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Global DC Closed Orbit Correction Experiment on the NSLS X-ray Ring

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Abstract

In this note are described the global DC closed orbit correction experiments conducted on the X-ray ring at National Synchrotron Light Source (NSLS). The beam response matrix, defined as beam motion at BPM locations per unit kick by corrector magnets, was measured and then inverted using the technique of singular value decomposition (SVD). The product of the inverted matrix and the difference orbit gives the incremental kick strengths necessary to correct the orbit. As a result, the r.m.s. orbit error around the ring was reduced from 208 μm to 61 μm.

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1. Introduction

The third generation synchrotron light sources, such as the APS, are characterized by low emittance of the charged particle beams and high brightness of the photon beams radiated from insertion devices. Transverse stability of the particle beams is a crucial element in achieving these goals and the APS will implement extensive beam position feedback systems, which include 320 corrector magnets, 360 positron beam position monitors distributed around the storage ring, miniature BPMs for insertion device beam lines, and photon beam position monitors at the end of the X-ray beam lines.

The beam position feedback systems can largely be divided into the global and local feedback systems according to the extent of correction, and the DC and AC feedback systems according to the bandwidth of correction. DC correction of the beam positions, as the name implies, is a slow process with sub-Hz bandwidth and is typically done with an integral control algorithm with unity gain for full correction and less-than-unity gain for partial correction. In contrast, AC correction is a fast process with wide bandwidth (typically 10 – 100 Hz), and the APS will employ the proportional, integral, and derivative (PID) control algorithm to ensure stability with minimal noise infiltration.[1,2]

In this note, we will present the results of global DC beam position feedback experiments conducted on the X-ray ring of the National Synchrotron Light Source (NSLS). Integral control with full correction was used, and the technique of singular value decomposition (SVD) was used to invert the response matrix. The product of the inverted matrix and the difference orbit gives the incremental kick strengths necessary to correct the orbit.

The rest of this note will consist of description of the theory of SVD for application to global beam position correction in Section 2 and the measurement results in Section 3. Summary will be given in Section 4.

2. Theory

Global correction of the closed orbit is done with a set of corrector magnets distributed around the ring and a set of beam position monitors (BPMs). Let M be the number of BPMs and let N be the number of correctors. Changes in the corrector strengths $\Delta\theta$ bring about changes in the closed orbit Δx, and we assume that they are linearly related through the response matrix R_{ij} by

$$\Delta x_i = \sum_{j=1}^{N} R_{ij}\, \Delta\theta_j. \quad (\text{for } 1 \le i \le M) \tag{2.1}$$

Δx_i is the beam motion at the i-th BPM and $\Delta\theta_j$ is the increment in the angular kick by the j-th corrector. The response matrix R_{ij} can be written in terms of the betatron functions β and ψ at the locations of the BPMs and correctors as

$$R_{ij} = \frac{\sqrt{\beta_i \beta_{cj}}}{2 \sin \pi\nu} \cos\left(\left|\psi_i - \psi_{cj}\right| - \pi\nu\right), \tag{2.2}$$

where β_i and ψ_i (β_{cj} and ψ_{cj}) are the betatron amplitude and phase functions of the i-th BPM (j-th corrector). ν is the tune.

The response matrix R_{ij} in Eq. (2.1) can be directly measured by changing the strength of the j-th magnet by a small amount and then measuring the resulting beam motion at all BPMs and repeating the same procedure for all correctors. Global correction of the closed orbit is then equivalent to inverting this process. Writing Eq. (2.1) in matrix form, we have

$$\mathbf{\Delta x} = \mathbf{R} \cdot \mathbf{\Delta\theta}. \tag{2.3}$$

The inverse matrix of $\mathbf{R}$, which we call $\mathbf{R}_{inv}$, uniquely exists such that

$$\mathbf{R} \cdot \mathbf{R}_{inv} = \mathbf{R}_{inv} \cdot \mathbf{R} = \mathbf{1} \tag{2.4}$$

if $M = N$ and if the matrix is not singular. $\mathbf{1}$ is the identity matrix.

Even if $M \neq N$ or if the matrix is singular, an inverse of the matrix can still be obtained, though with some restrictions, using the technique of singular value decomposition (SVD).[3–5] Any $M \times N$ matrix $\mathbf{R}$ can be written as[6]

$$\mathbf{R} = \mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^T, \tag{2.5}$$

where $\mathbf{U}$ is an $M \times M$ unitary matrix ($\mathbf{U}^T \cdot \mathbf{U} = \mathbf{U} \cdot \mathbf{U}^T = \mathbf{1}$), $\mathbf{W}$ is an $M \times N$ diagonal matrix with positive or zero elements, and $\mathbf{V}$ is an $N \times N$ unitary matrix ($\mathbf{V}^T \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{V}^T = \mathbf{1}$). The representation given in Eq. (2.5) is unique only to a certain extent, and there are other ways of decomposing the matrix $\mathbf{R}$.[7,8]

