

Evolution of Parton Fragmentation Functions at Finite Temperature

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The first order correction to the parton fragmentation functions in a thermal medium is derived in the leading logarithmic approximation in the framework of thermal field theory. The medium-modified evolution equations of the parton fragmentation functions are also derived. It is shown that all infrared divergences, both linear and logarithmic, in the real processes are canceled among themselves and by corresponding virtual corrections. The evolution of the quark number and the energy loss (or gain) induced by the thermal medium are investigated.

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I. INTRODUCTION

Parton production in hard processes is normally followed by final-state radiation and subsequent hadronization, giving rise to collimated jets of hadrons as observed in experiments. The hadron distributions inside a jet known as jet fragmentation functions can be defined as the vacuum expectation values of the parton fields and the hadronic interpolating fields [1]. Though these fragmentation functions are non-perturbative and currently can only be measured experimentally, their evolution with the probing scale can be calculated within perturbative QCD. The resultant Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [2] evolution equations have been stringently tested against experiments and now can even be used to measure the scale-dependence of the running strong coupling constant [3]. If the hard parton is produced amid a thermal QCD medium, as most likely occurs in high-energy heavy-ion collisions, the subsequent final-state radiation and parton cascade must be modified by the presence of the medium. One not only has to consider parton emission but also parton absorption [4] in describing the evolution of the fragmentation functions. Such a medium effect is closely related to the radiative parton energy loss, which also leads to modified fragmentation functions [5]. In most of the studies of parton energy loss, one relies on a Debye-screened potential model [6–9] for parton-medium interaction. One then has to introduce additional parameters such as the mean free path, which is ill-defined when the magnetic part of the one-gluon exchange interaction is included. One can replace the mean free path by a transport parameter [7] and eliminate the infrared problem by using a hard-thermal-loop (HTL) resummed gluon propagator [10]. However, it is still theoretically interesting to study the problem within the framework of field theory at finite temperature.

This paper is our first attempt to study medium-modified fragmentation functions within QCD field theory at finite temperature. We will first extend the definition of the fragmentation functions to include the scenario of parton propagation inside a thermal bath. We will calculate the first order corrections to the parton fragmentation functions in a thermal medium in the leading logarithmic approximation and derive the corresponding modified evolution equations. We will find that the modified evolution equation can be cast into a similar form as the DGLAP equations in the vacuum. However, the modified splitting functions depend explicitly on the temperature and the partons' initial energy. We will study the structure of different contributions and their physical interpretations. We will demonstrate that all infrared (both linear and logarithmic) divergences in radiative corrections cancel either among themselves or with the virtual corrections. We will also study the evolution of the net quark number and energy loss (or gain).

We should emphasize that our calculation in this paper includes only the first order corrections in the leading logarithmic approximation. At this order, we can only consider parton emission and absorption. By solving the evolution equations, one can effectively resum radiative corrections associated with the leading parton. However, collision-induced radiation is not considered at the first order of perturbation. Furthermore, we cannot include any interference effect like the Landau-Pomeranchuk-Migdal (LPM) interference [11] in medium-induced bremsstrahlung in the leading log approximation. One has to include the spectral function of the HTL resummed gluon propagator and go beyond the leading log approximation in order to consider radiation induced by multiple scattering and the LPM interference effect. We leave this to future investigation.

The remainder of the paper is organized as follows. In Sec. II, we will first review the definitions of the fragmentation functions in the vacuum and the basic physical processes that lead to the DGLAP evolution equations of the fragmentation functions. We then extend the definition to include the case in which a parton propagates through a

thermal QCD medium. In Sec. III, we derive the radiative corrections to the fragmentation functions in a thermal medium based on finite temperature field theory. We will discuss various physical processes that contribute to the radiative corrections and the cancellation of all infrared divergences. In Sec. IV, the QCD evolution equations are derived from the calculated radiative corrections. As examples of the application of the evolution equations, we derive the evolution equations for the net quark number and calculate the energy loss (gain) induced by the medium. Finally, we give a summary and conclusion in Sec. V.

II. FRAGMENTATION FUNCTIONS AT ZERO AND FINITE TEMPERATURE

A. Fragmentation Function at Zero Temperature

Semi-inclusive cross sections can generally be factorized into the convolution of a parton fragmentation function and a hard partonic cross section in the collinear (leading twist) approximation. Fragmentation functions are defined as the vacuum expectation values of parton fields and hadronic interpolating fields. The e^+e^- annihilation process is an ideal framework in which to define the quark and anti-quark fragmentation functions. The total hadronic cross section for this process can be expressed as

$$\sigma_{e^+e^- \rightarrow X} = \frac{2\pi e^4}{s} L_{\mu\nu}(p_a, p_b) W^{\mu\nu}(q), \quad (1)$$

where $s = (p_a + p_b)^2$ is the invariant mass of electron-positron system and $q = p_a + p_b$ the momentum of virtual photon (γ^*). The leptonic tensor $L_{\mu\nu}$ is given by

$$L_{\mu\nu}(p_a, p_b) = \frac{1}{4} \text{Tr}(\gamma_\mu \not{p}_a \gamma_\nu \not{p}_b), \quad (2)$$

where $1/4$ comes from the average over spin polarization of the initial e^+e^- state. The hadronic tensor is defined as

$$\begin{aligned} W^{\mu\nu}(q) &= \frac{1}{4\pi} \sum_X \langle 0 | J^\mu(0) | X \rangle \langle X | J^\nu(0) | 0 \rangle (2\pi)^4 \delta^4(p_X - q) \\ &= \frac{1}{4\pi} \int d^4x e^{-iq \cdot x} \langle 0 | J^\mu(0) J^\nu(x) | 0 \rangle, \end{aligned} \quad (3)$$

where $\sum_X$ runs over all possible intermediate states and the quark electromagnetic current is $J_\mu = \sum_q Q_q \bar{\psi}_q \gamma_\mu \psi_q$. Here, Q_q is the electric charge of the quark in units of the proton charge.

At the lowest order in pQCD, the electron-positron pair annihilates, forming a virtual photon which then decays into a quark anti-quark pair. After a sufficiently long time, the partons undergo a nonperturbative fragmentation process and emerge from the scattering center in the form of hadronic jets. The two partons fragment independently of one another in the leading twist approximation, allowing us to define process-independent fragmentation functions. In terms of these fragmentation functions, the semi-inclusive cross section $\sigma_{e^+e^- \rightarrow h}$ can be shown to have the form

$$\frac{d\sigma_{e^+e^- \rightarrow h}}{dz_h} = \sum_q \sigma_0^{q\bar{q}} [D_{\bar{q} \rightarrow h}(z_h) + D_{q \rightarrow h}(z_h)] \quad (4)$$

in the center-of-mass frame. Here,

$$\sigma_0^{q\bar{q}} \equiv N_c \frac{4\pi\alpha^2 Q_q^2}{3s} \quad (5)$$

is the perturbative cross section in the lowest order for $e^+e^- \rightarrow q\bar{q}$ and $D_{q(\bar{q}) \rightarrow h}(z_h)$ is the probability that a quark (anti-quark) will decay into a hadron h with a fraction z_h of its energy. N_c is the number of quark colors. The energy of the quark (anti-quark) is just $E_{q(\bar{q})} = \sqrt{s}/2$, so the observed hadron has energy $p_h^0 = z_h \sqrt{s}/2$. This relationship can also be written in the invariant form

$$z_h = \frac{2p_h \cdot q}{q^2}. \quad (6)$$

Applying the collinear approximation to Eq. (3), one can obtain a formal definition of the fragmentation functions [1]:

$$\begin{aligned}
D_{q(\bar{q}) \rightarrow h}(z_h) &= \frac{z_h^3}{2} T_{q(\bar{q})}(z_h) \\
&= \frac{z_h^3}{4} \int \frac{d^4 k}{(2\pi)^4} \delta\left(z_h - \frac{p_h \cdot n}{k \cdot n}\right) \text{Tr} \left[\frac{\gamma \cdot n}{p_h \cdot n} \hat{T}_{q(\bar{q})}(k, p_h) \right],
\end{aligned} \tag{7}$$

where the Dirac operators $\hat{T}_{q(\bar{q})}(k, p_h)$ are given by

$$(\hat{T}_q)_{\alpha\beta}(k, p_h) = \int d^4 x e^{-ik \cdot x} \sum_S \langle 0 | \psi_\alpha(0) | p_h, S \rangle \langle p_h, S | \bar{\psi}_\beta(x) | 0 \rangle, \tag{8a}$$

$$(\hat{T}_{\bar{q}})_{\alpha\beta}(k, p_h) = \int d^4 x e^{-ik \cdot x} \sum_S \langle 0 | \bar{\psi}_\beta(0) | p_h, S \rangle \langle p_h, S | \psi_\alpha(x) | 0 \rangle. \tag{8b}$$

Here, the sums go over all physical states S and sums over quark colors are implicitly averaged. The light-like vector $n^\mu \equiv [n^+, n^-, n_\perp] = [0, 1, 0_\perp]$ is taken conjugate to the momentum of the observed hadron in the sense that its spatial components are antiparallel to the spatial momentum of the hadron. This implies $n \cdot p_h = p_h^+ = (p_h^0 + |\vec{p}_h|)/2$. The gauge links required to make this expression gauge-invariant have been suppressed since they do not contribute to the leading-twist fragmentation functions in the light-cone gauge, $n \cdot A = 0$.

