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BUOYANCY INDUCED BOUNDARY LAYER FLOWS IN GEOTHERMAL RESERVOIRS

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Most of the theoretical study on heat and mass transfer in geothermal reservoirs has been based on numerical method. Recently at the 1975 NSF Workshop on Geothermal Reservoir Engineering, Cheng (1) presented a number of analytical solutions based on boundary layer approximations which are valid for porous media at high Rayleigh numbers. According to various estimates the Rayleigh number for the Wairakei geothermal field in New Zealand is in the range of 1000-5000, which is typical for a viable geothermal field consisting of a highly permeable formation and a heat source at sufficiently high temperature.

The basic assumption of the boundary layer theory is that heat convective heat transfer takes place in a thin porous layer adjacent to heated or cooled surfaces. Indeed, numerical solutions suggest that temperature and velocity boundary layers do exist in porous media at high Rayleigh numbers (2). It is worth mentioning that the large velocity gradient existing near the heated or cooled surfaces is not due to viscosity but is induced by the buoyancy effects. The present paper is a summary of the work that we have done on the analytical solutions of heat and mass transfer in a porous medium based on the boundary layer approximations since the 1975 Workshop.

Similarity Solutions to Boundary Layer Equations

Free Convection about a Vertical Impermeable Surface with Uniform Heat Flux

The solution for free convection about a vertical impermeable surface with wall temperature being a power function of distance, i.e., $T_w = T + Ax^{\lambda}$ for $x > 0$ is given by Cheng and Minkowycz (3). The constant heat flux solution can be obtained by a simple transformation of variables and by setting $\lambda = 1/3$ in Ref. 3. The

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expressions for local Nusselt number, the mean Nusselt number, and thermal boundary layer thickness are

$$Nu_x/[Ra_x^*]^{\frac{1}{3}} = 0.7723, \qquad (1)$$

$$\overline{Nu}_L/[Ra_L^*]^{\frac{1}{3}} = 1.029, \qquad (2)$$

$$\frac{\delta_T}{x} = 4.8/[Ra_x^*]^{\frac{1}{3}}, \qquad (3)$$

with $Nu_x \equiv \frac{hx}{k} = \frac{qx}{k(T_W-T_\infty)}$, $\overline{Nu}_L \equiv \frac{qL}{\kappa(\overline{T_W-T_\infty})} = \frac{\overline{h}L}{\kappa}$, $Ra_x^* \equiv \rho_\infty g\beta K q x^2/\mu\alpha k$, $Ra_L^* \equiv \rho_\infty g\beta K q L^2/\mu\alpha k$, and $\overline{T_W-T_\infty} \equiv \frac{1}{L}\int_0^L (T_W-T_\infty)dx$, where ρ_∞ is

the density of the fluid at infinity; g the gravitational acceleration; μ and β the viscosity and the thermal expansion coefficient of the fluid; K the permeability of the porous medium; L the length of the plate; q the surface heat flux; α and κ the equivalent thermal diffusivity and the thermal conductivity of the saturated porous medium. The equivalence of Eqs. (1) through (3) and the corresponding expressions given by Cheng & Minkowycz (3) was shown recently by Cheng (4).

Free Convection about a Horizontal Impermeable Surface with Uniform Heat Flux

The constant heat flux solution for a horizontal impermeable surface can be obtained by a simple transformation of variables and by setting $\lambda = 1/2$ in the solution given by Cheng and Chang (5). The expressions for local Nusselt number, the mean Nusselt number, and thermal boundary layer thickness for the present problem are

$$Nu_x/[Ra_x^*]^{\frac{1}{4}} = 0.8588, \qquad (4)$$

$$\overline{Nu}_L/[Ra_L^*]^{\frac{1}{4}} = 1.288, \qquad (5)$$

$$\frac{\delta_T}{x} = \frac{4.0}{[Ra_x^*]^{\frac{1}{4}}} \qquad (6)$$

Mixed Convection from a Vertical Isothermal Impermeable Surface

The problem of mixed convection from a vertical impermeable surface with a step increase in wall temperature (i.e., $T_w = T_\infty + A$ for $x \geq 0$), embedded in a porous medium is considered by Cheng (6). The expressions for local Nusselt number, average Nusselt number, and thermal boundary layer thickness are

