi$

$$\beta(x) = -\frac{11}{4}x^2\left[1 + 2.31818x + 8.11648x^2 + 41.5383x^3 + \underline{\underline{R_4}}\,x^4 + \ldots\right]$$

$[1 + R_1 x + R_2 x^2 + R_3 x^3]$ sufficient to predict $[2|1]$ and $[1|2]$ approximants

$$\beta_{[2|1]} = -\frac{11}{4}x^2\left[\frac{1 - 2.7996x - 3.7475x^2}{1 - 5.1178x}\right]$$

IRFP? Positive Numerator Zero: $x = 0.2639\ (= \alpha_s/\pi)$

Denominator Zero: $x = 0.1954$

Consistent with Kogan - Shifman Scenario

$$\beta_{[1|2]} = -\frac{11x^2}{4}\left[\frac{1 - 5.9672x}{1 - 8.2854x + 11.091x^2}\right]$$

IRFP? Numerator Zero: $x = 0.1676$

First Positive Denominator Zero: $x = 0.1514$

Again, consistent with Kogan - Shifman Scenario

⑩ ELIAS

Gluodynamics Cont'd:

Asym. Error Formula Predicts $R_4 = 302.2$

$[1 + R_1 x + R_2 x^2 + R_3 x^3 + R_4 x^4]$ Sufficient

to predict [2|2] approximant:

$$\beta_{[2|2]} = -\frac{11}{4} x^2 \left[\frac{1 - 9.6296x + 4.3327x^2}{1 - 11.9477x + 23.913x^2}\right]$$

IRFP? First Positive Numerator Zero: $x = 0.1092$

First Positive Denominator Zero: $x = 0.1063$

Kogan–Shifman Scenario!

Let R_4 be <u>arbitrary</u>...

$$\beta_{[2|2]} = \frac{-11}{4} x^2 \left[\frac{1 + a_1 x + a_2 x^2}{1 + b_1 x + b_2 x^2}\right]$$

$a_1 = 13.403 - 0.076215 R_4$

$a_2 = -22.915 + 0.090166 R_4$

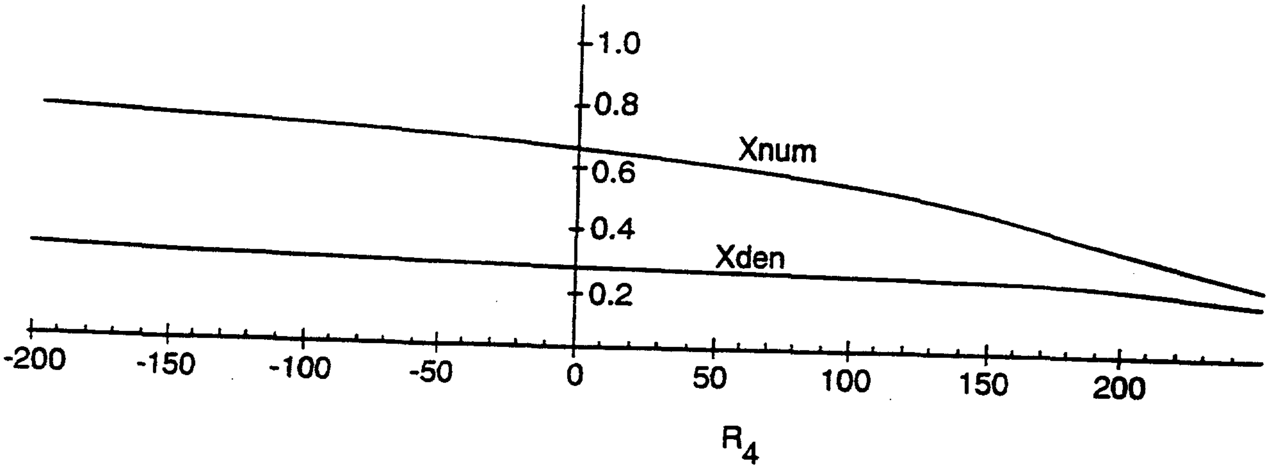
$b_1 = 11.084 - 0.076215 R_4$

$b_2 = -56.727 + 0.26685 R_4$

(11) ELIAS

$n_f = 0$ QCD

zeros of $\beta_{[2/2]}$ numerator and denominator:



For all values of R_4, the first positive zero of the denominator precedes the first positive zero of the numerator, consistent with Kogan-Shifman scenario

Topological effects in Applications

Topological Defects, Baryogenesis and CP odd Bubbles in QCD.

Ariel R. Zhitnitsky

(The University of British Columbia, Vancouver, Canada)

•1 We generalize the large N_c Di Vecchia-Veneziano-Witten effective chiral Lagrangian to the case of finite N_c.

•2 The picture of θ dependence in QCD which follows from this Lagrangian is more complicated than suggested by the large N_c approach or instanton arguments.

•3 Generically, the vacuum energy $E_{vac}(\theta)$ is a multi-valued function of θ admitting the existence of metastable states and domain walls separating these metastable vacua from the lowest energy vacuum.

•4 We discuss applications of the obtained results to the axion physics. Specifically, we argue, that in general, an arbitrary $|\theta\rangle$- state would be created in the heavy- ion collision, similarly to the creation of the disoriented chiral condensate (DCC) with an arbitrary direction. Therefore, the heavy -ion collisons give us a unique chance for a new axion search experiment.

•5 We propose a new mechanism for baryogenesis which takes place at the QCD scale and is based on the existence of domain walls. We argue that these new objects (B-shells) might be also responsible for the origin of the dark matter in the Universe. We emphasise that the suggested mechanism can be, in principle, experimentally tested at RHIC.

Relevant papers on the subject.

1) BARYOGENESIS WITH QCD DOMAIN WALLS. By R. Brandenberger, I. Halperin, A. Zhitnitsky hep-ph/9808471.

2) DOMAIN WALLS AND Θ DEPENDENCE IN QCD WITH AN EFFECTIVE LAGRANGIAN APPROACH. By Todd Fugleberg, Igor Halperin, Ariel Zhitnitsky hep-ph/9808469

3) AXION POTENTIAL, TOPOLOGICAL DEFECTS AND CP ODD BUBBLES IN QCD. By Igor Halperin, Ariel Zhitnitsky hep-ph/9807335, to appear in *Phys. Lett. B*

4) ANOMALOUS EFFECTIVE LAGRANGIAN AND Θ DEPENDENCE IN QCD AT FINITE N(C). By Igor Halperin, Ariel Zhitnitsky hep-ph/9803301, to appear in *Phys. Rev. Lett. 1998*

5) CAN $\frac{\Theta}{N}$ DEPENDENCE FOR GLUODYNAMICS BE COMPATIBLE WITH 2π PERIODICITY IN Θ? By Igor Halperin, Ariel Zhitnitsky Published in *Phys.Rev.D58:1998*, hep-ph/9711398

2. Effective Lagrangian in QCD after integrating Vh- fields

$$e^{-W(\varphi,\theta)\cdot V} = \sum_{l} e^{VE\cos\left[-\frac{q}{p}(\theta - i\log\det U) + \frac{2}{p}\right.} \quad + \frac{1}{2} V\, Tr\,(mU + mU^{+})$$

$m = m_q |\langle\bar{\psi}\psi\rangle|$ $\qquad l = 0, \ldots p-1$

$\frac{q}{p} = \frac{8}{3b} \sim \frac{1}{N_c}$, $\quad b = \frac{11}{3}N_c - \frac{2}{3}N_f$, $\quad E = \langle \frac{b\alpha_s}{32\pi} G_{\mu\nu}^2 \rangle$,

a) Lagrangian has p-different branches (p minima)

b) $\theta \to \theta + 2\pi$ is explicit

c) θ/N_c is explicit

d) all results do not depend on specific ratios m_i/m_j for $m_i \ll \Lambda_Q$

e) for large N_c, the VVW Lagrangian is reproduced!

$\cos\frac{q}{p}(\theta - i\log\det U) \to a/N_c^2\,(\theta - i\log\det U)^2$

f) There existance of p- minima implies the classification $|\theta, l\rangle$

g) $\Delta E \sim m_q \to 0$ There are domain walls

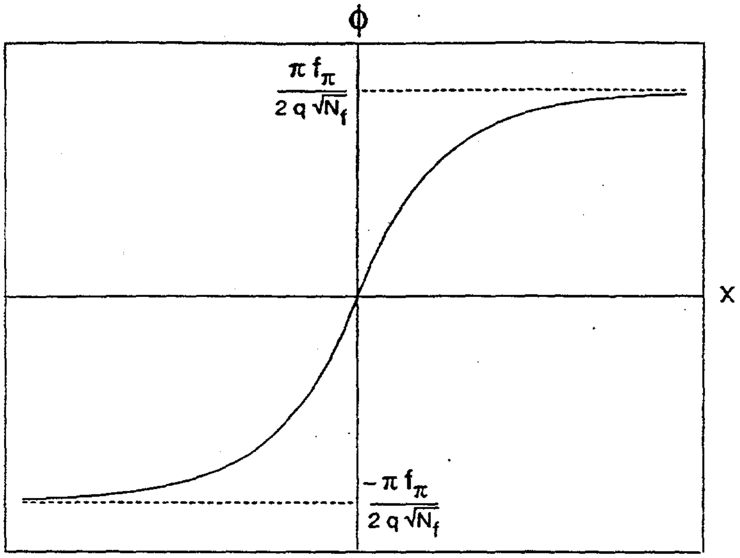


Figure 5: Domain wall profile.

$$\varphi_\pm = \frac{p f_\pi}{2q\sqrt{N_f}} \left[\pm \frac{\pi}{p} \mp 4 \arctan \left\{ \tan\left(\frac{\pi}{4p}\right) e^{\mp \mu (x - x_0)} \right. \right]$$

$$\varphi_+ = \varphi(x) \quad x > x_0$$

$$\varphi_- = \varphi(x) \quad x < x_0$$

$$\boxed{f_{\eta'}^2 m_{\eta'}^2 = \frac{8}{96} N_f < \frac{\alpha_s}{\pi} G_{\mu\nu}^2}$$

$$\mu = \frac{2q\sqrt{N_f}\sqrt{E}}{p f_\pi} \leftarrow \boxed{\text{exactly coincides with } m_{\eta'}}$$

$$\sigma \text{ (the wall surface tension)} = \frac{4\pi f_\pi}{q\sqrt{N_f}} \sqrt{< \frac{6\alpha_s}{32\pi} G_{\mu} \left(1 - \cos\frac{\pi}{2p}\right.}$$

$$\Gamma \sim \exp\left(-\frac{27\pi^2 \sigma^4}{2(\Delta E)^3}\right)$$

V Applications

(QCD scale and its role in development of the early Universe)

1. $|\theta\rangle$-state at RHIC - ?

We expect that, in general, $|\theta\rangle$-state would be created: $(E_\theta - E_0) \sim m_q$

2. Axion physics.

Axion is a particle which solves the strong CP-problem. Axion - is a dark matter candidate with $m_a \sim 10^{-5} eV$.

Axions will be produced at RHIC when $|\theta\rangle$-state is relaxing to $\theta = 0$.

3. Baryogenesis. $\frac{n_B - \bar{n}_B}{\gamma} \sim 10^{-10}$

This number is difficult to explain because it requires large CP-violation (Sakharov) which is known to be small in our world.

VII Charge separation effect.

1. $\mathcal{L} = \bar{N} i \partial\!\!\!/ N - m_N \bar{N}_L \mathcal{U} N_R - m_N \bar{N}_R \mathcal{U}^+ N_L$

Domain wall configuration $\mathcal{U} = e^{i\alpha(z)}$

$\Delta\alpha = \alpha(+\infty) - \alpha(-\infty) = -\frac{2\pi}{q}$

2. Because of the planar symmetry, the problem is reduced to the 2D-problem

$N_L = \frac{1}{2}\begin{pmatrix} X_+ \\ X_+ \end{pmatrix}$, $N_R = \frac{1}{2}\begin{pmatrix} X_- \\ -X_- \end{pmatrix}$.

$X_+ = \frac{1}{\sqrt{S}} \epsilon_{\alpha a} \chi$, $X_- = \frac{1}{\sqrt{S}} \epsilon_{\alpha a} \xi$

$$\boxed{\mathcal{L}_2 = \bar{\Psi}\left(i\partial\!\!\!/ - m_N e^{i\alpha(z)\gamma_5}\right)\Psi}, \quad \mu = 0, z,$$

$\Psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}$, $\gamma_0 = \tau_1$, $\gamma_1 = -i\tau_2$, $\gamma_5 = \tau_3$

$$B^{(4)} = \int \bar{N}\gamma_0 N \, d^3x \rightarrow \int \bar{\Psi}\gamma_0\Psi dz = B^{(2)}$$

$$B^{(4)} = B^{(2)} \cdot g \int \frac{d^2x \, d^2p}{(2\pi\hbar)^2} = B^{(2)} N$$, N - number of quantum states

$$B^{(2)} = \frac{\Delta\alpha}{2\pi} = -\frac{1}{q}$$

(Analogy with quantum Hall effect, where fractional charge is residing on the topol. soliton)

VIII Baryogenesis (qualitative estimates)

1. $Q = S T_c^2 \cdot \alpha_1$ $\alpha_1 = \frac{\sigma^{2/3}}{q\, T_c^2}\left(\frac{g \cdot g}{4\pi}\right)^{1/3} \sim 10^{-2}.$

Q - the induced fractional baryon charge residing on the domain wall area S.