Since both $\mathbf{U}$ and $\mathbf{V}$ are unitary, they represent orthonormal transformations from one frame to another. The $\mathbf{V}$ matrix rotates the N-dimensional orthogonal coordinate system, with each axis corresponding to a corrector magnet, to another N-dimensional orthogonal coordinate system and generates a new set of N transformed correctors (or t-correctors). The **U** matrix operates similarly on an M-dimensional BPM space and generates a new set of M transformed BPMs (or t-BPMs). Let us define $\Delta\mathbf{x}^t$ and $\Delta\boldsymbol{\theta}^t$ as

$$\Delta\mathbf{x}^t = \mathbf{U}^T \cdot \Delta\mathbf{x} \quad \text{and} \quad \Delta\boldsymbol{\theta}^t = \mathbf{V}^T \cdot \Delta\boldsymbol{\theta}, \tag{2.6}$$

where "t" denotes "transformed". Then from Eqs. (2.3), (2.5), and (2.6), we have

$$\Delta\mathbf{x}^t = \mathbf{W} \cdot \Delta\boldsymbol{\theta}^t. \tag{2.7}$$

Comparing Eqs. (2.3) and (2.7), we see that SVD diagonalized the matrix **R** into **W**. We can write the matrix **W**, with the indices i and j in the ranges $1 \le i \le M$ and $1 \le j \le N$, as

$$W_{ij} = \begin{cases} w_i\, \delta_{ij}, & (M \le N) \\ w_j\, \delta_{ij}, & (M \ge N) \end{cases} \tag{2.8}$$

where δ_{ij} is the Kronecker delta. The diagonal elements w_i's (or w_j's), or the eigenvalues, are non-negative, and the number of them are equal to the lesser of M and N. Associated with the eigenvalues are the mutually orthogonal eigenvectors $\{\mathbf{v}_j \mid 1 \le j \le N\}$ spanning the space of the t-correctors. Similarly, we have $\{\mathbf{u}_i \mid 1 \le i \le M\}$, a set of mutually orthogonal unit vectors spanning the t-BPM space. These eigenvectors are related by

$$\mathbf{R} \cdot \mathbf{v}_i = w_i\, \mathbf{u}_i\,, \quad 1 \le i \le \min\,(M, N). \tag{2.9}$$

Thus, the eigenvalues represent the coupling efficiency between the t-correctors and t-BPMs.

Since the **W** matrix has dimension $M \times N$, some of the columns are all zeroes when $M < N$ and some of the rows are all zeroes when $M > N$. Let us first consider the case when $M < N$. We can see immediately that there are at least $(N - M)$ t-correctors which have no corresponding non-zero eigenvalues and contribute nothing as far as orbit correction is concerned. Therefore, these t-correctors can be set to any arbitrary values, but for the purpose of minimizing the norm of the vector $\Delta\boldsymbol{\theta}$, they are set to zero. Simiarly, if $M > N$, then we have at least $(M - N)$ t-BPMs which cannot be changed since they have no coupling to the correctors. This imposes a limitation on how much the closed orbit can be corrected with a given number of correctors. The finite capacity of the power supplies is another limiting factor and will be discussed later.

One great advantage of SVD is that we can know in advance whether a given matrix is singular or not before trying to invert the matrix and remove the singularities if so desired. From Eq. (2.5), we write the inverse of the matrix **R** as

$$\mathbf{R}_{inv} = \mathbf{V}\cdot\mathbf{W}_{inv}\cdot\mathbf{U}^T, \qquad (2.10)$$

where the $N \times M$ matrix $\mathbf{W}_{inv}$ is constructed by inverting the eigenvalues and then taking the transpose of the matrix. If $M \leq N$, $\mathbf{W}\cdot\mathbf{W}_{inv}$ is equal to the the $M \times M$ identity matrix, but $\mathbf{W}_{inv}\cdot\mathbf{W}$ has only M unity elements in the diagonal axis and all others are equal to zero.

If any of the eigenvalues is equal to zero, that is, if the matrix **R** is singular, these singularities can be removed simply by putting

$$\frac{1}{w_j} \rightarrow 0 \qquad (2.11)$$

rather than a large number in the inverse matrix $\mathbf{W}_{inv}$. This technique can be extended to the cases when the matrix is nearly singular, that is, when some of the eigenvalues satisfy

$$w_j < \varepsilon\, w_{max} \qquad (2.12)$$

where the singularity criterion ε is a preset small number and represents the desired accuracy of feedback. w_{max} is the greatest of the eigenvalues. We now write $\mathbf{W}_{inv}$, with the indices i and j in the ranges $1 \leq i \leq N$ and $1 \leq j \leq M$, as

$$W_{inv,ij} = \begin{cases} q_j\, \delta_{ij}, & (M \leq N) \\ q_i\, \delta_{ij}, & (M \geq N) \end{cases} \qquad (2.13)$$

where

$$q_j = \begin{cases} 0, & w_j \leq \varepsilon w_{max} \\ \dfrac{1}{w_j}, & \text{otherwise} \end{cases} \qquad (2.14)$$

and similarly for q_i. For a given matrix **R**, let us define $\varepsilon_m(\mathbf{R})$ of the matrix **R** as

$$\varepsilon_m(\mathbf{R}) = \max\, \{\varepsilon \mid w_j > \varepsilon\, w_{max} \text{ for all } w_j \neq 0\}. \qquad (2.15)$$