$$\frac{Nu_x}{[Re_x Pr]^{1/2}} = [-\theta'(0)] , \qquad (7)$$

$$\frac{\overline{Nu}_L}{[Re_L Pr]^{1/2}} = 2[-\theta'(0)] , \qquad (8)$$

$$\frac{\delta_T}{x} = \frac{\eta_T}{[Re_x Pr]^{1/2}} , \qquad (9)$$

with $Re_x \equiv \frac{U_\infty x}{\nu}$ and $Pr \equiv \frac{\nu}{\alpha}$ where U_∞ is the velocity outside the boundary layer. The values of $Nu_x/[Re_x Pr]^{\frac{1}{2}}$ and $[Re_x Pr]^{\frac{1}{2}}\delta_T/x$ as a function of Gr_x/Re_x are shown in Figs. 1 and 2, where the corresponding values for free convection about a vertical isothermal plate as given by Cheng and Minkowycz (3) can be rewritten as

$$Nu_x/[Re_x Pr]^{\frac{1}{2}} = 0.444 \left[\frac{Gr_x}{Re_x}\right]^{\frac{1}{2}} , \qquad (10)$$

$$[Re_x Pr]^{\frac{1}{2}}\delta_T/x = \frac{6.31}{[Gr_x/Re_x]^{\frac{1}{2}}} . \qquad (11)$$

Mixed Convection from a Horizontal Impermeable Surface with Uniform Surface Heat Flux

The expressions for local Nusselt number, average Nusselt number, and thermal boundary layer thickness are given by (4,7)

$$\frac{Nu_x}{[Re_x Pr]^{\frac{1}{2}}} = \frac{1}{[-\phi(0)]} , \qquad (12)$$

$$\frac{\overline{Nu_L}}{[Re_L Pr]^{\frac{1}{2}}} = \frac{3}{2[-\phi(0)]}, \qquad (13)$$

$$\frac{\delta_T}{x} = \frac{\eta_T}{[Re_x Pr]^{\frac{1}{2}}}. \qquad (14)$$

The values of $Nu_x/[Re_x Pr]^{\frac{1}{2}}$ and $[Re_x Pr]^{\frac{1}{2}}\delta_T/x$ as a function of $Ra_x^*/(Re_x Pr)^2$ are given in Ref. 4, where the asymtotes for free convection about a horizontal plate with uniform heat flux are given by Eqs. (4) and (6), which can be rewritten as

$$Nu_x/[Re_x Pr]^{\frac{1}{2}} = 0.8588\ [Ra_x^*/(Re_x Pr)^2]^{\frac{1}{4}}, \qquad (15)$$

$$[Re_x Pr]^{\frac{1}{2}}\delta_T/x = 4.0/[Ra_x^*/(Re_x Pr)^2]^{\frac{1}{4}}. \qquad (16)$$

The Effect of U_∞ on Heat Transfer Rate and the Size of Hot-Water Zone

To gain some feeling on the order of magnitude of various physical quantities in a geothermal reservoir, consider a vertical impermeable surface at 215°C is embedded in an aquifer at 15°C. If there is a pressure gradient in the aquifer such that groundwater is flowing vertically upward, the values of heat transfer rate and the size of the hot water zone can be determined from Figs. 1 and 2. The results of the computations for U_∞ varying from 0.01 cm/hr to 10 cm/hr are plotted in Figs. 3 and 4 where it is shown that the total heat transfer rate for a vertical surface, 1 km by 1 km, increases from 20 MW to 110 MW, while the boundary layer thickness at x = 1 km decreases from 130 m to 20 m.

Validity of Boundary Layer Approximations

The validity of the boundary layer approximations can be accessed by a comparison of results obtained by similarity solutions to that of numerical solutions of exact partial differential equations, or to experimental data. For free convection in a porous medium between parallel vertical plates separated by a distance H, the correlation equation given by Bories and Combarnous (8) is

$$\overline{Nu}_H = 0.245\ (Ra_H)^{0.625}\ (\frac{H}{L})^{0.397}, \qquad (17)$$

where L is the length of the plate, $\overline{Nu}_H = \frac{\overline{h}H}{k}$ and $Ra_H = \rho g \beta K (T_W - T_L)H/\mu\alpha$. Eq. (17) is valid for Ra_H from 10^2 to 10^3 and for H/L between 0.05 and 0.15[8]. On the other hand, the heat transfer rate as obtained from boundary layer approximations for an isothermal vertical plate (3) is

$$\overline{Nu}_L = 0.888\ (Ra_L)^{0.5}, \qquad (18)$$

which can be rewritten as

$$\overline{Nu}_H = 0.888\ (Ra_H)^{0.5}\ (\frac{H}{L})^{0.5}, \qquad (19)$$

Eqs. (17) and (19) for H/L = 0.05 and 0.15 are plotted in Fig. 5 for comparison. It is shown that they are in good agreement, especially at high Rayleigh numbers where the boundary layer approximations are valid.