2. Total area is

$$\langle S \rangle \simeq \frac{V}{\xi}$$

Kibble, 1976, ξ - correlation length
V - Hubble volume

3. After $T_d < T_c$ the domain wall network will break up into bubble walls surrounding regions of the false vacuum.

4. When $\theta \equiv 0$ a total number of B and $\bar{B}$ is equal. However, for any $\theta \to 0$ at $T < T_d$ only one kind of domains will remain: $\sim e^{-\theta \cdot \frac{\Lambda_{QCD}^4 \cdot \xi^3}{T}} \sim e^{-\theta \frac{\Lambda_{QCD} (10^7)^3}{T}} \ll 1.$

$\theta > 10^{-22}$

5. $\bar{B}$s are concentrated on the domain walls which eventually become $\bar{B}$-shells.

6. Why the QCD scale is relevant?

Let ρ - density of B-shells in the Univers

Q - their charges

M - their masses

a) $\boxed{\frac{n_B}{S} \sim \rho \frac{Q}{g^* T_D^3}}$ g^* - number of degrees of freedom; $g^* \sim 10$

(This object scales like matter $\sim T^3$, not like radiation $\sim T^4$)

b) $\boxed{\Omega_B \sim \rho \frac{M}{g^* T_D^3 T_{eq}}}$ T_{eq} - is the temperature of equal matter and radiation

$\frac{T_{eq}}{T_D} \sim \frac{T_{eq}}{T_c} \sim 10^9$ - well known number

$\frac{n_B}{n_s} \sim 10^{-10}$

We combine two equations: a) b):

$$\Longrightarrow \left(\frac{T_{eq}}{T_D}\right)\left(\frac{n_B}{S}\right)\frac{1}{\Omega_B} \simeq \frac{Q}{M} \cdot T_D \Longrightarrow \frac{M}{Q} \simeq 1\,\mathrm{GeV}$$

Moral: B-shells will redshift as matter.

Hence, they can contribute to the dark matter of the Universe.

BNL Workshop on "QCD Phase Transitions" '98

Production of baryons as topological defects in chiral symmetry restoring phase transition

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Institute of Nuclear Physcis, Cracow, Poland

Based on J. Dziarmaga, MPS, hep-ph/9809313

R. D. Pisarski and F. Wilczek, PRD**29**(1984)338
F. Wilczek, Int.J.Mod.P.**A7**(1992)3911
K. Rajagopal and F. Wilczek, NP**399**(1993)395

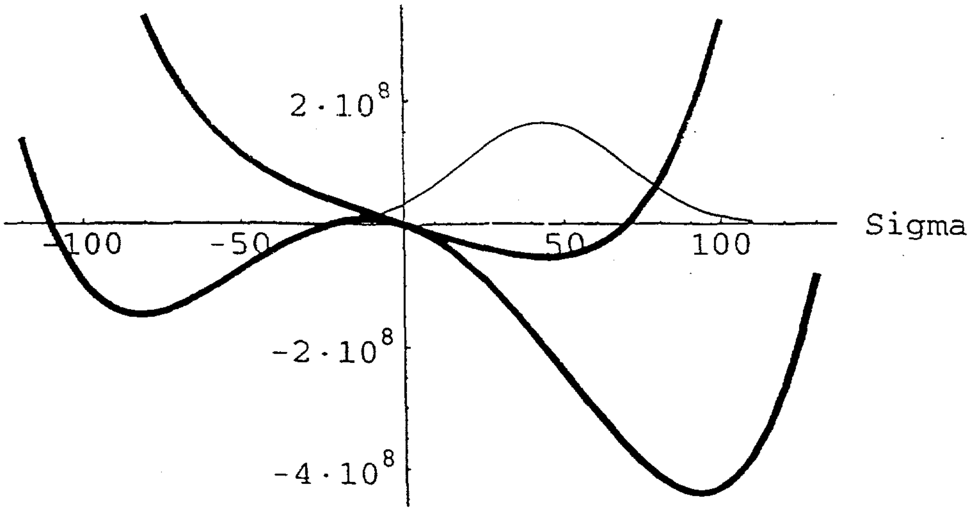
We cosider $N_f = 2$ case.

$$F = \int d^3x \left\{ \frac{1}{2}\partial^i\Phi^\alpha\partial_i\Phi_\alpha + \frac{\mu^2}{2}\Phi^\alpha\Phi_\alpha + \frac{\lambda}{4}(\Phi^\alpha\Phi_\alpha)^2 - H\sigma \right\}$$

$$\Phi^\alpha = (\sigma, \vec{\pi})$$

$$\mu^2(T_{ch}) = 0, \lambda > 0, H > 0$$

tential at T=0, 160 MeV and Fluctuations



- The QGP comes to thermal equilibrium around $\tau_I = 1$ fm.
- Partons are relevant degrees of freedom above T_{conf}
- Chiral fields are relevant degrees of freedom below T_{conf}
- $T_{conf} \approx T_{ch}$

The chiral fields enter the $T < T_{ch}$ stage in the state of thermal equilibrium.

Flactuation squared of $\bar{\sigma}$ around its avarage:

$$s^2 = \langle[\bar{\sigma} - \sigma_0(T_{ch})]^2\rangle = \int_0^{M_\sigma(T_{ch})} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{T_{ch}}{M_\sigma^2 + k^2} =$$

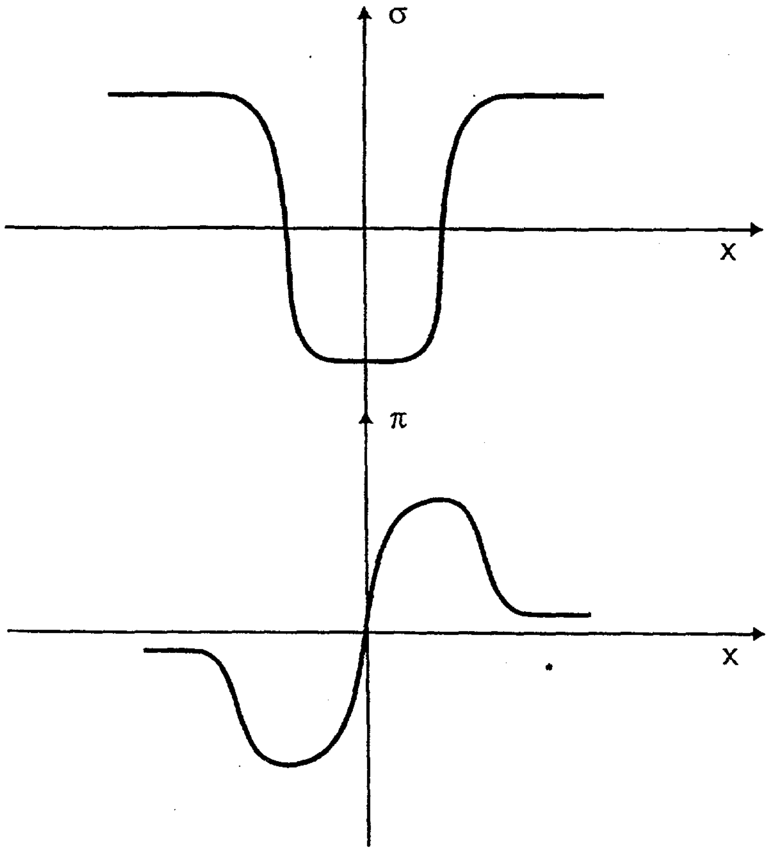
$$= \frac{T_{ch} M_\sigma(T_{ch})}{2\pi}(1 - \frac{\pi}{4})$$

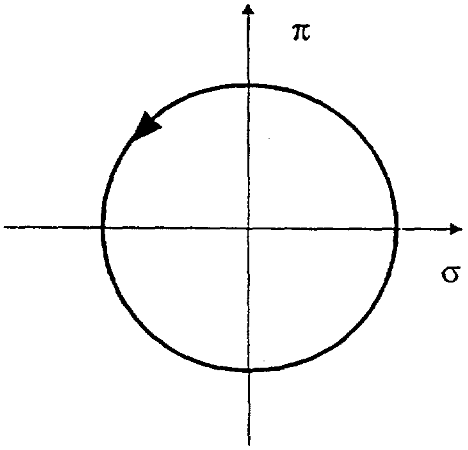
$\bar{\sigma}$ is an avarage of σ over its correlation domain

Gaussian distribution:

$$f(\bar{\sigma}) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{(\bar{\sigma} - \sigma_0(T_{ch}))^2}{2s^2}\right]$$

where $\bar{\sigma}$ is an avarage of σ over its correlation domain.





If the domain avaraged $\bar{\vec{\pi}} = 0$ and the domain avaraged $\bar{\sigma}$ is to the left of the top of the zero temperature potential the fields roll down to the bottom of the sombrero potential in the $(-1, \vec{0})$ direction.

$$N_{\bar{B}} = \frac{1}{2} P[\bar{\sigma} < \sigma_{top}] \rho[\bar{\vec{\pi}} = 0]$$

$$P[\bar{\sigma} < \sigma_{top}] = \int_{-\infty}^{\sigma_{top}} f[\bar{\sigma}; s]$$

Numerical estimation

At T=0:

$T = 0$: H=(119MeV)^3, $\lambda = 20$, $\mu^2 = \lambda(87.4\text{MeV})^2$

At $T_{ch} = 160$ MeV:

H(T_{ch})=H, $\lambda(T_{ch}) = \lambda$, $\mu^2(T_{ch}) = 0$

Correlation lengthes:

$\xi_\pi = 1$ fm and $\xi_\sigma = 0.6$ fm.

Quench approximation:

$$P[\bar{\sigma} < -11.2\text{MeV}] = \int_{-\infty}^{-11.2} f(\bar{\sigma};\mu)d\bar{\sigma} = 0.01$$

Halperin formula for density of zeros:

$$\rho[\vec{\bar{\pi}} = 0] = \frac{1}{\pi^2}\{\frac{< -\vec{\bar{\pi}}(\vec{x})\ \nabla^2\ \vec{\bar{\pi}}(\vec{x}) >}{< \vec{\bar{\pi}}(\vec{x})\ \vec{\bar{\pi}}(\vec{x}) >}\}^{3/2} \approx \frac{0.04}{\xi_\pi^3}$$

$$N_{\bar{B}} \approx 10^{-4}\frac{\text{antibaryons}}{\text{fm}^3}$$

Conclusions

- The probability of the topological defects formation is exponentialy suppresed by the explicit symmetry breaking parameter presents in the free energy describing the phase transition
- The light quarks massess break chiral symmetry. Hardly any antibaryons will be then produced in the chiral restoring phase transition.
- We expect 10^{-4} antibaryons/fm^3

Presentation is based on J. Dziarmaga, M. Sadzikowski, hep-ph/9809313

Spontaneous violation of P and CP invariances in hot QCD and its experimental signatures

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In a recent work with Rob Pisarski and Michel Tytgat, we argue that for QCD in the limit of a large number of colors, the axial U(1) symmetry of massless quarks is effectively restored at the deconfining phase transition. We find that as a consequence of this, metastable states in which parity and CP invariances are spontaneously broken can appear in the hadronic phase. In this talk, I discuss also possible manifestations of this phenomenon in future experiments at Relativistic Heavy Ion Collider.

References

[1] D. Kharzeev, R.D. Pisarski and M.H.G. Tytgat, Phys.Rev.Lett.81:512-515,1998.

[2] D. Kharzeev, R.D. Pisarski and M.H.G. Tytgat, HEP-PH 9808366.

Outline

1. Brief reminder about the $U_A(1)$ problem

mini-
(reminder about $U_A(1)$:

$\mathcal{L}_{QCD}$ is invariant under $q \rightarrow e^{i\gamma_5 \alpha} q$;
where are the parity doublets in hadron spectrum?
if broken, where is the Goldstone boson?)

