UCRL-JRNL-217364

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November 29, 2005

Physics of Plasmas

Disclaimer

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Abstract

The Translation-Confinement-Sustainment facility has been operated in the "translation-formation" mode in which a plasma is ejected at high-speed from a θ-pinch-like source into a confinement chamber where it settles into a field-reversed-configuration state. Measurements of the poloidal and toroidal field have been the basis of modeling to infer the safety factor. It is found that the edge safety factor exceeds two, and that there is strong forward magnetic shear. The high-q arises because the large elongation compensates for the modest ratio of toroidal-to-poloidal field in the plasma. This is the first known instance of a very high-β plasma with a safety factor greater than unity. Two-fluid modeling of the measurements also indicate several other significant features: a broad "transition layer" at the plasma boundary with probable line-tying effects, complex high-speed flows, and the appearance of a two-fluid minimum-energy state in the plasma core. All these features may contribute to both the stability and good confinement of the plasma.

PACS numbers: 52.30.Ex, 52.55.Lf

I. Introduction

Field-reversed configurations (FRC) are a subclass of Compact Toroid (CT). By definition, CTs have no toroidal field coils, and no center column. In some experiments a thin center column has been added, but it does not carry an axial current (i.e. no toroidal field coil). FRCs have a relatively low toroidal field and consequently very high β (ratio of plasma to magnetic pressure). Many FRC experiments are highly elongated.

Compact toroids have been on interest to fusion research for at least thirty years primarily because of their obvious engineering advantages, in particular simpler magnetics (one set of coils), and simpler geometry (no center column). However, physics of CTs is less understood than in other toroidal configurations such as tokamaks. Throughout the history of CT research, the most prominent physics issue has been global stability. This paper attempts to show that at least one FRC experiment has exhibited properties that bode well for global and local stability, both in present experiments and in larger, hotter fusion relevant plasmas.

The key question addressed here is–does *low* toroidal field in an FRC mean *negligible* toroidal field. Measurements from the Translation-Confinement-Sustainment (TCS) experiment at the University of Washington operated in the "translation" mode, form the basis for addressing this question.[Ref. 1] The measurements show that the equilibrium FRC formed in TCS has a non-zero toroidal field with a peak value of about 1/3 the poloidal field at the separatrix. This is low enough that the FRC still has high-β ~ 90 percent (engineering β based on the external magnetic field). However, despite the relative smallness of the toroidal field, the safety factor exceeds unity over much of the plasma, and especially so at the edge. This surprising fact results from the large elongation of the FRC formed in the TCS-translation experiments. Thus, surprisingly, this FRC has both very-high β and a safety factor greater than unity. It is believed that this is the first demonstration of both high-q and very high-β in the same plasma.

The outline of the paper is as follows. Section II introduces the TCS-translation experiment and presents observations relevant to the safety factor. Section III explores the safety factor in the context of CTs and describes the models used to interpret the magnetic field measurements. Section IV discusses the stability implications of high-q in FRC plasmas. Section V briefly presents other surprising conclusions about the FRC in TCS using theoretical modeling to interpret the observations. Section VI concludes the paper with a summary.

II. TCS-translation experiment

The TCS experiment has been operated so as for form FRCs (?) by two different methods. The method described here is the *TCS-translation* technique to be described shortly. Alternately the *TCS-RMF* experiments use rotating-magnetic-field (RMF) current drive to form the FRC, as described elsewhere (see e.g. Ref. 2). Figure 1 is a schematic of the TCS facility. The translation-formation sequence is (as?) follows [Ref. 1]. An initial bias magnetic field is set up in the source section (LSX/mod). Then the field is rapidly reversed, creating an elongated plasmoid in the source. Unequal magnetic fields at the two ends of the source eject the plasmoid at high speed (~200 km/s) into and through the "transition" section and thence into the "TCS" confinement section. After successive reflections from magnetic mirrors at each end of this chamber, the plasmoid settles into a quiescent FRC equilibrium state. The time from the rapid field reversal to the equilibrium state is about 100 μs, during which the plasmoid travels more than four meters from source section to the confinement section.

