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April 12, 2005

International Journal for Numerical and Analytical Methods in Geomechanics

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Submitted to : *International Journal for Numerical and Analytical Methods in Geomechanics*

Geomechanical analysis with rigorous error estimates for a double-porosity reservoir model

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Keywords: double-porosity, up-scaling, multiscale modeling, semi-analytical methods

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SUMMARY

A model of random polycrystals of porous laminates is introduced to provide a means for studying geomechanical properties of double-porosity reservoirs. Calculations on the resulting earth reservoir model can proceed semi-analytically for studies of either the poroelastic or transport coefficients. Rigorous bounds of the Hashin-Shtrikman type provide estimates of overall bulk and shear moduli, and thereby also provide rigorous error estimates for geomechanical constants obtained from up-scaling based on a self-consistent effective medium method. The influence of hidden (or presumed unknown) microstructure on the final results can then be evaluated quantitatively. Detailed descriptions of the use of the model and some numerical examples showing typical results for the double-porosity poroelastic coefficients of a heterogeneous reservoir are presented.

1. INTRODUCTION

Rapid progress in development of rigorous bounding methods for material coefficients in heterogeneous media [1, 2] has been made over the last fifty years. Effective medium theory, although very useful in many practical circumstances, nevertheless has not made such rapid progress. So a question that naturally arises is whether it might be possible to construct new effective medium formulas directly from the known bounds? Skeptics will immediately ask: Why do we need to do this at all if bounds are available? But the answer to this question is most apparent in poromechanics, where the bounds are frequently too far apart to be of much use in engineering and, especially, in field applications.

Hill [3] was actually the first to try constructing estimates from bounds. First he showed that the Voigt [4] and Reuss [5] averages/estimates in elasticity were in fact upper and lower bounds, respectively. Then he proceeded to suggest that estimates of reasonable accuracy were given by the arithmetic or geometric means obtained by averaging these two bounds together. Thus, the

Voigt-Reuss-Hill estimates were born. Better bounds than the Voigt and Reuss bounds are now known and no doubt some attempts to update Hill's approach have been made. However, to make a direct connection to traditional approaches of effective medium theory, we apply a more technical procedure here in order to obtain estimates of up-scaled constants using the known analytical structure of the bounds, especially for Hashin-Shtrikman [6] bounds. When this mathematical structure is not known — as might be the case if the bounds are expressed algorithmically rather than as analytical formulas — then we will see that it proves very worthwhile to expend the additional effort required to determine this structure. Whenever it is possible to carry the analysis further than has been done in the published literature, a self-consistent effective medium formula is fairly straightforward to obtain from the resulting expressions. The self-consistent predictions then lie within the bounds, as might be expected and desired.

In Section 2, results from double-porosity geomechanics analysis are presented. These results are general, and do not depend explicitly on the spatial arrangement or microstructure of the two porous constituents. Microstructure enters these formulas only through the overall drained bulk modulus K_d^*. Then, in Section 3, a preferred model microstructure — that of a locally layered medium — is imposed. This microstructure has the advantage that it forms hexagonal (or transversely isotropic) "crystals" locally. Then, if we assume these crystals, or grains, are jumbled together randomly to form an overall isotropic medium, we have the "random polycrystal of porous laminates" reservoir model. Hashin-Shtrikman bounds are known for such polycrystals composed of grains having hexagonal symmetry. So bounds are easily found. From the form of the bounds, we also obtain estimates of both overall bulk modulus and shear modulus, thus completing the semi-analytical poromechanics model. Examples are computed in Section 4, and results summarized in the final section.

Although the language we use here tends to emphasize the analogy to polycrystals of laminates, the reader should keep in mind that the equations of elasticity — and for present purposes (we

do not treat permeability here) also the equations of poroelasticity — are scale invariant. So the mathematics is the same whether the layering we are considering takes place at the scale of microns, meters, or kilometers. However, there is an implicit limitation that the scale considered cannot be so small that the continuum hypothesis fails to be valid.

2. DOUBLE-POROSITY GEOMECHANICS

The main results used here can be derived using uniform expansion, or self-similar, methods analogous to ideas used in thermoelasticity by Cribb [7] and in single-porosity poroelasticity by Berryman and Milton [8]. Cribb's method provided a simpler and more intuitive derivation of Levin's earlier results on thermoelastic expansion coefficients [9]. Our results also provide a simpler derivation of results obtained by Berryman and Pride [10] for the double-porosity coefficients. Related methods in other applications to micromechanics are called "the theory of uniform fields" by some authors [11].

First assume two distinct phases at the macroscopic level: a porous matrix phase with the effective properties $K_d^{(1)}$, $G_d^{(1)}$, $K_m^{(1)}$, $\phi^{(1)}$ (which are drained bulk and shear moduli, grain/mineral bulk modulus, and porosity of phase 1 with analogous definitions for phase 2), occupying volume fraction $V^{(1)}/V = v^{(1)}$ of the total volume and a macroscopic crack or joint phase occupying the remaining fraction of the volume $V^{(2)}/V = v^{(2)} = 1 - v^{(1)}$. The key feature distinguishing the two phases — and therefore requiring this analysis — is the very high fluid permeability of the crack or joint phase and the relatively lower permeability (but higher fluid volume content) of the matrix phase.