Concluding Remarks

As in the classical convective heat transfer theory, boundary layer approximations in porous layer flows can result in analytical solutions. Mathematically, the approximations are the first-order terms of an asymptotic expansion which is valid for high Rayleigh numbers. Comparison with experimental data and numerical solutions show that the approximations are also accurate at moderate values of Rayleigh numbers. For problems with low Rayleigh numbers where boundary layer is thick, higher-order approximations must be used (9).

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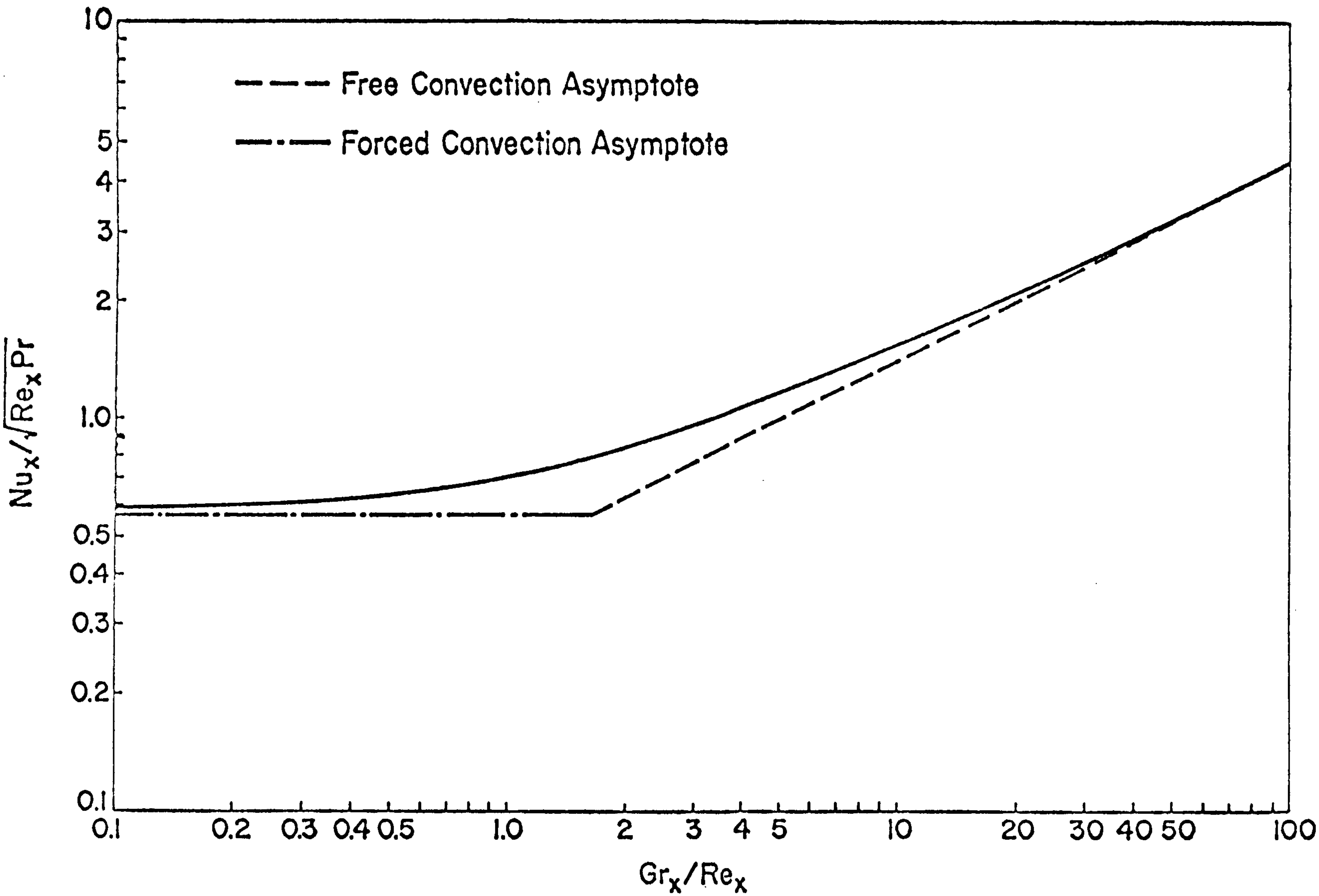
FIGURE 1. HEAT TRANSFER RESULTS FOR MIXED CONVECTION FROM AN ISOTHERMAL VERTICAL PLATE.