Table I shows typical experimental parameters for equilibrium FRCs produced in this way. The average "engineering" β is about 90%. Note that the elongation is $E \approx 4.5$. This is the *length-to-diameter* ratio of the separatrix. It should be distinguished from the elongation κ used in toroidal configurations, which is the *length-to-width* ratio of particular magnetic surfaces. In typical CTs, the $\kappa \approx 2E$ for surfaces near the magnetic axis. The energy diffusivity of $\chi_E \sim 8\ \mathrm{m^2/s}$ is remarkably low considering that the major radius is only about 7 cm. [Is this consistent with R=16 cm quoted in Table 1?]

The principal diagnostics applied on TCS are as follows. An internal magnetic probe array is inserted at the midplane of the confinement section. These measure B_x and B_z vs x. Examples considered for modeling have well-centered FRC so that B_x is interpreted as the toroidal magnetic field (B_θ in cylindrical coordinates) and x as r (cylindrical radius). An interferometer path on the diameter measures the line-integrated density $\int ndl$ at the midplane. A spectroscopic array measures $\langle u_x \rangle$ vs y, which can be unfolded to infer the toroidal flow velocity u_θ of a C^{++} impurity line. Finally, an excluded-flux array arranged along the confinement section maps the axial (z) structure of the separatrix.

The time history of three quantities from the midplane measurements is shown in Fig. 2: the poloidal flux ϕ_p; the external ("vertical") field B_e; and the integrated density $\int ndl$. The figure also indicates the time scales involved. The plasmoid first arrives at the midplane at $t \approx 30$ μs, after which extreme dynamical oscillations appear on the time traces. By $t \approx 100$ μs the plasma has settled into an equilibrium state. The modeling of the equilibrium which follows in Sec. III is applied to conditions at $t = 140$ μs, by which time the FRC is well into its slow decaying phase. For reference, the radial Alfven time is about 3 μs, and the axial Alfven time (given the large elongation) is about 13 μs. The multiple curves on Fig. 2 illustrate good shot-to-shot repeatability. Observe that when the internal magnetic probes are inserted (discharge #6371), the plasma decays slightly faster.

The radial structure of the magnetic field components from the internal probe measurements at the midplane are shown in Fig. 3. Again, the multiple curves illustrate the shot-to-shot repeatability. The poloidal field profiles are quite conventional for an FRC. The toroidal field is clearly nonzero with a peak value ~ 1/3 the poloidal field at the separatrix. The toroidal field goes to zero at the edge (as it should for a CT); observe that the edge has an inboard "surface" at the geometric axis $r = 0$, and an outboard surface at $r_s \approx 22$ cm (midplane). The toroidal field profile shows one surprising feature: the peak field occurs at quite small radius $r \sim r_s/3$. This contrasts with the normal position in static equilibria where the peak is near $0.6r_s$. This feature will prove significant shortly.

The axial structure of the FRC is shown in Fig. 4. This is the r vs z shape of the separatrix as inferred from the excluded flux array data (shown with error bars). Also shown is a fitting curve $r(z) = (z/z_s)^{2N}$, where $z = z_s$ is the x-point location. This is a least squares fit to the data points excluding the last two (largest z) where the measurement is affected by the finite width of the "divertor" plasma beyond the x-point. The data fit gives $N = 1.03$, very close to an elliptical ($N = 1$) separatrix shape. Note that in a CT, the x-points appear directly on the geometric axis, $r = 0$ and as such are a true points. By contrast toroidal plasmas with divertors have off-axis x-points, which are thus actually "x-rings" of finite radius.

III. Modelling to infer safety factor from TCS observations

A. Safety factor in a CT

Attention now shifts to assessing the significance of the nonzero toroidal field in TCS. Its measured value is still small enough that the plasma retains its very high-β. The question is whether this modest toroidal field is large enough to be important in other ways. In particular, does it give the FRC a significant safety factor value, defined as

$$q(\psi) = \frac{1}{2\pi}\oint_{\psi}\frac{B_\theta d\ell}{B_p r} \qquad (1)$$

Here cylindrical coordinates (r,θ,z) are employed so that r is the radius from the geometric axis and B_θ is the toroidal field. The line integral is around a closed poloidal surface $\psi = const$. The magnitude of q and its profile has a strong effect on both global kink and local instability modes.

