SGP-TR-141-23

PROCEEDINGS

SEVENTEENTH WORKSHOP
GEOTHERMAL RESERVOIR ENGINEERING

January 29-31, 1992

Henry J. Ramey, Jr., Roland N. Horne,
Paul Kruger, Frank G. Miller,
William E. Brigham, Jean W. Cook
Stanford Geothermal Program
Workshop Report SGP-TR-141

DISCLAIMER

PROCEEDINGS, Seventeenth Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 29-31, 1992
SGP-TR-141

A DUAL-POROSITY RESERVOIR MODEL WITH AN IMPROVED COUPLING TERM

Robert W. Zimmerman, Gang Chen, and Gudmundur S. Bodvarsson

Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

Abstract

A new dual-porosity model is developed for single-phase flow in fractured/porous media. As in the commonly-used approach, flow is assumed to take place through the fracture network, and between the fractures and matrix blocks. The matrix blocks are treated in a lumped-parameter manner, with a single average pressure used for each matrix block. However, instead of assuming that fracture/matrix flux is proportional to the difference between the fracture pressure and matrix pressure at each point, as in the Warren-Root model, a nonlinear equation is used which accurately models the flux at both early and late times. This flux equation is verified against analytical solutions for spherical blocks with prescribed pressure variations on their boundaries. This equation is then used as a source/sink term in the numerical simulator TOUGH. The modified code allows more accurate simulations than the conventional Warren-Root method, and with a large savings in computational time compared to methods which explicitly discretize the matrix blocks.

Introduction

Numerical simulation of flow processes in fractured rocks is a formidable task, due to the often complex geological and hydrological characteristics of such formations. The specific specific geometry and other characteristics of the fracture system is generally not known, so it is not possible to explicitly model individual fractures or individual matrix blocks. To circumvent this difficulty, so-called "double-porosity" models are often used. In double(or dual)-porosity models, knowledge of the actual geometric and hydrological features of the fracture network are not required, but instead only "average" properties, such as a typical fracture spacing, are needed. In a numerical simulation of a flow process in a dual-porosity system, the individual computational cells are assumed to be sufficiently large so that it is meaningful to assign suitably-averaged "effective" properties to them. Despite this simplification, numerical modeling of dual-porosity reservoirs is still a complicated and costly process. In general, fairly fine spatial discretization is needed in the matrix blocks - typically five to ten "matrix" cells are required for each "fracture" cell. Hence modeling of a fractured reservoir will require five-to-ten times as many computational cells as would be needed for a porous medium simulation of a reservoir of the same overall size.

Although most geothermal reservoirs reside in fractured rocks, most models that have been developed to analyze their behavior have been based on porous medium approximations. It is well-known, however, that porous medium models are poorly suited for predicting certain aspects of the behavior of geothermal wells, especially enthalpy transients, thermal front migration due to injection, or chemical tracer movement. Nevertheless, in many cases the porous medium approximation must be invoked, due to constraints of time or cost. There is consequently a great need for improved numerical capabilities for the modeling of fractured geothermal reservoirs, using accurate and appropriate models.

In this paper we present a new method for modeling fractured reservoirs that can simulate reservoir behavior more efficiently and economically. The method involves analytical treatment of fracture/matrix interflow, eliminating the need for discretization of the matrix blocks. This allows accurate dual-porosity simulations, using a substantially smaller number of cells than would be needed in a fully-discretized simulation. Although at this time we can simulate only single-phase, isothermal processes with our semi-analytical approach, our intention is to extend the basic approach to the treatment of two-phase, non-isothermal processes.

Dual-Porosity Models

When a single-phase, slightly compressible fluid flows through a macroscopically-homogeneous fractured medium, the fluid pressure in the fractures is governed by the usual diffusion equation used in reservoir engineering (Matthews and Russell, 1967):

$$\phi_f c_f \frac{\partial P_f(x_f,t)}{\partial t} = \frac{k_f}{\mu}\nabla^2 P_f(x_f,t) + Q(x_f,t). \qquad (1)$$

In this equation, t is the time, x_f is the position vector of a point in the fracture continuum, k_f is the absolute permeability of the fracture continuum, ϕ_f is the total fracture porosity, and c_f is the total compressibility of the fractures and the fluid within them. Q is a source term that represents the net addition of fluid to the fracture system from the matrix blocks, per unit of total volume. The pressure P_f represents the fluid pressure in the fractures, averaged over some suitably large representative elementary volume (REV; see Chen, 1989). The Laplacian operator ∇^2 represents the divergence of the gradient, and takes on different specific

forms for each type of coordinate system (*i.e.*, Cartesian, cylindrical, or spherical). The fracture continuum is assumed to occupy all of the physical space spanned by the variable x_f, with the actual pore volume of the fractures accounted for by the porosity factor.