That is, ε_m is the largest possible value for ε in order to retain all non-zero eigenvalues. Now that removal of singularities has become trivial, we will assume for simplicity that **R** is not singular in the following discussion unless noted otherwise. When ε is equal to 0, all the non-zero eigenvalues are kept, and we will have the most accurate feedback. However, this comes at the cost of more robust power supplies for the corrector magnets. In the other extreme case, when ε is equal to 1, $\mathbf{R}_{inv}$ is identically zero, and there is no feedback.

Now, from the above consideration, the pseudo-inverse of $\mathbf{R}$ defined in Eq. (2.10) satisfies

$$\mathbf{R}\cdot\mathbf{R}_{inv}\cdot\mathbf{R} = \mathbf{R} \quad (\varepsilon \le \varepsilon_m) \quad \text{and} \quad \mathbf{R}_{inv}\cdot\mathbf{R}\cdot\mathbf{R}_{inv} = \mathbf{R}_{inv} \quad (\text{for all } \varepsilon). \tag{2.16}$$

In addition, with $\varepsilon \le \varepsilon_m$,

$$\mathbf{R}_{inv}\cdot\mathbf{R} = \mathbf{1} \text{ if } M \ge N \quad \text{and} \quad \mathbf{R}\cdot\mathbf{R}_{inv} = \mathbf{1} \text{ if } M \le N. \qquad (\varepsilon \le \varepsilon_m) \tag{2.17}$$

By removing the eigenvalues satisfying Eq. (2.12) with $\varepsilon > \varepsilon_m$, the inverse matrix $\mathbf{R}_{inv}$ will be less accurate than it would otherwise be, but the vector norm of the solution can be significantly smaller. This is very desirable when certain limitations exist on the magnitude of the vector components. In our application, the solution vector is the change in the corrector strengths, which cannot be arbitrarily large because of the finite capacity of the power supplies.

Given the current orbit $\mathbf{x}_m$ measured by the BPMs and the desired reference orbit $\mathbf{x}_r$, let $\Delta\mathbf{x}_d$ be the difference orbit given by

$$\Delta\mathbf{x}_d = \mathbf{x}_r - \mathbf{x}_m. \tag{2.18}$$

We want to calculate back $\Delta\boldsymbol{\theta}_d$, the required changes in corrector strengths to bring the orbit to the desired reference orbit, which satisfies

$$\mathbf{R}\cdot\Delta\boldsymbol{\theta}_d = \Delta\mathbf{x}_d. \tag{2.19}$$

We may categorize this linear equation according to the relative sizes of M and N as follows:

$$\begin{cases} M > N & \rightarrow \text{overdetermined, no exact solutions} \\ M = N & \rightarrow \text{uniquely determined, a unique solution} \\ M < N & \rightarrow \text{underdetermined, many solutions} \end{cases} \tag{2.20}$$

Now, the inverse matrix $\mathbf{R}_{inv}$ obtained in Eq. (2.10) using SVD gives a solution as

$$\Delta\boldsymbol{\theta}_d = \mathbf{R}_{inv}\cdot\Delta\mathbf{x}_d. \tag{2.21}$$

In case M > N, this solution does not satisfy Eq. (2.19) exactly but minimizes the difference $\left|\mathbf{R}\cdot\Delta\boldsymbol{\theta}_d - \Delta\mathbf{x}_d\right|$. Consider

$$\left|\mathbf{R}\cdot\Delta\boldsymbol{\theta}_d - \Delta\mathbf{x}_d\right| = \left|\mathbf{W}\cdot\Delta\boldsymbol{\theta}_d^t - \Delta\mathbf{x}_d^t\right| = \left(\sum_{i=C+1}^{M} \left|\Delta x_{d,i}^t\right|^2\right)^{1/2}. \tag{2.22}$$

due to Eqs. (2.5) and (2.6). C is the number of coupled t-BPMs (or t-correctors). The index i between C + 1 and M corresponds to decoupled t-BPMs, for which the initial difference $\Delta x_{d,i}^t$ cannot be changed. The coupled t-BPMs will change to the reference values; therefore, Eq. (2.22) is the minimum difference and this is the best we can get.

On the other hand, when M < N, there are many (actually an infinite number of) solutions, and SVD picks the solution that minimizes $\left|\Delta\boldsymbol{\theta}_d\right|$ by setting the decoupled t-correctors to zero. That is,

$$\left|\Delta\boldsymbol{\theta}_d\right| = \left|\Delta\boldsymbol{\theta}_d^t\right| = \left(\sum_{j=1}^{C} \left|\Delta\theta_{d,j}^t\right|^2\right)^{1/2}. \tag{2.23}$$

is the absolute minimum among all solutions, with $\Delta\theta_{d,j}^t = 0$ for $C + 1 \leq j \leq N$. These decoupled t-correctors do not affect orbit correction at all. So, Eq. (2.23) is the best we can get, since the overall changes in corrector strengths will be the smallest possible.