In the double-porosity model, there are three distinct pressures: confining pressure δp_c, pore-fluid pressure $\delta p_f^{(1)}$ [for the storage porosity], and joint-fluid pressure $\delta p_f^{(2)}$ [for the transport porosity]. Treating $\delta p_c, \delta p_f^{(1)}$, and $\delta p_f^{(2)}$ as the independent variables in our double porosity theory, we de-

fine the dependent variables $\delta e \equiv \delta V/V$, $\delta\zeta^{(1)} = (\delta V_\phi^{(1)} - \delta V_f^{(1)})/V$, and $\delta\zeta^{(2)} = (\delta V_\phi^{(2)} - \delta V_f^{(2)})/V$, which are respectively the total volume dilatation, the increment of fluid content in the matrix phase, and the increment of fluid content in the joints. The fluid in the matrix is the same as that in the cracks or joints, but the two fluid regions may be in different states of average stress and, therefore, need to be distinguished by their respective superscripts.

Linear relations among strain, fluid content, and pressure take the symmetric form

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}, \tag{1}$$

following Berryman and Wang [12] and Lewallen and Wang [13]. It is easy to check that $a_{11} = 1/K_d^*$, where K_d^* is the overall drained bulk modulus of the system. We now find analytical expressions for the remaining five constants for a binary composite system.

The components of the system are themselves porous materials 1 and 2, but each is assumed to be what we call a "Gassmann material" satisfying

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta\zeta^{(1)}/v^{(1)} \end{pmatrix} = \frac{1}{K_d^{(1)}} \begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix} \begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix} \tag{2}$$

for material 1 and a similar expression for material 2. The new constants appearing on the right are the drained bulk modulus $K_d^{(1)}$ of material 1, the corresponding Biot-Willis [14] coefficient $\alpha^{(1)}$, and the Skempton [15] coefficient $B^{(1)}$. The volume fraction $v^{(1)}$ appears here in order to correct for the difference between a global fluid content and the corresponding local variable for material 1. The main special characteristic of a Gassmann [16] porous material is that it is composed of only one type of solid constituent, so it is "microhomogeneous" in its solid component, and in addition the porosity is randomly, but fairly uniformly, distributed so there is a well-defined constant porosity $\phi^{(1)}$ associated with material 1, etc.

To proceed further, we ask this question: Is it possible to find combinations of $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}$, $\delta p_f^{(1)}$, and $\delta p_f^{(2)}$ so that the expansion or contraction of the system is spatially uniform or self-similar? Or equivalently, can we find uniform confining pressure δp_c, and pore-fluid pressures $\delta p_f^{(1)}$ and $\delta p_f^{(2)}$, so all these scalar conditions can be met simultaneously? If so, then results for system constants can be obtained purely algebraically without ever having to solve equilibrium equations of the mechanics. We initially set $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}$, as this condition of uniform confining pressure is clearly a requirement for the self-similar thought experiment to be a valid solution of stress equilibrium equations.

So, the first condition to be considered is the equality of the strains of the two constituents:

$$\delta e^{(1)} = -\frac{1}{K_d^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)}) = \delta e^{(2)} = -\frac{1}{K_d^{(2)}}(\delta p_c - \alpha^{(2)}\delta p_f^{(2)}). \tag{3}$$

If this condition is satisfied, then the two constituents are expanding or contracting at the same rate and it is clear that self-similarity prevails, since

$$\delta e = v^{(1)}\delta e^{(1)} + v^{(2)}\delta e^{(2)} = \delta e^{(1)} = \delta e^{(2)}. \tag{4}$$

If we imagine that δp_c and $\delta p_f^{(1)}$ are fixed, then we need an appropriate value of $\delta p_f^{(2)}$, so that (3) is satisfied. This requires that

$$\delta p_f^{(2)} = \delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)}) = \frac{1 - K_d^{(2)}/K_d^{(1)}}{\alpha^{(2)}}\delta p_c + \frac{\alpha^{(1)}K_d^{(2)}}{\alpha^{(2)}K_d^{(1)}}\delta p_f^{(1)}, \tag{5}$$

showing that, for undrained conditions, $\delta p_f^{(2)}$ can almost always be chosen so the uniform expansion takes place.

Using (5), we now eliminate $\delta p_f^{(2)}$ from the remaining equality so

$$\delta e = -\left[a_{11}\delta p_c + a_{12}\delta p_f^{(1)} + a_{13}\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})\right] = \delta e^{(1)} = -\frac{1}{K_d^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)}), \tag{6}$$

where $\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$ is given by (5). Making the substitution and then noting that δp_c and $\delta p_f^{(1)}$ were chosen independently and arbitrarily, we find the resulting coefficients must each vanish. The

two equations we obtain are

$$a_{11} + a_{13}\left(1 - K_d^{(2)}/K_d^{(1)}\right)/\alpha^{(2)} = 1/K_d^{(1)} \tag{7}$$

and

$$a_{12} + a_{13}\left(\alpha^{(1)}K_d^{(2)}/\alpha^{(2)}K_d^{(1)}\right) = -\alpha^{(1)}/K_d^{(1)}. \tag{8}$$