A dual-porosity model can be formulated by first imagining that, at each point x_f, there is located a matrix block of some specified shape. Inside each block the fluid pressure P_m will, in general, vary from point to point. Two position variables are needed to identify a point inside a matrix block; x_m will locate the point within the block, relative to, say, the block's center of gravity, while x_f is needed as a label to fix the location of that particular block within the fracture continuum. Fluid flow *within* each matrix block is governed by an equation of a similar form as (1), which can be written as

$$\phi_m c_m \frac{\partial P_m(x_m, t; x_f)}{\partial t} = \frac{k_m}{\mu} \nabla^2 P_m(x_m, t; x_f) . \qquad (2)$$

In this equation, the parameters have meanings analogous to those in eq. (1). The derivatives implicit in the operator ∇^2 are taken with respect to the local variable x_m, while the variable x_f is merely used as a label. The fracture/matrix interflow term Q does not appear explicitly in eq. (2) since, whereas the interflow is assumed to be *distributed* throughout the fracture continuum as a source/sink term, the interflow enters the matrix blocks only at their *boundaries*. The pressures at the outer boundary of a given matrix block located at point x_f in the fracture continuum are always assumed to be equal to the fracture pressure at that point - *i.e.*, if x_m is on the boundary of the matrix block, then $P_m(x_m, t; x_f) = P_f(x_f, t)$.

The system of equations (1) and (2) actually represent a single equation for the fracture continuum, along with a family of equations for the matrix blocks that are located at each point x_f. These equations are coupled through the term Q, which can be found by integrating the flux out of the boundary of each matrix block, using Darcy's law (see Duguid and Lee, 1977):

$$Q(x_f, t) = \frac{-1}{V_m} \int_{\partial V_m} \frac{k_m}{\mu} \frac{\partial P_m}{\partial n} dA , \qquad (3)$$

where the derivative of P_m is taken in the direction of the outward unit normal vector to the boundary ∂V_m of the block, and the integral is taken over the entire boundary. A well-posed boundary-value problem for the system of equations (1-3) would typically require initial conditions for P_m and P_f, as well as boundary conditions for the pressures at the outer boundary of the macroscopic region under investigation, *i.e.*, at the outer boundary of the x_f domain. If the initial state were one of local equilibrium, as would often be the case, we would have $P_f(x_f, t=0) = P_m(x_m, t=0; x_f)$ at each point x_f.

Dual-porosity models of the type discussed above, in which diffusion equations are solved in both the fracture and the matrix systems, are sometimes used in numerical simulations. An example is the MINC method (Pruess and Narasimhan, 1985), in which the matrix blocks are discretized into nested shell-like cells. In order to achieve high accuracy over all time scales, however, we have found that about ten computational cells are needed in each matrix block. In some cases, only one cell is used to model each matrix block; this approach is then basically a numerical implementation of the Warren-Root model, which is discussed below. As is well known, when solving problems in dual-porosity media, the Warren-Root model is inaccurate during a certain intermediate time regime. The MINC method approaches the exact response as the number of nested matrix shells increases. The method we have developed, which treats fracture/matrix flow with a nonlinear ordinary differential equation, is reasonably accurate over all time scales.

Warren-Root Lumped-Parameter Models

The Warren and Root (1963) model is a simplified form of dual-porosity model in which no attempt is made to solve the diffusion equation within each block, but the blocks are instead treated in a "lumped parameter" fashion. The pressure in the matrix blocks is then governed by an ordinary, rather than partial, differential equation. If implemented into numerical simulators in the form of a source/sink term for the fracture elements, the amount of computational time spent on solving for the matrix block pressure, and the fluid-interaction term Q, becomes negligible compared to the time spent solving the diffusion equation (1) in the fracture continuum. This model can be derived by first replacing the pressure distribution in each block, $P_m(x_m, t; x_f)$, by the average pressure within the block,

$$\bar{P}_m(x_f, t) = \frac{1}{V_m} \int_{V_m} P_m(x_m, t; x_f) dV . \qquad (4)$$