Once $\Delta\boldsymbol{\theta}_d$ as given by Eq. (2.21) is applied, the closed orbit will move to a new orbit given by

$$\mathbf{x}_m' = \mathbf{x}_m + \mathbf{R}\cdot\Delta\boldsymbol{\theta}_d. \tag{2.24}$$

When ε is larger than ε_m, this new orbit will not necessarily be equal to the reference orbit $\mathbf{x}_r$, since $\mathbf{R}\cdot\mathbf{R}_{inv}$ is not necessarily equal to $\mathbf{1}$. However, as long as ε is not changed, which keeps $\mathbf{R}_{inv}$ the same, there cannot be any further correction of the closed orbit. The new difference orbit $\Delta\mathbf{x}_d'$, from Eqs. (2.21) and (2.22), is given by

$$\Delta\mathbf{x}_d' = \mathbf{x}_r - \mathbf{x}_m' = (\mathbf{1} - \mathbf{R}\cdot\mathbf{R}_{inv})\cdot\Delta\mathbf{x}_d. \tag{2.25}$$

The new corrector strength change $\Delta\boldsymbol{\theta}_d'$ then vanishes, since

$$\Delta\boldsymbol{\theta}_d' = \mathbf{R}_{inv}\cdot\Delta\mathbf{x}_d' = (\mathbf{R}_{inv} - \mathbf{R}_{inv}\cdot\mathbf{R}\cdot\mathbf{R}_{inv})\cdot\Delta\mathbf{x}_d = \mathbf{0} \tag{2.26}$$

according to Eq. (2.16). In reality, due to the error in the measurement of the response matrix $\mathbf{R}$, changes in the machine condition, and external perturbations, there will remain some residue in the closed orbit error which still needs to be corrected. Elimination of this residue in the orbit error will be done by fast AC closed loop feedback with appropriate bandwidth, as is discussed in Refs. 1 and 2.

Let us then consider optimization of orbit correction by adjusting the corrector strengths such that the total vector length is minimized. In Figs. 2.1(a) and 2.1(b), three points representing the uncorrected orbit, the reference orbit, and the current orbit are shown. Δx_2 is the difference between the reference orbit and the uncorrected orbit. The current orbit was established by applying corrector strength $\Delta\theta_1$ to the uncorrected orbit, which gives the current difference orbit as

$$\Delta x_3 = \Delta x_2 - \mathbf{R}\cdot\Delta\theta_1. \qquad (2.27)$$

Let $\Delta x_2'$ and $\Delta x_3'$ be the corresponding residual difference orbits after correction. Then we have

$$\Delta x_3' = (1 - \mathbf{R}\cdot\mathbf{R}_{inv})\cdot\Delta x_3 = \Delta x_2' - (\mathbf{R} - \mathbf{R}\cdot\mathbf{R}_{inv}\cdot\mathbf{R})\cdot\Delta\theta_1. \qquad (2.28)$$

This shows that the residual difference orbit depends on the current orbit and is not unique in general. Only when $\varepsilon \le \varepsilon_m$, we have $\Delta x_3' = \Delta x_2'$, according to Eq. (2.16).

A similar result can be derived for the corrector strength. The incremental corrector strength $\Delta\theta_3$ for correction of the current orbit is given by

$$\Delta\theta_3 = \mathbf{R}_{inv}\cdot\Delta x_3 = \Delta\theta_2 - \mathbf{R}_{inv}\cdot\mathbf{R}\cdot\Delta\theta_1, \qquad (2.29)$$

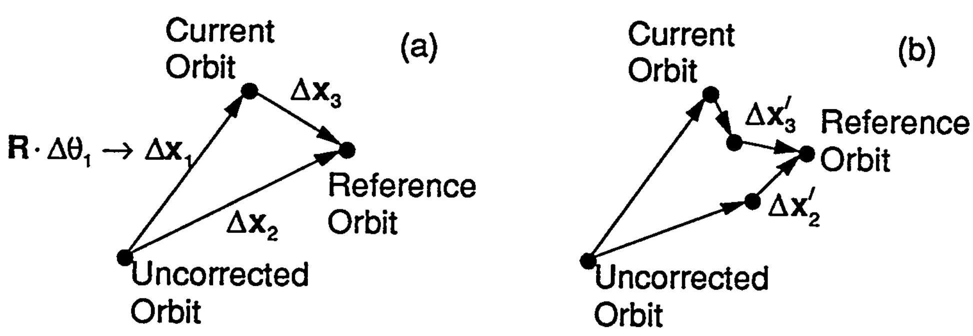


Fig. 2.1: Optimization of orbit correction. For $M \le N$, when ε is small enough such that $\mathbf{W}\cdot\mathbf{W}_{inv} = \mathbf{1}$, we have $\Delta x_3' = \Delta x_2'$ and $\Delta\theta_2 = \Delta\theta_1 + \Delta\theta_3$.