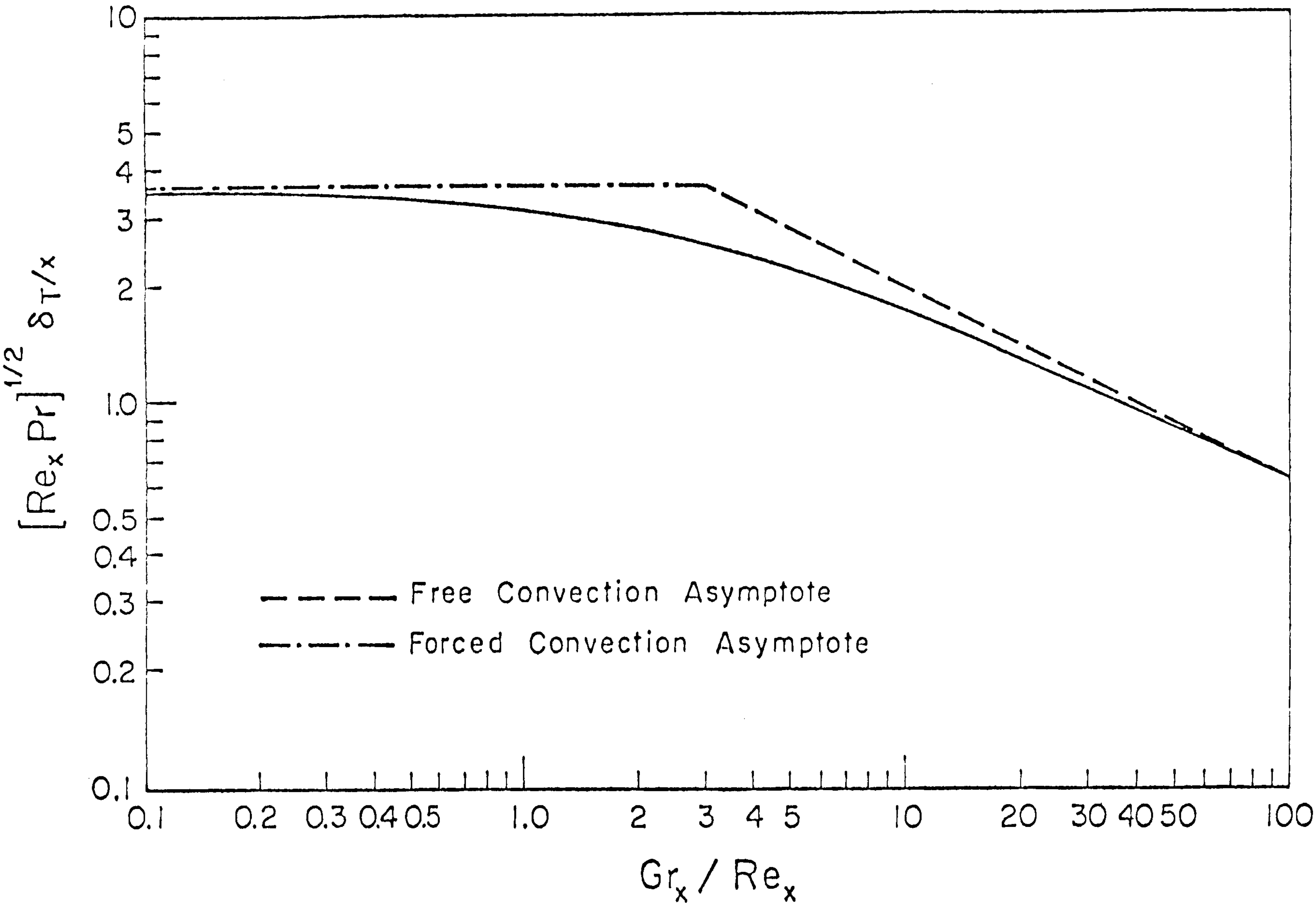