2. θ - vacua

3. Large N effective Lagrangian

4. Finite temperatures:
possibility of spontaneous P, CP violation ?!

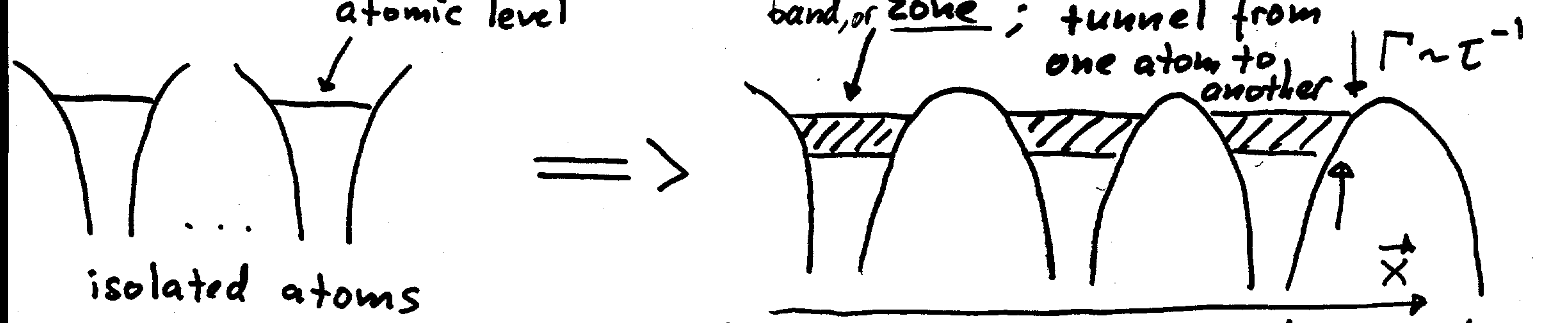
$$\Downarrow$$

5. Signatures at RHIC

2. θ - Worlds

Non-conservation of $Q_5 \Longleftrightarrow$ Existence of vacua with different "winding numbers" ν

Analogy: consider a crystal:

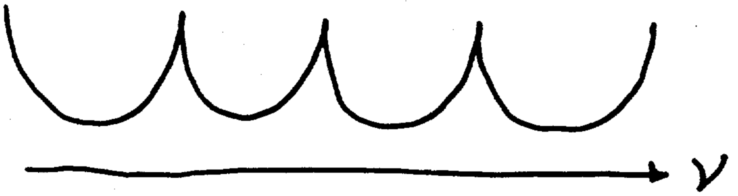


The wave function of the ground state:

$$|\vec{k}\rangle = \sum_{\vec{x}} e^{i\vec{k}\vec{x}} \, |\vec{x}\rangle$$

"quasi-momentum"

QCD:



$$|\theta\rangle = \sum_{\nu} e^{i\theta\nu} |\nu\rangle$$

To compute an observable, use

$$\langle O \rangle_{\Omega} = \frac{\sum_{\nu} e^{i\theta\nu} \int_{\nu} [d\varphi] \exp\left(i\int_{\Omega} d^4x \, \mathcal{L}\right) O(\varphi)}{\sum_{\nu} e^{i\theta\nu} \int_{\nu} [d\varphi] \exp\left(i\int_{\Omega} d^4x \, \mathcal{L}\right)}$$

$\Rightarrow$ this is equivalent to adding to the Lagrangian the term $\mathcal{L}_\theta \simeq \theta \cdot \int d^4x \, Tr(G_{\mu\nu}\tilde{G}^{\mu\nu})$

Example:

an effective Lagrangian including $U_A(1)$ term
(non-linear σ-model)

G. Veneziano,
P. di Vecchia;
E. Witten

$$\mathcal{L} = \frac{F_\pi^2}{2} \Big\{ \underbrace{\mathrm{Tr}\, \partial_\mu U \partial_\mu U^{-1}}_{U(3)\times U(3) \text{ invariant}} + \underbrace{(\mathrm{Tr}\, MU + \mathrm{Tr}\, MU^+)}_{\text{under } SU(3)\times SU(3) \text{ transforms as quark mass term}} -$$

$$- \underbrace{\frac{a}{N}\left(-i \ln \det U - \theta\right)^2}_{\text{preserves } SU(3)\times SU(3),\ \text{reflects } U_A(1) \text{ anomaly}} \Big\}$$

$$U = \exp\left(i \frac{\Phi}{F_\pi}\right)$$

$$a \sim \int d^4x \left\langle T\{ G_{\mu\nu} \tilde{G}^{\mu\nu}(x), G_{\mu\nu} \tilde{G}^{\mu\nu}(0) \} \right\rangle_{YM}$$

The angle θ is severely constrained

by D_n, $\eta \to \pi\pi$ $\quad |\theta| < 10^{-9}$

We will assume $\theta = 0$

B. Allés
M. D'Elia,
A. Di Giacomo
P.W. Stephenson
ep-lat/9808004

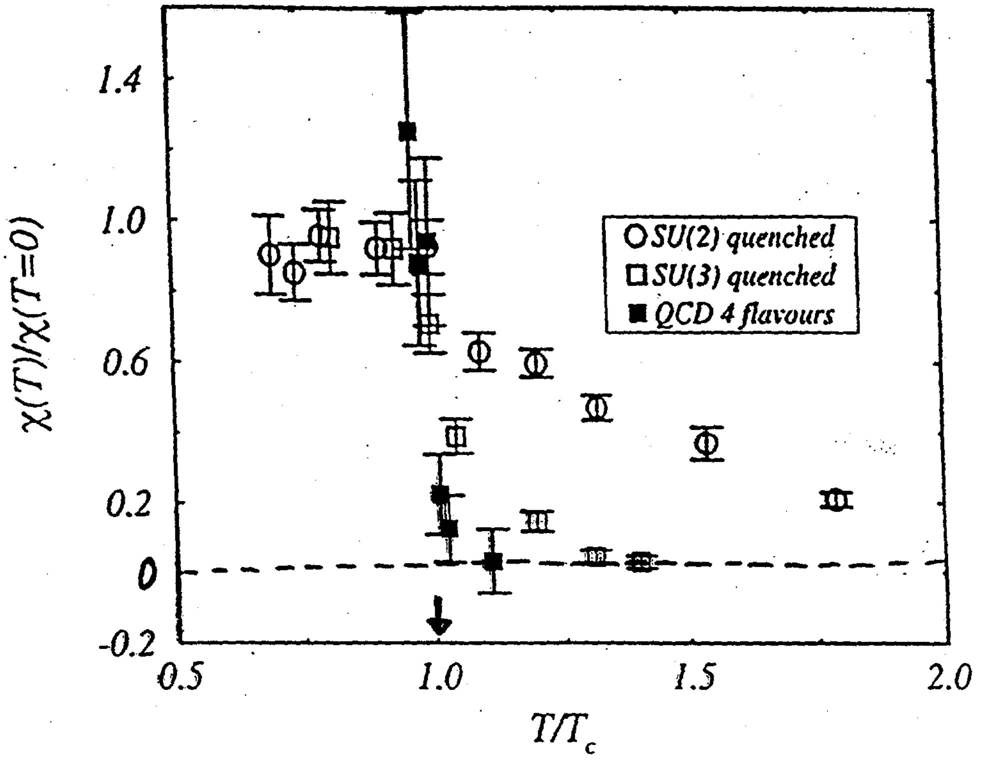


Figure 3. Behaviour of the topological susceptibility as a function of the normalized temperature T/T_c.

large N_c :

- below T_c, interactions are suppressed by $1/N_c$,
  \# of degrees of freedom $\sim N_c^0$
  $T_c \sim N_c^0$
  $\Rightarrow$ "cold" gas of glueballs and mesons
- above T_c, \# of degrees of freedom $\sim N_c^2$

$\Rightarrow$ huge change of the free energy at T_c

$\Downarrow$

any phase transition occurs at T_c

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Possibility of Spontaneous Parity Violation in Hot QCD

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(Received 3 April 1998)

We argue that for QCD in the limit of a large number of colors, the axial U(1) symmetry of massless quarks is effectively restored at the deconfining phase transition. If this transition is of second order, metastable states in which parity is spontaneously broken can appear in the hadronic phase. These metastable states have dramatic signatures, including enhanced production of η and η' mesons, which can decay through parity violating decay processes such as $\eta \to \pi^0\pi^0$, and global parity odd asymmetries for charged pions. [S0031-9007(98)06613-7]

PACS numbers: 11.10.Wx, 11.30.Er, 12.38.Mh, 12.39.Fe

Use QCD effective potential;

the simplest form

θ angle

$$U(\varphi_i) = f_\pi^2 \Big[-\sum_i \mu_i^2 \cos\varphi_i + a \Big(\sum_i \varphi_i - \theta \Big)^2 \Big]$$

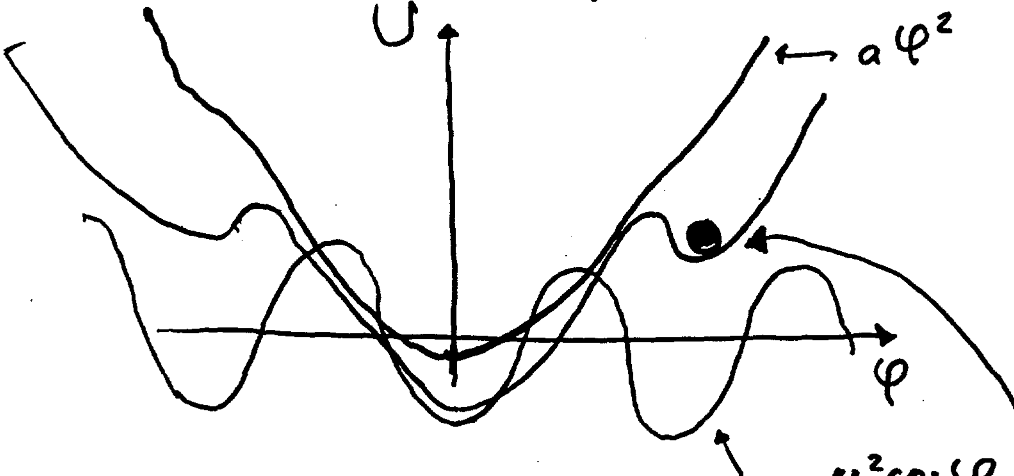
Goldstone fields

Goldstone masses

topological susceptibility
(measures the density of topological charges in QCD vacuum)

@ $T=0$ $|\theta| \lesssim 10^{-9}$ – constraints from d_n, $\eta \to \pi\pi$

a – can be deduced from $m_{\eta'}$; is <u>large</u>



But:

at least at large N_c,
$a \sim (T-T_c)^\alpha$ $T \to T_c$ $\Rightarrow$ metastable, CP-violating vacuum solution!

Signatures at RHIC

1) "False" vacua will decay with the emission of η, η' $\Rightarrow$ enhanced η, η' yields

J. Kapusta,
D.K.
L. McLerran;

Z. Huang
X.-N. Wang

How to detect?

$$\left.\begin{array}{l}\eta' \to \gamma\gamma \\ \eta \to \gamma\gamma\end{array}\right) \text{ difficult at small } p_T$$

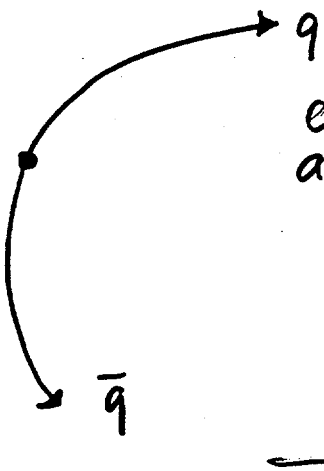
$\eta' \to \pi^+\pi^-\eta \Rightarrow$ HBT! S. Vance, T. Csörgo, D.K.

2) Parity-violating decays, e.g. $\eta \to \pi\pi$

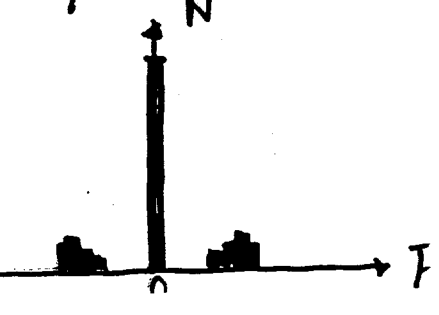
3) Global P, CP-odd observables,

$$\text{e.g.} \quad \mathcal{P} = \sum_{\pi^+\pi^-} \frac{[\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}] \cdot \vec{z}}{|\vec{p}_{\pi^+}|\,|\vec{p}_{\pi^-}|}$$

$$G\tilde{G} \sim \vec{E} \cdot \vec{H}$$



event-by-event analysis:

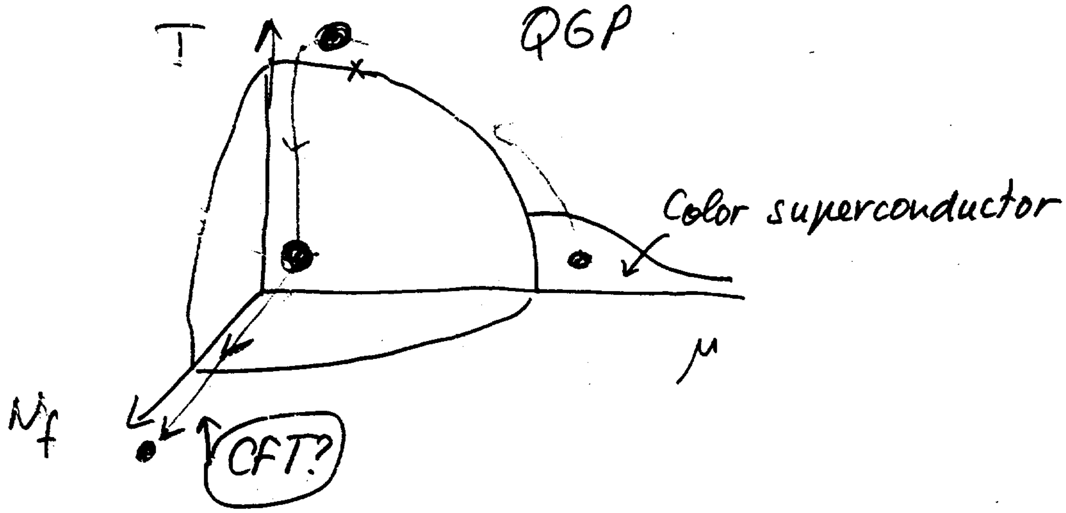


+ cosmological implications?