which gives the overall change $\Delta\theta_{31}$ as

$$\Delta\theta_{31} = \Delta\theta_3 + \Delta\theta_1 = \Delta\theta_2 + (\mathbf{1} - \mathbf{R}_{inv}\cdot\mathbf{R})\cdot\Delta\theta_1. \tag{2.30}$$

When $M < N$, $\Delta\theta_{31}$ always has larger vector norm than $\Delta\theta_2$, since $\Delta\theta_2$ is already optimized by SVD. When $M \geq N$, we can write $\mathbf{Q} = \mathbf{1} - \mathbf{R}_{inv}\cdot\mathbf{R}$, where

$$Q_{ij} = \begin{cases} 1, & i = j \text{ and } w_j \leq \varepsilon w_{max} \\ 0. & \text{otherwise} \end{cases} \tag{2.31}$$

Therefore, only when $\varepsilon \leq \varepsilon_m$, we have $\Delta\theta_{31} = \Delta\theta_2$, and the corrector strength is unique. Generally, when $\varepsilon > \varepsilon_m$, the corrector strength is not unique but we always have $|\Delta\theta_{31}| \geq |\Delta\theta_2|$, since $\Delta\theta_2\cdot\mathbf{Q}\cdot\Delta\theta_1 = \Delta\theta_2^t\cdot\mathbf{Q}\cdot\Delta\theta_1^t = 0$. Besides this, when optimizing the corrector strength, it also has to be considered that the magnet current should not exceed the limit for individual correctors.

3. Simulation of NSLS X-ray Ring

In this section we will discuss simulation of DC global beam position feedback on the NSLS X-ray ring using the model functions β and ψ, and in the next section we will present the measurement results. In Table 3.1 are shown the model β and ψ functions in the vertical direction at locations of BPMs (48) and corrector magnets (39). The nominal vertical tune of the machine is $\nu_v = 6.2$. From this table, we can construct the response matrix $\mathbf{R}$ as given by Eq. (2.2) and calculate the inverse matrix $\mathbf{R}_{inv}$ by using SVD. The result can be used to simulate beam position correction and estimate its efficiency in terms of the residual orbit error and the required changes in the corrector strength.

For a particle of momentum p, the relation between the angular deflection $\Delta\theta$ and the magnet current change ΔI is obtained from

$$\Delta\theta\ (\text{rad}) = \Delta I\ (\text{A}) \frac{0.2998}{p\ (\text{GeV/c})} \frac{B\ell\ (\text{T}\cdot\text{m})}{I\ (\text{A})}. \tag{3.1}$$

For the NSLS X-ray ring, $p = 2.528\ \frac{\text{GeV}}{c}$ and[9]

$$\frac{B\ell}{I} = \begin{cases} 4.68 \times 10^{-4}\ \frac{\text{T}\cdot\text{m}}{\text{A}}, & \text{V8 correctors} \\ 6.67 \times 10^{-4}\ \frac{\text{T}\cdot\text{m}}{\text{A}}. & \text{others} \end{cases} \tag{3.2}$$

Table 3.1: β and ψ functions (vertical) at the BPMs and correctors in the NSLS X-ray ring. The nominal tune is: $\nu_v = 6.2$. (* D: disabled, N: nonexistent)

BPM Name	β (m)	ψ (rad) / 2π
X1PUE1	1.553	0.1713
X1PUE2	15.9830	0.2498
X1PUE3	2.2850	0.3679
X1PUE4	5.3150	0.4660
X1PUE5	15.9830	0.5252
X1PUE6	14.0690	0.5504
X2PUE7	14.0690	0.9995
X2PUE8	15.9830	1.0248
X2PUE9	2.2850	1.1429
X2PUE10	5.3150	1.2410
X2PUE11	15.9830	1.3002
X2PUE12	14.0690	1.3254
X3PUE13	14.0690	1.7745
X3PUE14	15.9830	1.7998
X3PUE15	2.2850	1.9179
X3PUE16	5.3150	2.0160
X3PUE17	15.9830	2.0752
X3PUE18	14.0690	2.1004
X4PUE19	14.0690	2.5495
X4PUE20	15.9830	2.5748
X4PUE20	2.2850	2.6929
X4PUE22	5.3150	2.7910
X4PUE23	15.9830	2.8502
X4PUE24	14.0690	2.8754
X5PUE25	14.0690	3.3245
X5PUE26	15.9830	3.3498
X5PUE27	2.2850	3.4679
X5PUE28	5.3150	3.5660
X5PUE29	15.9830	3.6252
X5PUE30	14.0690	3.6504
X6PUE31	14.0690	4.0995
X6PUE32	15.9830	4.1248
X6PUE33	2.2850	4.2429
X6PUE34	5.3150	4.3410
X6PUE35	15.9830	4.4002
X6PUE36	14.0690	4.4254
X7PUE37	14.0690	4.8745
X7PUE38	15.9830	4.8998
X7PUE39	2.2850	5.0179
X7PUE40	5.3150	5.1160
X7PUE41	15.9830	5.1752
X7PUE42	14.0690	5.2004
X8PUE43	14.0690	5.6495
X8PUE44	15.9830	5.6748
X8PUE45	2.2850	5.7929
X8PUE46	5.3150	5.8910
X8PUE47	15.9830	5.9502
X8PUE48	14.0690	5.9754