A more rigorous definition of $\bar{P}_m$ would involve some sort of weighted average over the block, to account for the fact that the fluid compressibility varies with the thermodynamic state of the fluid. However, for isothermal single-phase flow, with moderate pressure variations, the fluid compressibility is nearly constant, and definition (4) suffices. Eq. (1) can still be used for the pressure within the fracture network, but eq. (2) governing the pressure distribution within the matrix blocks is no longer meaningful, since the pressure P_m is no longer defined at each point x_m within the matrix block. Instead, we integrate eq. (2) over an entire matrix block centered at point x_f, use the divergence theorem to convert the volume integral of $\nabla^2 P_m$ into a surface integral of $\partial P_m / \partial n$, and divide the resulting equation by V_m, to arrive at

$$\phi_m c_m \frac{\partial \bar{P}_m(x_f, t)}{\partial t} = \frac{1}{V_m} \int_{\partial V_m} \frac{k_m}{\mu} \frac{\partial P_m}{\partial n} dA . \qquad (5)$$

By comparison of eq. (5) with eq. (3), we see that the mean pressure in the matrix block is governed by the following ordinary differential equation:

$$\phi_m c_m \frac{d\bar{P}_m(x_f, t)}{dt} = -Q(x_f, t) . \qquad (6)$$

Equations (1) and (6) now govern the behavior of a lumped-parameter type dual-porosity model. Note that

since the local variable x_m within each matrix block has been integrated out, Q cannot be evaluated as in eq. (3), but must somehow be related to the two pressures P_f and $\bar{P}_m$.

In order to maintain the linearity and relative simplicity of the system of differential equations, Warren and Root (1963) chose to model the flux term Q by assuming that it is directly proportional to the difference between P_f and $\bar{P}_m$:

$$Q(x_f,t) = \frac{-\alpha k_m}{\mu}(P_f - \bar{P}_m) , \tag{7}$$

where α is a parameter that depends on block shape, and has dimensions of 1/Area. The governing equation (6) for $\bar{P}_m$ then takes the form

$$\phi_m c_m \frac{d\bar{P}_m(x_f,t)}{dt} = \frac{\alpha k_m}{\mu}(P_f - \bar{P}_m) . \tag{8}$$

Expressions (7) and (8) for the flux and the matrix pressure are often referred to as the "quasi-steady-state" approximation (Chen, 1989). This terminology reflects the fact that, under conditions of a step-function increase in pressure at the outer boundary of the block, the mean pressure in the block is governed by an equation of the form (8). For simple geometries, such as spheres or cubes, the parameter α can be related to the relaxation time of the most-slowly decaying Fourier component of the step-function response. For spherical blocks of radius a_m, for example, we find (Crank, 1975) that $\alpha = \pi^2/a_m^2$.

Potential difficulties with equations of the form (8) can be anticipated from the fact that this equation only strictly holds for large times, and even then only for step-function boundary conditions. The errors incurred by using eq. (8) will generally be quite large at "small" times, for any type of boundary condition. The aim of our work is to incorporate a modification of eq. (8) into a dual-porosity simulator, which will be accurate over all ranges of time scales, and for more general boundary conditions.

Fully-Transient Coupling Term

Our intention is to maintain the computational simplicity inherent in a lumped-parameter formulation of a dual-porosity model, but with equations (7) and (8) replaced by equations that more accurately account for fracture/matrix flow interactions. This approach requires the derivation of an equation for Q, which depends on P_f and $\bar{P}_m$, as well as the various physical parameters of the problem, but which does not necessarily have the same exact form as eqs. (7) and (8). Since the Warren-Root interaction equation can be derived by differentiating the large-time approximation to the step-function pressure response, it might be thought that a more general interaction equation could be derived by differentiating the exact step-function pressure response, which is (Crank, 1975)

$$\frac{\bar{P}_m - P_i}{P_o - P_i} = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-n^2\pi^2 k_m t/\phi_m \mu c_m a_m^2), \tag{9}$$

where P_i is the initial pressure in the block, and P_o is the pressure imposed at the outer boundary of the block at $t = 0$. Unfortunately, if we attempt this procedure, it is not possible to eliminate t from explicitly appearing in the resulting differential equation. A related approach is to first find an algebraically simple approximation to the step-function response, and then find the first-order differential equation that it satisfies. To do this, we start with the observation by Vermeulen (1953) that the step-function pressure response (9) can be approximated, over all time scales, by

$$\frac{\bar{P}_m - P_i}{P_o - P_i} = \left[1 - \exp(-\pi^2 k_m t/\phi_m \mu c_m a_m^2)\right]^{1/2} . \tag{10}$$