Since a_{11} is assumed to be known, (7) can be solved directly, giving

$$a_{13} = -\frac{\alpha^{(2)}}{K_d^{(2)}}\left(\frac{1 - K_d^{(1)}/K_d^*}{1 - K_d^{(1)}/K_d^{(2)}}\right). \tag{9}$$

Similarly, with a_{13} known, substituting into (8) gives

$$a_{12} = -\frac{\alpha^{(1)}}{K_d^{(1)}}\left(\frac{1 - K_d^{(2)}/K_d^*}{1 - K_d^{(2)}/K_d^{(1)}}\right). \tag{10}$$

So, formulas for three of the six coefficients are now known. [Also, note the similarity of the formulas (9) and (10), *i.e.*, interchanging indices 1 and 2 on the right hand sides takes us from one expression to the other.]

To evaluate the remaining coefficients, we consider what happens to fluid increments during the self-similar expansion. We treat only material 1, but the equations for material 2 are completely analogous. From the preceding equations,

$$\begin{aligned}\delta\zeta^{(1)} &= a_{12}\delta p_c + a_{22}\delta p_f^{(1)} + a_{23}\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)}) \\ &= \frac{v^{(1)}}{K_d^{(1)}}x\left[-\alpha^{(1)}\delta p_c + (\alpha^{(1)}/B^{(1)})\delta p_f^{(1)}\right].\end{aligned} \tag{11}$$

Again substituting for $\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$ from (5) and noting that the resulting equation contains arbitrary values of δp_c and $\delta p_f^{(1)}$, the coefficients of these terms must vanish separately. Resulting equations are

$$a_{12} + a_{23}(1 - K_d^{(2)}/K_d^{(1)})/\alpha^{(2)} = -\alpha^{(1)}v^{(1)}/K_d^{(1)}, \tag{12}$$

and

$$a_{22} + a_{23}\left(\alpha^{(1)}K_d^{(2)}/\alpha^{(2)}K_d^{(1)}\right) = \alpha^{(1)}v^{(1)}/B^{(1)}K_d^{(1)}. \tag{13}$$

Solving these equations, we obtain

$$a_{23} = \frac{K_d^{(1)}K_d^{(2)}\alpha^{(1)}\alpha^{(2)}}{\left[K_d^{(2)} - K_d^{(1)}\right]^2}\left[\frac{v^{(1)}}{K_d^{(1)}} + \frac{v^{(2)}}{K_d^{(2)}} - \frac{1}{K_d^*}\right], \tag{14}$$

and

$$a_{22} = \frac{v^{(1)}\alpha^{(1)}}{B^{(1)}K_d^{(1)}} - \left(\frac{\alpha^{(1)}}{1 - K_d^{(1)}/K_d^{(2)}}\right)^2\left[\frac{v^{(1)}}{K_d^{(1)}} + \frac{v^{(2)}}{K_d^{(2)}} - \frac{1}{K_d^*}\right]. \tag{15}$$

Performing the corresponding calculation for $\delta\zeta^{(2)}$ produces formulas for a_{32} and a_{33}. Since (14) is already symmetric in component indices, the formula for a_{32} provides nothing new. The formula for a_{33} is easily seen to be identical in form to a_{22}, but indices 1 and 2 are interchanged.

Formulas for all five of the nontrivial coefficients of double porosity have now been determined. These results also show how the constituent properties K_d, α, B up-scale at the macrolevel for a two-constituent composite [10, 12]. We find

$$\alpha = -\frac{a_{12} + a_{13}}{a_{11}} = \frac{\alpha^{(1)}(K_d^* - K_d^{(2)}) + \alpha^{(2)}(K_d^{(1)} - K_d^*)}{K_d^{(1)} - K_d^{(2)}}, \tag{16}$$

and

$$\frac{1}{B} = -\frac{a_{22} + 2a_{23} + a_{33}}{a_{12} + a_{13}}. \tag{17}$$

Note that all the important formulas [(8),(9),(11)-(14)] depend on the overall drained bulk modulus K_d^* of the system. So far this quantity is unknown and therefore must still be determined independently either by experiment or by another analytical method.

It should also be clear that some parts (but not all) of the preceding analysis generalize to the multi-porosity problem (*i.e.*, more than two porosity types). A discussion of the issues surrounding solvability of the multiporosity problem has been presented elsewhere [17].

