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Luminosity Optimization With Offset, Crossing Angle, and Distortion*

Juhao Wu†, T.O. Raubenheimer, SLAC, Menlo Park, CA 94025, USA

Abstract

In a linear collider, sources of beam jitter due to kicker noise, quadrupole vibration and long-range transverse wakefields will lead to beam offsets and tilts at the Intersection Point (IP). In addition, sources of emittance dilution such as short-range transverse wakefields or dispersive errors will lead to internal beam distortions. When the IP disruption parameter is large, these beam imperfections will be amplified by a single bunch kink instability which will lead to luminosity loss. In this paper, we study the luminosity loss and then the optimization required to partially cancel the luminosity loss both analytically and with direct simulation.

Introduction

To achieve the desired luminosity in a future linear collider, the beams are focused to small spot sizes and the resulting beam-beam forces can be very large. With oppositely charged beams, the beam-beam forces will lead to a mutual focusing or pinch which further increases the beam densities and the luminosity and is referred to as the luminosity enhancement. In addition, if the beams are offset from each other, the attractive beam-beam force can bring the beams closer together possibly recovering some of the lost luminosity. Unfortunately, if the beam-beam force is too large, this attraction can lead to an instability much like a plasma two-stream instability which is referred to as a single bunch kink instability [1, 2], which can be parameterized with the disruption parameter: $D_{x(,y)} \equiv \sigma_z/[f_{x(,y)}] = 2N_b r_e \sigma_z/[\gamma\sigma_{x(,y)}(\sigma_x+\sigma_y)]$, where $f_{x(,y)}$ is the focal length due to the beam-beam force; $\sigma_{x(,y,z)}$ is the rms beam size, N_b the number of particle per beam, r_e the electron classical radius, and γ the Lorentz factor. We will follow closely to the approach in Ref. [2]

Equations of Motion

Suppose that two beams move towards each other with velocity v, the equations of motion read [1]

$$\left(\frac{\partial}{\partial t} \pm v\frac{\partial}{\partial s}\right)^2 y_{l(,r)} = -\frac{2\lambda r_e c^2\left[y_{l(,r)} - y_{r(,l)}\right]}{\sigma_y(\sigma_x+\sigma_y)\gamma}, \quad (1)$$

where, $y_{l(,r)}$ is the centroid displacements of the electron (positron) beam from the reference axis and λ is the line density. The readers may refer to Ref. [2] for details.

Initial Value Problem

The internal coordinate z is introduced to label the slice at a distance z from the head of the beam, and $0 < z < l$, where l is the beam length. We define $t = 0$ when the heads of the two beams collide. We also define $s = 0$ as the IP where the two beams first collide. The positive s direction is to the right. Hence in the left-coming beam, the slice z will be located at $s = vt - z$ at time t when the beam head is at the location $s = vt$. In the right-coming beam, we also introduce z to describe the distance between a certain slice and the head of the beam, and again $0 < z < l$. Hence, when the head of the right-coming beam is at location $s = -vt$ at time t, the slice z is at location $s = -vt+z$. We now use (s, z) as the independent variable pair, and we define $k_0^2 \equiv 2\lambda r_e/\left[\sigma_y(\sigma_x+\sigma_y)\gamma\right]$. The coordinate system is shown in Fig. 1. Now let us study two cases. For the first case, the right-coming beam has an initial offset y_{r0}, and the left-coming beam is undistorted and on-axis. The second case, the right-coming beam is crabbed, and the left-coming beam is perfect. The initial conditions are then

$$y_l(0,z) = 0 \;\text{ and }\; \left.\frac{\partial y_l(s,z)}{\partial s}\right|_{s=0} = 0\,, \quad (2)$$

for the left-coming electron beam. In the first case,

$$y_r(0,z) = y_{r0} \;\text{ and }\; \left.\frac{\partial y_r(s,z)}{\partial s}\right|_{s=0} = 0\,, \quad (3)$$

for the right-coming positron beam. In the second case,

$$y_r(0,z) = \theta_{r0} z \;\text{ and }\; \left.\frac{\partial y_r(s,z)}{\partial s}\right|_{s=0} = 0\,. \quad (4)$$