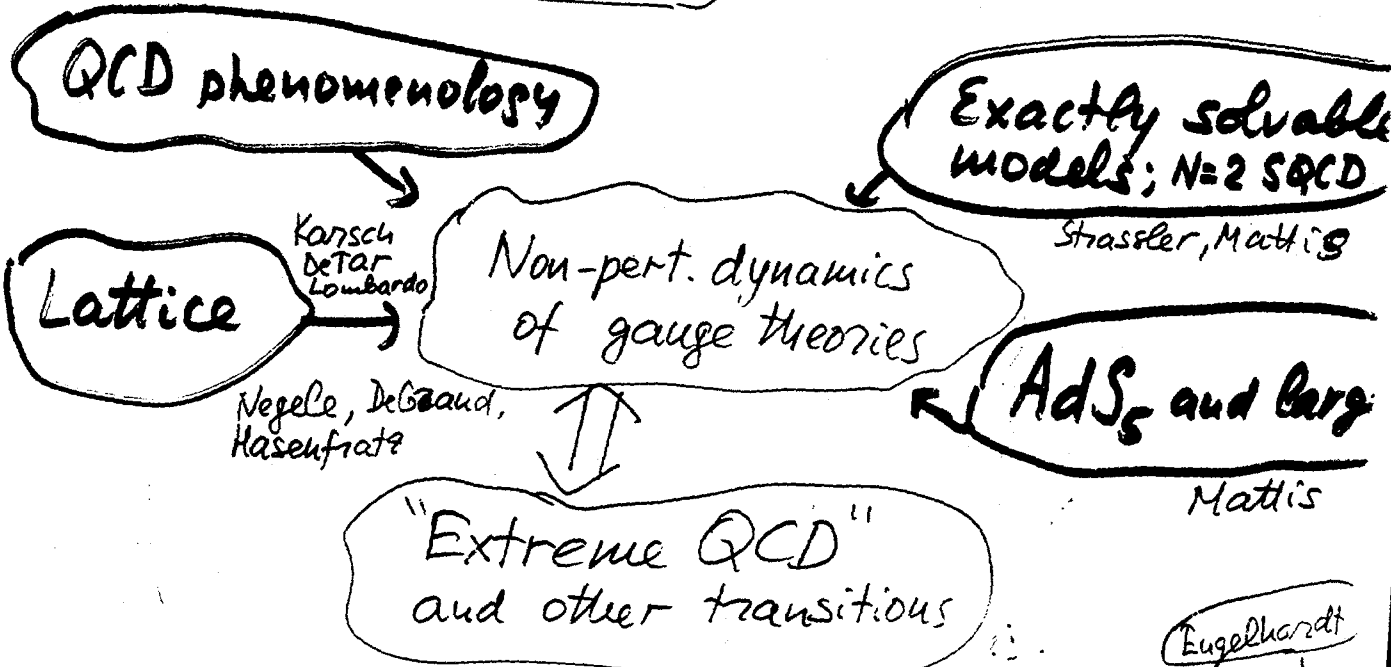
QCD Phase Transitions

Summary (E. Shuryak)

- Overview
- Large μ
- Large T
- Instantons in vacuum / phase transition
- Large N_f
- Supersymmetry



Overview



	I	Engelhardt ↓
1. Confinement/deconfinement	no	yes (?)
2. Chiral Breaking/restoration	yes	maybe ?
3. U(1) Breaking/"restoration"	yes	
4. Lightest hadrons ...	yes	no
5. Color superconductivity	yes	
6. Dynamics of SUSY QCD	yes	

How to separate ("quantum noise") from "signal" ?

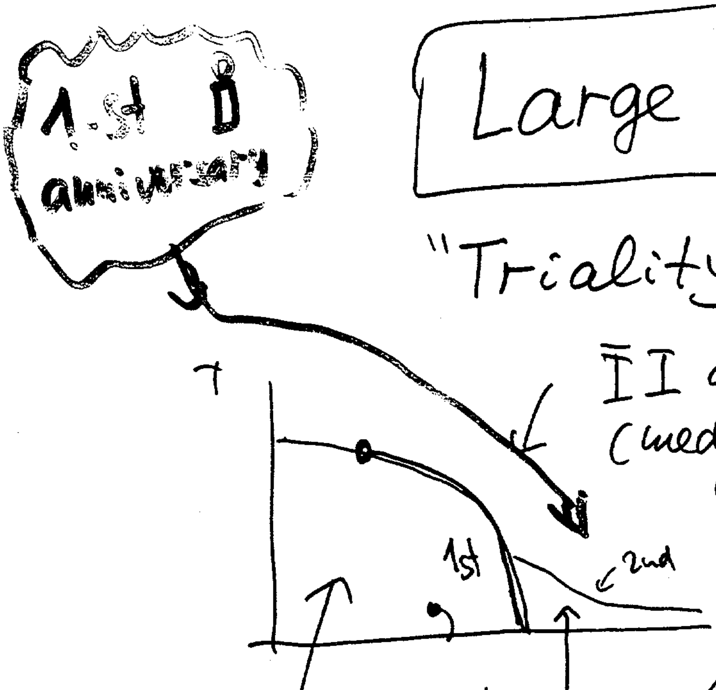
→ "Smoothen" lattice configurations ← Negele, DeGrand, Hasenf

→ go into supersymmetric world (diagrams cancel) ← Mattis

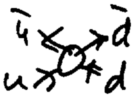
→ do saddle point for complex configurations ↑ Velkovsky

Large $\mu \leftrightarrow$ Color supercondu

"Triality"



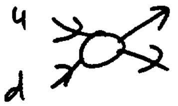
$\bar{I}I$ attraction
(mediated by quarks)
no condensates
R.Rapp



attractive
$(\bar{q}q)$ channel
(mediated by instantons)
$\Rightarrow$ condences

chiral symmetry is broken

attractive $qC\gamma_5 q$ channels
(mediated by instantons)
$\Rightarrow$ condenses



$\downarrow$

color symmetry is broken

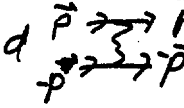
Refinements $\rightarrow$ Carter (formfactor
Schafer, Rapp

The story is much more interesting for u, d, s

Color-flavor locking
(Wilszek)

$$\langle q^i_a C\gamma_5 q^j_B\rangle = K_1 \delta^i_a \delta^j_B + K_2 \delta^i_B \delta^j_a,$$

ARW. considered
OGE



New developments $\rightarrow$ 1) instantons/molecules included
(RSSV)

2) C-f. locking + explicite chiral symmetry breaking: e.g.

3) For $m_s \neq 0$ there is continuous transition between two scenarios

---

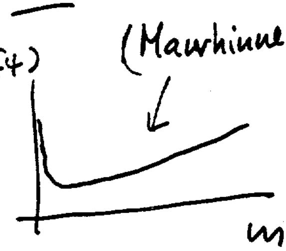
$\rightarrow$ - "nulcar" cooling problem is finally solve

Large T transitions

$\boxed{N_f=0} \longrightarrow \boxed{N_f=2} \longrightarrow \boxed{N_f=4}$

$N_f=0$

- $T_c \sim 260$ MeV 1-st order
- $\frac{T_c}{\sqrt{\sigma}} \approx \sqrt{\frac{3}{(d-2)\pi}}$ **Strings...**
- Chiral symmetry is <u>not</u> restored



(Domain wall fermions)

$N_f=2$

2nd?

$T_c \sim 150$ MeV.

$T_c \simeq \frac{1}{4\langle\rho\rangle} \ll \sqrt{\frac{3}{(d-2)\pi}} \cdot 6$

- Chiral condensate vanishes, clear change in Dirac eigenvalues → zero, or even forbidden zone develops
- Domain wall f's ⇒ small but nonzero $\chi_\pi - \chi_\delta$ above T_c U(1) "nearly" restored

$N_f=4$

T_c small?

$\langle\bar\psi\psi\rangle$ <u>is</u> small

Columbia

$\boxed{\text{Low } T}$

No$(\det \not{D}) \Rightarrow Q \neq 0$ [illegible] $O(V_4^{1/2})$

$\Rightarrow Q = N_+ - N_-$ <u>exactly zero</u> modes ← $\boxed{\text{Verbaarschot, Tener}}$

$$\rho(\lambda) = O(V_4^{1/2}) \cdot \delta(\lambda) + O(V) \log\lambda + \dots$$

$$\langle\bar\psi\psi(m)\rangle = \underset{\uparrow}{O(V^{1/2})}\underset{\uparrow}{(1/m)} + O(V) \underset{\uparrow}{\log m}$$

(unimportant V→∞) (very singular) (chiral logs are wrong)

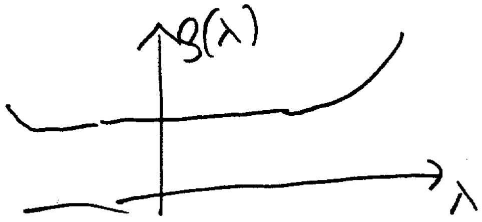
$\boxed{\text{High } T}$

Same as low T, only overlap Matrix elements not $O(\rho/R)$, but $\exp(-\pi T R)$

Too small compared to even (d.w.) "masses"

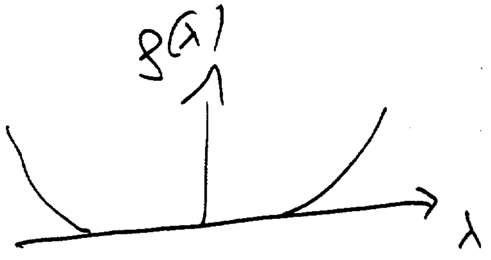
$\boxed{\sim \exp(-L}$

$\boxed{N_f = 2,\ T \simeq T_c}$



$T < T_c$

⇒



$T > T_c$

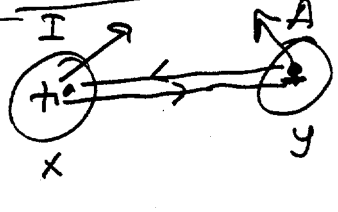
Why is the spectra changed?

"Molecules"? Ph. de Forcrand → partially only there are molecules, but the "movie"/further analysis is complic

What to do next?

- smaller m (domain wall fermions)
- more complicated clusters? $(\bar{I} I \bar{I} I)$?
- include correlation of SU(3) angles

Ilgenfritz et al analyzed $N_f = 0$ configurations



E→-E

$< G_{\mu\nu}(x) U G_{\mu\nu}(y$

$\simeq \cos\theta_{IA}$

$T < T_c$

random

$T \gtrsim T_c$

strongly correlated

So "molecules" can be formed even without quarks! (And even without ch. sym. restoration)

Phases at large N_f

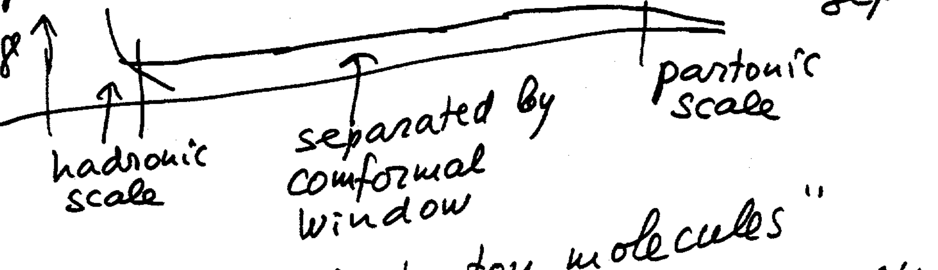
For $N_c=3$

$N_f=16.5$ → comformal ? $\langle\bar\psi\psi\rangle\neq 0$ ← $N_f=2,3,4$
confinement

Where do they meet?
(Or is there something in between?)

(Appelquist et al) at $N_f\sim 12 \to (\gamma=1)$ pert. th.
(Schafer/Shuryak) at $N_f\sim 5 \to$ instanton liquid disappears
(Iwasaki et al) at $N_f=7 \to$ lattice (with problems)

Sannino: (approaching from the right, $\langle\bar\psi\psi\rangle\to 0$)



Velkovsky: for $N_f>6$ "instanton molecules"
lead to oscillating vacuum energy
even $N_f \to 0$; odd $\to$ + − + − etc

Instantons in vacuum

- ⊛ For $(N_f=0)$ the top. sus. $\chi_t = \frac{\langle Q^2\rangle}{V_4}$ is finally stable, and dominated by instantons
- ⊛ $\langle \bar\psi\psi\rangle = \pi\rho(0)$ is also dominated by zero modes related to instantons
- ⊛ π correlator → Negele et al Very good
  → DeGrand et al Very bad ?
  lattice spacing? KS vs W?

Lattice artifacts (e.g. Wilson term $O(a^2/\rho^2)$) work <u>against collectivization of zero modes</u> (but help to locate / count instantons via $\psi_\lambda(x)$ without cooling → A. Hasenfratz)

- ⊛ <u>Size distribution</u> $dn/d\rho$ is strongly distorted by smoothening. Even for ideal action (preserving single I), there is a general growth of size with "time".
  (extrapolate back? $\psi_\lambda(x)$?)
- ⊛ "Instanton density" cannot be uniquely defined because of close $\bar{I}I$ pairs

"molecule"

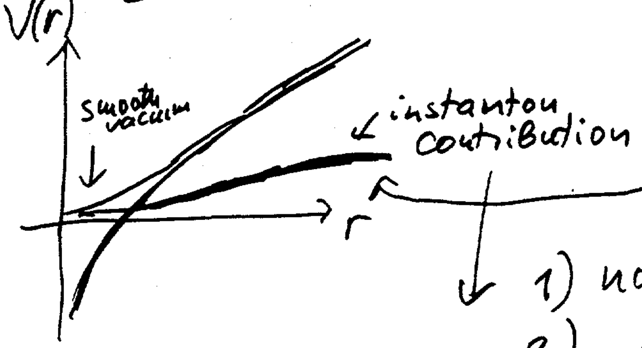
"flucton"

→ no boundary to pert. th

↑ can be still defined, and even treated semiclassically (action ~ S_0)

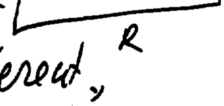
→ One should study the whole "$\bar{I}I$ valley" → config. are <u>known</u>. (Yung, Verbaar.)