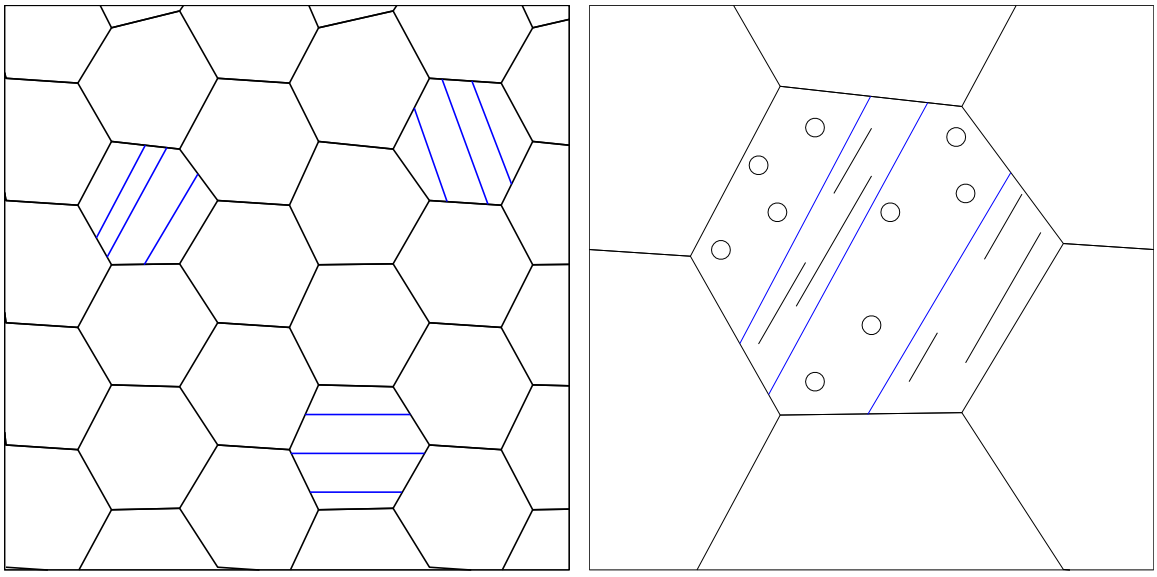

FIG. 1: Schematics of the random polycrystals of laminates model (on the left) and a blowup (on the right) showing a detail that illustrates how each one of the grains is composed of two very different types of porous materials: one being a storage material (high porosity and low permeability) and one a transport material (low porosity and high permeability).

3. UP-SCALING MODEL FOR GEOMECHANICS OF RESERVOIRS

3.1 *Elasticity of layered materials*

Next, to determine the overall drained (or undrained) bulk and shear moduli of the reservoir, assume a typical building block of the random system is a small (relative to the size of the reservoir) "grain" of laminate material whose elastic response for a transversely isotropic (hexagonal) system

can be described locally by:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & 2c_{44} & & \\ & & & & 2c_{44} & \\ & & & & & 2c_{66} \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix}, \tag{18}$$

where σ_{ij} are the usual stress components for $i, j = 1 - 3$ in Cartesian coordinates, with 3 (or z) being the axis of symmetry (the lamination direction for such a layered material). Displacement u_i is then related to strain component e_{ij} by $e_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$. This definition introduces some convenient factors of two into the $44, 55, 66$ components of the stiffness matrix shown in (18).

For definiteness we also assume that the stiffness matrix in (18) arises from the lamination of N isotropic constituents having bulk and shear moduli K_n, μ_n, in the $N > 1$ layers present in each building block. It is important that the thicknesses d_n always be in the same proportion in each of these laminated blocks, so that $f_n = d_n / \sum_{n'} d_{n'}$. But the order in which layers were added to the blocks is not important, as Backus's formulas [18] for the constants show. For the overall quasistatic (long wavelength) behavior of the system we are studying, Backus's results (also see [1, 19–21]) state that

$$\begin{aligned} c_{33} &= \left\langle \tfrac{1}{K+4\mu/3} \right\rangle^{-1}, & c_{13} &= c_{33} \left\langle \tfrac{K-2\mu/3}{K+4\mu/3} \right\rangle, \\ c_{44} &= \left\langle \tfrac{1}{\mu} \right\rangle^{-1}, & c_{66} &= \langle \mu \rangle, \\ c_{11} &= \tfrac{c_{13}^2}{c_{33}} + 4c_{66} - 4 \left\langle \tfrac{\mu^2}{K+4\mu/3} \right\rangle, & c_{12} &= c_{11} - 2c_{66}. \end{aligned} \tag{19}$$

This bracket notation can be correctly viewed as a line integral along the symmetry axis x_3. The bulk modulus K_n and shear modulus μ_n displayed in these averages can be either the drained or the undrained moduli for the individual layers. For the undrained case, the results are inherently assumed either to apply at very high frequencies, such as ultrasonic frequencies in laboratory

experiments, or to situations wherein each layer is physically isolated so that fluid increments cannot move from one porous layer to the next.

The bulk modulus for each laminated grain is that given by the compressional Reuss average K_R of the corresponding compliance matrix s_{ij} [the inverse of the usual stiffness matrix c_{ij}, whose nonzero components are shown in (18)]. The result is $e = e_{11} + e_{22} + e_{33} = \sigma/K_{\rm eff}$, where $1/K_{\rm eff} = 1/K_R = 2s_{11} + 2s_{12} + 4s_{13} + s_{33}$.

Even though $K_{\rm eff} = K_R$ is the same for every grain, since the grains themselves are not isotropic, the overall bulk modulus K^* of the random polycrystal does not necessarily have the same value as K_R for the individual grains [3]. Hashin-Shtrikman bounds on K^* for random polycrystals whose grains have hexagonal symmetry [22, 23] show in fact that the K_R value lies outside the bounds in many situations [21].

3.2 *Bounds for random polycrystals*

3.2.1 Voigt and Reuss bounds: hexagonal symmetry

For hexagonal symmetry, the nonzero stiffness constants are: c_{11}, c_{12}, $c_{13} = c_{23}$, c_{33}, $c_{44} = c_{55}$, and $c_{66} = (c_{11} - c_{12})/2$.