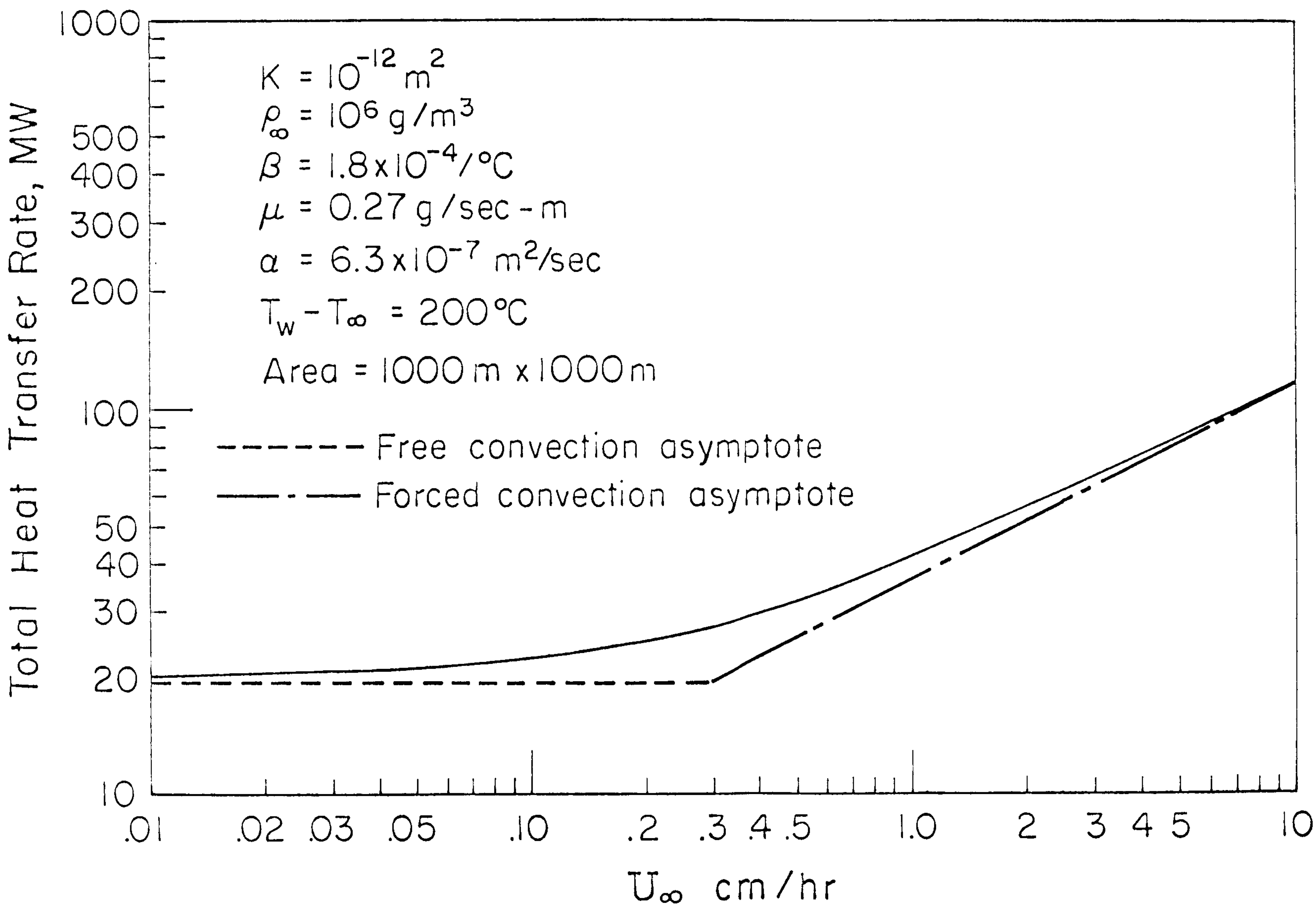