Instantons and Confinement

* 

DeGrand
Negele, Chen, E.S.
Diakonov Petrov

1) not confining
2) ~ 10% of what observed

[Side comment: if one measure in t  and t 
Wilson loops results are very different,
Only the second is right → But it was never well done for R ~ 1fm on the latti

* Even if large-ρ tail would make large $V(r)$,
it ① would not produce small-size strings
② would only be seen in few config. → contrary to observation

* Phenomenological argument:
$\pi, \rho, N \ldots$ and $J/\psi, \Upsilon \ldots$ <u>"do not speak to each other"</u>

Example 1

$\rho' \to \rho\pi\pi$
$\Delta m \sim 700$ MeV
$\pi\pi$ show 0^+ peak σ

$\psi' \to \psi\pi\pi$ $\quad \frac{\Gamma_{\psi'\to\psi\pi\pi}}{\Gamma_{\rho'\to\rho\pi\pi}} \sim 10^{-3}$!!
Δm same
invar. mass peaked at the largest $m_{\pi\pi}$, resonance σ absent, seen another one $m \sim 1.3$ or 1.6 GeV (?) (Glueball?)

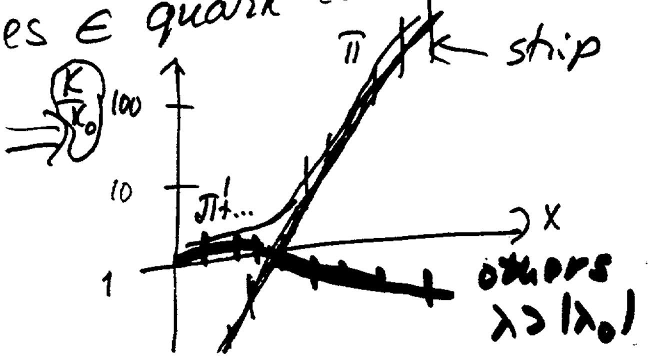
Example 2 Lepage gets good results for ψ/Υ spectra (no instantons!)

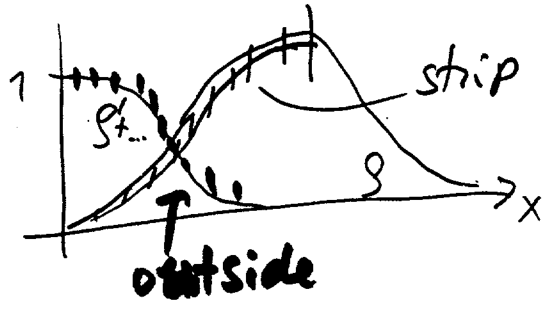
Instantons and Hadrons

"Lowest" (or collective)	"Others"	"Heavy"
π (138)	π' (~1300) etc	ψ...
η (547)	η_{next} (1440)	Υ...
ρ (770)	ρ' (1450)	Coulomb + string explain even $\vec{S}\vec{S}$, $\vec{S}\cdot\vec{L}$ ...
ω (770)	ω' (1420)	
η' (958)	η^{*}_{next} (1400)	
σ (~600)	σ' (or ε) (1300)	? (nearly) independent on instantons
100% instanton-dependent	~ 10% instanton dependent	

Ivanenko, Negele → let us take narrow $\delta|\lambda|$ strip λ_0 ("fermionic states ∈ quark condensate")

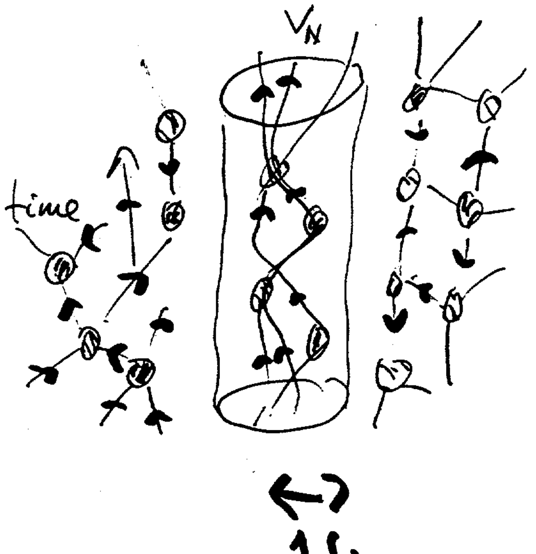
$$= \Big(\underbrace{\sum_{\lambda\in strip} \frac{1}{i\lambda+m}}_{\sim 100\ terms} + \underbrace{\sum_{outside} \frac{1}{i\lambda+m}}_{\sim 10^4\ terms}\Big)^2$$





How one can reconcile

non-suppression of $\langle G^2 \rangle$ with **complete suppression** of $\langle \bar q q \rangle$?



The density of instantons is the same,

but all instantons are busy making valence quarks massive

Pauli blocking (at zero-mode level) prevent sea quarks inside

Compare: MIT bag model (Donoghue - Nappi PL 168B (86) 105)

$\langle \bar u u \rangle_{sea} = \langle \bar d d \rangle_{sea} = \langle \bar s s \rangle_{sea}$

while lattice give the first 2 $\langle N | \bar u u + \bar d d | N \rangle \sim \boxed{9}$

while $\langle N | \bar s s | N \rangle \simeq \boxed{1 \div 1.5}$

→ s quarks are not blocked

(does the instanton density/distribution somehow changes!) Trento!

N=2 SUSY gauge theory

$$\begin{tikzcd} & \lambda^a_\alpha \arrow[dr] & \\ G^a_{\mu\nu} \arrow[ur] \arrow[dr] & & \varphi^a \\ & \psi^a_\alpha \arrow[ur] & \end{tikzcd}$$

(two Majorana fermions)

has multiple non-equivalent vacua, numerated by $u =$ [redacted] $= \frac{1}{2} T_R \langle \varphi^2 \rangle$ or a - Higgs VEV

Seiberg and Witten (94) have solved it, they have found <u>explicitly</u> "prepotential" $\mathcal{F}(\phi)$

$$\mathcal{F}''(a) = \tau(a) = \frac{4\pi i}{g^2(a)} + \frac{\theta}{2\pi}$$ "running coupling"

$\mathcal{F}'''(a)$ is "anomalous magnetic moment" $\mathcal{F}'''(\lambda\, \sigma_{\mu\nu}\, \psi)$

$\mathcal{F}''''(a)$ is 't Hooft vertex strength $\mathcal{F}''' \psi^2 \lambda^2$

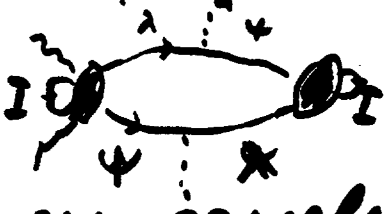
How the result looks like in the <u>weak coupling</u> $(a \gg \Lambda)$?

$$\frac{8\pi^2}{g^2(a)} = \underset{\substack{\uparrow \\ \boxed{\text{one loop } \beta \text{ function}}}}{4 \log\left(\frac{2a^2}{\Lambda^2}\right)} - 12\left(\frac{\Lambda}{a}\right)^4 - (3\cdot 4\cdot 5\cdot 7)\left(\frac{\Lambda}{a}\right)^8 + \underset{\uparrow}{\ldots}$$

① <u>Only</u> $\left(\frac{\Lambda}{a}\right)^{4n}$ appear → instantons (!!!

② These two have been directly calculated

Finnel + Pouliot 95



Dorey, Kh
Mattis

③ The instanton sum cancels the

R. Rattazzi, L. Randall, E.S., → in progres
hep-ph/98032

Effective charges

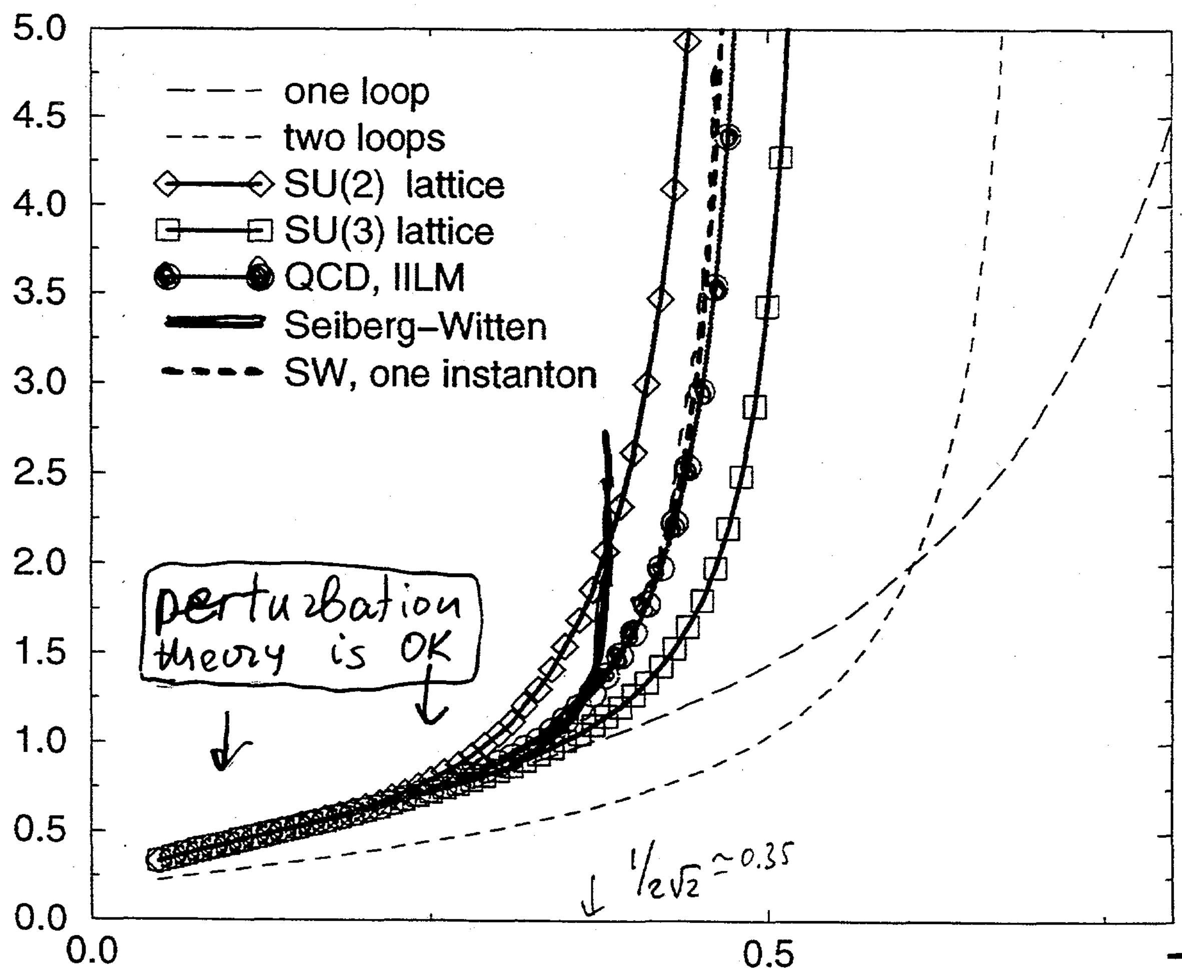
in **QCD** (a la Callan, Dashen, Gross
vs **N=2 SUSY QCD** (a la Witten - Seiberg)

integrate <d
up to instant
size $a = \rho_{ma}$
$\rho < a = \rho_{m}$

very similar picture!

$$b = \frac{11}{3}N_c - \frac{2}{3}N_f \quad QCD$$
$$b = 4 \quad N=2 \text{ SUSY QCD}$$

$$\frac{b g_{eff}^2}{8\pi^2}$$



perturbation theory is OK

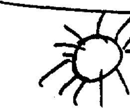
$1/2\sqrt{2} \simeq 0.35$

scale : a [fm]
$\frac{\Lambda}{2u}$ } both blow up perturbative at 1

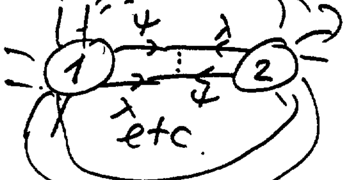
$u = \frac{1}{2}\langle\varphi^2\rangle$
Higgs VEV

Instantons and SUSY

$\boxed{\mathcal{N}=4}$ Mattis Conformal theory $\beta(g)=0$
no scale (no Λ), 4 fermions adjoint + scalars

1) Each instanton has  "16 legs"