The Voigt [4] average for bulk modulus of hexagonal systems is well-known to be

$$K_V = \left[2(c_{11} + c_{12}) + 4c_{13} + c_{33}\right]/9. \tag{20}$$

Similarly, for the overall shear modulus G^*, we have

$$G_V = \frac{1}{5}\left(G^v_{\rm eff} + 2c_{44} + 2c_{66}\right), \tag{21}$$

where the new term appearing here is essentially defined by (21) and given explicitly by

$$G^v_{\rm eff} = (c_{11} + c_{33} - 2c_{13} - c_{66})/3. \tag{22}$$

The quantity $G^v_{\rm eff}$ is the energy per unit volume in a grain when a "pure uniaxial shear" *strain* of

unit magnitude [*i.e.*, $(e_{11}, e_{22}, e_{33}) = (1, 1, -2)/\sqrt{6}$], whose main compressive strain is applied to the grain along its axis of symmetry [21, 24].

Note that the concept of "pure uniaxial shear" strain (or stress) is based on the observation that if a uniaxial principal strain (or stress) of magnitude 3 is applied along the symmetry axis, it can be decomposed according to $(0, 0, 3)^T = (1, 1, 1)^T - (1, 1, -2)^T$ into a pure compression and a pure shear contribution, which is then called for the sake of brevity the "pure uniaxial shear."

The Reuss [5] average K_R for bulk modulus can also be written in terms of stiffness coefficients as

$$\frac{1}{K_R - c_{13}} = \frac{1}{c_{11} - c_{66} - c_{13}} + \frac{1}{c_{33} - c_{13}}. \tag{23}$$

The Reuss average for shear is

$$G_R = \left[\frac{1}{5}\left(\frac{1}{G^r_{\text{eff}}} + \frac{2}{c_{44}} + \frac{2}{c_{66}}\right)\right]^{-1}, \tag{24}$$

that defines G^r_{eff} – *i.e.*, the energy per unit volume in a grain when a pure uniaxial shear *stress* of unit magnitude [*i.e.*, $(\sigma_{11}, \sigma_{22}, \sigma_{33}) = (1, 1, -2)/\sqrt{6}$], whose main compressive pressure is applied to a grain along its axis of symmetry.

For each grain having hexagonal symmetry, two product formulas found by Berryman [24] hold: $3K_R G^v_{\text{eff}} = 3K_V G^r_{\text{eff}} = \omega_+\omega_-/2 = c_{33}(c_{11} - c_{66}) - c_{13}^2$. The symbols $\omega_\pm$ stand for the quasi-compressional and quasi-uniaxial-shear eigenvalues for the crystalline grains. Thus, it follows that

$$G^r_{\text{eff}} = K_R G^v_{\text{eff}}/K_V \tag{25}$$

is a general formula, true for hexagonal symmetry.

3.2.2 Hashin-Shtrikman bounds

It has been shown elsewhere [21, 24] that the Peselnick-Meister-Watt [22, 23] bounds for bulk modulus of a random polycrystal composed of hexagonal (or transversely isotropic) grains are given

by

$$K_{PM}^{\pm} = \frac{K_V(G_{\text{eff}}^r + \zeta_\pm)}{(G_{\text{eff}}^v + \zeta_\pm)} = \frac{K_R G_{\text{eff}}^v + K_V \zeta_\pm}{G_{\text{eff}}^v + \zeta_\pm}, \tag{26}$$

where G_{eff}^v (G_{eff}^v) is the uniaxial shear energy per unit volume for a unit applied shear strain (stress). The second equality follows directly from the product formula (25). Parameters $\zeta_\pm$ are defined by

$$\zeta_\pm = \frac{G_\pm}{6}\left(\frac{9K_\pm + 8G_\pm}{K_\pm + 2G_\pm}\right). \tag{27}$$

In (27), values of $G_\pm$ (shear moduli of isotropic comparison materials) are given by inequalities

$$0 \le G_- \le \min(c_{44}, G_{\text{eff}}^r, c_{66}), \tag{28}$$

and

$$\max(c_{44}, G_{\text{eff}}^v, c_{66}) \le G_+ \le \infty. \tag{29}$$

The values of $K_\pm$ (bulk moduli of isotropic comparison materials) are then given by algorithmic equalities

$$K_\pm = \frac{K_V(G_{\text{eff}}^r - G_\pm)}{(G_{\text{eff}}^v - G_\pm)}, \tag{30}$$

derived by [22] and [23]. Also see [21].

Bounds $G_{\text{hex}}^\pm$ (+ is upper bound, − is the lower bound) on the shear moduli for random polycrystals of hexagonal crystals are then given by

$$\frac{1}{G_{\text{hex}}^\pm + \zeta_\pm} = \frac{1}{5}\Big[\frac{1 + \gamma_\pm(K_V - K_\pm)}{G_{\text{eff}}^v + \zeta_\pm + \delta_\pm(K_V - K_\pm)} + \frac{2}{c_{44} + \zeta_\pm} + \frac{2}{c_{66} + \zeta_\pm}\Big], \tag{31}$$

where $\gamma_\pm$ and $\delta_\pm$ are given by

$$\gamma_\pm = \frac{1}{K_\pm + 4G_\pm/3}, \quad \text{and} \quad \delta_\pm = \frac{5G_\pm/2}{K_\pm + 2G_\pm}. \tag{32}$$

K_V is the Voigt average of the bulk modulus as defined previously.