A taxonomy of fusion concepts can be defined in terms of the q-profile. At the high end is a typical *tokamak* with minimum $q(0)$ at the magnetic axis near unity, rising toward an edge value q_{edge} of several. (Since $B_p \to 0$ at x-points, q is logarithmically singular at the separatrix; the usual quoted values is q_{95}, the value at $\psi = 0.05\psi_{max}$, observing the convention that $\psi \to 0$ at the separatrix.) Next, coming down in q is the spheromak with typical $q(0) \sim 0.6$ falling to a lower value at the edge. Next lower is the RFP with typical $q(0) \sim 0.2$ falling and actually reversing slightly at the edge (small negative q_{edge}). Last and lowest is the FRC, the traditional paradigm of which has zero toroidal field so that $q = 0$ over its entire profile. The question then is whether the modest toroidal field in the TCS experiments significantly modifies the placement of the FRC in the standard taxonomy.

The safety factor, Eq. (1) has the following approximate scaling.

$$q \sim \frac{2E}{\pi}\frac{B_{tor}}{B_{pol}} \qquad (2)$$

If the nominal ratio of toroidal to poloidal field is not too large, say 1/3, then the engineering β can exceed 90%. However, high elongation E can compensate for low B_{tor}/B_{pol} and possibly lead to a q exceeding unity. In short, high-β and high q might occur at the same time in elongated FRCs with a modest toroidal field. In what follows the safety factor in the TCS experiments will be inferred.

B. Inferring the safety factor in TCS

The conventional method for inferring q is to fit a smooth magnetic structure to the internal field measurements. This is practical if the magnetic field is known over a two-dimensional array (r,z) of locations. If such detailed measurements are lacking and the field is known only on a one-dimensional array, e.g. (r), *and* the separatrix shape is known, then a Grad-Shafranov (GS) equilibrium solver can be applied and adjusted to match the observations. The q profile is then inferred from the GS solution. ~~Unfortunately~~ However (?), the FRC plasma in TCS differs markedly from GS equilibria in one important respect.

In GS (static) equilibria, the product rB_θ is a surface function, i.e. it depends only on the flux variable ψ. If B_θ vs r is known at the midplane for $0 \le r \le r_s$; then the trajectory rB_θ vs ψ can be plotted. If it is a static equilibrium, then the *inboard* branch of the trajectory, $0 \le r \le R$ (radius of the magnetic axis), will exactly overlay the outboard branch, $R \le r \le r_s$. Figure 5 shows the rB_θ vs ψ trajectory found from the TCS measurements. The inboard branch lies far above the outboard branch. Evidently this plasma is *not* a static equilibrium. The peculiar toroidal field structure must arise from effects not contained in the static plasma model. As discussed later, these features *can* be explained, but only in the context of a flowing, two-fluid equilibrium. Thus one cannot use GS modeling per se to infer q. At best, it might be used to bracket the q profile.

A flowing two-fluid equilibrium model is under development [Ref. 3] but only a one-dimensional version of it is presently available [Ref. 4]. The full, two-dimensional equilibrium structure is needed in order to infer the safety factor. Two-dimensional GS solvers are available and can bracket the q-profile as follows. A nominal *lower* bound on q is found by finding the GS solution with the same poloidal current at observed in TCS. This solution has a lower peak B_θ of about 4 mT at a radius $r \approx 0.6r_s$. This should give a reasonable estimate of $q(0)$ but will be too low elsewhere. A nominal *upper* bound on q can be found by finding the GS solution that matches the observed dB_θ/dr at the geometric axis. This should give a reasonable estimate of q_{edge} but be too high elsewhere.