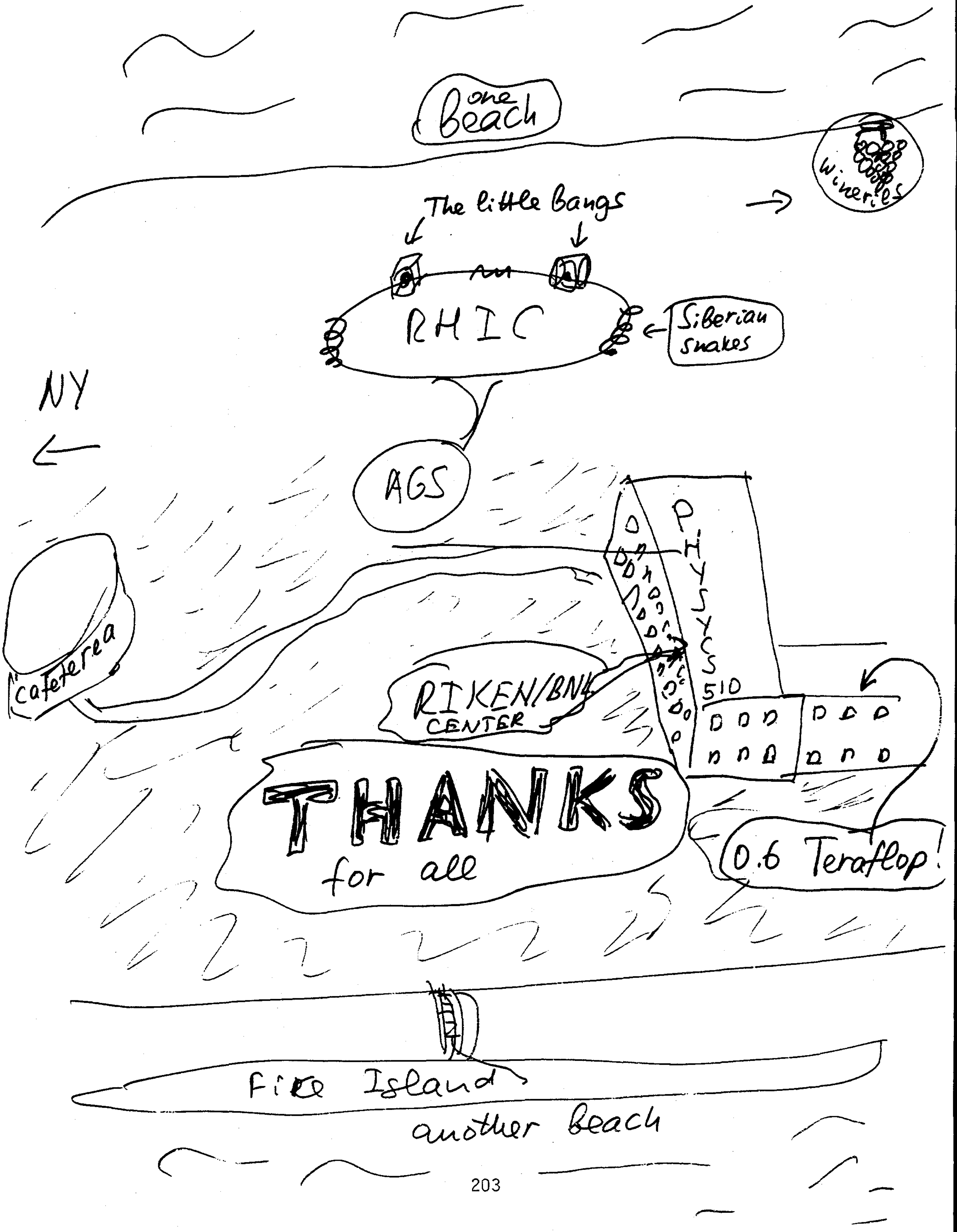
2) Can be coupled to each other (even without $\bar{I}$)



3) In the large N_c limit (and finite # of inst.'s) the "molecule" is simply made of K "atoms" all at the same place/size

$$\sum_K \left[e^{-\frac{8\pi^2}{g^2}} \underbrace{\left(\frac{d^4x\, d\rho}{\rho^5}\right)}_{AdS_5} \underbrace{\left(\underset{\text{diquarks}}{d\overset{3}{p}\; d\overset{2}{q}}\right)}_{S_5} \underbrace{S(x,\rho)\ldots S\ldots}_{\text{legs to outside points } x_1 \ldots x_n} \right]^{N_c}$$

De-mystification of Maldacen
conjecture



one Beach
wineries
The little Bangs
RHIC
Siberian snakes
NY
AGS
PHYSYCS
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RIKEN/BNL CENTER
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SUMMARY OF THE WORKSHOP "THE QCD PHASE TRANSITIONS" Brookhaven National Laboratory, November 4-7 1998

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1 Overview

As it was already mentioned in the Introduction, the subject of the meeting was non-perturbative dynamics of gauge theories, manifesting itself in form of various "phases" such theories have.

For QCD one of the main source of "input" remains experimental data about hadrons. The second, now nearly as important as the first, is provided by numerical lattice simulations. Those can also consider various flavor contents, change the quark masses, easily access finite temperatures (finite density remains so far a problem). Furthermore, they can study observables not in average, but on configuration-by-configuration basis, and reveal more details about a dynamics. The third major input is provided by exactly solvable (or partially solvable) models, mostly the Super-symmetric (SUSY) ones.

Let me on the onset indicate some similarity between various approaches discussed on the workshop. Many (if not most) of the talks in this way or another separate "quantum noise" (the perturbative phenomena) from "smooth" or even classical fields, related to non-perturbative dynamics. The tools used for this general aim are however very different: (i) Blocking lattice configurations, or "cooling" them; (ii) Considering super-symmetric theories in which many diagrams cancel; (iii) Considering large N_c limit, in which there should be some "master field" dominating the path integrals (Mattis again); (iv) going to complex-valued configurations, which are some non-trivial saddle points (Velkovsky).

But whatever the tools, the classical configurations themselves revealed in those analysis happened to be nothing else but our old friend, the *instanton*. Their ensemble saturates the topological susceptibility, solving the U(1) problem[1]. They also do saturate the lowest Dirac eigenmodes, explaining chiral symmetry breaking (again quantitatively, producing accurate value for the quark condensate) and even hadronic correlators, see recent review [1]. I will argue below that instantons explain also the origin of the famous "chiral scale" 1 GeV in QCD [5]. Furthermore, recently instantons emerged as the main driving force in Color Superconductivity.

Instantons also provide few exact results for SUSY theories. They reproduce expansion of the Seiberg-Witten "elliptic curve" for N=2 SUSY QCD [4], and also provide the "master field" of the N=4 theory [6], as discussed here by Mattis.

However many properties of the instanton ensemble are far from being clear. The major example (discussed especially by de Forcrand) is complicated behavior near the critical temperature T_c: qualitative changes in their ensemble are obvious but the structure above T_c is not yet understood.

The only exceptional non-perturbative phenomenon which instantons do *not* explain is confinement [7, 8]: this issue was discussed by Negele.

2 High density QCD

The field of high density QCD was mostly dormant since late-70's-early 80's, when implications of perturbative QCD for this case was worked out. However realization last year (simultaneously by "Stony Brook" and "Princeton" groups [17, 18]) that instantons can induced not only strong pairing of quarks with anti-quark in vacuum and break chiral symmetry, but also a quark-quark pairing at high density, has created a splash of activity. Such

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[1] Not "in principle" (which 't Hooft did back in 1976), but for real, quantitatively reproduces the value needed to explain correct η' mass.

Color Super-Conducting (CSC) phase was under very intense discussion at the workshop.

It was introduced in the first review talk by F.Wilczek (Princeton), who emphasized the so called color-flavor locking phase [19] which appears for three massless quarks ($N_f = 3$). Discussion of its rather unusual qualitative features was continued by T.Schafer (Princeton), who has presented some quantitative results [20] following from account for instanton interaction. One important result was a demonstration that, as one increase the mass of the strange quark and goes back to the $N_f = 2$ theory, no phase transitions actually happens and interpolation between two different structures of CSC is in fact continuous. Another interesting issue, for $N_f = 3$ case, is whether there can in principle be a continuous transition from hadronic to CSC phase. Schaefer and Wilczek [22] suggested that the answer is positive.

G.Carter (Copenhagen) had further discussed the $N_f = 2$ case in the instanton model in some details [21], including correct instanton-induced formfactors. R.Rapp (Stony Brook) have provided another view on this subject [20], using statistical rather than mean field description of the instanton ensemble, and discussing the role of instanton-anti-instanton molecules in this transition.

After the workshop an interesting paper written by Son [23] have shown that in the high density (weak coupling) limit (when the instantons are Debye-screened) the leading behavior is not provided by electric (Coulomb) part of the one-gluon exchange, but by a magnetic one.

The talks have so many details that I would not go into it. In summary, QCD demonstrate a kind of "triality". There are three major phases of QCD: (i) hadronic, dominated by $\bar{q}q$ attraction leading to chiral symmetry breaking; (ii) CSC at high density, dominated by qq attraction and condensation, and (iii) QGP at high T, in which there are no condensates but instantons and anti-instantons themselves are bound by a fermion-induced forces.

A complementary approach to high density QCD, now based on random matrix model, was reviewed by M.Stephanov (Stony Brook). He outlined what exactly goes wrong in "quenched" QCD at finite density, and also how the correct behavior of the Dirac eigenvalue at increasing μ should look like: the resulting picture resembles "a dividing chromosome", rather than a "cloud" coming from quenched theory. He also pointed out the existence of the tri-critical point at the phase diagram of the random matrix model [24], as well as importance and even possible ways to search for it in heavy ion collisions [25].

Various ideas of how one can proceed to study the high density on the lattice were also discussed. At the end of the talk F.Karsch described new approach, with finite baryon density (instead of chemical potential). M.Alford (MIT) has described possible analytic continuation to complex chemical potential.

Finally M-P.Lombardo (Gran Sasso) had presented very interesting data for 2-color QCD. In this theory the determinant is real even with chemical potential, and so the usual lattice calculations are possible. The results are consistent with CSC phase being developed.

3 High temperature QCD

Lattice results on finite temperature transitions were reviewed by F.Karsch (Bielefeld) and also by C.DeTar (U. of Utah) . Excellent data for pure gauge theories exist by now, and they show transition at $T_c \approx 260MeV$. The ratio to the string tension $T_c/\sigma^{1/2}$ is close to $(3/(d-2)\pi)^{1/2}$ as predicted by the string model of deconfinement. M.Wingate (RIKEN/BNL) has presented new data for deconfinement in 4-color gauge theory, which also support this trend.

However, as it is well known by now, QCD with light quarks show *much smaller* critical temperature T_c. This suggests that it has nothing to do with deconfinement, as it is described by the string model.

For 2 light quarks ($N_f = 2$) $T_c \approx 150MeV$ and is driven by chiral symmetry restoration. The order of the transition in the $N_f = 2$ theory is second, as expected, but "current analysis did not reproduced the expected critical behavior for a system in the universality class of O(4)-symmetric spin models", Karsch concluded. The situation remains to be quite confusing, the current set of indices do not fit into any of the established universality classes. Maybe the issue is complicated by "approximate restoration of the U(1) symmetry" [32] which add 4 more light (although still massive) modes. If so, the transition may be driven to weak first order instead. DeTar have also shown how lattice artifacts present for $N_t = 4$ and creating doubts about relevance of this case for continuous limit, are actually dissolves for larger values of N_t (up to 12) studied.

DeTar also mentioned interesting simulations by Kogut et al [33] who found weak first order in a simulation in which on top of standard lattice action a small 4-fermion term was added. Let me comment on it: Kogut et al have considered this interaction as a pure methodical tool, they did not specified or speculated about its possible structure. I have however made a point that in fact there is the natural reason why such small interaction should exist: there are small-size ($\rho \sim a$) instantons which "fall through the lattice". Their contribution should therefore be explicitly added, as another operator into the lattice action.

For the $N_f = 4$ theory, discussed by Mawhinney, the condensate is so small that the critical temperature is not even measured yet. It however supports a prediction of the instanton liquid model [1] that instanton-induced chiral symmetry breaking should be small at $N_f = 4$ and gone by $N_f = 5$, even at $T = 0$.

The central part of the talk by R.Mawhinney (Columbia) was first results on chiral restoration phase transition using new "domain wall" lattice fermions [34]. The first result is that in this case the chiral symmetry is very accurate[2], and so one can clearly recognize some zero modes of instantons.

In particularly, he discussed also an old question: *what happens in the quenched (pure gauge) theory above T_c?*. Without a determinant, there is no reason for the instantons to be strongly correlated, and if they are more or less random the chiral symmetry should *not* be restored. That contradicted to earlier lattice data, who concluded that chiral symmetry is restored above the deconfinement transition.

One well-understood issue arise here, which may affect recent (not so large-volume) simulations. The total topological charge of the configuration with randomly placed instantons is Q=$|N_+ - N_-| \sim \sqrt{N_+ + N_-}$. Therefore spectrum of the Dirac eigenmodes of quenched configurations should have a term

$$\frac{dN}{d\lambda} = \delta(\lambda) * O(V_4^{1/2})$$

where V_4 is the 4-volume. According to Banks-Casher formula $dN/d\lambda(0) = \pi| < \bar{q}q > |/V_4$, but this density does not lead to infinite condensate because it drops out in the thermodynamical limit.

New Columbia data shown by Mawhinney are consistent with this interpretation for $T < T_c$, but *above* T_c the comparison for few volumes available

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[2] It is broken only by an exponentially small tails of the fermionic wave functions, bound to "plus" and "minus" walls.

suggested that the coefficient was actually $O(V)$, and the contribution to the condensate therefore is there. He concluded that $< \bar{q}q >$ is in fact *infinite* above T_c, not zero as people have claimed before. This is in sharp contrast to earlier works: the measured condensate has changed from 0 to ∞!