The equations of motion together with the initial conditions yield the following integral representation of the solution

$$\begin{aligned} y_l(s,z) &= y_l(z/2,z)\cos\left[k_0(s-z/2)\right] \\ &+ \left.\frac{\partial y_l(s,z)}{\partial s}\right|_{s=z/2}\frac{\sin\left[k_0(s-z/2)\right]}{k_0} \\ &+ k_0\int_{z/2}^{s} ds' y_r(-s', 2s'-z)\sin[k_0(s-s')]\,. \end{aligned} \quad (5)$$

Similarly, for the right-coming beam, we have

$$\begin{aligned} y_r(s,z) &= y_r(-z/2,z)\cos\left[k_0(s+z/2)\right] \\ &+ \left.\frac{\partial y_r(s,z)}{\partial s}\right|_{s=-z/2}\frac{\sin\left[k_0(s+z/2)\right]}{k_0} \\ &+ k_0\int_{-z/2}^{s} ds' y_l(-s', -2s'-z)\sin[k_0(s-s')]. \end{aligned} \quad (6)$$

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† jhwu@SLAC.Stanford.EDU

Series Solution

Let us solve the above set of Eqs. (5) and (6) via a series solution approach [2, 3, 4]. We expand $y_{r(,l)}(s,z)$ in a series of powers in k_0

$$y_{r(,l)}(s,z) = \sum_{n=0}^{\infty} y_{r(,l)}^{(n)}(s,z) \ , \tag{7}$$

and obtain the nth-order term from the $(n-1)$th-order term. According to Eq. (5), for $n = 1, 2, 3, \cdots$, $y_l(s,z)$ would be

$$y_l^{(n)}(s,z) = k_0 \int_{z/2}^{s} ds' y_r^{(n-1)}(-s', 2s'-z) \sin[k_0(s-s')]; \tag{8}$$

and similarly, according to Eq. (6), for $y_r(s,z)$, we have

$$y_r^{(n)}(s,z) = k_0 \int_{-z/2}^{s} ds' y_l^{(n-1)}(-s', -2s'-z) \sin[k_0(s-s')]. \tag{9}$$

According to Eqs. (5) and (6) with the initial conditions in Eqs. (2), (3), and (4), we can get series solution as Ref. [2]. Here, we only give explicit asymptotic solution.

Offset For the first case, *i.e.*, the right coming beam has an initial offset, we have

$$\begin{aligned} y_r(s,z) &\approx -\frac{i y_{r0}}{2} \sqrt{\frac{|2s+z|}{z}} J_1\left(ik_0\sqrt{z|2s+z|}/2\right) \\ &\times \sin[k_0(s+z)] \qquad (10) \\ &\approx \frac{y_{r0}}{2}\sqrt{\frac{|2s+z|}{z}} \left(\pi k_0 \sqrt{z|2s+z|}\right)^{-1/2} \\ &\times \exp\left\{k_0\sqrt{z|2s+z|}/2\right\} \sin[k_0(s+z)], \end{aligned}$$

for $-z/2 > s > -(l+z)/2$; and

$$\begin{aligned} y_l(s,z) &\approx -\frac{y_{r0}}{2} J_0\left(ik_0\sqrt{z(2s-z)}/2\right) \cos[k_0(s-z)] \\ &\approx -\frac{y_{r0}}{2} \left(\pi k_0\sqrt{z(2s-z)}\right)^{-1/2} \\ &\times \exp\left\{k_0\sqrt{z(2s-z)}/2\right\} \cos[k_0(s-z)], (11) \end{aligned}$$

for $(l+z)/2 > s > z/2$.

Crabbed beam For the second case, *i.e.*, the right coming beam is crabbed, we have

$$\begin{aligned} y_r(s,z) &\approx \frac{i\theta_{r0}}{k_0}\sqrt{\frac{|2s+z|}{z}} J_1\left(ik_0\sqrt{z|2s+z|}/2\right) \\ &\times \cos[k_0(s+z)] \qquad (12) \\ &\approx -\frac{\theta_{r0}}{k_0}\sqrt{\frac{|2s+z|}{z}} \left(\pi k_0\sqrt{z|2s+z|}\right)^{-1/2} \\ &\times \exp\left\{k_0\sqrt{z|2s+z|}/2\right\} \cos[k_0(s+z)], \end{aligned}$$

for $-z/2 > s > -(l+z)/2$; and

$$\begin{aligned} y_l(s,z) &\approx -\frac{\theta_{r0}}{k_0} J_0\left(ik_0\sqrt{z(2s-z)}/2\right) \sin[k_0(s-z)] \\ &\approx -\frac{\theta_{r0}}{k_0}\left(\pi k_0\sqrt{z(2s-z)}\right)^{-1/2} \\ &\times \exp\left\{k_0\sqrt{z(2s-z)}/2\right\} \sin[k_0(s-z)], (13) \end{aligned}$$