Corrector Name	β (m)	ψ (rad) / 2π	*
X1V3	12.3800	0.2345	D
X1V5	26.4170	0.2559	
X1V8	3.4200	0.3343	
X1V14	26.4170	0.5191	
X1V16	12.3800	0.5405	
X2V3	12.3800	1.0095	
X2V5	26.4170	1.0309	
X2V8	3.4200	1.1093	
X2V14	26.4170	1.2940	
X2V16	12.3800	1.3155	
X3V3	12.3800	1.7845	
X3V5	26.4170	1.8059	
X3V8	3.4200	1.8843	
X3V14	26.4170	2.0691	
X3V16	12.3800	2.0905	
X4V3	12.3800	2.5595	
X4V5	26.4170	2.5809	
X4V8	3.4200	2.6593	
X4V14	26.4170	2.8440	
X4V16	12.3800	2.8655	
X4V17	7.3259	2.8853	N
X5V3	12.3800	3.3345	
X5V5	26.4170	3.3559	
X5V8	3.4200	3.4343	
X5V14	26.4170	3.6191	
X5V16	12.3800	3.6405	
X5V17	7.3259	3.6603	N
X6V3	12.3800	4.1095	
X6V5	26.4170	4.1309	
X6V8	3.4200	4.2093	
X6V14	26.4170	4.3941	
X6V16	12.3800	4.4155	
X7V3	12.3800	4.8845	
X7V5	26.4170	4.9059	
X7V8	3.4200	4.9843	
X7V14	26.4170	5.1691	
X7V16	12.3800	5.1905	
X8V3	12.3800	5.6595	
X8V5	26.4170	5.6809	
X8V8	3.4200	5.7593	
X8V14	26.4170	5.9440	
X8V16	12.3800	5.9655	

Table 3.2: Simulation results of DC global beam position correction on the NSLS X-ray ring. $\mathbf{R}_{inv}$ was obtained for different values of ε for comparison. $\varepsilon_m = 0.00234$. The initial Δx_{rms} is 207.8 μm.

ε	ΔI_{min} (A)	ΔI_{max} (A)	ΔI_{rms} (A)	Δx_{rms} (μm)
0.001	-4.10	3.39	1.54	46.8
0.002	-4.10	3.39	1.54	46.8
0.003	-2.64	1.32	0.85	72.2
0.005	-1.27	0.74	0.46	85.2
0.01	-1.15	0.27	0.33	91.6
0.02	-0.08	0.07	0.04	128.7
0.1	-0.08	0.07	0.04	128.7
0.2	-0.04	0.04	0.02	162.3
0.3	-0.01	0.01	0.01	183.1
1.0	0.00	0.00	-0.00	207.8

Therefore, we have from Eqs. (3.1) and (3.2)

$$\Delta\theta \text{ (rad)} = \begin{cases} 5.55 \times 10^{-5}\, \Delta I \text{ (A)}, & \text{V8 correctors} \\ 7.91 \times 10^{-5}\, \Delta I \text{ (A)}. & \text{others} \end{cases} \tag{3.3}$$

The increment in the magnet current ΔI can then be obtained from Eqs. (2.21) and (3.3) in terms of the difference orbit Δx.

For simulation of beam position correction on the NSLS X-ray ring, a sample of uncorrected orbit with r.m.s. orbit error of 208 μm relative to the reference orbit "orbit44" was taken as the initial state. The response matrix **R** was calculated from the betatron functions listed in Table 3.1. For different values of ε, the pseudo-inverse matrix $\mathbf{R}_{inv}$ was then calculated, which gives changes in the magnet current and resulting reduction in orbit error. The result is summarized in Table 3.2. For smaller ε, the r.m.s. orbit error is smaller, but the price is the larger changes in the corrector strengths.

4. Measurement Results

In this section, we will present the results of global vertical orbit correction experiments on the X-ray ring of NSLS. All of the 48 BPMs and 39 correctors as listed in Table 3.1 were used, except for the X1V3, X4V17, and X5V17 correctors.