The gluon fragmentation function $D_{g \rightarrow h}(z_h)$ are defined in a similar way:

$$\begin{aligned}
D_{g \rightarrow h}(z_h) &= \frac{z_h}{2} T_g(z_h) \\
&= \frac{z_h^2}{2} \int \frac{d^4 k}{(2\pi)^4} \delta\left(z_h - \frac{p_h \cdot n}{k \cdot n}\right) d_{\mu\nu}(k) \hat{T}_g^{\mu\nu}(k, p_h),
\end{aligned} \tag{9}$$

with

$$\hat{T}_g^{\mu\nu}(k, p_h) = \int d^4 x e^{-ik \cdot x} \sum_S \langle 0 | A^\mu(0) | p_h, S \rangle \langle p_h, S | A^\nu(x) | 0 \rangle. \tag{10}$$

Similarly, gluonic color indices are implicitly averaged over. The gluon polarization vectors $\varepsilon^\mu(k)$ satisfy

$$\begin{aligned}
d_{\mu\nu}(k) &\equiv \sum_{\lambda=1,2} \varepsilon_\mu(k, \lambda) \varepsilon_\nu(k, \lambda) \\
&= -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k}
\end{aligned} \tag{11}$$

in the light-cone gauge. The field-strength tensor, $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$, allows us to write the gluon fragmentation function (9) in the manifestly gauge-invariant form

$$D_{g \rightarrow h}(z_h) = -\frac{z_h^2}{2p_h^+} \sum_S \int \frac{dx^-}{2\pi} e^{-ip_h^+ x^- / z_h} \langle 0 | G^{+\mu}(0) | p_h, S \rangle \langle p_h, S | G^+_\mu(x^-) | 0 \rangle. \tag{12}$$

Once again, the gauge links necessary to make this expression gauge-invariant do not contribute to our functions in the light-cone gauge.

Using the ‘cut vertices’ technique introduced by A. Mueller [12], we can calculate the scale dependence of these functions in perturbation theory. The Feynman diagrams illustrating this calculation are shown at leading order in Fig. 1. The corresponding bare quark and gluon cut vertices are

$$\frac{\gamma \cdot n}{2p_h \cdot n} \delta\left(z_h - \frac{p_h \cdot n}{k \cdot n}\right) \tag{13a}$$

and

$$z_h d_{\mu\nu}(k) \delta\left(z_h - \frac{p_h \cdot n}{k \cdot n}\right), \tag{13b}$$

respectively, in the light-cone gauge.

At next-to-leading order, $\mathcal{O}(\alpha_s)$, the gluon bremsstrahlung process represented in Fig. 2 gives a positive contribution to the quark fragmentation function. This is due to the fact that more options are open to the fragmenting quark

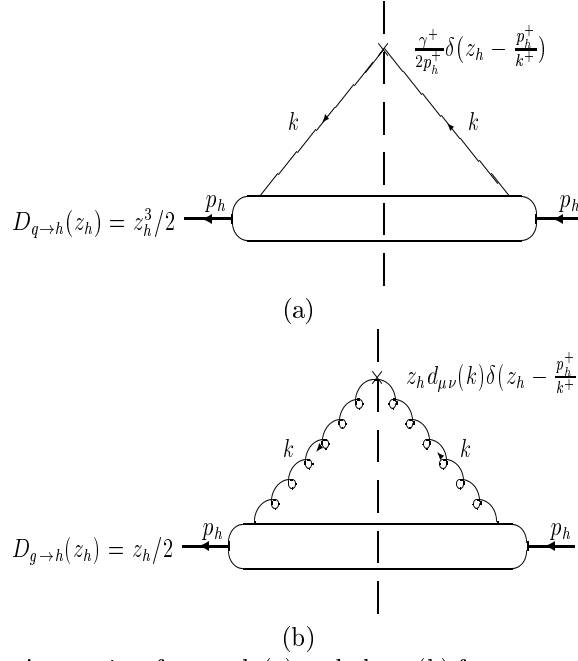


FIG. 1. Cutting vertices for quark (a) and gluon (b) fragmentation.

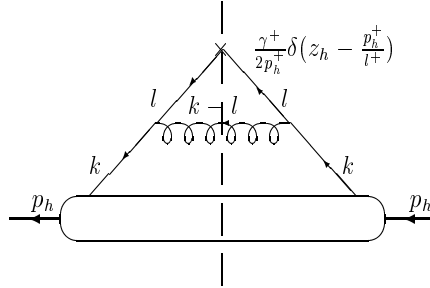


FIG. 2. Real correction ($q \rightarrow q + g$) to the quark fragmentation.

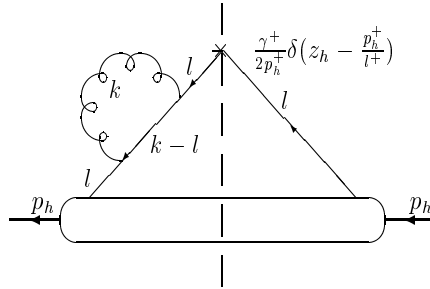


FIG. 3. Virtual correction to the quark fragmentation.

at this order: it can fragment immediately, or radiate a gluon first. This bremsstrahlung process becomes easier as the energy of the radiated gluon decreases, leading to a divergence as the energy goes to zero. Physically, this divergence is meaningless. If the quark radiates no energy, its fragmentation should not be altered. This idea is realized mathematically by the self-energy contribution represented in Fig. 3. This correction removes the double-counting inherent in the above argument, and ensures that each decay channel is counted only once. For this reason, the divergent contribution from zero-energy bremsstrahlung is entirely canceled.

All next-to-leading order fragmentation sub-processes lead to similar corrections to the fragmentation functions. We will list a complete set of Feynman ‘cut’ diagrams later. Apart from the spurious infrared divergences in some of the diagrams, each correction contains a collinear divergence as the transverse momentum of the loop approaches zero. These divergences signal an unavoidable scale dependence in our corrections. Physically, this scale dependence represents the fact that partons appear different when probed at different scales. We can handle these divergences mathematically by computing only the difference between our fragmentation functions probed at different scales. Since the asymptotic contribution to our loop integrals cannot depend on the external scales of the process, the collinear divergences will always cancel in such differences. With this in mind, we write the results

$$D_{q \rightarrow h}(z_h, Q^2) = D_{q \rightarrow h}(z_h, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_h}^1 \frac{dz}{z} \left[\gamma_{qq}(z) D_{q \rightarrow h}\left(\frac{z_h}{z}, \mu^2\right) + \gamma_{qg}(z) D_{g \rightarrow h}\left(\frac{z_h}{z}, \mu^2\right) \right], \quad (14a)$$

$$D_{g \rightarrow h}(z_h, Q^2) = D_{g \rightarrow h}(z_h, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_h}^1 \frac{dz}{z} \left[\gamma_{gq}(z) D_{s \rightarrow h}\left(\frac{z_h}{z}, \mu^2\right) + \gamma_{gg}(z) D_{g \rightarrow h}\left(\frac{z_h}{z}, \mu^2\right) \right], \quad (14b)$$

of the calculation. Here, we have defined the singlet quark fragmentation function as,

$$D_{s \rightarrow h}(z, \mu^2) \equiv \sum_q (D_{q \rightarrow h}(z, \mu^2) + D_{\bar{q} \rightarrow h}(z, \mu^2)) \quad (15)$$

The integration kernels, or splitting functions, are given by [13]:

$$\gamma_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right], \quad (16a)$$

$$\gamma_{qg}(z) = C_F \frac{1+(1-z)^2}{z}, \quad (16b)$$

$$\gamma_{gq}(z) = T_F [z^2 + (1-z)^2], \quad (16c)$$

$$\gamma_{gg}(z) = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(1-z) \left[\frac{11}{6} C_A - \frac{2}{3} n_f T_F \right]. \quad (16d)$$

Here, n_f is the number of active quark flavors and the $SU(N_c)$ Casimirs are given by $C_F = (N_c^2 - 1)/2N_c$, $C_A = N_c$ and $T_F = 1/2$; $N_c = 3$. The ‘+’-function is defined such that the replacement

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z) - f(1)}{1-z} \quad (17)$$

is valid for any function $f(z)$ that is continuous at $z = 1$.

The scale dependence of the fragmentation functions is then determined by the DGLAP evolution equation [2],

$$Q^2 \frac{d}{dQ^2} D_{q \rightarrow h}(z_h, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[\gamma_{qq}(z) D_{q \rightarrow h}(z_h/z, Q^2) + \gamma_{qg}(z) D_{g \rightarrow h}(z_h/z, Q^2) \right], \quad (18a)$$

$$Q^2 \frac{d}{dQ^2} D_{g \rightarrow h}(z_h, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[\gamma_{gq}(z) D_{s \rightarrow h}(z_h/z, Q^2) + \gamma_{gg}(z) D_{g \rightarrow h}(z_h/z, Q^2) \right], \quad (18b)$$

as can be shown by repeated iteration of two-particle-irreducible diagrams similar to those shown in Figs. 2 and 3. Eq.(14) is analogous to the ‘Born’ approximation of (18). Since we have calculated our splitting functions only at leading order in α_s , our evolution equations are not correct to all orders. However, it can be shown [13] that Eq. (18) contains the highest power of $\log(Q^2/\mu^2)$ present at each order in α_s . For this reason, it is known as the leading log approximation (LLA) to the full evolution equation.

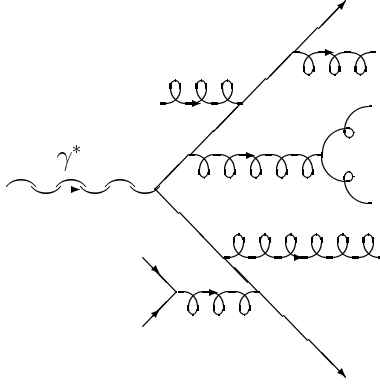


FIG. 4. Parton emission and absorption by propagating jets in a medium.