A reliable inference of the safety factor can be made at the edge (where ?) a simple expression for q_{edge} is available. Since $B_\theta = 0$ at the edge of a CT, the *outer* leg of the separatrix contributes nothing to q since $B_\theta/r = 0$. However, the *inner* leg of the edge flux surface lies on the geometric axis where the limit of B_θ/r remains finite. Thus q_{edge} is given by a simple expression

$$q_{edge} = \frac{z_s}{\pi}\left\langle \frac{1}{B_p}\frac{dB_\theta}{dr}\right\rangle_{r=0} \tag{3}$$

where the average is along the axis between the two x-points. In TCS $(dB_\theta/dr)/B_p$ is measured at the midplane. Because the equilibrium is highly elongated, $(dB_\theta/dr)/B_p$ is roughly constant along the axis, although both numerator and denominator fall to zero at the x-points. Applying this to TCS yields a value of $q_{edge} \approx 2.2$. Thus the FRC produced in the TCS-translation experiments has quite a large safety factor, at least at the edge.

Results from the various models for the q profile are shown in Fig. 6. The edge value from Eq. (3) is shown as a square symbol. The fourth model uses a magnetic structure with the GS-like solution for the poloidal field, but with a toroidal field structure that matches that measured at the midplane on TCS:

$$\psi = -\frac{1}{2}B_{z0}r^2\left(1-\frac{r^2}{r_s^2}-\frac{z^{2N}}{z_s^{2N}}\right) \tag{4}$$

$$B_\theta = B_{\theta 0}\frac{r}{r_s}\left(1-\frac{r^2}{r_s^2}-\frac{z^{2N}}{z_s^{2N}}\right)e^{-\alpha r^2/r_s^2} \tag{5}$$

where B_{z0}, $B_{\theta 0}$, r_s, z_s, N, and α are fitting parameters. The fitted field structure Eqs. (4,5) should give the most reliable overall q-profile. Observe that the safety factor falls from more than two at the edge to an axis value in the neighborhood of 1/2. The critical $q = 1$ surface is well away from the edge.

IV. Implications of *q*-profile in TCS

"Traditional" FRCs, i.e. static plasmas with zero toroidal field have been heavily studied with standard MHD stability theory. The main focus has been the internal tilt mode, which is the CT version of the lowest-order kink. On the other hand, there are no known studies of $q > 1$ FRC equilibria. This is not surprising since a high-q FRC was hardly expected. It would not be difficult to carry out such studies since only conventional techniques are required. However, something beyond conventional methods is needed, especially since two-fluid effects and flow appear to be essential feature of FRCs. A hint of this appears in Fig. 5, where the toroidal field profile departs radically from the static plasma paradigm. Recent analysis of FRC equilibrium also showed the importance of both two-fluid effects and flow.[Ref.3,4] Unfortunately, a practical stability theory including both two-fluid effects and flow is still under development. In the meantime one must be content with simple indicators of stability, such as the q profile.

The indicators drawn from the q profile are as follows. The appearance of $q_{edge} \sim 2$ meets the Kruskal condition for global kink stability. In the case of the FRC this is primarily the dreaded tilt mode. At the magnetic axis, $q(0) < 1$, in fact near 1/2. In this case, internal relaxation activity is expected near the m=1/n=1 and m=1/n=2 rational surfaces. However, both surfaces are well inside the separatrix and the latter is quite close to the magnetic axis. The resulting internal mode activity may take the form of a benign relaxation akin to sawtoothing. Further, FRCs have a relatively small field magnitude in the interior: thus the smearing effect of finite-Larmor-radius (FLR) near rational surfaces may limit such activity. The most probable q-profile in Fig. 6 has significant magnetic shear $dq/d\psi$. This should help suppress local modes. One more advantage of having even a small toroidal field in FRC is related to the fact that such a field restores a concept of magnetic flux surfaces and suppresses the wandering of the magnetic field lines caused by magnetic fluctuations or field errors [D.D. Ryutov, J. Kesner, and M.E. Mauel. Phys. Plasmas, 11, 2318 (2004)].