This result can probably be resolved as follows[3]. At high T the overlap matrix elements between instantons are qualitatively different: instead of decreasing with distance as R^{-3} (as at T=0), there appear exponential suppression $exp(-\pi Tr)$ for spatial distance r. Therefore, the whole zone of instanton-related modes shrinks and it looks as $O(V)\delta(\lambda)$ if the quark mass is not small compared to its width.

True shape of the the zone based on weakly overlapping instantons and anti-instantons[4] was discussed by Verbaarschot Stony Brook). His result [36] (recently also confirmed by M.Teper et al[37]) is that in quenched QCD the eigenvalue density actually does grow indefinitely at the origin, but as $dN/d\lambda = O(V) log \lambda$.

What this means for Columbia results is that for sufficiently small masses (or large length in the 5-th dimension) the singularity in the condensate is going to change from 1/m to log(m). The same behavior should also be there at low T as well, so the quenched theory always has an infinite condensate.

I.Zahed (Stony Brook) has discussed new ideas [35] about "chiral disorder", connecting motion of light quarks in the QCD vacuum to that of electrons in "dirty metals". He also proposed two potentially possible regimes for chiral restoration (i) fractal support for the chiral condensate; (ii) either some intermediate phase or specific places on the phase diagram where finite $< \bar{q}q >$ (density of eigenvalues) coexist with zero $F_\pi = 0$ (no conductivity) due to eigenmodes localization.

J.Verbaarschot (Stony Brook) have discussed a number of topics about the Dirac eigenvalues. The main point was that zero-momentum sector reduces to Chiral Random Matrix Theory, but it deviates from it at larger eigenvalues [38] He disagreed with Zahed on his last point, arguing (following Parisi) that the localized modes are independent and therefore the fermionic determinant should be a product of the eigenvalues. It strongly mis-favored by any unquenched theory due to smallness of the fermionic determinant,

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[3] This comment was made in the discussion by T.Schaefer.

[4] It is better to consider the case when their number is exactly the same, Q=0, so that there are no exactly zero topological modes.

and so he concluded localization scenario is not viable.

M.Engelhardt (Tubingen) have argued that the deconfinement in pure gauge theory can be described de to vortex percolation, rather than monopoles.

4 Lattice instantons at zero and non-zero T

The issue was reviewed by J.Negele (MIT), see [13]. He shown that topological susceptibility is stabilized in many simulations, and the value (dominated by instantons) agrees well with Witten-Veneziano formula. The measurements of the size, defined by extrapolation to the uncooled vacuum, give $\rho = .39 \pm 0.05$ fm. This number, as well as the shape of the size distribution, agrees well with the phenomenology and the instanton liquid calculations. For finite T the size decreases by about 25% by $T = 1.3T_c$, and shrinks at higher T, also in good agreement with the Debye screening mechanism [15, 16].

Negele has shown that most of the smallest fermionic zero modes are related to instantons, both in quenched and full simulations. The important conclusion is that the quark condensate is definitely completely dominated by instantons. Furthermore, restricting the quark propagator to contribution of the lowest modes only, one actually reproduces the correlation functions, not only for such "collective mode" as pions but also for other channels, in particularly ρ. Again, this is in agreement which we have found previously by doing correlators in the instanton liquid models.

Another issue Negele discussed based on [7] was the role of instantons in the heavy quark potential and confinement. The conclusion is that the "instanton liquid" does not confine, and contribute to heavy quark potential at the 10-20 % level. The potential found agrees well with other numerical calculations done before, and with analytical one due to Diakonov and Petrov.

There are however three extra points which can be made in connection to this issue. One is that we have found during this investigation that the potential is sensitive to the shape of the Wilson loop, and only if its time dimension T is much larger than spatial one L one gets a correct potential. Diakonov and Petrov recently wrote a rather provocative paper[8], arguing that all existing lattice measurements of the confinement at distances above 1 fm are actually from loops with $L >> T$, and are therefore suspicious. Unfortunately, simple statistical argument shows that it is practically impossible to go to large enough L in a correct way.

The second point is related with another idea, suggested by Diakonov et al [40], namely that a tail of the distribution at the large-size side may decrease as $dN/d\rho \sim \rho^{-3}$ and lead to infinite confining potential. I think it cannot work, or rather in any way explain what we know about confinement from the lattice. One basic reason is that it would not generate small-size strings, and also generate long-range gluonic correlators. The other is that huge configuration-per-configuration fluctuations of the string tension would be the case, again contrary to to observations.

My third comment is a phenomenological observation, which is by no means new but I think reveal something profoundly important. It is found that quarkonia made of heavy quarks (c,b) and related to confining (and Coulomb) potential have surprisingly small interactions with light quark hadrons. Examples are numerous, let me give one only. Compare two decays with the same quantum numbers of the participants and about the same released energy. $\rho' \rightarrow \rho\pi\pi$ and $\psi' \rightarrow \psi\pi\pi$. The ratio of widths is about a factor 1000! Where this huge factor come from? Only from very different nature of light-quark hadrons (collective excitations of the quark condensate, in a way, as Negele demonstrated) and quarkonia, bound by the confining strings. Why this interaction is *so* small remains unknown.

T.DeGrand and A.Hasenfratz (Boulder) have presented different aspects of their extensive studies of lattice instantons using improved actions [29].

DeGrand reached conclusions similar to Negele's about instantons dominating the smallest eigenvalues, but has shown that instantons alone lead to bad results for the correlators, even the pion one. The difference should be due to different lattice fermions (KS in his work, Wilson in Negele's): in the debate to follow I made a point that in KS case lattice artifacts forbid "collectivisation" of eigenmodes (leading to a scenario similar to what was advocated by Zahed).

A.Hasenfratz (Boulder) described the current status of their work aimed to used "perfect lattice actions" to revealed the true soft content of the quantum configurations. Impressive results for topological observables such as instanton size distribution were presented. The instanton sizes were shown to drift upward, presumably due to mutual attraction, and so the "extrapolation back" seem like a good idea. She had also demonstrated that maybe the best way to "hunt for instantons" is not via very noisy gauge fields, but from lowest fermionic eigenmodes.

One issue discussed in connection to this talks was related to what we actually mean by " total" instanton density. It is clear that as it is done it depends on particular program recognizing instantons. Closed $\bar{I}I$ pairs (or "fluctons" as I have called them in studies of tunneling in quantum mechanics [30]) can only be separated from perturbative fluctuations by some *ad hoc* condition, since there is no real difference between the two. Still, let me point out, to a large extent such pairs can still be well described by semi-classical fields: only instead of the classical fields (minima of the action) we should look at the "streamline" configurations. Their shapes (and references to the previous works) can be found in [39]: those can well be used for "flucton recognition".

In summary: the instanton-antiinstanton pairs form the famous valley of Q=0 configurations, going smoothly to zero field one. Its population in the vacuum may and can be studied, especially in connection to the long-pending question about understanding of "non-perturbative" aspects of high-order perturbative terms. However, those close pairs do not provide the main object of the instanton physics, the lowest Dirac eigenmodes, and so they would be simply ignored by any fermionic algorithms (like the one discussed by Hasenfratz).

Ph.de Forcrand (Zurich) had also described his version of the "improved cooling" as a way to look for the instantons. He has also observed good agreement between Banks-Casher relation used for the instanton eigenmodes, and the value of the quark condensate. The main topic of his talk however is related with a puzzling question, *what happens at* $T > T_c$ for QCD with dynamical quarks?

The proposal by Ilgenfritz and myself [26] was that the ensemble of instantons is broken into so called instanton-anti-instanton molecules. This idea has worked well in the instanton liquid model simulations, see review [1].

However, de Forcrand et al results [27] neither disprove nor completely supported this scenario. On the *pro* side, de Forcrand had demonstrated us that all configuration there have Q=0, and that the Dirac eigenvalue spectrum even develops something like a forbidden gap. Many of the smallest eigenmodes do indeed display two maxima in space-time, corresponding to instanton and anti-instanton. There is also some support to our prediction that the molecules should be predominantly oriented in time direction. However, on the *con* side, as seen from de Forcrand's movie displaying instantons at different T, pure inspection of the action does not provide any clear identification of the $\bar{I}I$ pairs or other clusters in this ensemble. Therefore a change in the spectrum remains a mystery.

In connection to this issue, let me recall recent work by Ilgenfritz and Thurner [28]. Although for quenched configurations only, they have developed a way to correlate relative color orientations of instanton and anti-instanton. They have measured distribution of the following quantity[5]

$$\text{"}cos\theta\text{"} = \frac{< G_{mu\nu}(z_I) U G_{mu\nu}(z_{\bar{I}}) U^+ >}{|G_{mu\nu}(z_I)||G_{mu\nu}(z_{\bar{I}})|}$$

where U is transport between centers $z_I, z_{\bar{I}}$. The surprising result is that the distribution is very different at low T and $T > T_c$: the former correspond to random distribution, with cosθ peaked around 0, while in the latter case it is peaked at 1 and -1. It probably means, that *even in quenched theory without the determinant* there is some formation of the "molecules".

Let me summarize the somewhat puzzling situation once again: de Forcrand et al have found only marginal support for the molecular scenario in *full* theory (where it was predicted), while Ilgenfritz and Thurner seem to find them in *quenched* theory (where we did not expected to find them). New simulations, with smaller quark masses (or better, with domain wall fermions) and new way of analysis are needed to clarify it.

5 QCD at larger number of flavors

This is one more direction of the QCD phase diagram, in which we expect chiral symmetry restoration. As it is well known, right below the line at which asymptotic freedom disappears ($N_f = 11 * N_c/2$) the new phase must be a conformal theory because the beta function crosses zero and therefore the theory has an infrared fixed point. We do not however know till what N_f this phase exists, and whether its disappearance and the appearance appearance of the usual hadronic phase (with confinement and chiral symmetry breaking) is actually the same line, or some intermediate phase may also exist in between.

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[5] In fact in order it to be non=zero, it is also necessary to flip sign of the electric component in one of the fields.

F.Sannino (Yale)[6] has started this discussion. Based on the gap equation with the one-gluon exchange, Appelquist and collaborators [41] have argued that it should happen close to the line $N_f = 4N_c$, or 12 flavors in SU(3). Another idea suggested by Appelquist et al is the so called "thermodynamical inequality", according to which the number of massless hadronic degrees of freedom $N(T = 0)$ can never be larger than the number of fundamental degrees of freedom $N(T = \infty)$. The corresponding numbers at temperature T are defined as

$$N = -F(T) * (90/\pi^2 T^4)$$

If the saturation of it, $N(0) = N(\infty)$, indicates the boundary of hadronic world[7], one can compare the number of pions $N_\pi = (N_f^2 - 1)$ to the number of gluons and quarks (taken with the coefficient 7/8) and get the same boundary as above.

One may compare these ideas to the boundary found by Seiberg based on his duality considerations and 't Hooft matching anomaly conditions. According to those, the lower boundary of the conformal phase in N=1 SUSY QCD[8] is at $N_f = (3/2)N_c$. The "thermodynamical inequality" of Appelquist remarkably reproduces it!

However (as pointed out by Appelquist et al themselves) he one gluon exchange gap equation actually indicate a *different* point, and, even more important, a completely different pattern of massless particles. The gap equation leads to quark and gluino chiral condensation, but the Seiberg phase has a different set of massless hadrons which are *not* Goldstones, related to chiral symmetry breaking. It probably means that this approach is too naive. Len me made a suggestion here: one can also get gap equations for the channels favored by Seiberg and see if those can make massless hadrons instead.

As we already mentioned in the section about finite T transition, the instantons can restore chiral symmetry by breaking the random liquid into finite clusters, e.g. $\bar{I}I$ molecules. With increasing N_f this is also happens: it is easy to see if one consider any fermionic line between them as a kind of

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[6] He partially presenting his own talk and also substituted T.Appelquist who got ill right before the talk

[7] Although I do not understand the reasoning here, sorry. It may somehow be related to 't Hooft matching anomaly conditions, but I was not able to work it out.

[8] Of course, the ordinary and SUSY QCD have different multiplets and beta functions, so we do not mean compare the numbers literally.

additional chemical binding bond. At some critical number of those, the entropy of the random phase is no longer able to compensate for binding energy. Explicit simulations suggest it to be at $N_f = 5$, above which the instanton-induced chiral symmetry breaking disappears. This number agrees with a rapid change of the condensate value between $N_f = 3$ and 4 (Mawhinney) and it is also much closer to lattice indications (Iwasaki et al) to the critical point at $N_f = 7$. On the other hand, formation of instanton molecules by no means prevents chiral symmetry breaking by a gluon exchange or any other mechanism (confinement?), and so strictly speaking there is no direct contradiction between two approaches. One may have a strong decrease in a condensate, but not to zero at such $N_f = 5 - 7$.

M.Velkovsky (BNL) discussed a calculation [31] of the vacuum energy density due to such $\bar{I}I$ molecules. He concluded that for $N_f > 6$ there is a difference between even and odd N_f: while for the former the contribution vanishes, for the later it oscillate, changing the sign. It may lead to different (or even alternating) phases at some intermediate N_f.

A very interesting question discussed by Sannino [42](see also [43]) was a question about behavior near the conformal phase boundary. He emphasized that the transition should be infinite order, with not just few but *all* hadronic masses going to zero (see also [43]).