Differentiating eq. (10) with respect to t, and then eliminating t from the result, leads to

$$\frac{d\bar{P}_m}{dt} = \frac{\pi^2 k_m[(P_o - P_i)^2 - (\bar{P}_m - P_i)^2]}{2\phi_m \mu c_m a_m^2(\bar{P}_m - P_i)} . \tag{11}$$

We now generalize eq. (11) by assuming that P_o represents the fracture pressure P_f, even if P_f varies with time:

$$\frac{d\bar{P}_m}{dt} = \frac{\pi^2 k_m[(P_f - P_i)^2 - (\bar{P}_m - P_i)^2]}{2\phi_m \mu c_m a_m^2(\bar{P}_m - P_i)} . \tag{12}$$

For the step-function boundary conditions, eq. (12) integrates to eq. (10), which is a very close approximation to the exact step-function response, eq. (9). Using the value $\alpha = \pi^2/a_m^2$ that is appropriate for a spherical block, the Warren-Root equation (8) can be integrated to yield the following step-function response:

$$\frac{\bar{P}_m - P_i}{P_o - P_i} = 1 - \exp(-\pi^2 k_m t/\phi_m \mu c_m a_m^2) . \tag{13}$$

The Warren-Root step-function response is compared in Fig. 1 to the exact response, and to the response predicted by the Vermeulen equation. While both the Warren-Root and Vermeulen approximations are valid as $t \to \infty$, the Warren-Root step-function response is very inaccurate at small times, whereas the Vermeulen equation is accurate for all values of t. Note that various authors have used slightly different values for the parameter α, particularly for the case of cubical matrix blocks (cf., deSwaan, 1990). Such choices cannot remedy the fact that a Warren-Root-type equation will predict the incorrect exponent for the time-dependence of the pressure in the small-time limit.

The superiority of the Vermeulen differential equation (12) over the Warren-Root differential equation (8), for step-function boundary conditions, is to be expected, since eq. (12) was derived for those conditions. However, we have (fortuitously) found that the Vermeulen equation is also more accurate than Warren-Root under very general types of boundary conditions. For example, consider a ramp-function increase in P_f, which can be specified by

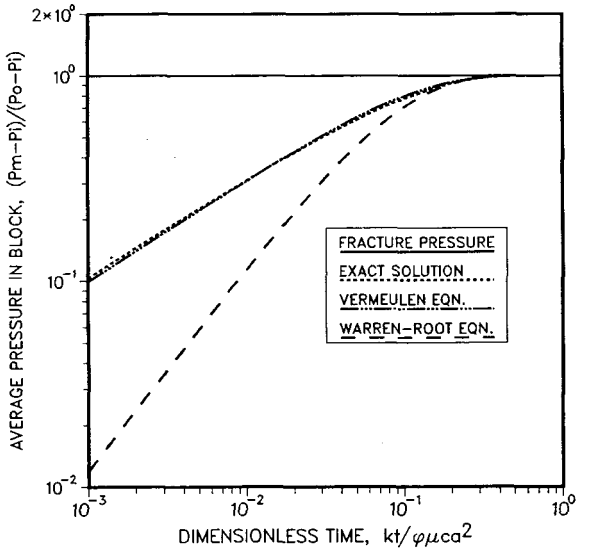

Fig. 1. Normalized average matrix pressure for a spherical block subjected to a step-function increase in the pressure at its boundary, as given by the exact solution (9), the Vermeulen prediction (10), and the Warren-Root prediction (13). For comparison, the pressure at the boundary (*i.e.*, in the fractures) is also shown.