Perhaps the most important aspect of high-q in an FRC is that it does not scale away. This stands in contrast to the common explanation of the robust stability of experimental FRCs, i.e. that the tilt and other modes are stabilized by FLR effects. Unfortunately the importance of FLR weakens as the plasma is scaled up and become ineffective at fusion-relevant sizes. On the other hand, the q-profile has no size scaling; a favorable q-profile need not scale away.

V. Other observations from modeling of TCS

A. Broad boundary plasma

The inference of high safety factor is not the only new and surprising property to be found in the TCS-translation plasmas. Other important properties emerge from the data, including the measurement of strong toroidal flow. It has been shown that a two fluid plasma has *two* sets of characteristic surfaces, one associated with the electron flow, and one associated with the ion flow [Ref.5]. The former are identical to the magnetic surfaces $\psi = const$ assuming massless electrons. The ion surfaces are given by

$$Y = \psi + (m_i c/e) r u_\theta = const \qquad (6)$$

The parameter characterizing two-fluid effects is $\varepsilon \equiv \ell_i/L$ where ℓ_i is the ion skin depth (c/ω_{pi}) and L is the characteristic length in the system, e.g. r_s in an FRC. In the TCS-translation FRC the two-fluid parameter is $\varepsilon = 0.22$. Two-fluid effects are negligible only if $\varepsilon << 1$. The existence of two sets of characteristics contrasts with the standard "single-fluid" magnetohydrodynamic model which legislates only one set of surfaces $\psi = const$.

A critical manifestation of the two-fluid effect is that the ion flow separatrix Y = 0 and the electron (and magnetic) separatrix $\psi = 0$ do not coincide. In fact, if the rotational flow at the separatrix is in the ion diamagnetic direction, the former lies outside the latter. Figure 7 illustrates this. The flow separatrix is displayed as a solid line with radius r_f at the midplane. The magnetic separatrix is displayed as a dashed line with radius r_s at the midplane. The *transition* layer is the shaded region within which flowing ions sample open magnetic field lines on the outboard side. The thickness of the transition layer's outboard leg is $\Delta_{tr} = (u_\theta/\omega_{ci})_{edge}$. For TCS $\Delta_{tr} \sim 2$ cm, which is fully a tenth of the separatrix radius. The inner leg of the transition layer is even broader; in TCS its radius is ~$0.4r_s$.

The physics in the transition layer is interesting, to say the least. (1) The ion fluid is magnetically confined, i.e. it has closed flow surfaces. On the other hand, the electrons, at least on the outboard leg of the transition layer, are electrostatically confined by ambipolar fields. (2) "Line-tying" of electrons on the outboard leg should affect ions throughout the transition layer, since the ions in the layer sample the electrically-shorted region. (3) Because of line-tying, the transition layer may serve as quasi-rigid boundary to suppress global kink modes.

B. Complex flows

The toroidal flow u_θ is found by unfolding spectroscopic data from TCS, which measures the chord-integrated emission from C^{++} as a function of the chord displacement from the diameter. The poloidal flow at the midplane u_z is found by applying the one-dimensional two-fluid equilibrium model to the TCS data [Ref. 4]. Figure 8 shows the flow structures inferred by these means. Here the flow speeds are shown as a fraction of the reference Alfven speed, 72 km/s. The results are surprisingly complex. FRCs have long been known to rotate, but it has generally been thought that the flow is roughly rigid and much less than the Alfven speed. In fact *both* toroidal and poloidal flows exhibit maximum values near half the Alfven speed. Moreover, both flow components display significant zonal structure. These highly-sheared flows are likely to play a stabilizing role and possibly a transport-reducing role.

C. Minimum-energy state

In two-fluid equilibria, the ion and electron stream functions are *surface functions* of their respective surface variables, ψ_i(Y) and $\psi_e(\psi)$.[Ref. 5] In two-fluid minimum-energy states these surface functions are *linear* [Ref. 6]. This is analogous to constant λ (eigenvalue) in a Taylor state. Two-fluid modeling of the TCS plasma produces the surface functions shown in Fig. 9. Note that both surface functions have a significant linear region in the plasma interior, the "core" region surrounding the magnetic axis. On the other hand, toward the edge, both depart significantly from the core-linear behavior. This departure is analogous to varying λ in a force-free plasma. The presence of a broad minimum-energy core of the FRC should promote both good global stability and low transport rate in that region.

