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Full Wave Analysis of RF Signal Attenuation in a Lossy Rough Surface Cave using a High Order Time Domain Vector Finite Element Method

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Full Wave Analysis of RF Signal Attenuation in a Lossy Rough Surface Cave using a High Order Time Domain Vector Finite Element Method *

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Abstract

We present a computational study of signal propagation and attenuation of a 200 MHz planar loop antenna in a cave environment. The cave is modeled as a straight and lossy random rough wall. To simulate a broad frequency band, the full wave Maxwell equations are solved directly in the time domain via a high order vector finite element discretization using the massively parallel CEM code EMSolve. The numerical technique is first verified against theoretical results for a planar loop antenna in a smooth lossy cave. The simulation is then performed for a series of random rough surface meshes in order to generate statistical data for the propagation and attenuation properties of the antenna in a cave environment. Results for the mean and variance of the power spectral density of the electric field are presented and discussed.

1 Introduction

The study of electromagnetic wave propagation in caves and tunnels is of great practical interest to antenna engineers due to the increasing demands for reliable wireless communications systems in such environments. Current wireless radio frequency (RF) communication systems were not designed to operate reliably in enclosed environments such as caves and tunnels, and signal quality is severely compromised due to the rough and lossy surfaces of the cave. Today there is limited ability to maintain communications in cave-like structures, tunnels or subways, prohibiting the quick deployment of wireless systems in caves and tunnels. If the electromagnetic properties of the tunnel could be characterized (e.g. dissipation, dispersion, fading, etc.), then a more robust communication system could be designed specifically for operation in such environments, hence full wave EM simulations of propagation in this type of environment are very useful.

Much theoretical work in this field has been done in order to develop a better understanding of the RF propagation channel. Dudley recently studied models for propagation in lossy circular tunnels [1]. He produced expressions for the fields in terms of a Fourier transform over the axial variables, and presented the numerical results for the field intensity both as a function of axial distance and as a function of radial distance. However, this work only involved smooth tunnel walls and not the more realistic situation of rough wall tunnels. In this case, the electromagnetic fields are no longer deterministic, but random variables that need to be solved using statistical methods. Recently, Pao and Kasey have investigated the statistical properties of wave propagation in straight, rough-walled tunnels [2], [3]. This work assumes a perfect electrical conductor (PEC) boundary at the rough wall / air interface. A more realistic model needs to take into account the lossy nature of the rough walls and the cave material (typically granite or some sort of earth like material with electrical conductivities on the order of $0.1 S/m$).

In this paper we use a high order finite element discretization to solve the full wave Maxwell equations directly in the time domain for the case of a planar loop antenna placed at the mouth of a straight, lossy rough walled tunnel. We chose a time domain simulation in order to efficiently compute the response over a broad frequency band. We begin with a brief description of the numerical method employed for this problem (as implemented in the EMSolve code [4]). We then verify the numerical method against the theoretical results of Dudley [1] for a planar loop antenna in a smooth lossy cave and discuss the limitations of the numerical model. Finally, we proceed to solve the electromagnetic wave equation on a sequence of randomly generated meshes to determine statistical properties for the power spectral density of the electric field.

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2   Numerical Formulation

We begin with the second order time dependent wave equation for the electric field in a 3 dimensional domain Ω

$$
\begin{aligned}
\epsilon \frac{\partial^2}{\partial t^2}\mathbf{E} &= -\nabla \times (\mu^{-1}\nabla \times \mathbf{E}) - \sigma\frac{\partial}{\partial t}\mathbf{E} - \frac{\partial}{\partial t}\mathbf{J} & \text{in } \Omega \\
\nabla \cdot (\epsilon\mathbf{E}) &= 0 & \text{in } \Omega \\
\hat{\mathbf{n}} \times \mathbf{E} &= \mathbf{E}_{bc} & \text{on } \partial\Omega
\end{aligned}
\tag{1}
$$

where $\partial\Omega$ is the two dimensional boundary of the domain, $\hat{\mathbf{n}}$ is the outwardly directed unit normal of this boundary and $\mathbf{J}$ is a free current density source that can be added to drive the problem. The value $\mathbf{E}_{bc}$ represents an arbitrary boundary condition imposed on the electric field intensity while ϵ, μ and σ denote the dielectric permittivity, magnetic permeability and electrical conductivity of the materials contained in the domain Ω respectively.