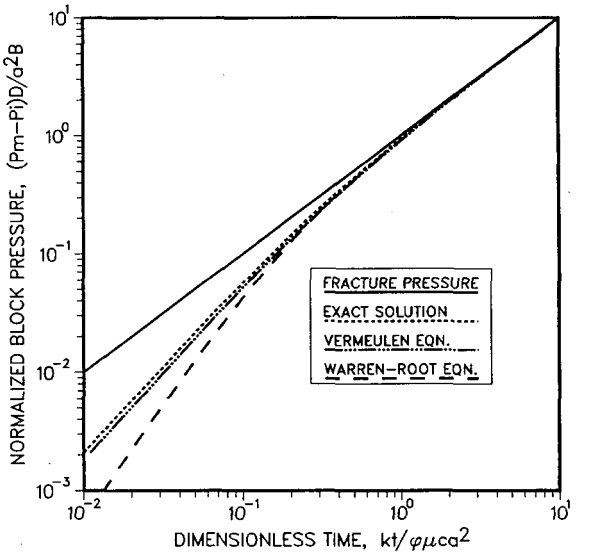

Fig. 2. Same as Fig. 1, for a ramp-function increase in the boundary pressure. The diffusion coefficient D is defined as $k_m/\phi_m \mu c_m$.

$$\bar{P}_m(t=0) = P_i \ , \tag{14}$$

$$P_f(t>0) = P_i + Bt \ , \tag{15}$$

where B is some constant with dimensions of pressure/time. The exact solution for $\bar{P}_m$ in this case is (Crank, 1975)

$$\frac{k_m(\bar{P}_m - P_i)}{\phi_m \mu c_m a_m^2 B} = \frac{k_m t}{\phi_m \mu c_m a_m^2} - \frac{1}{15}$$

$$+ \frac{6}{\pi^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \exp(-n^2 \pi^2 k_m t / \phi_m \mu c_m a_m^2) . \tag{16}$$

The ramp-function response predicted by the Warren-Root equation can be found by solving eq. (8) subject to conditions (14) and (15), to yield

$$\frac{k_m(\bar{P}_m - P_i)}{\phi_m \mu c_m a_m^2 B} = \frac{k_m t}{\phi_m \mu c_m a_m^2} - \frac{1}{\pi^2}$$

$$+ \frac{1}{\pi^2} \exp(-\pi^2 k_m t / \phi_m \mu c_m a_m^2) \ . \tag{17}$$

The Vermeulen equation cannot be solved in closed-form for the ramp-function boundary condition, but can be integrated numerically to yield the results plotted in Fig. 2. As was the case for the step-function boundary condition, the Vermeulen equation is considerably more accurate than the Warren-Root equation in predicting the matrix block pressures. Furthermore, it can also be shown that the Vermeulen differential equation *always* predicts the correct exponent in the time-dependence for the pressure in the small-time limit, whereas the Warren-Root equation always predicts an *incorrect* exponent, for arbitrary variations in P_f. Hence, it seems that eq. (12) can be used to model the mean matrix block pressure, in cases where the pressure at the boundary of the block is specified as a function of time. With this in mind, we have modified an existing numerical simulator so as to use eq. (12) to compute fracture/matrix flow. This modified simulator can be used to solve large-scale reservoir problems in which, in general, the fracture pressures are not known in advance, and must be found, in a coupled manner along with the matrix block pressures, as part of the solution.

Coupled Dual-Porosity Simulator

Numerical reservoir simulators used for single-continuum systems typically solve eq. (1) by discretizing the reservoir into a number of computational cells, and use some numerical scheme such as finite-differences (Huyakorn and Pinder, 1983), finite elements (Pinder and Gray, 1977), or integral finite-differences (Edwards, 1972; Narasimhan and Witherspoon, 1976), to reduce the partial differential equation to a set of algebraic equations. These algebraic equations are solved at each time-step, t_n, in order to yield the pressures in each cell at the next time-step, $t_{n+1} = t_n + \Delta t$. Our approach is to assign to the computational cells those properties that correspond to the fractured continuum, averaged over a suitably-large REV. Fluid that enters or leaves the fracture system from the matrix blocks is then treated as a source/sink term. This approach requires minimal modifications to existing simulators, which typically allow for sources/sinks of various kinds. A certain number of matrix blocks will

be associated with each computational cell, with physical properties $\{k_m, \phi_m, a_m, \text{and } c_m\}$ that must be entered as input for each cell. Each computational cell will have associated with it a new variable, $\bar{P}_m$, which represent the average matrix pressure in those matrix blocks that are contained in that cell.