B. Fragmentation Function at Finite Temperature

In the finite-temperature case, we consider the fragmentation of a parton in a thermal medium with temperature T . In such a thermal environment, it will interact with the medium through emission and absorption as illustrated in Fig. 4. Assuming N quarks and gluons are absorbed and M quarks and gluons are emitted, we can express the hadronic tensor [Eq. (3)] at finite temperature as

$$\begin{aligned}
W_{T \neq 0}^{\mu\nu}(q) = & \frac{1}{4\pi} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \cdots \frac{d^3 p_N}{(2\pi)^3 2E_N} \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \cdots \frac{d^3 p'_M}{(2\pi)^3 2E'_M} (2\pi)^4 \delta\left(\sum_{i=1}^M p'_i - \sum_{i=1}^N p_i - q\right) \\
& \times \langle p_1, \dots, p_N | J^\mu(0) | p'_1 \cdots p'_M \rangle \langle p'_1 \cdots p'_M | J^\nu(0) | p_1, \dots, p_N \rangle \\
& \times \frac{1}{e^{\beta E_1} \mp 1} \cdots \frac{1}{e^{\beta E_N} \mp 1} \left[1 \pm \frac{1}{e^{\beta E'_1} \mp 1}\right] \cdots \left[1 \pm \frac{1}{e^{\beta E'_M} \mp 1}\right].
\end{aligned} \tag{19}$$

The phase space integrals of the quarks and gluons are weighted by Bose-Einstein or Fermi-Dirac thermal distributions for gluon and quark absorption, and by the Bose-Einstein enhancement and Pauli blocking factor for emission. As in the derivation of the emission rate for photons and dileptons from a quark gluon plasma by McLerran and Toimela [14], the hadron tensor in Eq. (19) can be expressed as the thermal expectation value of a partonic electromagnetic current-current correlation function,

$$W_{T \neq 0}^{\mu\nu}(q) = \frac{1}{4\pi} \int d^4 x e^{-iq \cdot x} \langle J^\mu(0) J^\nu(x) \rangle, \tag{20}$$

where $\langle \cdots \rangle$ stands for the thermal expectation value,

$$\langle \mathcal{O} \rangle = \frac{\text{Tr}[e^{-\beta \hat{H}} \mathcal{O}]}{\text{Tr} e^{-\beta \hat{H}}}. \tag{21}$$

Comparing to the hadronic tensor of Eq. (3) defined at zero temperature, the hadronic tensor in Eq. (20) at finite temperature can be obtained by replacing the vacuum expectation value in Eq. (3) by a thermal expectation value. Applying this idea to the fragmentation functions is slightly more complicated. First, the conventional interpretation of the fragmentation functions as probabilities for a parton to fragment into a hadron needs to be modified. The thermal QCD medium is defined to be color deconfined, so the concept of hadrons inside such a thermal medium is ill-defined. Hadrons should only emerge after the hadronization of the medium. Therefore, the fragmentation functions in a thermal medium are defined in such a way that they give the probabilities of producing a hadron as the partons hadronize together with the medium. Another useful scenario is to consider how an energetic parton with high virtuality produces secondary partons. In this case, the fragmentation functions can be defined as the probabilities of finding a secondary parton inside a parton jet. One can then study how such parton production from a jet is modified in a thermal medium.

At zero temperature, the absence of a scale implies that the fragmentation functions depend only on the ratio of the hadron and parton energies. This leads one to consider the fraction of parton momentum carried by the hadron rather than the absolute momentum. At finite temperature, the thermal average introduces dependence on temperature and therefore the absolute momenta of the initial partons. This implies that a fragmentation function defined in the spirit

of the zero temperature functions cannot be complete. In particular, fragmentation functions relating to different initial parton energies will mix with each other under renormalization, resulting in a very complicated evolution equation.

A more straightforward way to approach the problem of fragmentation at finite temperature is to consider the momenta of the fragmenting parton and the final state hadron separately. In particular,

$$F_{h/q}(y; x) = \frac{y}{4x} \int \frac{d^4 k}{(2\pi)^4} \delta(k^+ - x) \int d^4 z e^{-ikz} \sum_{\tilde{S}} \text{Tr} \left[\langle \psi(0) | \tilde{S}, p_h \rangle \gamma^+ \langle \tilde{S}, p_h | \bar{\psi}(z) \rangle \right] \quad (22)$$

represents the probability for a quark of momentum $p^+ = x$ to decay into a hadron of momentum $p_h^+ = y$. This expression is to be understood as a thermal average over the quark field operators in which a particular intermediate state, $|p_h\rangle$, is singled out. The set of states $\{|\tilde{S}\rangle\}$ includes the thermal phase space indicated in Eq. (19) and the quark color is implicitly averaged. Note that in the following discussion of the newly defined fragmentation functions at finite temperature, all variables, x, y, z , etc. represent absolute '+'-momentum and thus carry the dimension of energy. We will denote fractional momentum by these variables with a subscript (z_f , for example).

This new function is quite similar in form to the quark fragmentation function in the vacuum. Defining

$$\tilde{D}_{q \rightarrow h}(z_f; x) \equiv F_{h/q}(xz_f; x) \quad , \quad (23)$$

we find that $\tilde{D}_{q \rightarrow h}(z_f; x)|_{T=0} = D_{q \rightarrow h}(z_f)$. However, the interpretation of $F_{h/q}$ is entirely different. By tracking the absolute momentum of both the initial parton and the observed hadron, this new function is able to clearly express the dependence of fragmentation on the temperature and the initial parton energy. We will see below that this separation of fragmenting parton and observed hadron is essential to a transparent evolution equation at finite temperature where fragmentation functions with different initial parton energies mix.

Fragmentation functions for anti-quarks and gluons can be defined in a similar way. We have

$$F_{h/\bar{q}}(y; x) = \frac{y}{4x} \int \frac{d^4 k}{(2\pi)^4} \delta(k^+ - x) \int d^4 z e^{-ikz} \sum_{\tilde{S}} \langle \bar{\psi}(0) | \tilde{S}, p_h \rangle \gamma^+ \langle \tilde{S}, p_h | \psi(z) \rangle \quad (24a)$$

$$F_{h/g}(y; x) = -\frac{y}{2x^2} \int \frac{d^4 k}{(2\pi)^4} \delta(k^+ - x) \int d^4 z e^{-ikz} \sum_{\tilde{S}} \langle G^{+\mu}(0) | \tilde{S}, p_h \rangle \langle \tilde{S}, p_h | G_{\mu}^+(z) \rangle \quad , \quad (24b)$$

where colors are implicitly averaged over, as before.

III. RADIATIVE CORRECTIONS

In this section we calculate the first order correction to the fragmentation functions at finite temperature. The result will be used to derive the corresponding evolution equations in the next section. Using

$$\frac{y}{x} \delta(k^+ - x) = z_h^3 \frac{1}{p_h \cdot n} \delta\left(z_h - \frac{p_h \cdot n}{k \cdot n}\right) \quad , \quad (25)$$

where $z_h = y/x$ and $p_h \cdot n = y$, we see that the form of this calculation is quite similar to that of the zero-temperature case. The only practical difference between the two cases lies in the form of the propagators, both cut and uncut. For uncut (intermediate state) propagators, the expressions

$$S(k) = \left[\frac{i}{k^2 + i\eta} - 2\pi f(k^+) \delta(k^2) \right] \not{k} \quad , \quad (26a)$$

$$\Delta_{\mu\nu}(k) = \left[\frac{i}{k^2 + i\eta} + 2\pi n(k^+) \delta(k^2) \right] d_{\mu\nu}(k) \quad , \quad (26b)$$

replace the ordinary zero-temperature propagators. The distribution functions

$$f(x) = \frac{1}{e^{|x|/T} + 1} \quad , \quad (27a)$$

$$n(x) = \frac{1}{e^{|x|/T} - 1} \quad , \quad (27b)$$

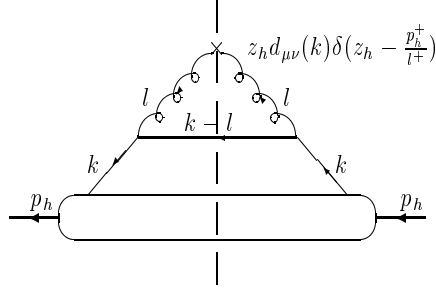


FIG. 5. Real correction ($q \rightarrow g + q$) to the quark fragmentation.

represent the effect of the thermal medium on free propagation.

Cut (final state) propagators of thermal partons are represented by

$$S^{cut}(k) = \left[\theta(k^+) - f(k^+) \right] k 2\pi \delta(k^2), \quad (28a)$$

$$\Delta_{\mu\nu}^{cut}(k) = \left[\theta(k^+) + n(k^+) \right] d_{\mu\nu}(k) 2\pi \delta(k^2), \quad (28b)$$

where $\theta(x)$ is the step function:

$$\theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0. \end{cases} \quad (29)$$

As before, the vertices and propagators on the right side of the cut are the complex conjugates of the usual ones.

A. First Order Correction to Quark Fragmentation Functions

The first order corrections to the quark fragmentation function are illustrated in Figs. 2, 3 and 5. Figs. 2 and 5 represent real gluon and quark emission and absorptive processes, which give a positive contribution to the fragmentation function. Fig. 3 represents the virtual self-energy correction, which removes the double-counting inherent in the real corrections.

The correction due to real gluon emission/absorption shown in Fig. 2 can be expressed as

$$\begin{aligned} \Delta F_{h/q}^{(q)}(y; x) &= \frac{y^3}{4x^3} \int dz'_f \frac{d^4 l}{(2\pi)^4} \delta\left(z'_f - \frac{p_h^+}{l^+}\right) \\ &\times \int d^4 \xi e^{-i l \xi} \sum_{\tilde{S}} \text{Tr} \left[\langle \psi(0) | \tilde{S}, p_h \rangle \frac{\gamma^+}{p_h^+} \langle \tilde{S}, p_h | \bar{\psi}(\xi) \rangle \right] \text{Tr} \left[\not{p}_h \hat{H}_q^{(q)}(l) \right], \end{aligned} \quad (30)$$

where the contribution from the loop is

$$\begin{aligned} \hat{H}_q^{(q)}(l) &= C_F g^2 \int \frac{d^4 k}{(2\pi)^4} (-i \gamma_\mu) \frac{i \not{k}}{k^2 + i\eta} \left[\frac{\gamma^+}{2p_h^+} \delta\left(\frac{y}{x} - \frac{p_h^+}{k^+}\right) \right] \frac{-i \not{k}}{k^2 - i\eta} (i \gamma_\nu) \\ &\times d^{\mu\nu}(k-l) [\theta(k^+ - l^+) + n(k^+ - l^+)] 2\pi \delta[(k-l)^2]. \end{aligned} \quad (31)$$

This expression describes a quark of momentum k emitting a gluon of momentum $k-l$ (or absorbing a gluon of momentum $l-k$), then fragmenting into a hadron of momentum p_h such that $p_h^+/k^+ = y/x$. Note that the thermal part of the uncut quark propagators on each side of the cut does not contribute because the quark lines are off-shell ($k^2 \neq 0$) before the gluon radiation. This will be the case for all other diagrams including the virtual corrections after carefully taking the collinear approximation limit.