Applying an arbitrary order Galerkin finite element discretization to the wave equation (the details of which can be found in [5], [6]) yields the following semi-discrete system of ordinary differential equations

$$
M_\epsilon \frac{\partial^2}{\partial t^2}e = -S_\mu e - M_\sigma \frac{\partial}{\partial t}e - \frac{\partial}{\partial t}j
\tag{2}
$$

where M_ϵ, M_σ are finite element mass matrices, S_μ is a finite element stiffness matrix and e, j are discrete arrays of finite element degrees of freedom. Applying a backward difference approximation for the first order time derivative in (2) and a central difference approximation for the second order time derivative yields the following fully discrete linear system of equations

$$
M_\epsilon \, e_{n+1} = (2M_\epsilon - \Delta t^2 S_\mu - \Delta t M_\sigma)e_n + (\Delta t M_\sigma - M_\epsilon)e_{n-1} - j'
\tag{3}
$$

where Δt is the discrete time step, the integer n denotes the current time step and the time derivative of the free current source has been directly incorporated into the new source term j'.

For the results shown in section 4 below we employ second-order interpolatory $H(curl)$ basis functions along with custom quadrature rules that yield a diagonal "mass" matrix M_ϵ. The details of this discretization are presented in [7]. This method is much more accurate than standard FDTD, the numerical dispersion for this method is $O(h^4)$ rather than $O(h^2)$ as it is for FDTD. Compared to higher-order FDTD schemes, this method is better at modeling the jump discontinuity of fields across the air-earth interface.

3   Verification

Before we proceed to simulations of a random rough surface, we begin by verifying our numerical method with known theoretical values for the case of a smooth, lossy tunnel. These results were computed by Dudley according to the procedures described in [1]. We consider the case of an axially symmetric circular current loop of radius $b = 0.2m$ placed at the mouth of a circular tunnel of length $60m$ and radius $a = 2.0m$. The current loop is driven by a time harmonic source of frequency $f = 200MHz$. The electric field is "measured" along the length of the tunnel at a radial observation point ρ such that $\rho/a = 0.3$ The tunnel has a relative dielectric constant of $\epsilon_r = 5.0$. We consider two cases, a tunnel with an electric conductivity $\sigma_{low} = 0.02S/m$ and $\sigma_{high} = 0.1S/m$.

For the numerical model, we discretize the tunnel domain in two different ways using both a Cartesian (or "stair-step") approximation to the smooth tunnel wall and a more accurate conforming cylindrical mesh (see Figure 1 and Figure 2). For both cases, a planar loop of current of radius $0.2m$ is placed at one end of the tunnel, while a simple absorbing boundary condition (ABC) is placed at the other. The ABC is imperfect for anything other than plane waves at normal incidence, hence we make our tunnel mesh $75m$ long and ignore field data from the last 20% of the tunnel mesh. A perfect electric conductor (PEC) boundary condition is applied to the outer most walls of the cave to fully define the problem. For both the Cartesian and Cylindrical meshes, the outermost PEC boundary is made sufficiently large to prevent spurious reflections. The temporal dependence of the current source is a Gaussian pulsed sine wave centered at 200 MHz with a 20% bandwidth. The simulation is performed using high order $p = 2$ basis functions to mitigate the effects of numerical dispersion.

In Figure 3 and Figure 4 we compare results for both numerical models (Cartesian and cylindrical meshes) to the theoretical results for both conductivity values. Note that the agreement between the theoretical model and the numerical model using the conforming cylindrical mesh are excellent, indicating that the proposed numerical method is working properly. The discrepancies between the Cartesian results and the theoretical results are due to
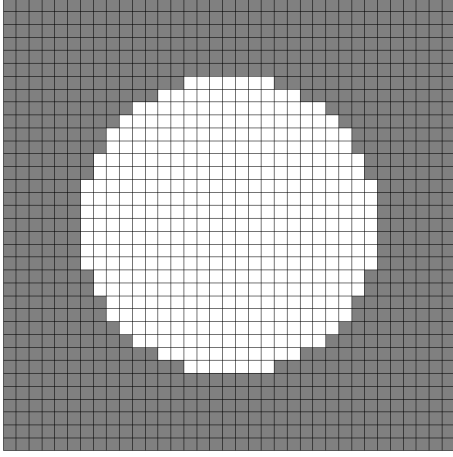
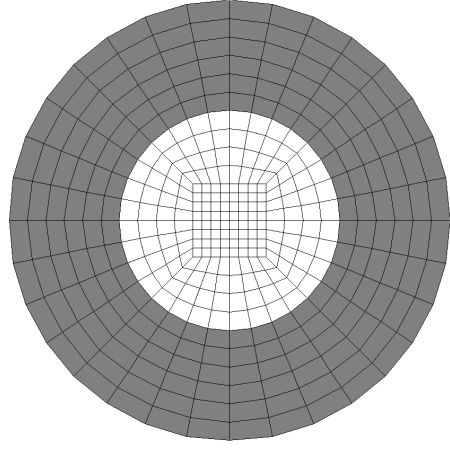Figure 1: Cross section of smooth tunnel Cartesian mesh.