One particular pair of the correlators was discussed by Sannino in particular: those are of two vector and axial correlators. In QCD they are related to rho and a1 excitations, with their parameters approximately related to each other by two Weinberg sum rules[9] should look like. He has shown that as one becomes close to the transition in question, there appear three separate momenta scales: (i) "partonic" one, $p > \Lambda$, (ii) "hadronic" one $p < | < \bar{q}q > |^{1/3}$, and (iii) conformal window in between. The contribution of the part (iii) to Weinberg sum rule, if non-zero, may deform the "hadronic" theory compared to the usual QCD.

V.Elias (U. of Western Ontario) using Pade-summation for beta function, in SUSY and non-SUSy theories E.Gardi (l'Ecole Polytechnique) to penetrate to the boundary of the conformal window, and how far in N_f/N_c can the perturbative theory can actually be used. He concluded that for low N_f such as zero Pade approximant show no indications fro infrared fixed point. He

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[9] Those have zero r.h.s. because QCD have no operators of dimension 2, and also because in the chiral limit the operator of the dimension 4, $G^2_{\mu\nu}$, cancels in the difference.

also discussed Kogan-Shifman scenario which appears due to a pole (rather than zero) in the beta function.

E.Gardi also considered the boundary of the conformal window, both in the ordinary and SUSY QCD. He emphasized that bottom of the window correspond to $\gamma = 0$. He concluded in particularly that QCD remains weakly couple in the whole window, which excluded dual description. In SUSY QCD, on the other hand, does become strongly coupled inside the window.

There was a discussion on how exactly people should look for this transition on the lattice. As the transition itself is of "infinite order" because the scale of chiral symmetry breaking is going to the infrared, it should look like rapid decrease of the condensate, with unusual extrapolation to zero. The demonstration of the "conformal window" is much however more straightforward, as it amounts to finding power-like correlators. One more way to see it is to study scaling and construct lattice beta function: it should vanish in the conformal window. In principle, it should converge to the same behavior in the infrared no matter what is the initial charge in the lattice Lagrangian. In reality, the closer it is to the fixed point the better.

6 Some lessons from Supersymmetric Theories

On the onset, let me emphasize one general point. SUSY theories are not a separate class of gauge theories, but rather a particular points on the phase diagrams. One can always enlarge this theories breaking the supersymmetry (e.g. consider the same fundamental fields but different coupling constants). Therefore all features which are not directly caused by SUSY should be true in general. Our general aim is to understand those general dynamical features, to the extent known results in SUSY points can help.

M.Mattis (Los Alamos) had reviewed the status of the instanton calculus for the super-symmetric theories. For N=2 SUSY QCD ("Seiberg-Witten theory") it agrees with expansion of the elliptic curves if $N_f < 2N_c$ but not for the case $N_f = 2N_c$.

Let me inject here a discussion of the amusing similarity between QCD and (its relative) the N=2 SUSY QCD have been recently demonstrated in [5]. It is related to the issue of already mentioned "chiral scale" 1 GeV. In

QCD it is phenomenologically known that this scale is not only the upper bound of effective theory but also the lower bound on parton model description. However, one cannot really see it from the perturbative logs: 1 GeV is several times larger than their natural scale, $\Lambda_{QCD} \sim 200MeV$. In the N=2 SUSY QCD the answer is known: effective theory at small a (known also as "magnetic" formulation) is separated from perturbative region of large a by a singularity, at which monopoles become massless and also the effective charge blows up. How it happens also follows from Seiberg-Witten solution, see Fig.2. Basically the perturbative log becomes cancelled by instanton effects, long before the charge blows up due to "Landau pole" at $p \sim \Lambda$. It happens "suddenly" because instanton terms have strong dependence on a: therefore perturbative analysis seems good nearly till this point.

For comparison, in QCD we have calculated effective charge with the instanton correction, as defined by Callan-Dashen-Gross expression. All we did was to put into it the present-day knowledge of the instanton density. The resulting curve is astonishingly similar to the one-instanton one in N=2 SUSY QCD. Note, that in this case as well, the "suddenly appearing" instanton effect blows up the charge, making perturbation theory inapplicable, and producing massless pions, the QCD "magnetic" objects. Moreover, it even happens at about the same place! (Which is probably a coincidence.)

The behavior is shown in Fig.1, where we have included both a curve which shows the full coupling (thick solid line), as well as a curve which illustrates only the one-instanton correction (thick dashed one). Because we will want to compare the running of the coupling in different theories, we have plotted $bg^2/8\pi^2$ (b=4 in this case is the one-loop coefficient of the beta function) and measure all quantities in units of Λ, so that the one-loop charge blows out at 1. The meaning of the scale can therefore be determined by what enters in the logarithm.

The title of Mattis talk is actually "The Physicist's proof of the Maldacena conjecture". In essence, this work [6] is a semi-classical calculation of some specific Green functions in $N = 4$ super-symmetric gauge theory[10], in the large number of colors limit. The multi-instanton "molecules" in this limit becomes dominated by a configuration in which all instantons are at the same place z and have the same size ρ: there is enough space in color space

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[10] E.g. in N=4 theory considered by Mattis all logs are gone and beta function is just zero.

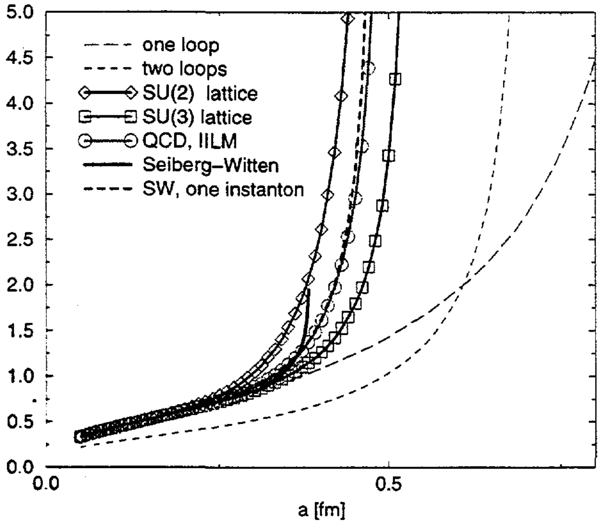

Figure 1: The effective charge $b\, g^2_{eff}(\mu)/8\pi^2$ (b is the coefficient of the one-loop beta function) versus normalization scale μ (in units of its value at which the one-loop charge blows up). The thick solid line correspond to exact solution [3] for the N=2 SUSY YM, the thick dashed line shows the one-instanton correction. Lines with symbols (as indicated on figure) stand for N=0 QCD-like theories, SU(2) and SU(3) pure gauge ones and QCD itself. Thin long-dashed and short-dashed lines are one and two-loop results.

not to worry about their overlap. So, instanton is the "master field" of this approach. The answer obtained is in perfect agreement with Maldacena conjecture and IIB SUGRA calculation, since it looks like classical Green function in which all field propagate from the origination points $x_1 ... x_n$ to a point in the AdS_5 space, which is nothing but[11] $d^4 z d\rho/\rho^5$. Additional S_5 also appears, but as a non-trivial space of diquark "condensates" created by such molecules.

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[11] Let me recall that when I found it, I had a feeling similar to the famous Mollier character, who just discovered that in all his previous life what he was saying and writing was "prose".

7 Topological effects in Applications

There were other workshops around (including two October RIVEN workshops and November one in Nordita) dealing with QGP and the phase transition as studied in heavy ion collisions. For that reason we only included in our workshop those talks which have significant overlap with other discussions, such as topology[12] and/or CP violating phases in the θ direction.

A.Zhitnitsky (Vancouver) had literally shocked the audience by his bold proposal that the baryon asymmetry of the Universe is *not* due to baryon number violation but rather a large scale *baryon charge separation* in the cosmological QCD transition [44]. He also proposed that all anti-quarks are get locked in the surface of what he calls B-shell, now making the dark matter. The reason it is locked is similar to domain wall fermions: it is a topological bound state resulting from different vacua inside and outside the ball. The sign of the charge is always the same, he explained, because the vacuum inside has a particular CP phase. This meta-stable vacuum related to the (so far rather murky) subject of "other brunches" of QCD vacua as a function of θ parameter.

This development is at its early stage, and it is not possible to tell if it can survive. In a very lovely discussion to follow, several critical comments were made. One of them I made ~~are~~ was related to safety issues related to fall on by one of those shells. According to some estimates presented, the baryon charge of the ball is about $B \sim 10^{20}$, or a mass of the order of a gram. If its energy is released in annihilation with matter, it is about an atomic bomb. However Zhitnitsky argued that because the B-shells are large bubbles of another vacuum, the probability of the annihilation should be small.

M.Sadzikowski [45] (Cracow) has demonstrated that earlier estimates of multiple production of baryons and anti-baryons in hadronic and nuclear collisions as a topological defects in chiral models was actually too optimistic. Including realistic quark masses and fluctuations in the same model significantly reduce the rate. His prediction for the rate is about 10^{-4} anti-baryons/fm^3.

D.Kharzeev (BNL) addressed the issue of the non-trivial vacuum bubbles with effectively different θ and CP violation [46]. Unlike Zhitnitsky, however, he discussed heavy ion collisions, not cosmology. He argued that high-degree

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[12] Not directly related to instantons, which are discussed in other sections.

of U(1) restoration may make it possible, although in small vicinity of T_c. The estimates of what the probability of such bubble production are very uncertain. However some ideas how one should look for it were discussed.

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WORKSHOP ON QCD PHASE TRANSITIONS
NOVEMBER 4-7, 1998
ORGANIZERS: THOMAS SCHAEFER AND EDWARD SHURYAK

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RIKEN BNL Research Center

Workshop on QCD Phase Transitions
Nov. 4–7, 1998
Physics Department - Large Seminar Room and Rm. 2-160
Organizers: T. Schäfer and E. Shuryak

AGENDA

Wednesday 4 November **Finite Temperature and Baryon Density**

Morning - Large Seminar Room

8:30		Registration - Lounge
9:00	TBA	Welcome Address
9:05	F. Wilczek	*Color-Flavor Locking vs Classic Color Superconductivity*
9:45	F. Karsch	*Results on Deconfinement and Chiral Symmetry Restoration from Lattice QCD*
10:30		Coffee Break
11:00	M. Stephanov	*Phase Diagram of QCD and Random Matrices*
11:45	T. Schäfer	*Extreme QCD in the Instanton Model*
12:30		Lunch

Afternoon - Room 2-160

14:00	M. Lombardo	*Phase Diagram of Two Color QCD*
14:30	G. Carter	*Symmetry Breaking by Instantons at Finite Density*
15:00	R. Rapp	*Chiral Restoration at Finite Density: Instanton vs. Cooper Pairs*
15:30		Coffee Break
16:00	M. Alford	*Imaginary Chemical Potential as a Tool for Lattice QCD*
16:20	TBA	Discussion, Additional Short Contributions (1.2hr)

Thursday 5 November **Instantons and Lattice QCD**

Morning - Large Seminar Room

9:00	J. Negele	*Recent Insight from the Lattice into Instantons at Zero and Nonzero Temperature*
9:45	P. DeForcrand	*Topological and Chiral Properties of QCD from Lattice Studies*
10:30		Coffee Break
11:00	T. DeGrand	*Distribution of Instantons in Lattice QCD*
11:45	A. Hasenfratz	*Fermion-Instanton Interaction in QCD: Lattice Studies*
12:30		Lunch

Afternoon - Room 2-160

14:00	R. Mawhinney	*Domain Wall Fermion Thermodynamics*
14:30	C. DeTar	*Critical Behavior at the High Temperature QCD Phase Transition*
15:00	I. Zahed	*Chiral Disorder in QCD and Phase Transitions*
15:30		Coffee Break
16:00	M. Wingate	*SU(4) Yang-Mills Theory: Phase Transition and String Tension* (30')
16:30	M. Englehardt	*Deconfinement in Yang-Mills Theory as a Vortex Percolation Transition*
16:50	TBA	Discussion, Additional Short Contributions (1.2hr)
19:00		Dinner - Berkner Hall

Friday 6 November

SUSY, Large N_f

Morning - Room 2-160

9:00	M. Strassler	*More on Phases in Supersymmetric Theories*
9:45	M. Mattis	*Instantons in SUSY Theories*
10:30		Coffee Break
11:00	T. Applequist	*The QCD Conformal Phase Transition*
11:45	F. Sannino	*From Super-QCD to QCD*
12:10	M. Velkovsky	*Vacuum Energy in Large N_f QCD and Instanton Molecules*
12:30		Lunch

Afternoon - Room 2-160

14:00	J. Verbaarschot	*QCD Phase Transitions and the Spectrum of the Dirac Operat*
14:30	E. Gardi	*The Conformal Window in QCD and SQCD*
15:00	V. Elias	*Pade Summation Representation of the MS Beta Function*
15:30		Coffee Break
16:00	TBA	Discussion, Additional Short Contributions (1.5hr)

Saturday 7 November

Theta Vacua and Other Topological Phenomena

Morning - Room 2-160

8:30		Breakfast
9:00	A. Zhitnitsky	*Topological Defects, Baryogenesis and CP-odd Bubbles in QCD*
9:45	M. Sadzikowski	*Production of Baryons as Topological Defects in Chiral Symmetry Restoring Phase Transition*
10:15		Coffee Break
10:45	D. Kharzeev	*Bubbles of Parity Odd Vacua?*
11:15		Lunch and Discussion
12:00	E. Shuryak	*Closing Remarks*

Afternoon - Excursion

Evening - Dinner