As it is, this expression cannot be written in terms of the simple collinear fragmentation function $F_{h/q}$ defined above. The integral over l couples its perturbative and nonperturbative parts in a way that $F_{h/q}$ is not sufficient to describe. However, it can be shown [15] that the contributions to this expression from $l_\perp$ and l^- are suppressed by powers of p_h^+ as $p_h^+ \rightarrow \infty$. At high energy, we can expand $\hat{H}(l)$ about $l = [l^+, 0, 0_\perp]$ and drop the power-suppressed corrections. This collinear approximation captures the complete leading-twist behavior of our fragmentation, so it can be used to extract the evolution of the finite-temperature fragmentation functions at the leading-twist level.

Using the collinear approximation,

$$l = l_c \equiv [y/z'_f, 0, 0_\perp], \quad (32)$$

in Eq. (31) decouples the integral over l in Eq. (30) from $\hat{H}(l_c)$. This allows us to write the correction as

$$\Delta F_{h/q}^{(q)}(y; x) = \int dz'_f \left(\frac{y}{xz'_f} \right)^3 F_{h/q}(y; y/z'_f) \text{Tr} \left[\not{p}_h \hat{H}_q^{(q)}(l_c) \right] . \quad (33)$$

Performing the change

$$z'_f = \frac{y}{z} \quad (34)$$

in $\hat{H}(l_c)$, we arrive at the result

$$\Delta F_{h/q}^{(q)}(y; x) = \frac{\alpha_s}{2\pi} \int \frac{dk_\perp^2}{k_\perp^2} \int_0^\infty \frac{dz}{z} P_{qq}(z/x) \epsilon(x-z) [\theta(x-z) + n(x-z)] F_{h/q}(y; z) . \quad (35)$$

Here, we have defined

$$\epsilon(x) = \begin{cases} -1, & x < 0 \\ +1, & x > 0 \end{cases} \quad (36)$$

and

$$P_{qq}(z_f) = C_F \frac{1 + z_f^2}{1 - z_f} . \quad (37)$$

The integral over z in Eq.(35) can be divided into two regions: $0 < z < x$ and $z > x$. In the first region, we have the physical situation in which the quark emits a gluon into the thermal bath before it fragments. The second region represents a physical absorption process. In either case, the plus-component of the intermediate quark momentum is z . At zero temperature, only the emission process contributes. Since the quark can only lose energy through evolution in this case, the ‘+’-momentum of the intermediate quark must be larger than that of the observed hadron as well as smaller than that of the initial quark, $y < z < x$. This leads to the limits on the zero-temperature evolution equation in Eq. (18). At finite temperature, the quark can gain energy through absorption. This process extends the region of integration in both directions, $z > x$ and $z < y$. The extension to $z > x$ is an obvious effect of absorption: the intermediate quark in an absorptive process has more energy than the initial quark. The extension $z < y$ is somewhat more subtle. At zero temperature, conservation of energy implies that the observed hadron cannot have more energy than its parent parton. At finite temperature, we have no such restriction. The thermal bath can provide the excess energy needed for a parton to fragment into a more energetic hadron. This can be thought of as a kind of ‘collective’ fragmentation of the parton and the bath. We can immediately see from Eq. (35) that the inclusion of this effect is necessary: even if we choose to take $F_{h/q}(y; x) = 0$ for $y > x$, Eq. (35) will generate a finite contribution in this region.

Our phenomenological interpretation of Eq. (35) leads to a somewhat fundamental difficulty with the definition of fragmentation functions at finite temperature. As the intermediate energy, z , decreases around and below the temperature, it becomes inconsistent to treat the parton as separate from the bath itself. Furthermore, it is impossible to distinguish the fragmentation of a low-energy parton in a thermal bath from the fragmentation of the bath itself. This difficulty can be resolved via a phenomenological separation between the fragmentation of a parton distinct from the bath and that of the bath itself, as we will discuss in the next section. For the remainder of this section, we will ignore our difficulty and take the radiative corrections at face value.

As in the zero-temperature case, our result for real emission/absorption is plagued with infrared divergences as $z \rightarrow x$. The divergences in Eq. (35) are made worse by the Bose-Einstein functions. As $z \rightarrow x$, these functions generate a linear divergence in addition to the logarithmic divergence present in the zero-temperature part. All of these divergences are unphysical in the sense that they represent overcounting of the relevant degrees of freedom. As explained above, they must be canceled by virtual corrections.

The virtual correction shown in Fig. 3 reads

$$\Delta F_{h/q}^{(q)}(y; x) = \frac{y^3}{4x^3} \int dz'_f \frac{d^4 l}{(2\pi)^4} \delta\left(z'_f - \frac{p_h^+}{l^+}\right) \times \int d^4 \xi e^{-i l \xi} \sum_{\tilde{S}} \text{Tr} \left[\langle \psi(0) | \tilde{S}, p_h \rangle \frac{\gamma^+}{p_h^+} \langle \tilde{S}, p_h | \bar{\psi}(\xi) \rangle \right] \text{Tr} \left[\not{p}_h \hat{H}_q^{(v)}(l) \right], \quad (38)$$

where

$$\begin{aligned} \hat{H}_q^{(v)}(l) &= \frac{1}{2} \left[\Sigma(l) \frac{i \not{l}}{l^2 + i\eta} + \Sigma^*(l) \frac{-i \not{l}}{l^2 - i\eta} \right] \frac{\gamma^+}{2p_h^+} \delta\left(\frac{y}{x} - \frac{p_h^+}{l^+}\right) \\ &= -\text{Im} \left[\frac{\Sigma(l)}{l^2 + i\eta} \right] \not{l} \frac{\gamma^+}{2p_h^+} \delta\left(\frac{y}{x} - \frac{p_h^+}{l^+}\right). \end{aligned} \quad (39)$$

Here, $\Sigma(l)$ is the quark self-energy. The prefactor of 1/2 comes from the renormalization of the initial state [16].

As before, we intend to use the collinear approximation in Eq. (32) to relate the nonperturbative part of this expression to $F_{h/q}$. In this case, however, direct substitution of $l = l_c$ leads to an indeterminate form, since $\Sigma(l)/l^2$ is not defined for $l^2 = l_c^2$. For this reason, we must carefully take the limit of Eq. (39) as $l \rightarrow l_c$. With this in mind, we can read the quark self-energy at one-loop from the diagram in Fig. 3:

$$\Sigma(l) = -g^2 C_F \int \frac{d^4 k}{(2\pi)^4} \gamma_\rho S(l-k) \gamma_\sigma \Delta^{\rho\sigma}(k). \quad (40)$$

After completing the k^- integral by contour, the collinear divergent part of this expression is given by

$$\begin{aligned} \lim_{l \rightarrow l_c} \frac{1}{l^2} \text{Tr} [\not{l} \Sigma(l) \not{l}] &= 2i \frac{\alpha_s}{\pi} l_c^+ \int \frac{dk_\perp^2}{k_\perp^2} \left\{ \int_{-\infty}^{\infty} \frac{dz}{x} P_{qq}(z/x) [\theta(z)\theta(x-z) + n(x-z)\epsilon(x-z) - f(z)\epsilon(z)] \right\} \\ &= 2i \frac{\alpha_s}{\pi} l_c^+ \int \frac{dk_\perp^2}{k_\perp^2} \left\{ \int_0^x \frac{dz}{x} P_{qq}(z/x) [(1+n(x-z))(1-f(z)) + n(x-z)f(z)] \right. \\ &\quad + \int_{-\infty}^0 \frac{dz}{x} P_{qq}(z/x) [(1+n(x-z))f(z) + n(x-z)(1-f(z))] \\ &\quad \left. + \int_x^\infty \frac{dz}{x} \frac{z}{x} P_{qq}(x/z) [n(z-x)(1-f(z)) + (1+n(z-x))f(z)] \right\}, \end{aligned} \quad (41)$$

where we have taken $l = l_c$ wherever it does not generate a singularity and used the property $P_{qq}(z_f) = -z_f P_{qq}(1/z_f)$ of the quark splitting function. This result can be understood in terms of physical decay and scattering processes at finite temperature [17]. In the second equality above, the first line corresponds to the decay of a quark into a quark and a gluon, as well as the inverse process. The second and third lines correspond to the forward and backward processes of Landau damping via scattering of the quark with another thermal quark or gluon from the thermal bath. Landau damping requires the presence of thermally excited states, and disappears at zero temperature. In each term, every incoming gluon (quark) from the thermal bath is weighted with a Bose-Einstein distribution function $n(k_i^+/T)$ [a Fermi-Dirac distribution function $f(k_i^+/T)$], and each outgoing gluon (quark) receives a Bose-Einstein enhancement factor $1 + n(k_i^+/T)$ [a Pauli blocking factor $1 - f(k_i^+/T)$].

Substituting Eqs.(39) and (41) into (38) and collecting terms, we arrive at the virtual correction

$$\Delta F_{h/q}^{(v)}(y; x) = -\frac{\alpha_s}{2\pi} F_{h/q}(y; x) \int \frac{dk_\perp^2}{k_\perp^2} \int_{-\infty}^{\infty} \frac{dz}{x} [\theta(z)\theta(x-z) + \epsilon(x-z)n(x-z) - \epsilon(z)f(z)] P_{qq}(z/x). \quad (42)$$

In order to see clearly that all infrared divergences in the real gluon radiation are canceled by the virtual correction, we combine Eqs.(35) and (42) and divide the integral into different regions:

$$\begin{aligned} \Delta F_{h/q}^{(q)}(y; x) + \Delta F_{h/q}^{(v)}(y; x) &= \frac{\alpha_s}{2\pi} \int \frac{dk_\perp^2}{k_\perp^2} \left\{ \int_0^x \frac{dz}{z} P_{q \rightarrow qg}(z/x) \underbrace{[1 + n(x-z)]}_{(a)} F_{h/q}(y; z) \right. \\ &\quad \left. + \int_x^\infty \frac{dz}{x} P_{qq}(x/z) \underbrace{n(z-x)}_{(b)} F_{h/q}(y; z) \right\} \end{aligned}$$

$$\begin{aligned}
& -F_{h/q}(y; x) \left[\int_0^x \frac{dz}{x} P_{qq}(z/x) \underbrace{[1 + n(x-z)]}_{(c)} - \underbrace{f(z)}_{(d)} \right. \\
& + \int_{-\infty}^0 \frac{dz}{x} P_{qq}(z/x) \underbrace{[n(x-z) + f(z)]}_{(e)} \\
& \left. + \int_x^\infty \frac{dz}{x} \frac{z}{x} P_{qq}(x/z) \underbrace{[n(z-x)]}_{(f)} + \underbrace{f(z)}_{(g)} \right] \Bigg\} . \tag{43}
\end{aligned}$$

As $z \rightarrow x$, the logarithmic divergences in terms (a), (b) and (d) are canceled by those in (c), (f) and (g), respectively. The linear infrared divergences in terms (a) and (b) are canceled by those in (c) and (f), leaving behind a residual logarithmic divergence. This divergence cancels in the sum of terms (a) and (b) as long as $F_{h/q}(y; z)$ is differentiable at $z = x$. Term (e) is finite.