TABLE 1. Input Parameters for Weber Sandstone Model of Double-Porosity System.

K_s (GPa)	$K_s^{(1)}$ (GPa)	$K_d^{(1)}$ (GPa)	$G_d^{(1)}$ (GPa)	$\phi^{(1)}$	$K_s^{(2)}$ (GPa)	$K_d^{(2)}$ (GPa)	$G_d^{(2)}$ (GPa)	$\phi^{(2)}$
27.9	28.0	19.3	20.2	0.095	27.0	0.24	0.60	0.095

Note: Porosity ϕ is dimensionless.

4. EXAMPLE: WEBER SANDSTONE

Weber sandstone is one possible host rock for which the required elastic constants have been measured by Coyner [25]. TABLE 1 displays the values needed in the double-porosity theory presented here. These values follow from Coyner's data if we assume the stiffer phase occupies about 92% of the volume and the more compliant phase the remaining 8% of total volume.

The drained bulk moduli of the storage and fracture phases are used in the effective medium theory of Section 3 to determine the overall drained and undrained bulk moduli of the random polycrystal of laminates system. Results for the self-consistent estimates [21] and the upper and lower bounds for the bulk moduli are all displayed in Figure 2. We see the undrained moduli are nearly indistinguishable, but the drained constants show some dispersion.

Similarly, we show bounds and self-consistent estimates for the overall shear modulus of this model reservoir in Figure 3. Both undrained and drained shear moduli show some dispersion.

Note that a correction must be applied to (31) before computing the self-consistent effective constants. The self-consistent estimates for bulk modulus are found correctly from the bounds (26) by taking $K_\pm \to K^*$, $G_\pm \to G^*$, and therefore $\zeta_\pm \to \zeta^*$. The resulting formula is

$$K^* = K_V \frac{(G^r_{\text{eff}} + \zeta^*)}{(G^v_{\text{eff}} + \zeta^*)}. \tag{33}$$

The self-consistent formula for shear modulus requires more effort. The difficulty is that the

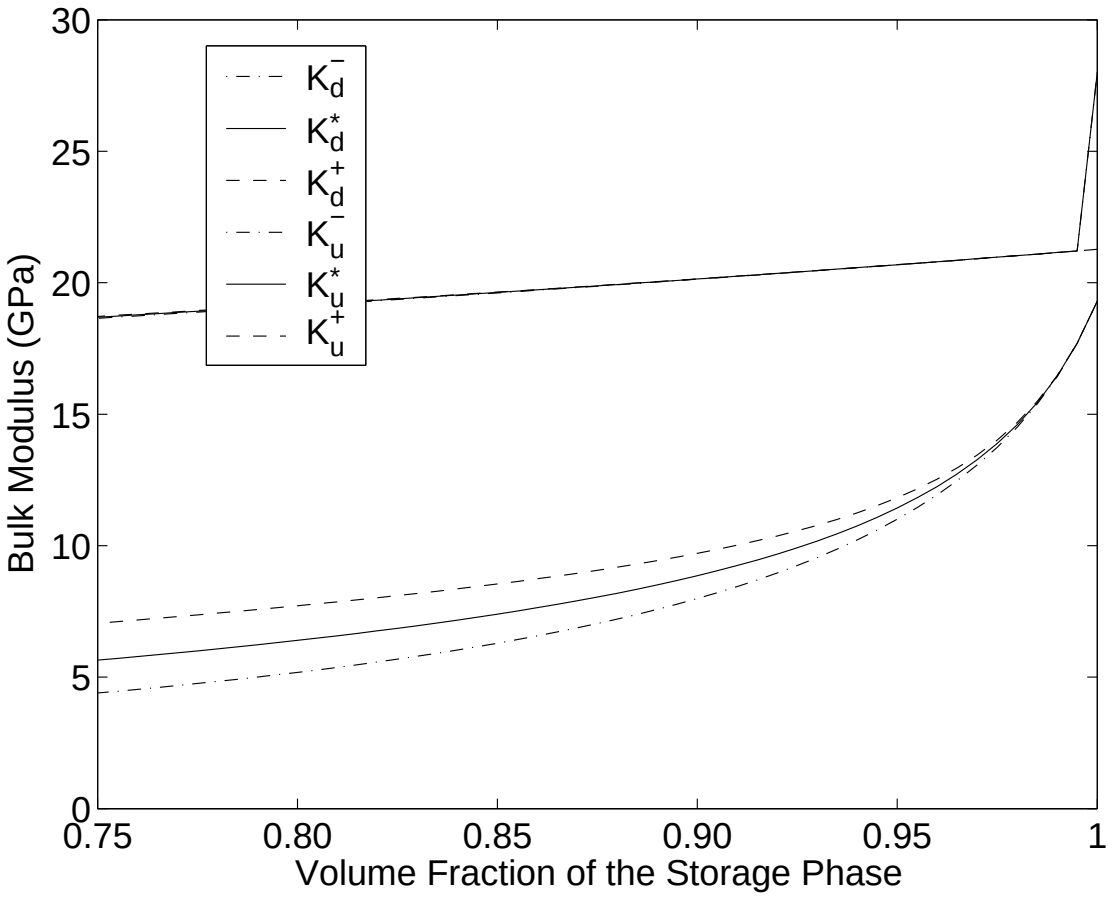

FIG. 2: Bulk modulus bounds and self-consistent estimates for the random polycrystal of porous laminates model of a Weber sandstone reservoir.