Fig. 2 Boundary Layer Thickness for Mixed Convection From An Isothermal Vertical Plate

FIGURE 3. THE EFFECT OF U_∞ ON TOTAL HEAT TRANSFER RATE FOR MIXED CONVECTION FROM AN ISOTHERMAL VERTICAL HEATED SURFACE IN A GEOTHERMAL RESERVOIR.



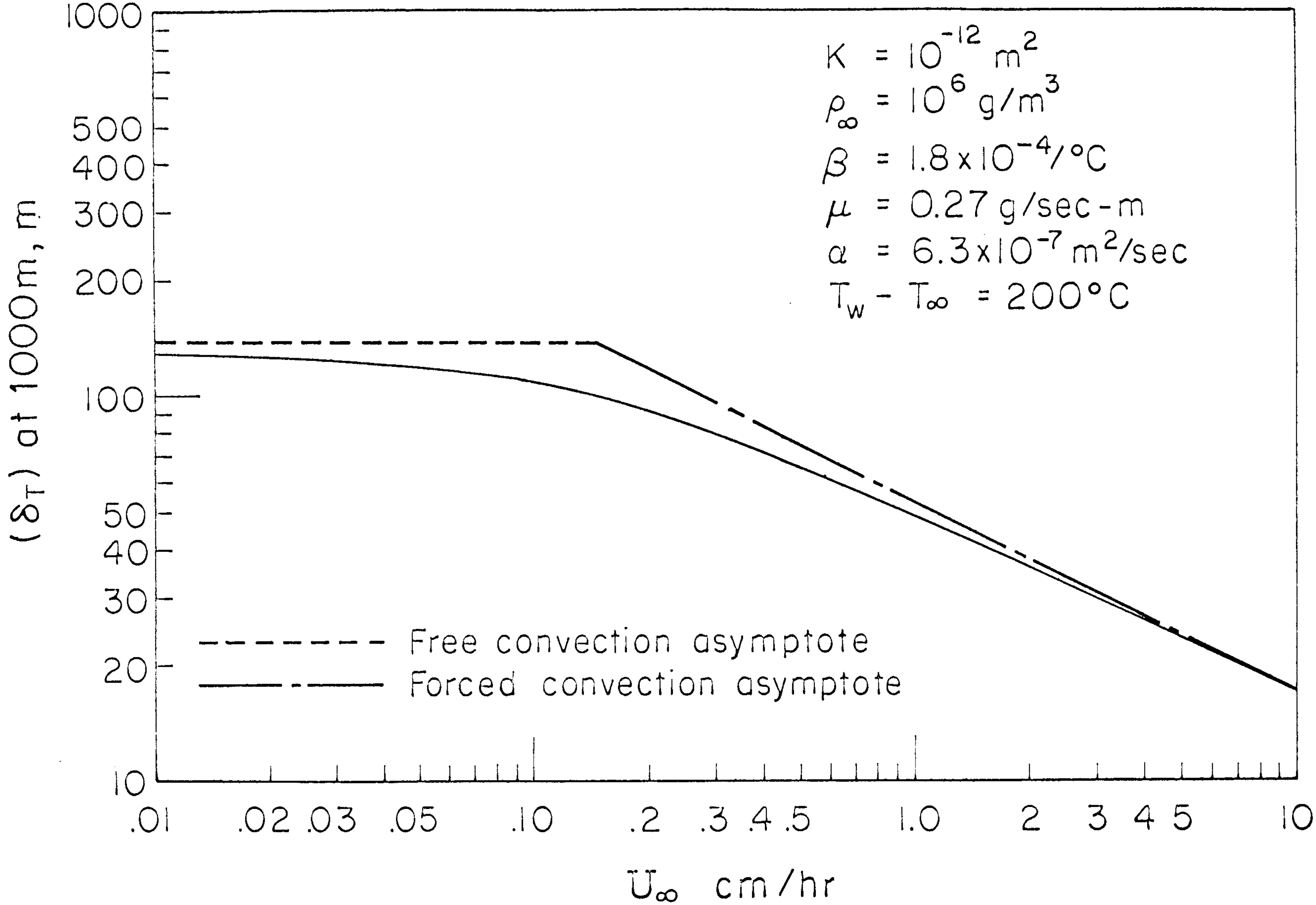


Fig. 4 The Effect of U_∞ on the Boundary Layer Thickness (Size of Hot Water Zone) for Mixed Convection from a Vertical Isothermal Heated Surface in a Geothermal Reservoir

FIGURE 5. COMPARISON OF THEORY AND EXPERIMENT FOR VERTICAL POROUS LAYERS.