Another real correction to quark fragmentation comes from Fig. 5,

$$\begin{aligned}
\Delta F_{h/q}^{(g)}(y; x) &= \frac{y}{4x^3} \int dz'_f \frac{d^4 l}{(2\pi)^4} \delta \left(z'_f - \frac{p_h^+}{l^+} \right) (-z'_f)^2 \\
&\times \int d^4 \xi e^{-i l \xi} \sum_{\tilde{S}} \langle G^{+\mu}(0) | \tilde{S}, p_h \rangle \langle \tilde{S}, p_h | G_\mu^+(\xi) \rangle d_{\mu\nu}(l) \hat{H}_q^{\mu\nu(g)}(l) , \tag{44}
\end{aligned}$$

Here, the loop gives the hard contribution

$$\begin{aligned}
\hat{H}_q^{\mu\nu(g)}(l_c) &= C_F g^2 \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[(-i\gamma^\mu) \frac{i \not{k}}{k^2 + i\eta} \frac{\gamma^+}{2p_h^+} \frac{-i \not{k}}{k^2 - i\eta} (i\gamma^\nu) (\not{k} - \not{l}_c) \right] \\
&\times [\theta(k^+ - l_c^+) - f(k^+ - l_c^+)] 2\pi \delta[k^2 - 2l_c \cdot k] \delta \left(\frac{y}{x} - \frac{p_h^+}{k^+} \right) . \tag{45}
\end{aligned}$$

Performing the integrals and simplifying, we obtain

$$\Delta F_{h/q}^{(g)}(y; x) = \frac{\alpha_s}{2\pi} \int \frac{dk_\perp^2}{k_\perp^2} \int_0^\infty \frac{dz}{z} P_{qq}(z/x) \epsilon(x-z) [\theta(x-z) - f(x-z)] F_{h/g}(y; z) . \tag{46}$$

The new splitting function,

$$P_{qq}(z_f) = \gamma_{qq}(z_f) = C_F \frac{1 + (1 - z_f)^2}{z_f} , \tag{47}$$

is related to $P_{qq}(z_f)$ via $z_f \rightarrow 1 - z_f$.

As in the zero temperature case, the scale dependence of the fragmentation functions is generated by the collinear divergences present in Eqs. (43) and (46). To remove the divergences, we consider the difference between fragmentation functions measured at two different scales:

$$\begin{aligned}
F_{h/q}(y; x, Q^2) - F_{h/q}(y; x, \mu^2) &= \Delta F_{h/q}^{(g)}(y; x, Q^2) + \Delta F_{h/q}^{(q)}(y; x, Q^2) + \Delta F_{h/q}^{(v)}(y; x, Q^2) \\
&= \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^\infty \frac{dz}{z} [\tilde{\gamma}_{qq}(z; x) F_{h/q}(y; z, \mu^2) + \tilde{\gamma}_{qg}(z; x) F_{h/g}(y; z, \mu^2)] . \tag{48}
\end{aligned}$$

The modified kernels are

$$\begin{aligned}
\tilde{\gamma}_{qq}(z; x) &= P_{qq}(z/x) \epsilon(x-z) [\theta(x-z) + n(x-z)] \\
&\quad - \delta(1-x/z) \int_{-\infty}^\infty \frac{d\omega}{x} P_{qq}(\omega/x) [\theta(\omega) \theta(x-\omega) + \epsilon(x-\omega) n(x-\omega) - \epsilon(\omega) f(\omega)] \tag{49a}
\end{aligned}$$

$$\tilde{\gamma}_{qg}(z; x) = P_{qg}(z/x) \epsilon(x-z) [\theta(x-z) - f(x-z)] , \tag{49b}$$

where the splitting functions are given by Eqs.(37) and (47). Obviously, the zero-temperature result is recovered as n and f approach zero.

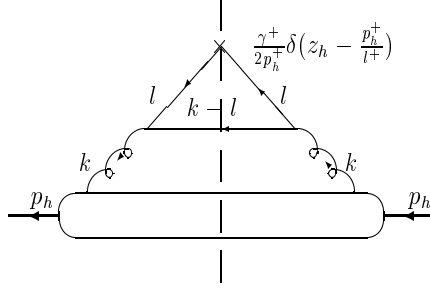


FIG. 6. Real correction ($g \rightarrow q + \bar{q}$) to the gluon fragmentation.

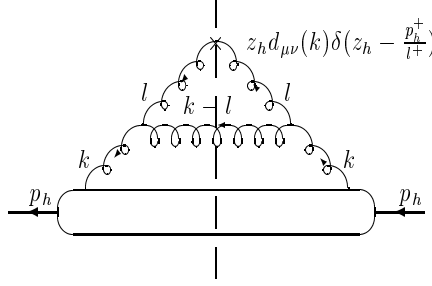


FIG. 7. Real correction ($g \rightarrow g + g$) to the gluon fragmentation.

B. First Order Correction to Gluon Fragmentation Functions

In the singlet sector, the calculation of first order correction to the gluon fragmentation function is quite similar. The first order real and virtual corrections to the gluon fragmentation function are illustrated by Feynman ‘cut’ diagrams in Figs. 6-9.

The real correction from Fig. 6 with gluon splitting into quark and anti-quark pair is,

$$\begin{aligned} \Delta F_{h/g}^{(q)}(y; x) = & \frac{y}{4x} \int dz'_f \frac{d^4 l}{(2\pi)^4} \delta\left(z'_f - \frac{p_h^+}{l^+}\right) \int d^4 \xi e^{-i l \xi} \sum_{q=1}^{n_f} \sum_{\tilde{S}} \left\{ \text{Tr} \left[\langle \psi(0) | \tilde{S}, p_h \rangle \frac{\gamma^+}{p_h^+} \langle \tilde{S}, p_h | \bar{\psi}(\xi) \rangle \right] \right. \\ & \left. + \langle \bar{\psi}(0) | \tilde{S}, p_h \rangle \frac{\gamma^+}{p_h^+} \langle \tilde{S}, p_h | \psi(\xi) \rangle \right\} \text{Tr} \left[\not{p}_h \hat{H}_g^{(q)}(l) \right], \end{aligned} \quad (50)$$

which contains the hard loop correction

$$\begin{aligned} \hat{H}_g^{(q)}(l_c) = & T_F g^2 \int \frac{d^4 k}{(2\pi)^4} (-i \gamma_\mu) (\not{k} - \not{l}_c) (i \gamma_\nu) \\ & \times [\theta(k^+ - l_c^+) - f(k^+ - l_c^+)] 2\pi \delta[k^2 - 2l_c \cdot k] \\ & \times \left(\frac{i d^{\mu\alpha}(k)}{k^2 + i\eta} \right) \left(\frac{-i d^{\nu\beta}(k)}{k^2 - i\eta} \right) \frac{y}{x} d_{\alpha\beta}(k) \delta\left(\frac{y}{x} - \frac{p_h^+}{k^+}\right). \end{aligned} \quad (51)$$

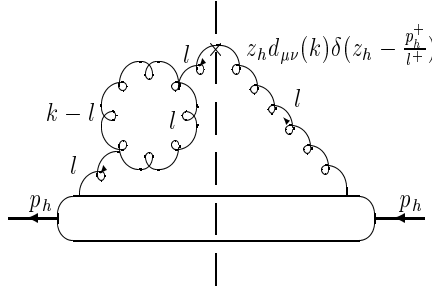


FIG. 8. Virtual correction (gluon-loop) to the gluon fragmentation.

Simplifying, we obtain

$$\begin{aligned}
d_{\alpha\beta}(l)V^{\beta\sigma\mu}(l,k-l,-k)V^{\alpha\lambda\nu}(-l,l-k,k)d_{\lambda\sigma}(k-l)d_{\mu\rho}(k)d^{\rho\tau}(k)d_{\tau\nu}(k) \\
= d_{\alpha\beta}(l)V^{\beta\sigma\mu}(l,k-l,-k)V^{\alpha\lambda\nu}(-l,l-k,k)d_{\lambda\sigma}(k-l)d_{\mu\nu}(k) \\
= -8k^2 \left[\frac{z_f}{1-z_f} + \frac{1-z_f}{z_f} + z_f(1-z_f) \right], \tag{60}
\end{aligned}$$

where $z_f = y/xz'_f$ as before. Performing the k^+ and k^- integrations leaves us with

$$\Delta F_{h/g}^{(g)}(y; z) = \frac{\alpha_s}{2\pi} \int \frac{dk_{\perp}^2}{k_{\perp}^2} \int_0^\infty \frac{dz}{z} P_{gg}(z/x) \epsilon(x-z) [\theta(x-z) + n(x-z)] F_{h/g}(y; z). \tag{61}$$

The splitting function

$$P_{gg}(z_f) = 2C_A \left[\frac{z_f}{1-z_f} + \frac{1-z_f}{z_f} + z_f(1-z_f) \right] \tag{62}$$

is familiar from the zero-temperature result.