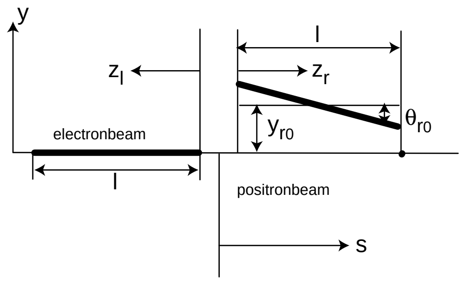

Figure 1: Schematic of the initial condition of the two beams and also the definition of the notations: s, $z_{r(,l)}$, l, and y_{r0}. In Fig. 1, $\theta_{r0} < 0$.

for $(l+z)/2 > s > z/2$.

Luminosity

The luminosity is defined as [5]

$$\mathcal{L} = 2N_b^2 v \int dxdydsdt \ n_l(x,y,z_l,t) n_r(x,y,z_r,t), \tag{14}$$

where $z_l = vt - s$ and $z_r = vt + s$ and we assume the same number population N_b in each beam and head-on collisions. The distribution function is normalized to unit, *i.e.*, $\int dxdyds \ n_{l(,r)}(x,y,z_{l(,r)},t) = 1$. Assuming Gaussian transversely and uniform longitudinally, and ignoring the luminosity enhancement due to beam-beam pinch, the 'geometric' luminosity is $\mathcal{L}_{00} = N_b^2/[4\pi\sigma_x\sigma_y]$. Finally, the nominal luminosity $\mathcal{L}_0$, including the effect of the luminosity enhancement, is found by multiplying by the enhancement factor H_D which is typically between 1 and 2 for flat beam collisions, *i.e.*, $\mathcal{L}_0 = \mathcal{L}_{00}H_D$. Now, we study the luminosity loss due to the beam-beam disruption. Given the solutions in Eqs. (10) $-$ (13), we compute the luminosity. On the other hand, we also simulate the luminosity loss via GuineaPig [6] for longitudinal Gaussian distribution.

Table 1: Summary of the parameters for the US Cold [7].

E (GeV)	N_b (10^{10})	σ_x (μm)
250	2.0	0.543
$\sigma_{x'}$ (μrad)	σ_y (nm)	$\sigma_{y'}$ (μrad)
36	5.7	14
σ_z (m m)	σ_δ (%)	D_y
0.3	0.1	22.0

Luminosity loss

Now let us illustrate how the beam-beam disruption leads to a large luminosity loss. We study the ILC US Cold [7] with the parameters in Table 1. In Figs. 2, 3, 4, and 5,

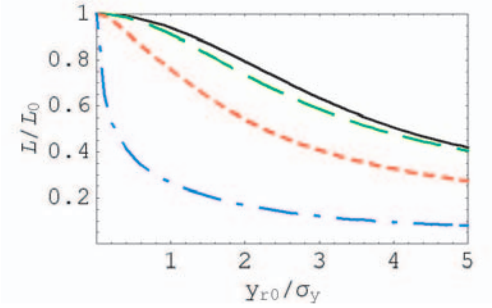

Figure 2: Luminosity loss as a function of offset y_{r0} for various $D_y = 1$ (black solid), 5 (green long-dashed), 10 (red dashed), and 50 (blue dash-dotted).

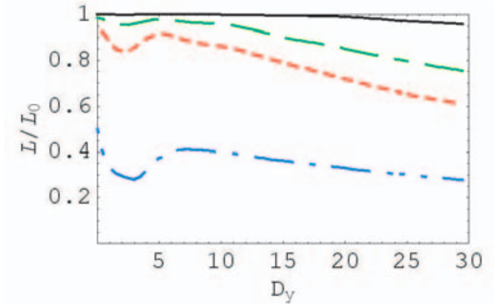

Figure 5: Luminosity loss as a function of D_y for various $\theta_{r0} = 1$ (black solid), 5 (green long-dashed), 10 (red dashed), and 50 (blue dash-dotted) μrad.