formula given in (31) has already made use of a constraint that is only true along the bounding curves defining the upper and lower bounds on shear modulus. Since the self-consistent estimate always falls at points away from this curve, a more general result must be employed. When the inappropriate constraint is replaced by the general formula and then (33) is substituted, we find instead that the self-consistent formula for shear modulus is given by

$$\frac{1}{G^*+\zeta^*} = \frac{1}{5}\left(\frac{1+\gamma^*(K_V-K^*)}{G^v_{\text{eff}}+\zeta^*} + \frac{2}{c_{44}+\zeta^*} + \frac{2}{c_{66}+\zeta^*}\right), \tag{34}$$

where $\gamma^* = 1/(K^* + 4G^*/3)$. The main difference is that the denominator of the first term on the right hand side is simpler than it is in the formulas for the shear modulus bounds.

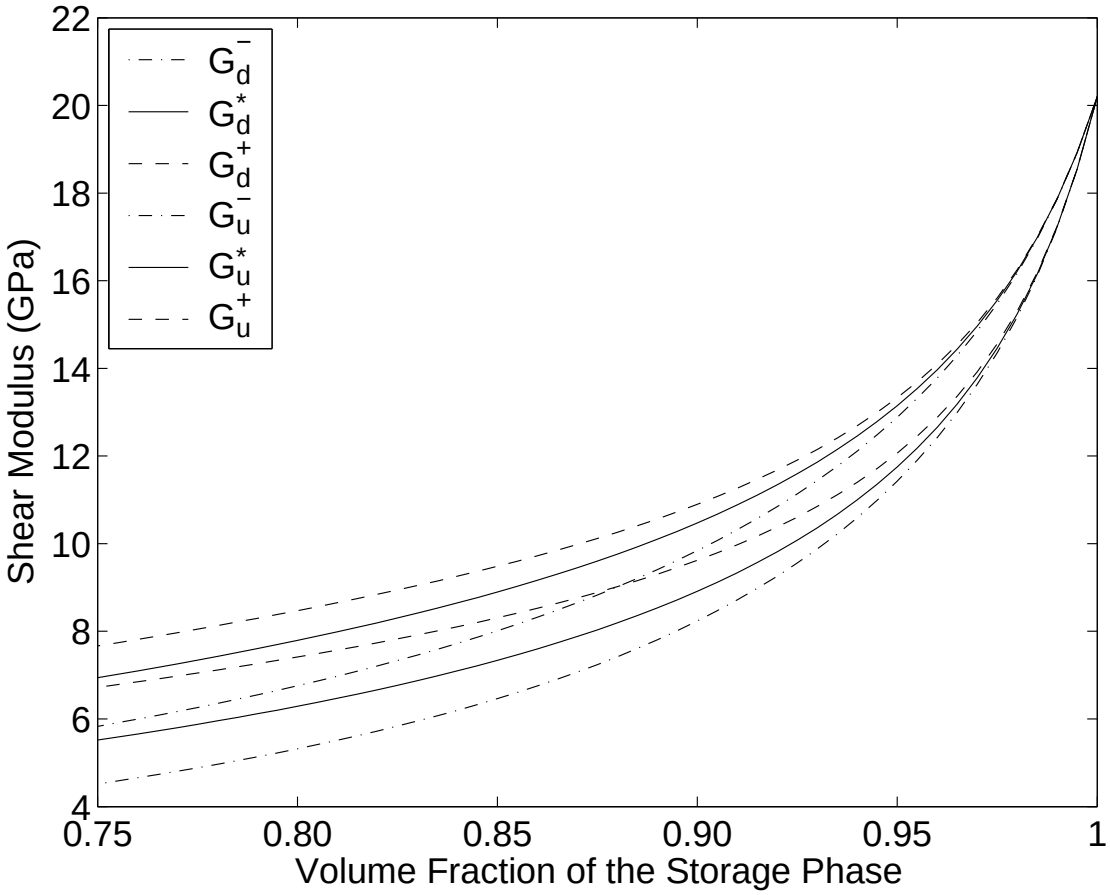

FIG. 3: Shear modulus bounds and self-consistent estimates for the random polycrystal of porous laminates model of a Weber sandstone reservoir.

Observed dispersion is small over the range of volume fractions considered. Then these drained values K_d^*, $K_d^{\pm}$ are used in the formulas of Sec. 2 to determine both estimates and bounds on the double-porosity coefficients. These results are then displayed in Figure 4, which is the main result of this paper. Note that the curves for a_{11} essentially repeat results shown in Figure 2, but for the compliance $1/K_d^*$, instead of the stiffness K_d^*.