As $z \rightarrow x$, our correction in Eq. (61) generates infrared divergences which must be canceled by virtual contributions. The loop in the diagram of Fig. 8 represents

$$\begin{aligned}
\hat{H}_g^{\mu\sigma(vg)}(l) &= \frac{1}{2} \left[\Pi_g^{\mu\nu}(l) \frac{i}{l^2 + i\eta} + \Pi_g^{\mu\nu*}(l) \frac{-i}{l^2 - i\eta} \right] \frac{y}{x} d_{\nu\rho}(l) d^{\rho\sigma}(l) \delta \left(\frac{y}{x} - \frac{p_h^+}{l^+} \right) \\
&= -\text{Im} \left[\frac{\Pi_g^{\mu\nu}(l)}{l^2 + i\eta} \right] \frac{y}{x} d_{\nu\rho}(l) d^{\rho\sigma}(l) \delta \left(\frac{y}{x} - \frac{p_h^+}{l^+} \right). \tag{63}
\end{aligned}$$

$\Pi_g^{\mu\nu}(l)$ is the non-Abelian contribution to the gluon self-energy, which can be written as

$$\Pi_g^{\mu\nu}(l) = \frac{1}{2} g^2 C_A \int \frac{d^4 k}{(2\pi)^4} V^{\beta\sigma\mu}(k, l-k, -l) V^{\alpha\lambda\nu}(-k, k-l, l) \Delta_{\alpha\beta}(k) \Delta_{\lambda\sigma}(l-k). \tag{64}$$

The symmetry factor 1/2 takes into account the indistinguishability of the intermediate gluons. As with the quark virtual correction, we must be careful when taking the limit $l \rightarrow l_c$. The result of the calculation contains the collinear divergence

$$\begin{aligned}
\lim_{l \rightarrow l_c} \frac{1}{l^2} d_{\sigma\mu}(l) [\Pi_g^{\mu\nu}(l)] d_{\nu\rho}(l) d^{\rho\sigma}(l) &= \lim_{l \rightarrow l_c} \frac{1}{l^2} [\Pi^{\mu\nu}(l)] d_{\nu\mu}(l) \\
&= i \frac{\alpha_s}{2\pi} \int_{-\infty}^{\infty} \frac{dz}{x} [\theta(z)\theta(x-z) + 2n(z)\epsilon(z)] P_{gg}(z/x) \int \frac{dk_{\perp}^2}{k_{\perp}^2}. \tag{65}
\end{aligned}$$

As in Eq.(41), $\Pi_g^{\mu\nu}(l)d_{\nu\mu}(l)/l^2$ can be divided into physical decay and scattering processes coming from Landau damping at finite temperature. This leads to the contribution

$$\Delta F_{h/g}^{(vg)}(y; x) = -\frac{\alpha_s}{4\pi} F_{h/g}(y; x) \int \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{-\infty}^{\infty} \frac{dz}{x} [\theta(z)\theta(x-z) + 2\epsilon(z)n(z)] P_{gg}(z/x). \tag{66}$$

Dividing this integral into different regions, as in Eq. (43), one can see that all of the infrared divergences present in Eq. (61) are canceled by those in Eq. (66).

An additional virtual correction to the gluon's fragmentation function is illustrated in Fig. 9. Although this correction contains no infrared divergence, it has the normal collinear divergence and thus will contribute to the scale dependence of the fragmentation function. This correction contains the hard part

$$\begin{aligned}
\hat{H}_g^{\mu\nu(vq)}(l) &= \frac{n_f}{2} \left[\Pi_q^{\mu\sigma}(l) \frac{i}{l^2 + i\eta} + \Pi_q^{\mu\sigma*}(l) \frac{-i}{l^2 - i\eta} \right] \frac{y}{x} d_{\sigma\tau}(l) d^{\tau\nu}(l) \delta \left(\frac{y}{x} - \frac{p_h^+}{l^+} \right) \\
&= -n_f \text{Im} \left[\frac{\Pi_q^{\mu\sigma}(l)}{l^2 + i\eta} \right] \frac{y}{x} d_{\sigma\tau}(l) d^{\tau\nu}(l) \delta \left(\frac{y}{x} - \frac{p_h^+}{l^+} \right). \tag{67}
\end{aligned}$$

Here, $\Pi_q^{\mu\nu}(l)$ is the quark contribution to the gluon self-energy. It has the form

$$\Pi_q^{\mu\sigma}(l) = g_s^2 T_F \int \frac{d^4 k}{(2\pi)^4} \text{Tr} [\gamma^\mu S(k) \gamma^\sigma S(k-l)] , \quad (68)$$

Standard manipulations lead to a collinear divergent term

$$\begin{aligned} \lim_{l \rightarrow l_c} \frac{1}{l^2} d_{\nu\mu}(l) [\Pi_q^{\mu\sigma}(l)] d_{\sigma\tau}(l) d^{\tau\nu}(l) &= \lim_{l \rightarrow l_c} \frac{1}{l^2} [\Pi_q^{\mu\sigma}(l) d_{\sigma\mu}(l)] \\ &= i \frac{\alpha_s}{\pi} \int_{-\infty}^{\infty} \frac{dz}{x} [\theta(z)\theta(x-z) - 2f(z)\epsilon(z)] P_{gq}(z/x) \int \frac{dk_\perp^2}{k_\perp^2} , \end{aligned} \quad (69)$$

which gives rise to the virtual correction

$$\Delta F_{h/g}^{(vq)}(y; x) = -\frac{\alpha_s n_f}{2\pi} F_{h/g}(y; x) \int \frac{dk_\perp^2}{k_\perp^2} \int_{-\infty}^{\infty} \frac{dz}{x} [\theta(z)\theta(x-z) - 2\epsilon(z)f(z)] P_{gq}(z/x) . \quad (70)$$

This correction does not contain an infrared divergence, as mentioned earlier, but the presence of a collinear divergence implies that it will still contribute to the scale dependence of the fragmentation function.

Putting all of these corrections together, we arrive at the expression

$$\begin{aligned} F_{h/g}(y; x, Q^2) - F_{h/g}(y; x, \mu^2) &= \Delta F_{h/g}^{(g)}(y; x, Q^2) + \Delta F_{h/g}^{(q)}(y; x, Q^2) + \Delta F_{h/g}^{(vg)}(y; x, Q^2) + \Delta F_{h/g}^{(vq)}(y; x, Q^2) \\ &= \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^\infty \frac{dz}{z} [\tilde{\gamma}_{gq}(z; x) F_{h/s}(y; z, \mu^2) + \tilde{\gamma}_{gg}(z; x) F_{h/g}(y; z, \mu^2)] \end{aligned} \quad (71)$$

for the renormalized gluon fragmentation function. The modified kernels are

$$\tilde{\gamma}_{gq}(z; x) = P_{gq}(z/x) \epsilon(x-z) [\theta(x-z) - f(x-z)] \quad (72a)$$

$$\begin{aligned} \tilde{\gamma}_{gg}(z; x) &= P_{gg}(z/x) \epsilon(x-z) [\theta(x-z) + n(x-z)] \\ &\quad - \frac{1}{2} \delta(1-x/z) \int_{-\infty}^{\infty} \frac{d\omega}{x} \{ P_{gg}(\omega/x) \epsilon(x-\omega) [\theta(\omega)\theta(x-\omega) + 2n(x-\omega)] \\ &\quad + 2n_f P_{gq}(\omega/x) \epsilon(\omega) [\theta(\omega)\theta(x-\omega) - 2f(\omega)] \} , \end{aligned} \quad (72b)$$

where the splitting functions $P_{ga}(z_f)$ have been given in Eqs.(54) and (62).

IV. EVOLUTION EQUATIONS AND ENERGY LOSS IN A THERMAL MEDIUM

The first order corrections to the fragmentation functions derived above can be used to resum the leading powers of $\alpha_s(\mu^2) \log(Q^2/\mu^2)$ in our perturbative expansion. In order to obtain these leading logarithms, the transverse momenta of the emitted and absorbed thermal partons must be ordered from largest to smallest as the quark propagates. Essentially, this fact arises from the form of the nested loop integrals. In order to obtain a double logarithm from

$$\int^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int^{Q^2} \frac{dl_\perp^2}{l_\perp^2 + k_\perp^2} , \quad (73)$$

we must have $l_\perp^2 > k_\perp^2$. At zero temperature, this ordering implies that the virtuality of the leading parton is similarly ordered. Absorption processes present in thermal media disallow this virtuality ordering, so only the minimum requirement of transverse-momentum ordering is observed by our corrections.

As in Eq.(18), the result of the resummation can be expressed in the compact form of an evolution equation:

$$Q^2 \frac{d}{dQ^2} F_{h/q}(y; x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{dz}{z} [\tilde{\gamma}_{qq}(z; x) F_{h/q}(y; z, Q^2) + \tilde{\gamma}_{qg}(z; x) F_{h/g}(y; z, Q^2)] \quad (74a)$$

$$Q^2 \frac{d}{dQ^2} F_{h/g}(y; x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{dz}{z} [\tilde{\gamma}_{gq}(z; x) F_{h/s}(y; z, Q^2) + \tilde{\gamma}_{gg}(z; x) F_{h/g}(y; z, Q^2)] . \quad (74b)$$

These equations express the change in our fragmentation functions as the probing scale is varied. At zero temperature, this dependence arises from the arbitrary distinction between a parton and its surrounding vacuum fluctuations. The theoretical process in which a quark emits a gluon with transverse momentum smaller than the probing scale cannot be distinguished from the process in which no such emission occurred. Hence, this state is considered part of the vacuum fluctuations inherent in the nature of the quark's propagation. On the other hand, if the transverse momentum between the quark and emitted gluon is larger than the probing scale, the final state is physically different from that of a single quark. In this case, the process represents a physical bremsstrahlung. Obviously, changes in the probing scale will give rise to changes in the effective fragmentation function.

At finite temperature, the probing scale also serves to separate the parton from the surrounding thermal bath. Just as we cannot completely separate a parton from its vacuum fluctuations, a parton in a heat bath cannot be completely separated from the fluctuations of the bath. The idea behind these two cases is quite similar, but there are some striking qualitative differences. In particular, the vacuum does not carry any energy. Therefore, the total energy of a fragmenting parton is conserved under changes of scale:

$$Q^2 \frac{d}{dQ^2} \sum_h \int dz z D_{q \rightarrow h}(z, Q^2) = 0 . \quad (75)$$

However, a thermal bath does carry energy. Hence the energy that is attributed to a jet in a thermal medium will depend on the scale used to probe the jet.