The flowchart of the algorithm for DC global beam position feedback is shown in Fig. 4.1. The first step is to measure the response matrix **R**. If it has already been done,

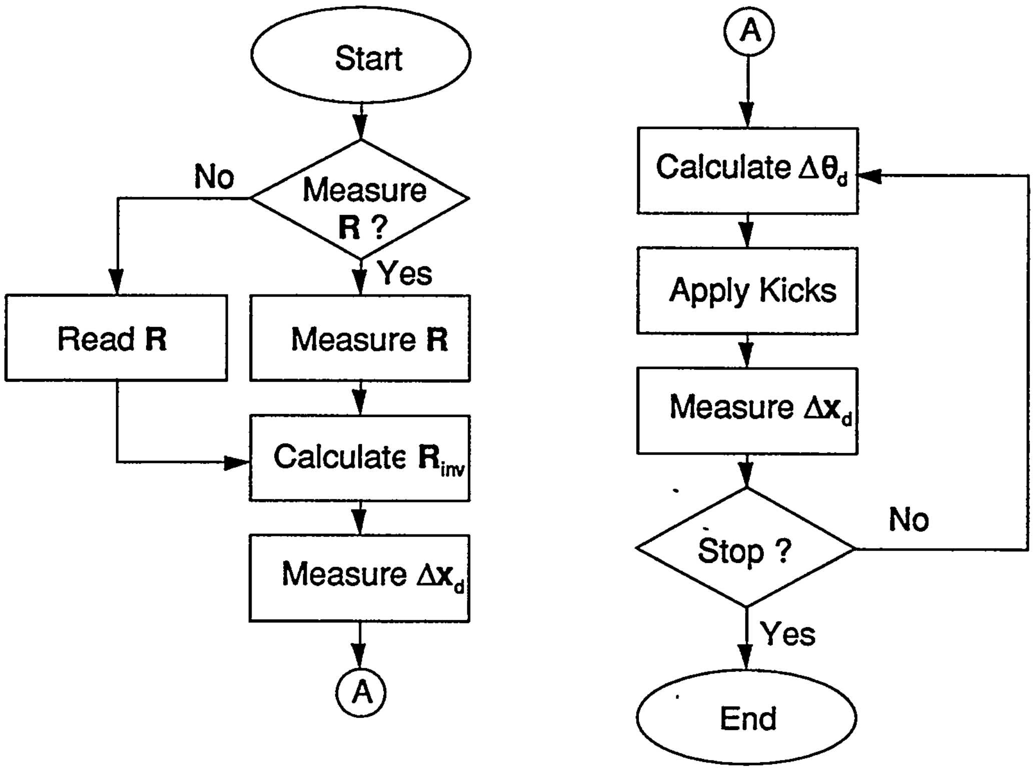


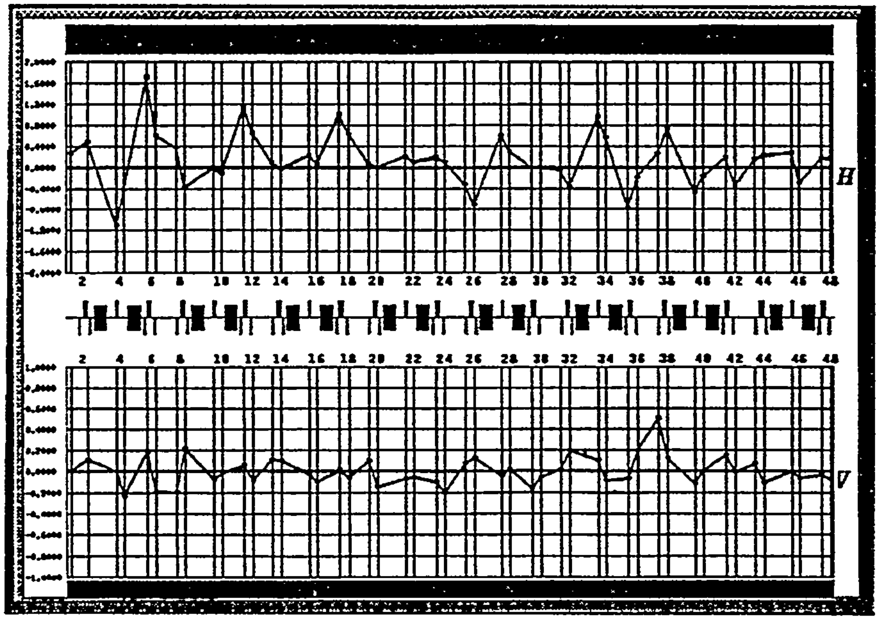
Fig. 4.1: Flowchart for DC global beam position feedback.

this step is skipped and the matrix is read from the disk. The pseudo-inverse matrix $\mathbf{R}_{inv}$ is then calculated using ε in Eq. (2.12) and stored in the memory. It is used to calculate the necessary kick strengths for the corrector magnets after each measurement of the difference orbit Δx_d, until it is requested by the user to stop the process.

The response matrix was measured by changing the strength of the correctors one by one and measuring the beam motion at all BPMs. From this raw response matrix, the betatron functions β and ψ were derived for the BPMs and correctors,[10] which again were used to reconstruct the response matrix, thereby reducing the measurement error. For this reconstructed matrix, ε_m was 0.00182.