The coefficients a_{12}, a_{22}, and a_{23} show little dispersion. This is natural for a_{12} and a_{22} because the storage material contains no fractures, and therefore is not sensitive to fracture compliance, whereas those mechanical effects on the overall reservoir response can be very large. The behavior of a_{23} also shows little dispersion as this value is always quite close to zero [10, 12]. The two

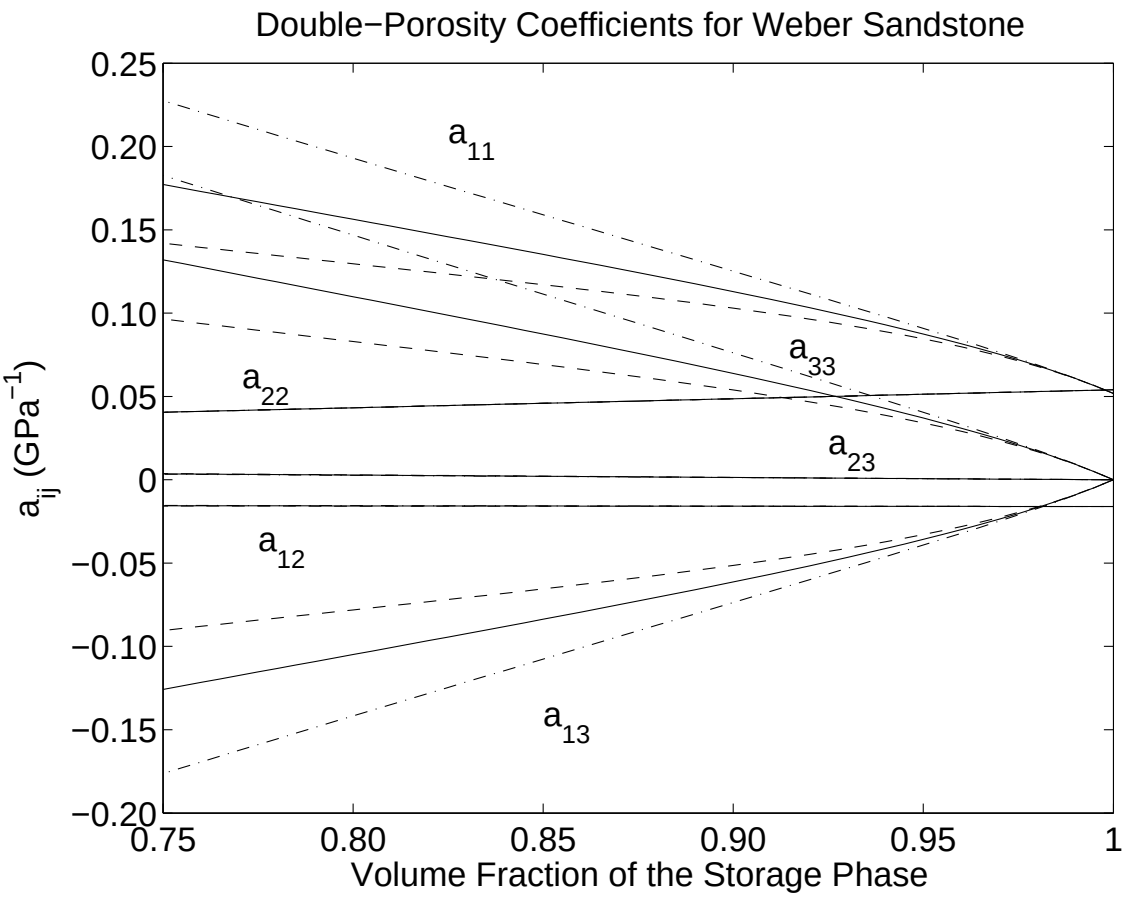

FIG. 4: Values of double-porosity coefficients a_{ij} for a system similar to Weber sandstone. Values used for the input parameters are listed in TABLE 1. For each coefficient, three curves are shown, depending on which estimate of the overall bulk modulus is used: lower bound (dot-dash line), self-consistent (solid line), or upper bound (dashed line).

remaining coefficients show a significant level of dispersion are a_{13} and a_{33}, where the third stress is the pore pressure $p_f^{(2)}$ of the fracture or joint phase. We generally expect that the joint phase is most tightly coupled to, and therefore most sensitive to, the fluctuations in overall drained bulk modulus K_d^*. So all these results are qualitatively consistent with our intuition.

Since we have analytical formulas for all the a_{ij}'s, it is straightforward to check that the observed dispersion in a_{13} and a_{33} is directly proportional to the dispersion in $1/K_d^*$ (or, equivalently, a_{11}).

5. CONCLUSIONS

The methods presented have been successfully applied to determine geomechanical parameters for one reservoir model assuming Weber sandstone is the host rock. Although the details differ, the general ideas used above for elastic and poroelastic constants can also be used to obtain bounds and estimates of electrical formation factor and fluid permeability for the same random polycrystal of porous laminates model.

The analysis for permeability for this model requires some extra care, and so we will defer this part of the work to a future contribution.

ACKNOWLEDGMENTS

Work performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48 and supported specifically by the Geosciences Research Program of the DOE Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences and Biosciences.

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