In order to analyze the effect of evolution on the fragmentation functions at finite temperature, we must first resolve the difficulty mentioned in Section III A: It is not truly consistent to treat the fragmentation of a low-energy parton without taking the fragmentation of the bath itself into account. To separate the contribution from distinct partons to our evolution equations from that of the bath, we introduce the 'assimilation functions' $p_{q,g}(z/T)$ and $\bar{p}_{q,g}(z/T) = 1 - p_{q,g}(z/T)$. Phenomenologically, $p_{q(g)}(z/T)$ represents the probability that a quark (gluon) of momentum z will thermalize with a bath of temperature T before it has time to fragment. Keeping track of only the first term in Eq. (74a), we write

$$\begin{aligned} Q^2 \frac{d}{dQ^2} F_{h/q}(y; x, Q^2) dy/x &= Q^2 \frac{d}{dQ^2} P(x \rightarrow (y, dy); Q^2) \\ &= \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{dz}{x} \tilde{\gamma}_{qq}(z; x) \bar{p}_q(z/T) P(z \rightarrow (y, dy); Q^2) \\ &\quad + \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{dz}{x} \tilde{\gamma}_{qg}(z; x) p_g(z/T) P(z \rightarrow (y, dy); Q^2) . \end{aligned} \quad (76)$$

Here, $P(x \rightarrow (y, dy); Q^2)$ is the probability that a quark of momentum x will decay into a hadron with momentum between y and $y + dy$ and transverse momentum $k_\perp^2 \leq Q^2$. In the first term on the right-hand side of the second equality, the quark maintains its identity after the bremsstrahlung. Hence the use of $F_{h/q}(y; z, Q^2)$ to describe the subsequent fragmentation is appropriate. In the second term, however, the quark thermalizes with the medium before the fragmentation takes place. Therefore, it is actually the thermal bath that produces the final-state hadron.

Although the latter contribution naturally appears when calculating the radiative corrections to the fragmentation functions at finite temperature as defined in Eq.(22), it is inconsistent with the idea of a distinct parton fragmenting into an observed hadron while propagating through a thermal bath. Experimentally, it belongs in the background and should be removed before the data are analyzed. For this reason, we retain only the first term in our evolution equation. This procedure does not come without a price. In particular, the fragmentation function described by our evolution equation is no longer rigorously associated with the thermal matrix element appearing in (22). However, this truncation is necessary in order to describe a phenomenologically meaningful function. Discrepancies between the truncated fragmentation function described below and that defined in Eq.(22) are important only when we consider hadrons which are easily generated by the bath. Since these hadrons are difficult to distinguish experimentally from the background, their contribution does not concern us here.

Extending the above analysis to the full evolution equation (74), we arrive at

$$Q^2 \frac{d}{dQ^2} F_{h/q}(y; x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{dz}{z} [\tilde{\gamma}_{qq}(z; x) \bar{p}_q(z/T) F_{h/q}(y; z, Q^2) + \tilde{\gamma}_{qg}(z; x) \bar{p}_g(z/T) F_{h/g}(y; z, Q^2)] \quad (77a)$$

$$Q^2 \frac{d}{dQ^2} F_{h/g}(y; x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{dz}{z} [\tilde{\gamma}_{gq}(z; x) \bar{p}_q(z/T) F_{h/s}(y; z, Q^2) + \tilde{\gamma}_{gg}(z; x) \bar{p}_g(z/T) F_{h/g}(y; z, Q^2)] . \quad (77b)$$

As explained above, these evolution equations contain contributions only from processes in which the parton propagating through the thermal medium remains independent of the medium.

The assimilation functions, $p_{q,g}(z/T)$, are easy to understand phenomenologically, but difficult to define rigorously. They can be determined from phenomenological models of parton thermalization, such as those used in Ref. [18]. All that is required for Eq. (77) to be well-defined mathematically is that $\bar{p}_q(\zeta) \rightarrow 0$ faster than ζ and $\bar{p}_g(\zeta) \rightarrow 0$ faster than ζ^3 as $\zeta \rightarrow 0$, along with the obvious restriction $0 \leq p_{q,g}(\zeta) \leq 1$. In addition, we impose the physical restriction $p_{q,g}(\zeta) \rightarrow 0$ as $\zeta \rightarrow \infty$.

At this point, we are ready to investigate the effects of evolution on simple observables of the parton jet. Defining the jet energies

$$E_q(x, Q^2) \equiv \sum_h \int_0^\infty \frac{y}{x} \frac{dy}{x} F_{h/q}(y; x, Q^2); \quad (78a)$$

$$E_g(x, Q^2) \equiv \sum_h \int_0^\infty \frac{y}{x} \frac{dy}{x} F_{h/g}(y; x, Q^2), \quad (78b)$$

we have

$$Q^2 \frac{d}{dQ^2} E_q(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{zdz}{x^2} [\tilde{\gamma}_{qq}(z; x) \bar{p}_q(z/T) E_q(z, Q^2) + \tilde{\gamma}_{qg}(z; x) \bar{p}_g(z/T) E_g(z, Q^2)]; \quad (79a)$$

$$Q^2 \frac{d}{dQ^2} E_g(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{zdz}{x^2} [\tilde{\gamma}_{gq}(z; x) \bar{p}_q(z/T) E_q(z, Q^2) + \tilde{\gamma}_{gg}(z; x) \bar{p}_g(z/T) E_g(z, Q^2)]. \quad (79b)$$

These functions represent the fraction of the initial parton energy carried by the final hadron jet. Given initial conditions, we can solve this set of equations numerically to obtain the scale dependence of the jet energy.

We can get some idea of the effect of evolution at large $x \gg T$ by considering $E_{q,g}(z, Q_0^2) = 1$ at some scale Q_0^2 . This is precisely the zero-temperature value of $E_{q,g}$, so it should be approximately valid at high energy. This ansatz leads to

$$Q^2 \frac{d}{dQ^2} E_q(x, Q^2) \Big|_{Q^2=Q_0^2} = \frac{\alpha_s(Q_0^2)}{2\pi} [\bar{\Gamma}_{qq}^{(2)}(x) + \bar{\Gamma}_{qg}^{(2)}(x)] \quad (80a)$$

$$Q^2 \frac{d}{dQ^2} E_g(x, Q^2) \Big|_{Q^2=Q_0^2} = \frac{\alpha_s(Q_0^2)}{2\pi} [2n_f \bar{\Gamma}_{gq}^{(2)}(x) + \bar{\Gamma}_{gg}^{(2)}(x)], \quad (80b)$$

where

$$\bar{\Gamma}_{pp'}^{(n)}(x) = \int_0^\infty \frac{dz}{x} \left(\frac{z}{x}\right)^{n-1} \tilde{\gamma}_{pp'}(z; x) \bar{p}_{p'}(z/T). \quad (81)$$

Contributions to Eq.(80a) can be understood in terms of the energy loss and gain to different partonic degrees of freedom due to the basic bremsstrahlung/absorption process. The first and second terms represent energy gain to the quark and gluon components of the fragmenting quark, respectively. Since the vacuum does not carry any energy, energy losses to the quark degrees of freedom must be compensated completely by gains in the gluon sector at zero temperature. Explicit calculation yields

$$\bar{\Gamma}_{qq}^{(2)}(x)|_{T=0} = -\bar{\Gamma}_{qg}^{(2)}(x)|_{T=0} = -\frac{4}{3} C_F, \quad (82)$$

in agreement with our expectations.

As the temperature is increased from zero, several effects work to alter this result. To analyze their contributions separately, it is useful to replace $\bar{p}_{q,g}(z/T)$ with $1 - p_{q,g}(z/T)$ in $\bar{\Gamma}_{q,g}^{(2)}(x)$. The ‘1’ contains the vacuum contributions described above, as well as the thermal effects of induced emission and absorption to $\bar{\Gamma}_{qq}^{(2)}(x)$ and those of Pauli blocking and quark annihilation to $\bar{\Gamma}_{qg}^{(2)}(x)$. The assimilation functions take the energy flow from explicit parton degrees of freedom to the heat bath into account. These contributions act to reduce the amount of energy available to both quark and gluon degrees of freedom in the jet.

Induced gluon emission and absorption allow the thermal bath to change explicitly the energy of the distinct parton degrees of freedom. These processes only involve thermal gluons, so they do not contribute to $\bar{\Gamma}_{qg}^{(2)}(x)$. The form of the splitting function $P_{qq}(z_f)$ guarantees that the absorption process wins out over that of induced emission, giving rise to the net fractional energy gain

$$\delta_1 \bar{\Gamma}_{qq}^{(2)}(x) = \frac{5}{6} C_F \left(\frac{\pi T}{x} \right)^2 + \mathcal{O}((T/x)^4) \quad (83)$$

to the quark degrees of freedom. The analogous processes of Pauli blocking and quark annihilation affecting the gluonic degrees of freedom cancel almost exactly, leaving a net energy gain from annihilation that is suppressed by $\exp(-x/T)$.

Our distinct parton degrees of freedom lose energy to the medium mainly through the direct bremsstrahlung process. In order to obtain an explicit expression for this contribution to the energy loss, we must take a specific form for the assimilation functions. For illustrative purposes, we choose the hard cut-off $p_{q,g}(\zeta) = \theta(\eta_{q,g} - \zeta)$, where $\eta_{q,g} \sim 1$ governs the thermalization of partons introduced to our thermal medium. This choice yields the fractional energy losses

$$\delta_2 \bar{\Gamma}_{qq}^{(2)}(x) = -\frac{1}{2} C_F \left(\frac{\eta_q T}{x} \right)^2 - \frac{1}{3} C_F \left(\frac{\eta_q T}{x} \right)^3 + \mathcal{O}((T/x)^4); \quad (84a)$$

$$\delta_2 \bar{\Gamma}_{gg}^{(2)}(x) = -2C_F \frac{\eta_g T}{x} + C_F \left(\frac{\eta_g T}{x} \right)^2 - \frac{1}{3} C_F \left(\frac{\eta_g T}{x} \right)^3 + \mathcal{O}(\exp(\eta_g - x/T)) . \quad (84b)$$

The appearance of a linear (in T/x) term in Eq. (84b) is quite striking since it dominates all other corrections as $x \rightarrow \infty$. This term is generated by the divergent behavior of the soft gluon emission rate. Since $P_{gg}(z_f) \rightarrow 2/z_f$ as $z_f \rightarrow 0$, the fractional energy carried by soft gluons saturates. This causes the amount of energy absorbed by the bath to be proportional to the size of the energy window that thermalizes with the bath, which is dictated by the temperature T . Such a linear term will appear in the asymptotic expansion (as $x \rightarrow \infty$) for any choice of assimilation function satisfying the requirements as we have given before.