We have implemented this approach using the TOUGH simulator (Pruess, 1987), an integral-finite-difference code that has been shown to accurately simulate three-dimensional, single-phase, isothermal flow processes such as those discussed in this paper (as well as non-isothermal and two-phase processes). The fracture/matrix interaction equation has been incorporated as an option in a subroutine which is normally used for sources/sinks that represent injection or withdrawal of fluid from a well, etc. As a test of the use of our modified dual-porosity code, consider the problem of linear one-dimensional flow from a boundary that is maintained at some pressure P_o, into a semi-infinite formation that is initially at pressure P_i. We have also tested the modified version of TOUGH on problems involving radial flow to a well, and under constant-flux boundary conditions. However, the problem discussed here seems to most clearly illustrate the different time regimes, and the effects of fracture/matrix flow. The boundary and initial conditions for this problem are

$$P_f(x_f, t=0) = \bar{P}_m(x_f, t=0) = P_i \ , \tag{18}$$

$$P_f(x_f = 0, t > 0) = P_o \ , \tag{19}$$

$$\lim_{x_f \to \infty} P_f(x_f, t) = P_i \ . \tag{20}$$

The results of the simulation using the new semi-analytical dual-porosity version of TOUGH, incorporating eq. (12) as the fluid coupling term, are presented in Fig. 3. The figure shows the flowrate from the inlet feeding the fractures, as a function of time. In the simulation, the permeabilities were taken as $k_f = 10^{-15}\,\text{m}^2$ and $k_m = 10^{-18}\,\text{m}^2$, the porosities were taken as $\phi_f = 0.001$ and $\phi_m = 0.1$, and the matrix block radii were taken to be $a_m = 1\,\text{m}$. The temperature was set at 20 °C, and the boundary and initial pressures were taken to be $P_i = 10\,\text{MPa}$ and $P_o = 11\,\text{MPa}$. Under these conditions, the viscosity of water is roughly $0.001\,\text{Pa·s}$, and the compressibility is roughly $4.5 \times 10^{-10}\,/\,\text{Pa}$, although the TOUGH code actually uses more accurate values that are computed at each temperature and pressure from empirically-derived equations of state. For simplicity, we assume that the rock is rigid, so that the compressibility term reflects only the compressibility of the water.

At small times, flow takes place primarily in the fractures, and the flux varies as $t^{-1/2}$, as is typical in a one-dimensional diffusion problem. However, as time progresses, the leakage of fluid into the matrix blocks has the effect of temporarily halting the decline of the flux into the system, as is seen in Fig. 3. According to the Warren-Root method, this leads to an intermediate-time regime in which the overall flux is essentially constant. However, Nitao and Buscheck (1991) have shown that in this intermediate regime, the flux actually decreases as $t^{-1/4}$, which is in agreement with the results of our semi-analytical dual-porosity calculation. Also

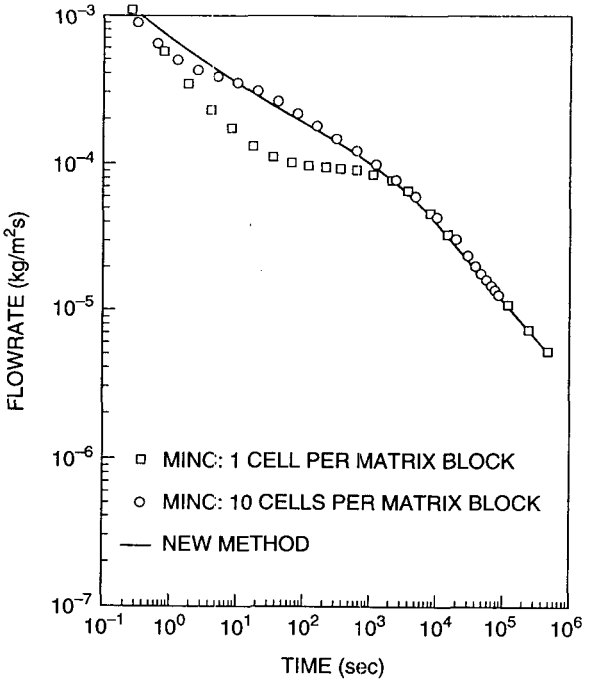

Fig. 3. Total instantaneous flux for one-dimensional flow into a dual-porosity formation with constant boundary pressure. The meaning of the parameters, and their values, are discussed in the text. MINC simulations were carried out using TOUGH code; "new method" simulation was carried out using modified TOUGH.