Figure 2: Cross section of smooth tunnel conforming cylindrical mesh.

the "stair-step" approximation to smooth surfaces, which is known to be problematic since such approximations fail to converge to a true cylindrical surface. While the Cartesian mesh is a bad choice for modeling smooth tunnels, it is sufficient for modeling rough surface tunnles, which we will use in the next section.
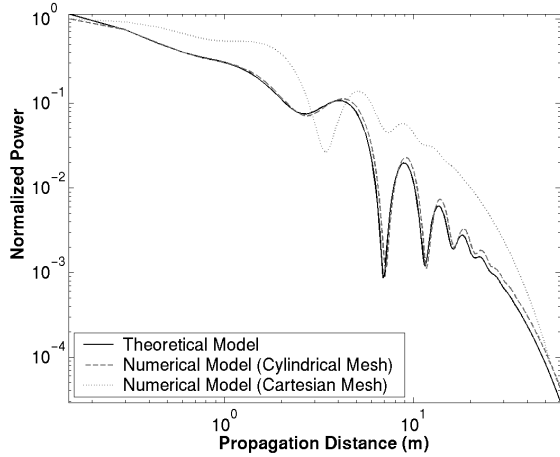

Figure 3: Comparison between theoretical model and two different numerical models at 200 MHz for a smooth cave with conductivity $\sigma_{low} = 0.02S/m$.
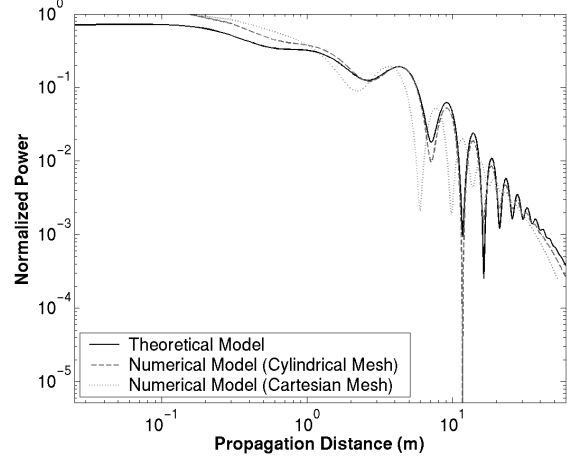

Figure 4: Comparison between theoretical model and two different numerical models at 200 MHz for a smooth cave with conductivity $\sigma_{high} = 0.1S/m$.

4 Computational Results

We now proceed to apply the same process discussed above to the more complicated case of random rough walled caves. The random rough surface is generated as follows. Step 1: Generate a cylindrical surface of appropriate radius and length, Step 2: Add a random perturbation of specified standard deviation to the surface, Step 3: Smooth the random surface (low pass filter) to introduce a surface correlation of a given length, Step 4: Generate a 3D Cartesian mesh, where the electrical conductivity of each element depends upon whether the element is inside the random surface (air) or outside the random surface (earth). For mesh elements that straddle the random surface, a volume-fraction is used to determine the conductivity. To model the dielectric and conducting properties of earth, we choose a uniformly varying electrical conductivity of 0.1-0.02S/m and a dielectric permittivity of 5

times the free space permittivity ϵ_0 (a typical value for granite). Each computational mesh consists of $583,200$ hexahedral elements, an example of which is shown in Figure 5. Note that the portion of the mesh representing the air has been removed to illustrate the random rough surface. The various simulation parameters for the random rough surface computations are summarized in Table 1
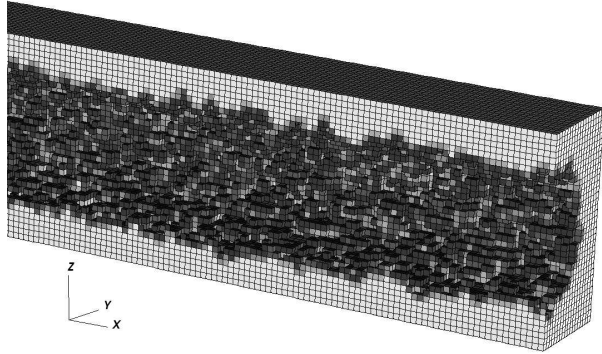

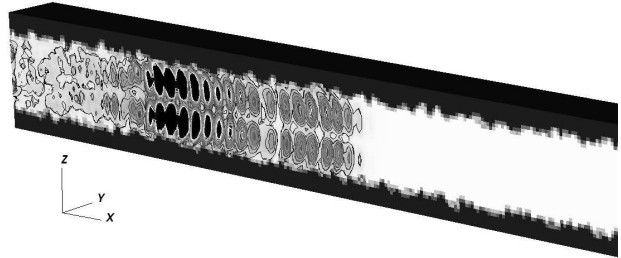Figure 5: Example of randomly generated cave mesh with interior removed (close-up view).