Figure 4.2(a) shows the horizontal (upper) and vertical (lower) closed orbits around the ring after applying the harmonic correction to the uncorrected orbit with vertical r.m.s. orbit error of 208 μm. The horizontal orbit was not corrected. After harmonic correction, the vertical r.m.s. orbit error was 138 μm. The SVD correction was applied to this orbit, with $\varepsilon = 0.002$, which further reduced the orbit error to 61 μm as shown in Fig. 4.2(b). A

(a)



(b)

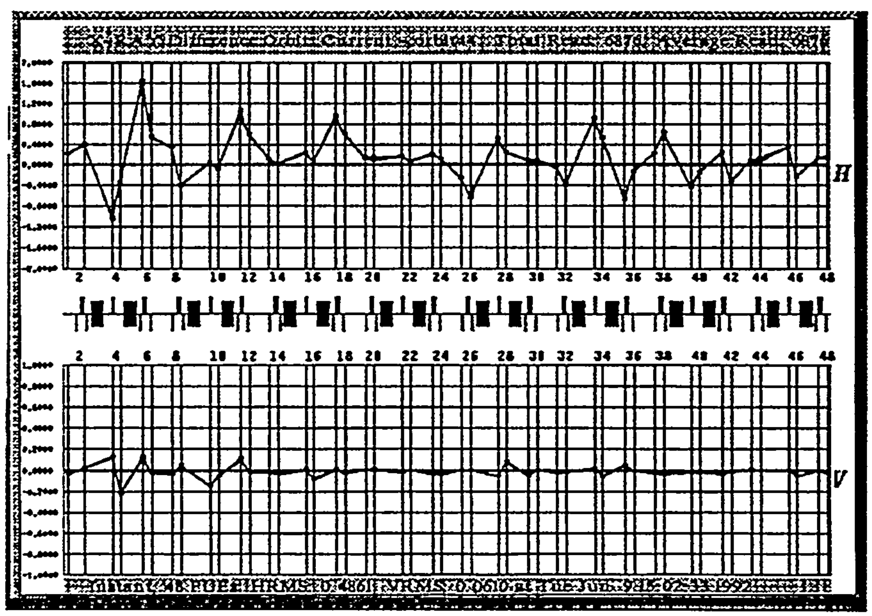


Fig. 4.2: Results of global orbit correction using (a) harmonic correction and (b) SVD correction. The uncorrected orbit had 208 μm r.m.s. error relative to the reference orbit, which was reduced to 138 μm by harmonic correction. SVD correction further reduced it to 61 μm using ε = 0.002.

few corrections were necessary before the r.m.s. error settled down to this value. This is due to the difference in the machine conditions when the matrix was measured and when the orbit correction was done. While the response matrix was measured with the X25 wiggler gap closed,[11] the orbit correction was done with the gap open. This slight difference, though minor, has resulted in a less than exact correction. This was confirmed by later measurement of the response matrix with the wiggler gap open.

The corrector strength change ranged from -4.85 A to 2.27 A, with the r.m.s. value of 1.41 A. Some of the corrector power supplies got close to, but did not reach, saturation at the maximum current of 10 A. With $\varepsilon_m = 0.00182$, reducing ε to 0.001 would trip off some of the power supplies and was not tried.

5. Summary

In this note, we presented the theory and application of the singular value decomposition (SVD) for DC global correction of the vertical closed orbit in the NSLS X-ray ring. The method is, in principle, equivalent to inversion of the matrix, and the matrix in our case is the response matrix, which is the ratio of orbit motion per unit change in the corrector strength. Using SVD, either the residual orbit error ($M \geq N$) or the r.m.s. corrector strength change ($M \leq N$) is absolutely minimized. This means that given the initial difference orbit, no other correction algorithm can further reduce these. This was proven by introducing the concepts of t-BPMs and t-correctors, which are appropriate linear combinations, or transforms, of the actual BPMs and correctors.

Considering the limitation on the corrector power supplies, the important parameter is the singularity criterion ε for SVD, which represents the degree of correction accuracy. For the most accurate correction, ε is set to less than ε_m, but this will result in large changes in the corrector strength. When this is unacceptably large due to the current limit of the power supply, ε must be increased to a value less than 1. When ε is equal to 1, there is no correction. Therefore, by adjusting ε, orbit corrections can be optimized in terms of the desired orbit error and the corrector strength limit.

As a result of the correction using SVD with $\varepsilon = 0.002$, the r.m.s. orbit error in the vertical plane was reduced to 61 μm from 138 μm due to harmonic correction. The uncorrected orbit had 208 μm orbit error. The corrector current change ranged from -4.85 A to 2.27 A, with the r.m.s. of 1.41 A.

The computer code used for this work is highly modularized so that it can be easily applied to closed orbit correction in other storage rings with proper I/O interface to the beam position monitors and corrector magnets. It also can be used for simulation and diagnosis of an orbit correction system if the response matrix, or alternatively the beta function, phase, and tune, is known. Such analysis for the APS storage ring is now being undertaken and will be published in the near future.

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