shown are the results calculated using a fully discretized MINC-type approach, in which each spherical matrix block is broken up into ten nested shells; the MINC simulation with one cell per matrix block corresponds to the Warren-Root model. Note that as the number of shells in the MINC simulation increases, the fluxes approach those calculated with our new semi-analytical approach. As expected, the Warren-Root method overestimates the time needed for flow into the matrix blocks to begin to appreciably influence the overall flowrate into the formation, and gives an inaccurate flowrate variation for intermediate times. At large times, the matrix blocks near the $x = 0$ inlet have been filled, and the overall response is similar to that of a single-porosity medium with an effective porosity of $\phi_m + \phi_f \approx \phi_m$, and an effective permeability of $k_m + k_f \approx k_f$. Hence at large times the flowrate again drops off as $t^{-1/2}$, but with a multiplicative constant that is larger by a factor of about $\sqrt{(\phi_m + \phi_f)/(k_m + k_f)} / \sqrt{\phi_f / k_f} \approx \sqrt{\phi_m / \phi_f}$.

Conclusions

We have developed a new dual-porosity model for single-phase flow in porous/fractured media. Instead of using a Warren-Root-type equation for fracture/matrix flow, in which the flux is proportional to the difference between the fracture pressure and the mean matrix pressure, we use a nonlinear differential equation. This equation is more accurate than the Warren-Root equation, for a wide variety of matrix block boundary conditions. This differential equation has been incorporated

into the numerical simulator TOUGH, to serve as a source/sink term for the discretized fracture continuum. For the test problems we have simulated, the modified TOUGH code is more accurate than the Warren-Root model, and is more computationally efficient than models which require discretization of the matrix blocks.

Acknowledgments

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Geothermal Division, U.S. Department of Energy, under Contract No. DE-AC03-76SF00098, with the Lawrence Berkeley Laboratory. The authors thank Marcelo Lippmann and Lea Cox of LBL for reviewing this paper.

References

Chen, Z.-X. (1989), "Transient flow of slightly compressible fluids through double-porosity, double-permeability systems - A state-of-the-art review," *Transp. Porous Media, 4,* 147-184.

Crank, J. (1975), *The Mathematics of Diffusion,* Clarendon Press, Oxford.

deSwaan, A. (1990), "Influence of shape and skin of matrix-rock blocks on pressure transients in fractured reservoirs," *Soc. Petrol. Eng. Format. Eval., 5,* 344-352.

Duguid, J. O., and Lee, P. C. Y. (1977), "Flow in fractured porous media," *Water Resour. Res., 13,* 558-566.

Edwards, A. L. (1972), TRUMP: A computer program for transient and steady state temperature distribution in multidimensional systems, *Rep. UCRL-14754, Rev. 3,* Lawrence Livermore National Laboratory, Livermore, Calif.

Huyakorn, P. S., and Pinder, G. F. (1983), *Computational Methods in Subsurface Flow,* Academic Press, San Diego, Calif.

Matthews, C. S., and Russell, D. G. (1967), *Pressure Buildup and Flow Tests in Wells,* Society of Petroleum Engineers, Dallas.

Narasimhan, T. N., and Witherspoon, P. A. (1976), "An integrated finite-difference method for analyzing fluid flow in porous media," *Water Resour. Res., 12,* 57-64.

Nitao, J. J., and Buscheck, T. A. (1991), "Infiltration of a liquid front in an unsaturated, fractured porous medium," *Water Resour. Res., 27,* 2099-2112.

Pinder, G. F., and Gray, W. G. (1977), *Finite Element Simulation in Surface and Subsurface Hydrology,* Academic Press, San Diego, Calif.

Pruess, K. (1987), TOUGH User's Guide, *Rep. LBL-20700,* Lawrence Berkeley Laboratory, Berkeley, Calif.

Pruess, K., and Narasimhan, T. N. (1985), "A practical method for modeling heat and fluid flow in fractured porous media," *Soc. Petrol. Eng. J., 25,* 14-26.

Vermeulen, T. (1953), "Theory of irreversible and constant-pattern solid diffusion," *Ind. Eng. Chem., 45,* 1664-1670.

Warren, J. E., and Root, P. J. (1963), "The behavior of naturally fractured reservoirs," *Soc. Petrol. Eng. J., 3,* 245-255.