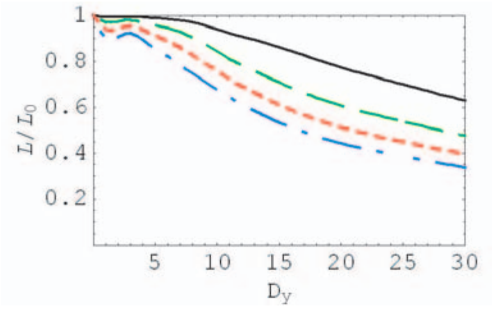

Figure 3: Luminosity loss as a function of D_y for various $y_{r0}/\sigma_y = 1/3$ (back solid), $2/3$ (green long-dashed), 1 (red dashed), and $4/3$ (blue dash-dotted).

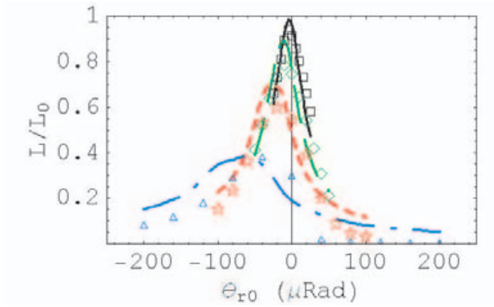

Figure 6: Luminosity loss as a function of initial crabbing angle θ_{r0} for various offset $y_{r0} = 1/3$ (black), 1 (green), 2 (red), and 5 (blue). Curves for analytical result and symbol for GuineaPig simulation.

we luminosity loss as a function of various parameters. All these plots show that when D_y is large, say $D_y > 10$, the beam-beam disruption tends to exponentially amplify initial offset as well for initial crabbing angle. This is similar to the modulation studied in Ref. [2].

Luminosity optimization

We now study possible optimization via partial cancellation among various imperfection. As an example, imaging that the two beams come in with an offset y_{r0}; naively, we would crab the right-coming beam with a negative angle, so that the beam-beam attraction will bend the beam opposite to its original angle. The configuration is shown in Fig. 1. Now, in Fig. 6, the right-coming beam has an initial offset of $y_{r0}/\sigma_y = 1/3$ (black), 1 (green), 2 (red), and 5 (blue). We vary the initial angle θ_{r0}. It is clearly shown that the optimization for each case comes at a negative θ_{r0}.

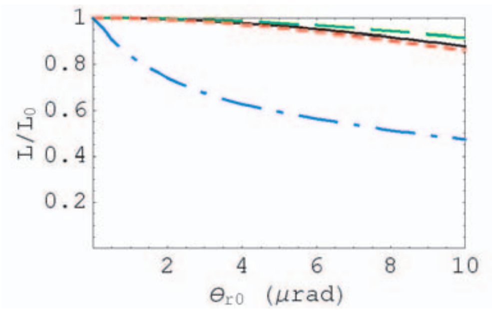

Figure 4: Luminosity loss as a function of θ_{r0} for various $D_y = 1$ (black solid), $D_y = 5$ (green long-dashed), $D_y = 10$ (red dashed), and $D_y = 50$ (blue dash-dotted).

Discussion

Due to the strong beam-beam disruption, initial imperfectness is exponentially amplified, which leads to substantial luminosity loss for $D_y > 10$. In reality, beams have offset, crossing angle, and also modulation [2], then a detailed optimization is necessary, besides feedback approach.

REFERENCES

[1] Y.H. Chin, DESY 87-011, Jan. 1987.

[2] J. Wu, T.O. Raubenheimer, A.W. Chao, A. Seryi, and C.K. Sramek, these PAC'05 proceedings.

[3] A.W. Chao *et al.*, *Nucl. Instrum. and Methods*, **178**, 1 (1980).

[4] T.O. Raubenheimer, F. Zimmermann, PRE **52**, 5487 (1995).

[5] C. Møller, K. Danske Vidensk. Selsk. Mat.-Fys. Medd. **23**, 1 (1945).

[6] D. Schulte, Ph.D. thesis, University of Hamburg 1996.

[7] http://www.slac.stanford.edu/xorg/accelops/.