Putting all of this together and performing similar calculations in the gluon sector, we rewrite Eq.(80) as

$$Q^2 \frac{d}{dQ^2} E_q(x, Q^2) \Big|_{Q^2=Q_0^2} = -\frac{\alpha_s(Q_0^2) C_F}{2\pi} \frac{T}{x} \left[2\eta_g \left(1 - \frac{\eta_g T}{2x} \right) + \eta_q \frac{\eta_q T}{2x} - \frac{5}{6} \pi^2 \frac{T}{x} + \mathcal{O}((T/x)^2) \right] \quad (85a)$$

$$Q^2 \frac{d}{dQ^2} E_g(x, Q^2) \Big|_{Q^2=Q_0^2} = -\frac{\alpha_s(Q_0^2)}{2\pi} \frac{T}{x} \left[2C_A \left(\eta_g - \eta_g \frac{\eta_g T}{x} - \frac{1}{3} \pi^2 \frac{T}{x} \right) + 2n_f T_F \left(\eta_q \frac{\eta_q T}{2x} - \frac{1}{6} \pi^2 \frac{T}{x} \right) + \mathcal{O}((T/x)^2) \right] . \quad (85b)$$

As before, the linear term in the gluon sector is generated by the divergence of the soft gluon bremsstrahlung rate. These expressions indicate that the parton energy which contributes to a hadronic jet is reduced by an amount approximately proportional to the temperature of the thermal medium as the transverse momentum cut-off of the jet is increased. The effect is slightly smaller at lower energies due to absorption processes. This phenomenon arises from the fact that increasing the momentum cut-off increases the amount of transverse phase space available to partonic sub-processes, which increases the amount of energy lost to the bath. Although our result was derived from a constant input distribution and rather simplistic assimilation functions, the qualitative effect of evolution is expected to be similar to that of a more realistic calculation. In particular, the dominant behavior of the thermalization mechanism over the phenomena of induced emission, absorption, annihilation and Pauli blocking is expected to persist in a complete calculation.

Fluctuations in the thermal medium can also screen the quark flavor of a jet. The valence structure of a jet is measured by the quark number

$$N_q(x, Q^2) = \sum_{h(q)} \int_0^\infty \frac{dy}{x} [F_{h/q}(y; x, Q^2) - F_{h/\bar{q}}(y; x, Q^2)] , \quad (86)$$

where the sum goes over all hadrons h whose valence structure includes q . Only fragmentation processes in which the leading quark contributes to the valence structure of the observed hadron can contribute to Eq. (86), since all others will cancel in the difference. In vacuum, $N_q(x, Q^2)$ represents the net flavor of the jet. Since QCD interactions conserve flavor,

$$N_q(x, Q^2)|_{T=0} = 1 \quad (87)$$

is independent of scale or parton energy. At finite temperature, the leading quark in a jet can scatter into the thermal bath or annihilate with an anti-quark in the bath. As before, these processes generate a dependence of the quark number on both scale and parton energy. Eq.(77a) leads to the evolution equation

$$Q^2 \frac{d}{dQ^2} N_q(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^\infty \frac{dz}{x} \tilde{\gamma}_{qq}(z; x) \bar{p}_q(z/T) N_q(z, Q^2) \quad (88)$$

for $N_q(x, Q^2)$.

As before, we illustrate the effect of this evolution equation at high energy by considering $N_q(x, Q_0^2) = 1$ at the scale Q_0^2 . This gives

$$Q^2 \frac{d}{dQ^2} N_q(x, Q^2) \Big|_{Q^2=Q_0^2} = \frac{\alpha_s(Q_0^2)}{2\pi} \bar{\Gamma}_{qq}^{(1)}(x) . \quad (89)$$

Using the same assimilation function as above, we arrive at the expression

$$\bar{\Gamma}_{qq}^{(1)}(x) = -C_F \frac{T}{x} \left[\eta_q + \eta_q \frac{\eta_q T}{2x} - \frac{1}{4} \pi^2 \frac{T}{x} + \mathcal{O}((T/x)^2) \right] , \quad (90)$$

which indicates a net transfer of flavor from the jet to the bath as the momentum scale is increased. It is interesting to note that while the effect of the medium on energy loss becomes larger with the energy of the leading parton, its effect on flavor loss diminishes as the energy is increased. This is essentially because the *fractional* rather than absolute energy loss is comparable to the net flavor loss.

We can also consider the energy loss of a quark propagating through a thermal medium from a slightly different point of view. Our interpretation of Eq.(74) indicates that

$$dP_{qq}(x \rightarrow z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \tilde{\gamma}_{qq}(z; x) \bar{p}_q(z/T) \frac{dz}{x} \frac{dQ^2}{Q^2} \quad (91)$$

represents the probability that a leading quark of energy x will split into a quark with energy between z and $(z + dz)$ and transverse momentum between Q^2 and $Q^2 + dQ^2$ and a gluon. The effective energy loss of the leading quark can be summarized as the expectation value of the emitted gluon's energy.

To obtain a number for this quantity, we must first determine a range of values for the probing scale. Since we have identified Q^2 phenomenologically with the transverse momentum $k_\perp^2$ between the quark and gluon, it is natural to use the kinematic limit

$$k_\perp^2 \leq 4z|x - z| , \quad (92)$$

as shown in Ref. [4]. This limit depends on the details of the process, so our result cannot be directly related to a renormalization group analysis. For this reason, the appearance of the running coupling in Eq.(91) is not fully consistent with the idea of renormalization. We will take the coupling evaluated at some fixed scale, Q_0^2 , characteristic of our process. The infrared scale μ^2 , given as the Debye screening mass, is taken as a lower limit of the k_T integral. These considerations lead to the quark fractional energy loss

$$\begin{aligned} \frac{\langle z_g \rangle}{x} &= \frac{\alpha_s(Q_0^2)}{2\pi} \int_0^\infty \frac{dz}{x} \left(1 - \frac{z}{x}\right) \int_{\mu^2}^{4z|x-z|} \frac{dk_\perp^2}{k_\perp^2} \tilde{\gamma}_{qq}(z; x) \bar{p}_q(z/T) \\ &= \frac{\alpha_s(Q_0^2) C_F}{2\pi} \left[\left(\frac{4}{3} \log \frac{4x^2}{\mu^2} - \frac{49}{18} \right) - \frac{\eta_q T}{x} \left(\log \frac{4x\eta_q T}{\mu^2} - 1 \right) \right. \\ &\quad \left. - \frac{2}{3} \left(\frac{\pi T}{x} \right)^2 \left(\log \frac{4xT}{\mu^2} + 2 - \gamma_E + \frac{6\zeta'(2)}{\pi^2} - \frac{3}{4} \frac{\eta_q^2}{\pi^2} \right) + \mathcal{O}((T/x)^3) \right] , \end{aligned} \quad (93)$$

where we have again used the hard cut-off form of $\bar{p}_q(z/T)$.

The first term in this expression represents quark energy loss due to vacuum bremsstrahlung, and is independent of the medium. Some of the energy lost to the quark through these processes re-appears in the gluon fragmentation, while the rest is lost to the medium. The difference is unimportant here since we do not track the gluon degrees of freedom. The second term represents the shift in net quark number of the jet as the scale varies. Its sign indicates that it represents energy *gain* to the quark when the energy is large. This is because we have considered the normalized energy loss to one quark, while the net quark number of the jet is actually less than one due to annihilation effects in the medium. The result is that the energy *per quark* increases. The third term corresponds to energy that comes from the medium directly either from absorption/emission processes or via soft quark thermalization. This contribution indicates that the absorptive processes in the plasma overcome the emissive processes in the absence of thermalization ($\eta_q = 0$), leading to a net energy gain. The details of the calculation depend on our choice of assimilation function, as well as transverse momentum cut-offs, but the results are nonetheless interesting. We note here that secondary scattering effects, such as those studied in Ref. [4], appear only at the next-to-leading-log approximation. These are not taken into account in our formulae.

V. CONCLUSION

In this paper we have taken a first step in the study of parton fragmentation functions in a thermal QCD medium via thermal field theory. We extended the definition of the parton fragmentation functions to the case of a thermal medium with finite temperature which will reduce to the conventional fragmentation functions in the zero temperature limit. We then calculated the radiative corrections to these fragmentation functions within the framework of perturbative QCD at finite temperature and derived the corresponding DGLAP evolution equations. The modified evolution equations can be written in the same form as at zero temperature. However, the effective (or medium modified) splitting functions depend explicitly on the temperature and contain terms corresponding to various physical processes such as radiation, absorption, backward and forward scattering, etc. The introduction of the temperature causes the effective splitting functions to depend explicitly on the value of the initial parton energy. This complicates the renormalization group analysis of the effective fragmentation functions and causes us to modify their interpretation slightly. However, we have shown that the modified evolution equations are self-consistent. There are both linear and logarithmic infrared divergences in the radiative corrections. Some cancel among themselves while others are canceled by the virtual corrections.

As an example of the application of the evolution equations, we also derived the evolution equations for the net quark number and the energy loss (gain). We find that one has to introduce 'assimilation functions' in order to separate the hadronization of the thermal bath from that of the initial parton jet. We find similarly as in an earlier study [4] that the absorption of thermal gluons wins out over the stimulated emission, due to the form of $q \rightarrow qg$ splitting function. This effective medium-induced energy gain will then reduce the total energy loss experienced by a propagating parton.

As we have emphasized in the Introduction, we have considered only the first order corrections in the leading logarithmic approximation. In this case we only included sequential parton emission or absorption. Collision-induced bremsstrahlung occurs beyond this order of perturbation theory. The intermediate thermal gluon will no longer be on-shell and its propagator will also develop an imaginary part. This will result in collision-induced radiation. When the formation time of the gluon radiation is larger than the collisional rate which is contained in the imaginary part of the propagator, there will be LPM suppression of the induced radiation [19]. We hope to address this problem in future studies.

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