VI. Summary

Measurements of FRCs formed by the translation method on the TCS facility have been combined with various models to infer the magnetic and flow structure of the plasma. These results indicate several surprising features. (1) The FRC has large safety factor (> 2 at the edge). (2) There is significant forward magnetic shear $dq/d\psi$ throughout the plasma. (3) The ion flow separatrix is significantly outside the magnetic separatrix, setting up a transition layer in which line-tying and end-shorting effects should be important. (4) Significant poloidal and toroidal flows appear with maximum speeds comparable to half the Alfven speed, and a significant zonal feature (flow reversal layer). (5) The core of the FRC displays the structure of a two-fluid minimum-energy state. All of these observations highlight factors that generally contribute to plasma stability, both local and global. Moreover, some of these features (zonal flow, minimum-energy state) generally help reduce the transport rate, perhaps dramatically. Since the discovery of these features in an FRC is so recent, and is first reported here, no relevant stability studies have yet been brought to bear. Even so, the five features noted here bode well for the stability of FRCs of this type. Moreover, the simultaneous appearance of very high-β with such favorable stability and transport indicators also bodes well for the prospects of the FRC as a fusion confinement system.

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This work was performed under the auspices of the U. S. Department of Energy by University of California, Lawrence Livermore National Laboratory under contract W-7405-Eng-48.

Fig. 1

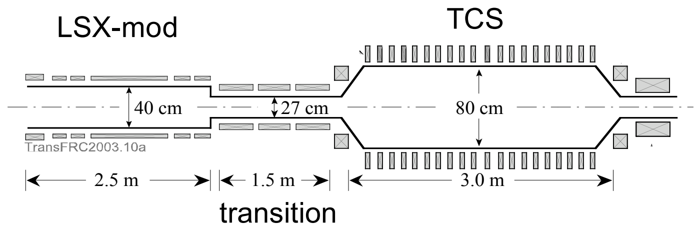

Fig. 1. Schematic of TCS-translation experiment

Fig. 2

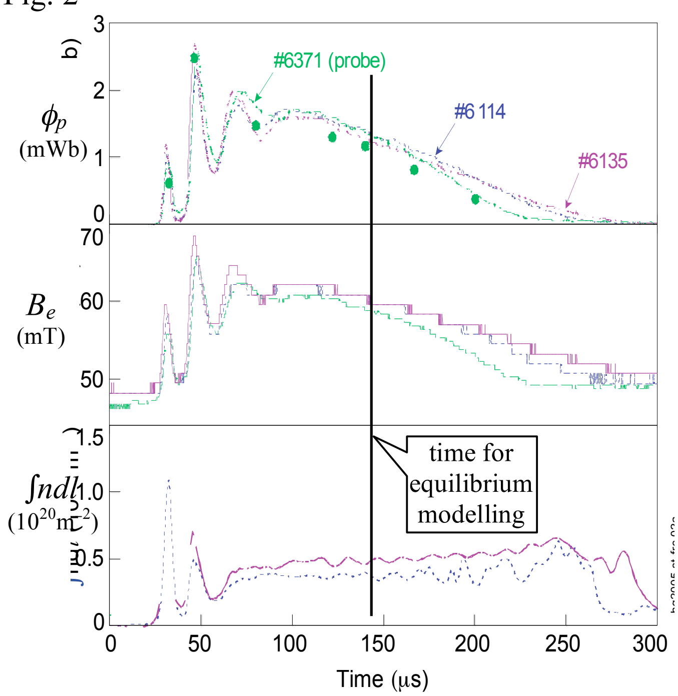

Fig. 2. The time history of poloidal flux, external magnetic field, and integrated density.