Figure 6: Snapshot of computed electric field magnitude at $t = 89.6ps$.

Avg. Cave Radius	$2\ m$ (1.333 λ)
Cave Length	$75\ m$ (50 λ)
Element Size (Δx)	$0.167\ m$ (0.111 λ)
Surface Roughness Variance	$0.3\ m$ (0.2 λ), Gauss. dist. centered at 0
Conductivity Variation	Uniform, $0.1 - 0.02\ S/m$
Signal Type	Modulated Gaussian pulse, planar loop antenna
Pulse Frequency	200 MHz, 20% Bandwidth
Gaussian Width, Delay	4.67e-9s, 2.50e-8 s
No. of Trials	10
No. Unknowns per Trial	~ 14 million
No. Parallel CPU's per Trial	192

Table 1: Summary of computational statistics for 10 random cave simulations.

A total of 10 random caves were simulated. A time history was recorded at each x, for all time steps. This data was used to find the spectrum at every spatial step. For each simulation the mean and variance of the power spectral density (PSD) and phase were extracted over the bandwidth of the signal. Each run was normalized by dividing by the total PSD magnitude at the first x-data point, thereby removing the characteristics of the input signal, but preserving the relative magnitudes vs. polarization. The last 20% of the spatial samples were removed to avoid reflections from the end of the cave (due to the imperfect nature of the ABC). The results for computed mean power at 5 different frequencies are shown in Figure 7. Note that in general, the scattered field from the rough surface walls fills in nulls which are created by destructive interference in the smooth case (see Figure 3 and Figure 4 for comparrison). In addition, note that the lower frequencies are attenuated more rapidly than the higher frequencies as expected. In Figure 8 we plot the computed variance in power across all 10 simulations.
Figure 8

5   Conclusions

We have applied the high order time domain vector finite element methods described in [5], [6] and [7] to the case of RF electric field propagation in a lossy rough wall tunnel. This particular calculation has proved difficult to solve using direct theoretical analysis. We have verified our numerical results by direct comparison to a theoretical model for propagation in a smooth lossy cave. We have presented statistical data for the power spectral density of
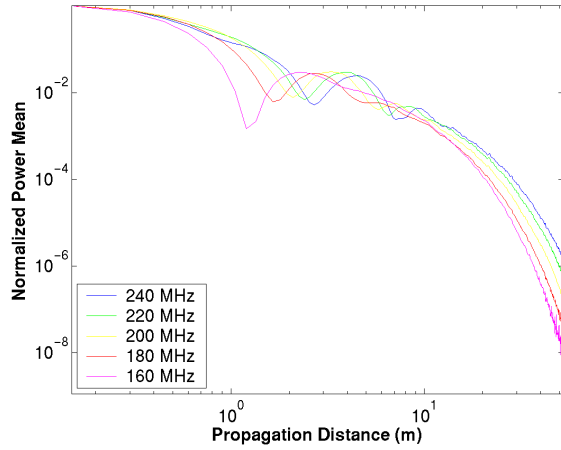
Figure 7: Mean power for 10 random cave simulations at 5 different frequencies.
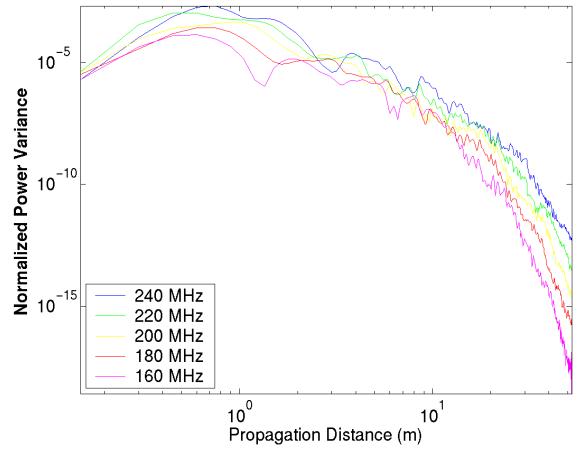

Figure 8: Variance of power for 10 random cave simulations at 5 different frequencies.

a 200 MHz planar loop antenna and have compared our data in a rough walled cave to one with a smooth surface. Further work will allow direct time domain modeling of more complicated cave structures with bends and forks that are too complex for theoretical techniques.

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