Fig. 3

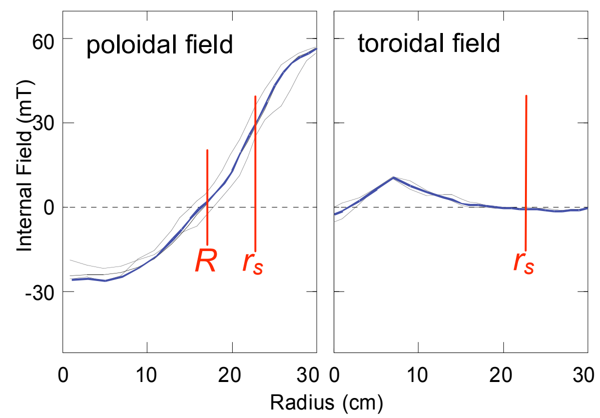

Fig. 3. Radial profiles of magnetic field components at midplane, t = 140 μs.

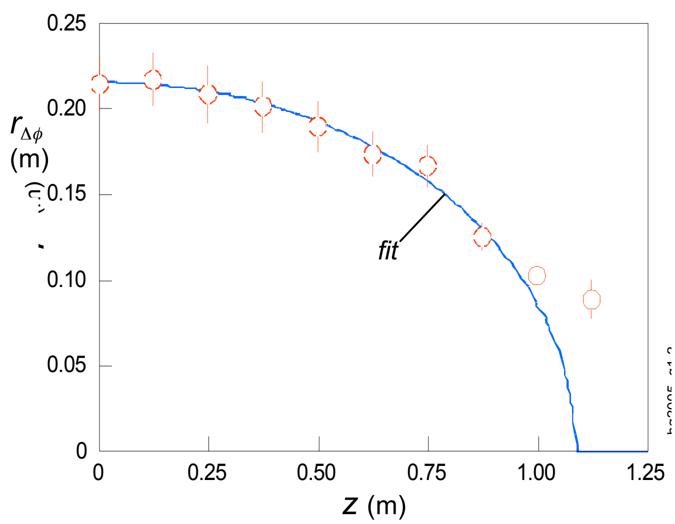

Fig. 4. Excluded flux radius axial structure.

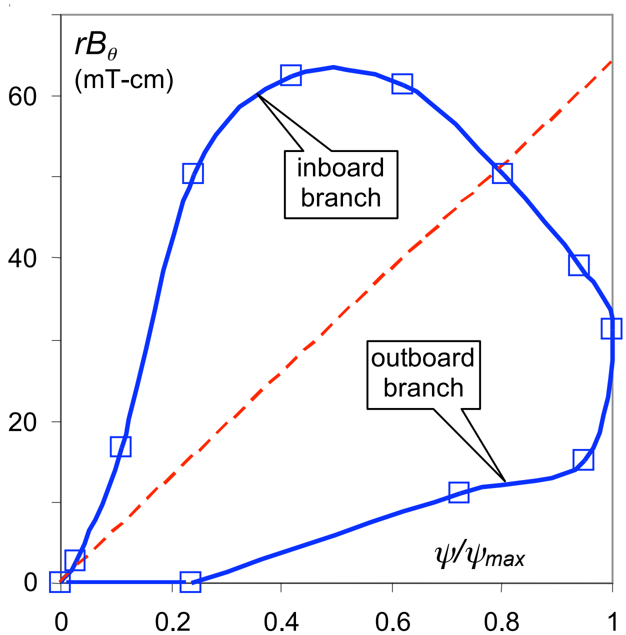

Fig. 5. Trajectory of rB_θ vs ψ.

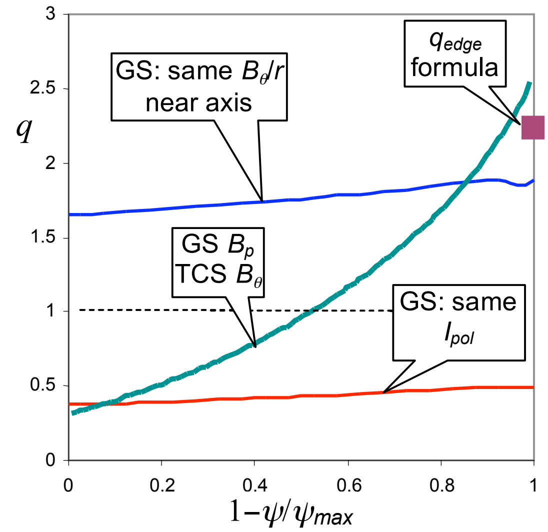

Fig. 6. Various models of the q profile

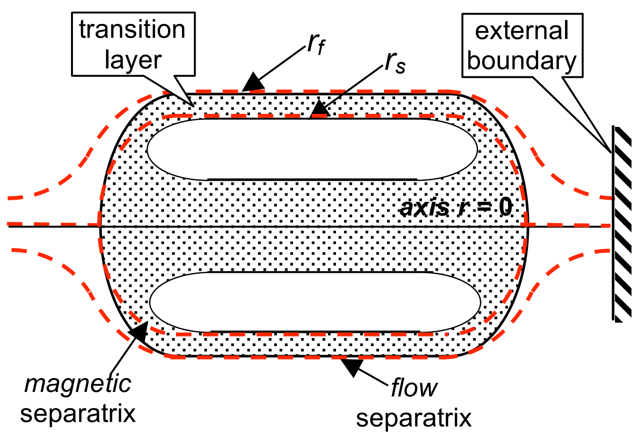

Fig. 7. Transition layer in rotating, two-fluid FRC.

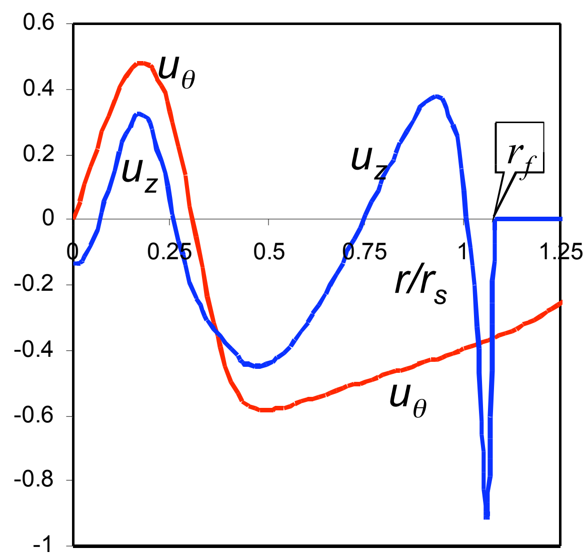

Fig. 8. Toroidal and poloidal flow velocity, normalized to reference Alfven speed, 72

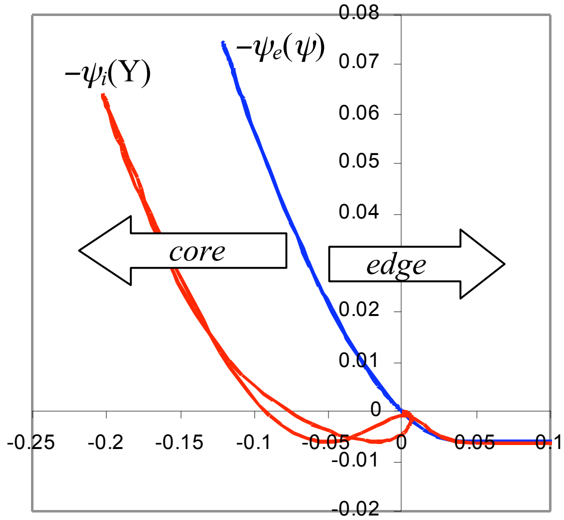

Fig. 9. Surface functions for ion and electron stream functions.

Table I. TCS-Translation parameters .

Major radius, R	0.16 m
Minor radius, $a = r_s - R$	0.07 m
Half-length, z_s	1.0 m
Poloidal field B_p (separatrix)	30 mT
Peak toroidal field B_T	9 mT
Average density, $\langle n \rangle$	$4{\times}10^{19}$ m^{-3}
Temperature, T_i,T_e	100 eV.